The QCD critical region and higher moments for three quark flavors

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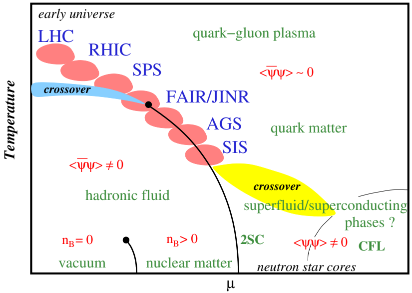
Outline

- **QCD Phase Diagram**
- **(Polyakov)-Quark-Meson Models**
- **Importance of fluctuations:**
  - ▷ Matter back-coupling to Yang-Mills sector
- **Anatomy of the critical region around the CEP**
- **Higher moments**

The conjectured QCD Phase Diagram



At densities/temperatures of interest
only model calculations available

➜ can one improve the model calculations?

➜ remove model parameter dependency?

Open issues:

related to chiral & deconfinement transition

- ▷ existence/location of CEP? How many? Additional CEPs?
- ▷ coincidence of both transitions at $\mu = 0$ and $\mu > 0$ (quarkyonic phase)?
- ▷ relation between chiral and deconfinement? chiral CEP/deconfinement CEP?
- ▷ are there finite volume effects? $\rightarrow$ lattice comparison
- ▷ mostly only MFA results
  effects of fluctuations?
  they are important!
  $\rightarrow$ **size of crit. region**
- ▷ What are good exp. signatures? $\rightarrow$ higher moments more sensitive

non-perturbative functional methods (FunMethods)

$\rightarrow$ complementary to lattice

- no sign problem $\mu > 0$
- chiral symmetry/fermions (small masses/chiral limit) etc...

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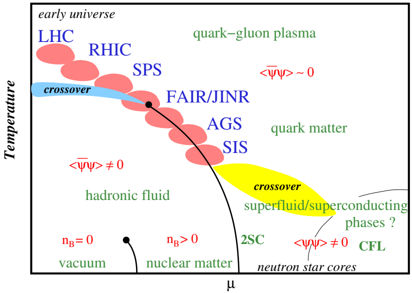

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Method of choice: Funtional Renormalization Group Method (FRG)
one needs a truncation: e.g. (Polyakov)-quark-meson model

• good description for chiral sector

• what can we learn about the deconfinement sector?

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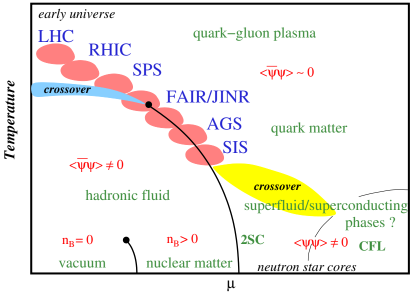

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test example: $N_c = 2$ QCD: lattice (no sign problem) versus FRG

$\rightarrow$ Polyakov-quark-meson-diquark (PQMD) model

[Strodthoff, BJS, von Smekal; in prep. '11]

$\mathbf{N_f = 3}$ Quark-Meson (QM) model

Let's start with the chiral sector:

- Model Lagrangian: $\mathcal{L}_{\text{qm}} = \mathcal{L}_{\text{quark}} + \mathcal{L}_{\text{meson}}$

Quark part with Yukawa coupling $\mathbf{h}$:

$$\mathcal{L}_{\text{quark}} = \bar{q}(i\partial\!\!\!/ - h\frac{\lambda_a}{2}(\sigma_a + i\gamma_5\pi_a))q$$

Meson part: scalar σ_a and pseudoscalar π_a nonet

meson fields: $M = \sum_{a=0}^{8} \frac{\lambda_a}{2}(\sigma_a + i\pi_a)$

$$\mathcal{L}_{\text{meson}} = \text{tr}[\partial_\mu M^\dagger \partial^\mu M] - m^2 \text{tr}[M^\dagger M] - \lambda_1 (\text{tr}[M^\dagger M])^2 - \lambda_2 \text{tr}[(M^\dagger M)^2] + c[\det(M) + \det(M^\dagger)]$$
$$+ \text{tr}[H(M + M^\dagger)]$$

- explicit symmetry breaking matrix: $H = \sum_a \frac{\lambda_a}{2} h_a$
- $U(1)_A$ symmetry breaking implemented by 't Hooft interaction

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- Mean-field approximation (MFA):

  Integrate over quarks and neglect mesonic fluctuations

Grand potential:

$$\Omega(T,\mu) = \Omega_{\text{vac}} + \Omega_{q\bar{q}} + U_{\text{class}}$$

no-sea MFA: neglect Ω_{vac}

cf talk K. Fukushima

$N_f = 2 + 1$ Phase diagram $(\mu \equiv \mu_q = \mu_s)$

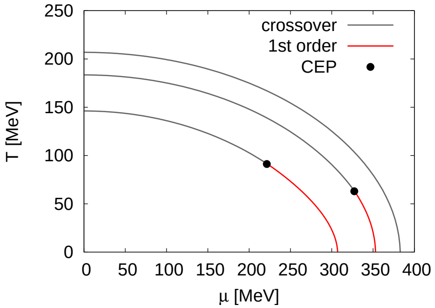
- Model parameter fitted to (pseudo)scalar meson spectrum:
- one parameter precarious: $f_0(600)$ 'particle' (sigma)$\rightarrow$ broad resonance

PDG: mass = $(400 \ldots 1200)$ MeV

➜ existence of CEP depends on m_σ!

Example: $m_\sigma = 600$ MeV (lower lines), 800 and 900 MeV (here no-sea MFA)

with $U(1)_A$ **[BJS, M. Wagner '09]**



- m_σ smaller $\rightarrow$ CEP towards T axis
- including vacuum term Ω_{vac} (beyond no-sea approximation)
  $\rightarrow$ CEP moves down
- including mesonic fluctuations (full FRG calculation):
  $\rightarrow$ CEP moves up again

$N_f = 3$ Polyakov-Quark-Meson (PQM) model

add now a statistical 'confinement' via Polyakov-loop (not yet dynamical)

- Lagrangian $\mathcal{L}_{\text{PQM}} = \mathcal{L}_{\text{qm}} + \mathcal{L}_{\text{pol}}$ with $\mathcal{L}_{\text{pol}} = -\bar{q}\gamma_0 A_0 q - \mathcal{U}(\phi, \bar{\phi})$

- **polynomial Polyakov loop potential:** **Polyakov 1978, Meisinger 1996, Pisarski 2000**

$$\frac{\mathcal{U}(\phi,\bar{\phi})}{T^4} = -\frac{b_2(T, T_0)}{2}\phi\bar{\phi} - \frac{b_3}{6}\left(\phi^3 + \bar{\phi}^3\right) + \frac{b_4}{16}\left(\phi\bar{\phi}\right)^2$$

$$b_2(T,T_0) = a_0 + a_1(T_0/T) + a_2(T_0/T)^2 + a_3(T_0/T)^3$$

beyond standard MFA:

improve this model by back-coupling of quarks (QCD matter sector) to the Yang-Mills sector

this yields to a N_f and μ-modifications in presence of dynamical quarks:

$T_0 = T_0(N_f, \mu, m_q)$ **BJS, Pawlowski, Wambach; 2007**

N_f	0	1	2	2+1	3
T_0 [MeV]	270	240	208	187	178

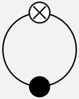
This becomes clear by considering the FRG

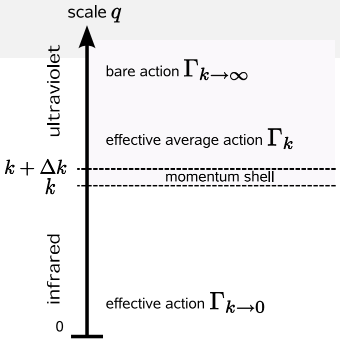
Functional RG Approach

$\Gamma_k[\phi]$ scale dependent effective action ; $t = \ln(k/\Lambda)$; R_k regulators

FRG (average effective action) **[Wetterich '93]**

$$\partial_t \Gamma_k[\phi] = \frac{1}{2} \mathrm{Tr}\, \partial_t R_k \left(\frac{1}{\Gamma_k^{(2)} + R_k} \right) \quad ; \quad \Gamma_k^{(2)} = \frac{\delta^2 \Gamma_k}{\delta\phi\delta\phi}$$

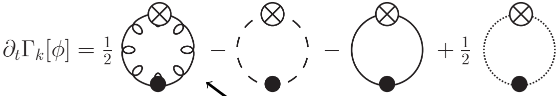
$k\partial_k \Gamma_k[\phi] \sim \frac{1}{2}$ 

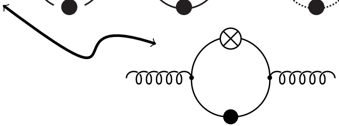


$T_0(N_f, \mu)$ modification

full dynamical QCD FRG flow: fluctuations of
gluon, ghost, quark and meson (via hadronization) fluctuations

[Braun, Haas, Marhauser, Pawlowski; '09]





in presence of dynamical quarks
gluon propagator modified:

$\rightarrow$ pure Yang Mills flow + these modifications

pure Yang Mills flow
replaced by effective Polyakov loop potential:
(fit to YM thermodynamics)

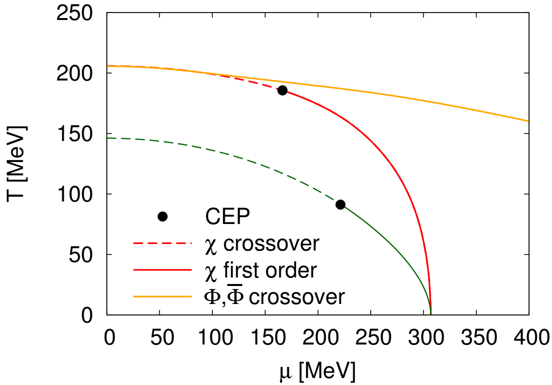
$$T_0 \leftrightarrow \Lambda_{QCD} \qquad : \qquad T_0 \rightarrow T_0(N_f, \mu, m_q)$$

[BJS, Pawlowski, Wambach; 2007]

$N_f = 2 + 1$ (P)QM phase diagrams

Summary of QM and PQM models in mean field approximation

chiral transition and 'deconfinement' coincide at small densities



- for PQM model

  (upper lines)

- for QM model

  (lower lines)

[BJS, M. Wagner; in prep. '11]

$N_f = 2$: [BJS, Pawlowski, Wambach; 2007]

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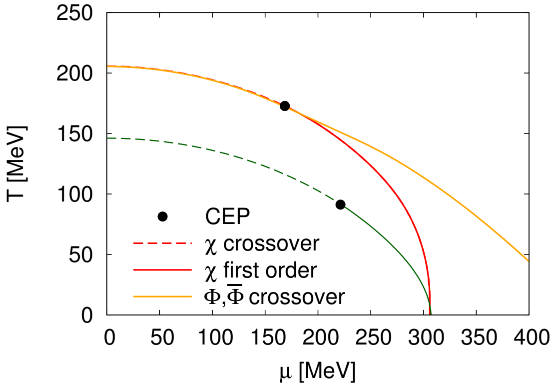


- for PQM model

  (upper lines)

  with
  matter back reaction in Polyakov loop potential
  i.e. $T_0(\mu)$

  $\rightarrow$ shrinking of possible quarkyonic phase

- for QM model

  (lower lines)

[BJS, M. Wagner; in prep. '11]

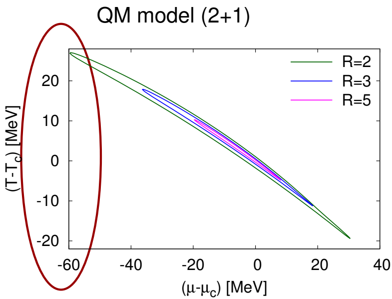
$N_f = 2$: [BJS, Pawlowski, Wambach; 2007]

Critical region

contour plot of **size of the critical region** around CEP

defined via fixed ratio of susceptibilities: $R = \chi_q / \chi_q^{\text{free}}$

➜ compressed with Polyakov loop



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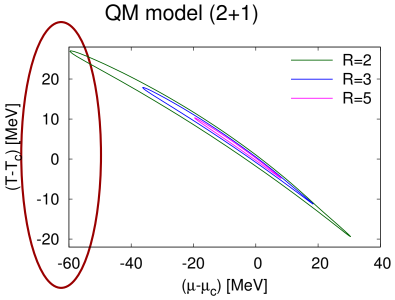
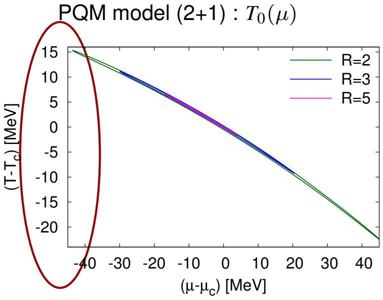
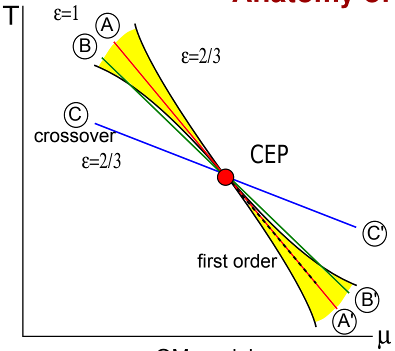




[BJS, M. Wagner; in preparation 11]

Anatomy of the critical region

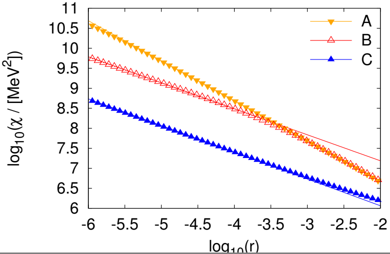


- crit. exponents depend on path (cf. different slopes)

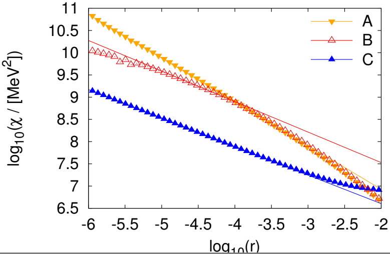
  → reason for the elongation

[BJS, M. Wagner; in preparation 11]

QM model



PQM model (T_0 const)



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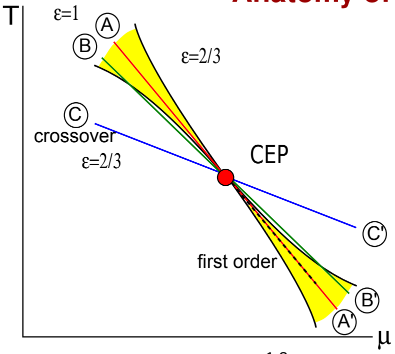
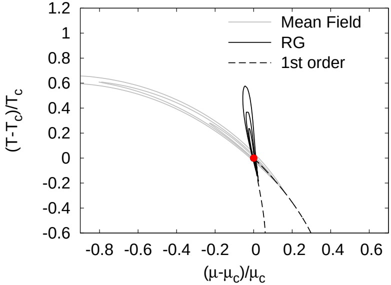


- crit. exponents depend on path (cf. different slopes)

  → reason for the elongation
- another influence: fluctuations
- (P)QM models with/without vacuum (zero modes) quark contribution
- Mean-field approximations vs. RG calculation

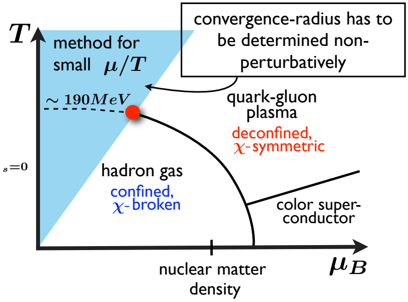


[BJS, J. Wambach; 06]

Taylor coefficients for $N_f = 2 + 1$ PQM model

Taylor expansion:

$$\frac{p(T,\mu)}{T^4} = \sum_{n=0}^{\infty} c_n(T) \left(\frac{\mu}{T}\right)^n \quad \text{with} \quad c_n(T) = \frac{1}{n!} \frac{\partial^n \left(p(T,\mu)/T^4\right)}{\partial \left(\mu/T\right)^n}\Bigg|_{\mu=0}$$



[C. Schmidt '09]

convergence radii:

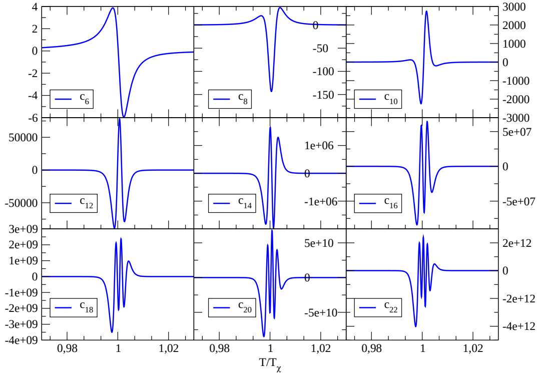
limited by first-order line?

$$\rho_{2n} = \left|\frac{c_2}{c_{2n}}\right|^{1/(2n-2)}$$

Taylor coefficients for $N_f = 2 + 1$ PQM model

New method: based on algorithmic differentiation [M. Wagner, A. Walther, BJS, CPC '10]

Taylor coefficients c_n numerically known to high order, e.g. $n = 22$



- ▷ this technique applied to PQM model
- ▷ investigation of convergence properties of Taylor series
- ▷ properties of c_n
  - oscillating
  - increasing amplitude
  - no numerical noise
  - small outside transition region
  - number of roots increasing
  - 26th order

[F. Karsch, BJS, M. Wagner, J. Wambach; arXiv:1009.5211]

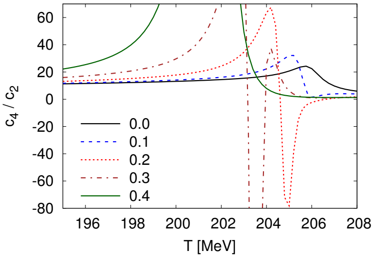
Can we use these coefficients to locate the CEP experimentally ?

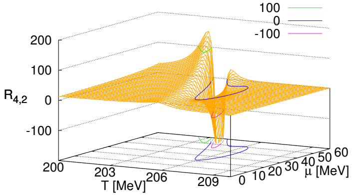
Generalized Susceptibilities $\equiv$ higher moments

cf. talks by V. Koch, F. Karsch, Ch. Redlich, S Gupta, V Skokov...

- Higher moments are increasingly sensitive to critical behavior even at $\mu = 0$
- Example: Kurtosis $R^B_{4,2} = \frac{1}{9}\frac{c_4}{c_2}$ $\rightarrow$ probe of deconfinement?

  It measures quark content of particles carrying baryon number B
- in HRG model $R_{4,2} = 1$ (always positive)

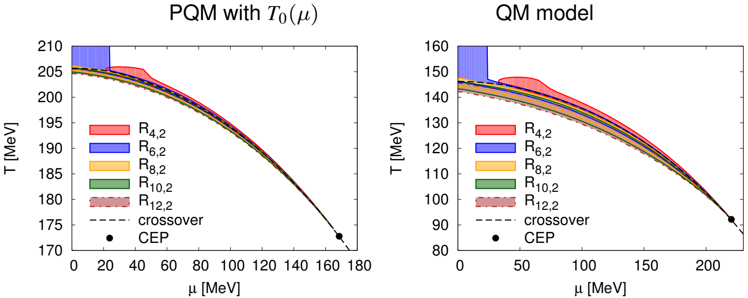




Generalized Susceptibilities $\equiv$ higher moments

Fluctuations of higher moments exhibit **strong variation from HRG model**

- $\rightarrow$ turn negative   [Karsch, Redlich; 11] see also [Friman, Karsch, Redlich, Skokov; 11]
- higher moments: $R^q_{n,m} = c_n / c_m$
- regions where $R_{n,m}$ are negative along crossover line in the phase diagram



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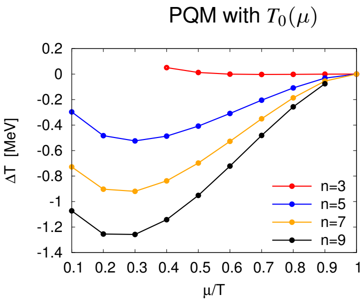
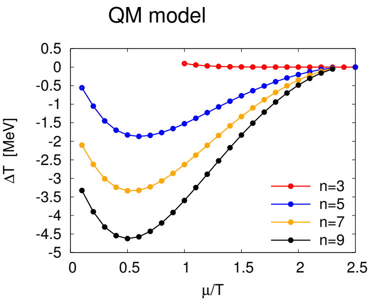
- Distance $\Delta T \equiv T_n - T_\chi$ of the first root in $R_{n,2}$ to the chiral temperature T_χ





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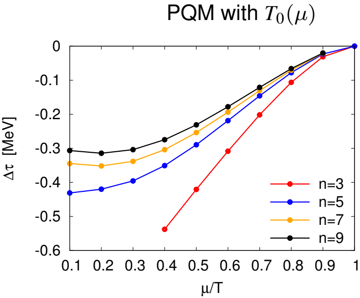

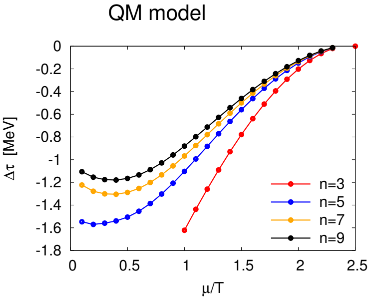

[BJS, M.Wagner; in prep. 11]

Summary and Outlook

- chiral (Polyakov)-quark-meson model study (three flavor)

➜ **role of fluctuations: they are important**

➜ location and existence of the CEP

➜ size of the critical region around CEP

functional approaches (such as the FRG) are suitable and controllable tools to investigate the QCD phase diagram and its phase boundaries

➜ **FunMethods guide the way towards full QCD**