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Hadron-Quark Phase Transition and Hybrid Stars with Dyson-Schwinger Quark Model

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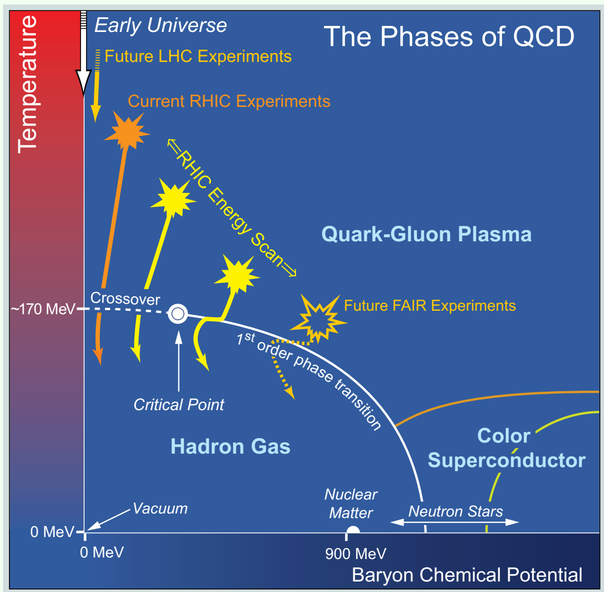
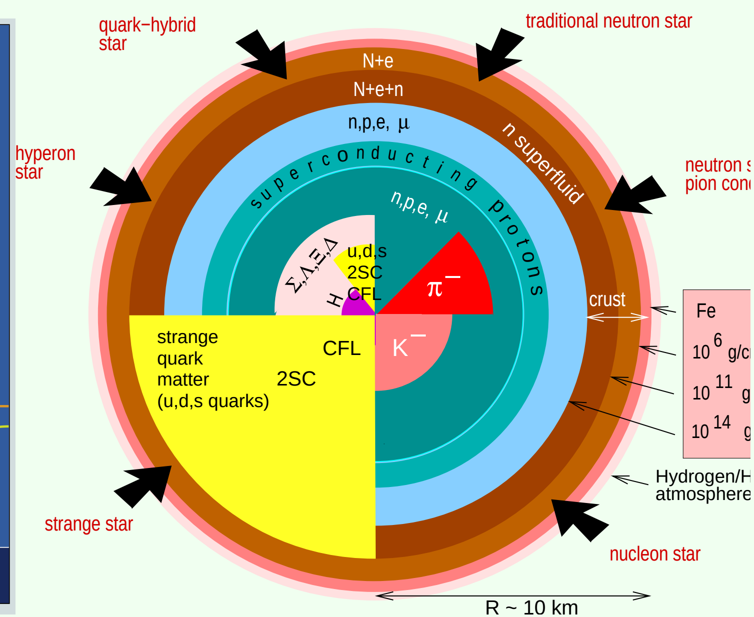
OUTLINE

- ⋇ Introduction
- ⋇ Dense quark matter at $T = 0$
- ⋇ Hadron-quark phase transition
- ⋇ Structure of hybrid stars
- ⋇ Summary and outlook

⋇ Phase structure and phase transitions of QCD





Taken from ’The Frontiers of Nuclear Science - A Long Range Plan’,2007(left), and ’F.Weber, et.al. astro-ph/0705.2708’(right).

⁂ Equation of states (EOS) at large density

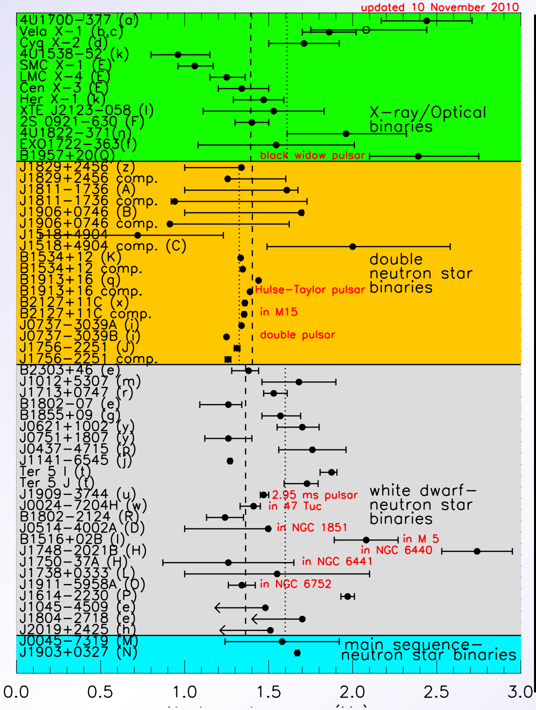
⁂ Hadron matter

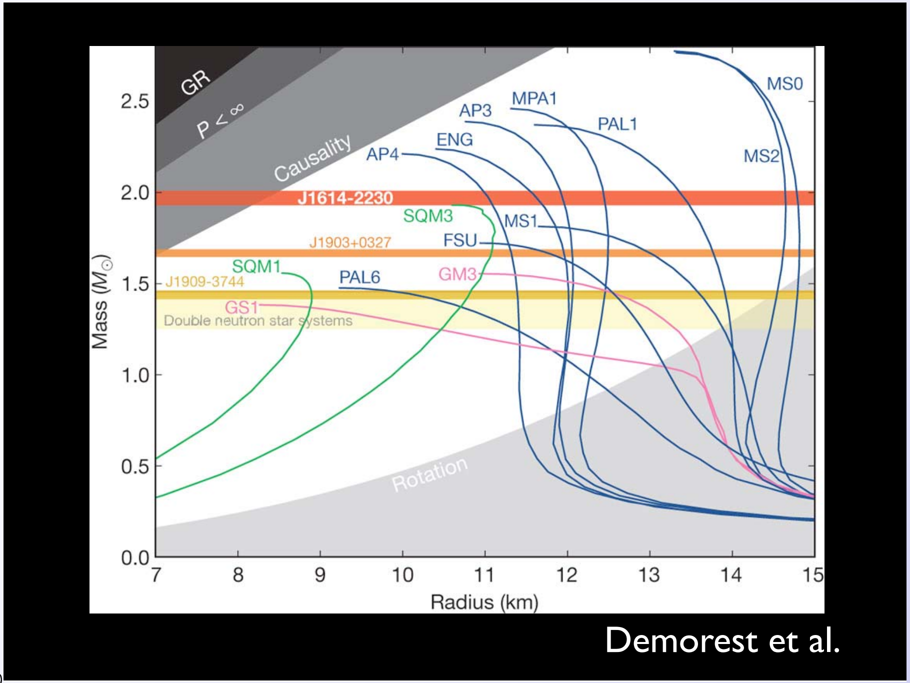
- ∗ Nuclear many body theories: Brueckner-Bethe-Goldstone(BBG)

⁂ Quark matter

- ∗ MIT bag model
- ∗ Dyson-Schwinger equations (DSE)

⁂ Observable: Masses of Neutron Stars





⋇ Quark phase: DSE

⋇ Gap equation

$$S(p;\mu)^{-1} = Z_2(i\vec{\gamma}\cdot\vec{p} + i\gamma_4(p_4 + i\mu) + m^{\rm bm}) + \Sigma(p;\mu)$$

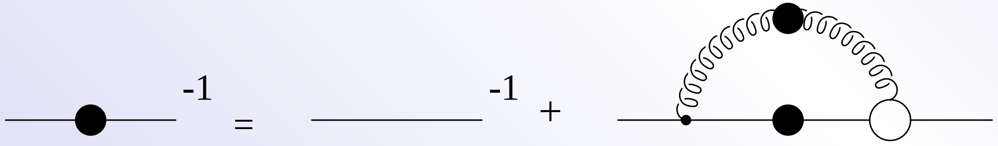
with

$$\Sigma(p;\mu) \;=\; Z_1 \int_q^\Lambda g^2(\mu) D_{\rho\sigma}(p-q;\mu)\frac{\lambda^a}{2}\gamma_\rho S(q;\mu)\Gamma^a_\sigma(q,p;\mu),$$

⋇ General structure of quark propagator

$$S(p;\mu)^{-1} = i\vec{\gamma}\cdot\vec{p}\,A(p^2, p\cdot u) + i\gamma_4\tilde{p}_4\,C(p^2, p\cdot u) + B(p^2, p\cdot u)$$

where $\tilde{p}_4 = p_4 + i\mu, u = (\vec{0}, i\mu)$



⋇ Quark-gluon vertex

⋇ Bare vertex (rainbow approximation)

$$\Gamma_\nu(q,k) = \gamma_\nu$$

⋇ Effective running coupling

$$Z_1 g^2 D_{\rho\sigma}(k) = \frac{\mathcal{G}(k^2)}{k^2}\,(\delta_{\rho\sigma} - \frac{k_\rho k_\sigma}{k^2})$$

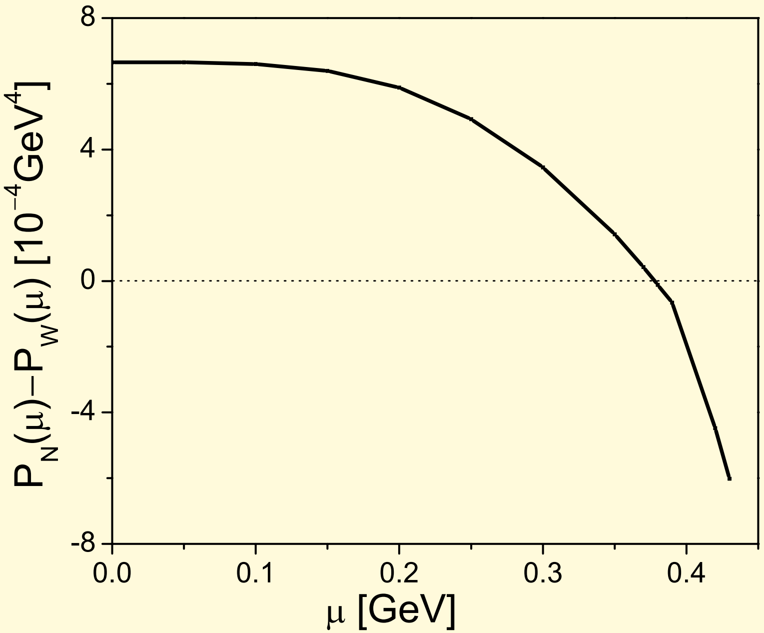
⋇ Modified Gaussian Model at finite chemical potential ($MG\alpha$):

$$4\pi\frac{\mathcal{G}(k^2)}{k^2} = 4\pi^2 d\,\mathrm{e}^{-\alpha\mu_q^2/\omega^2}\frac{k^2}{\omega^6}exp(-\frac{k^2}{\omega^2}) \quad (1)$$

with $\omega = 0.5GeV$, $d = 1GeV^2$, $m_{u,d} = 0$, $m_s = 0.115GeV$ and α as a phenomenological parameter, $\alpha = 0$: coupling is independent on chemical potential; $\alpha = \infty$: free quarks at finite chemical potential.

⁜ Nambu solution (phase): DCSB and confinement

⁜ Wigner solution (phase): chiral symmetric and deconfined



(H.Chen et.al, PRD(2008)) with 'steepest-descent' approximation ($\alpha = 0$, consistent with DSE):

$$P[S] = \mathrm{TrLn}\left[S^{-1}\right] - \frac{1}{2}\mathrm{Tr}\left[\Sigma\, S\right]$$

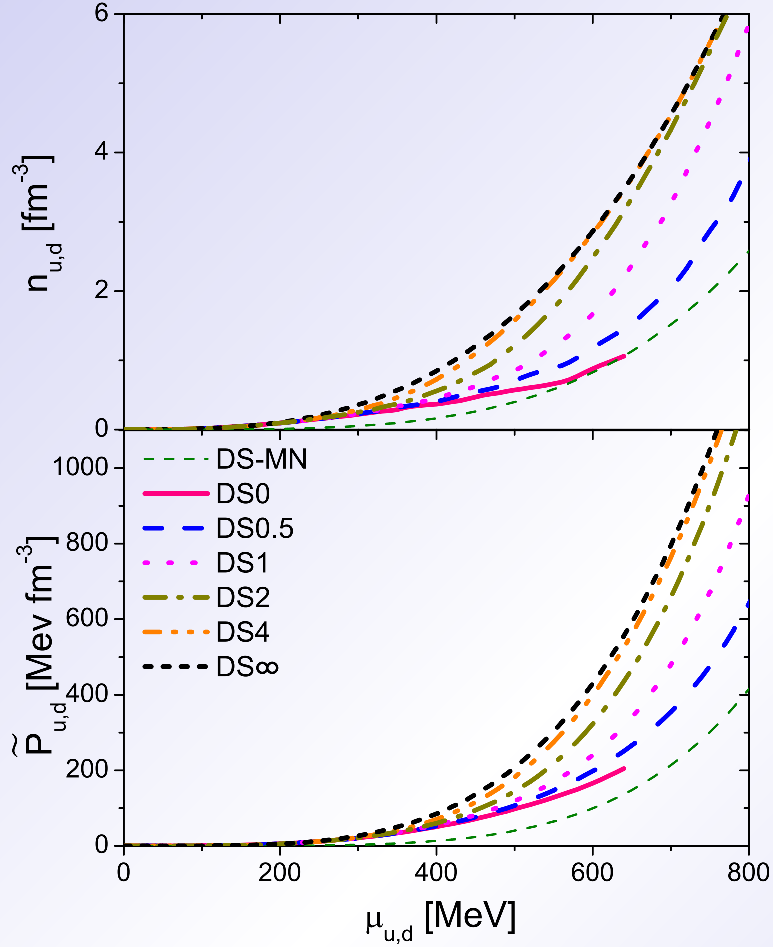
Note the (quark) Nambu phase is still in Vacuum $P_N(\mu_c) = 0$.

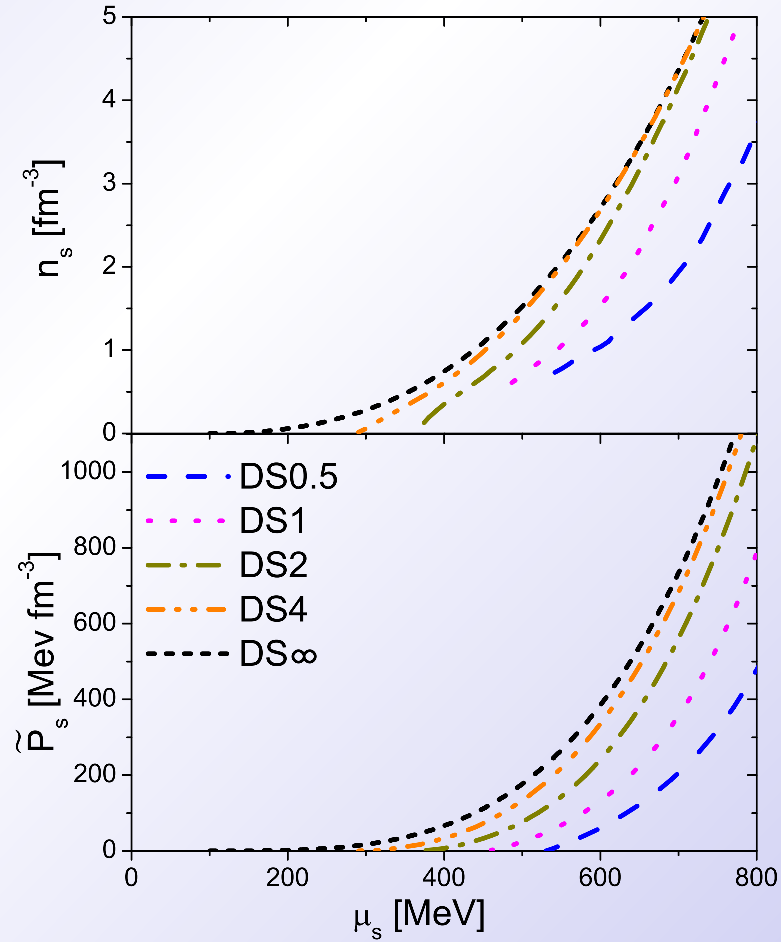
⋇ Thermodynamic

$$
\begin{aligned}
f_q(|\vec{p}|;\mu) &= \frac{1}{4\pi}\int_{-\infty}^{\infty} dp_4\,\mathrm{tr_D}\left[-\gamma_4 S_q(p;\mu)\right]\ , \\
n_q(\mu) &= 6\int \frac{d^3p}{(2\pi)^3}\, f_q(|\vec{p}|;\mu)\ , \\
P_q(\mu_q) &= P_q(\mu_{q,0}) + \int_{\mu_{q,0}}^{\mu_q} \mathrm{d}\mu\, n_q(\mu)\ . \\
\tilde{P}_q(\mu_q) &\equiv \int_{\mu_{q,0}}^{\mu_q} \mathrm{d}\mu\, n_q(\mu)\ , \\
B_{\mathrm{DS}} &\equiv -\sum_{q=u,d,s} P_q(\mu_{q,0})\ . \\
P_Q(\mu_u,\mu_d,\mu_s) &= \sum_{q=u,d,s} \tilde{P}_q(\mu_q) - B_{\mathrm{DS}}\ ,
\end{aligned}
$$

with $\mu_{u,0} = \mu_{d,0} = 0$ and $\mu_{s,0}$ at the chemical potential that the Wigner phase of s quark emerges. $B_{\mathrm{DS}} = 90 MeV\, fm^{-3}$

⋇ Density and (reduced) Pressure $\tilde{P}$ for single flavor quark





Compare to free quarks, $n_q(\mu)$ and $\tilde{P}(\mu)$ are suppressed, and so the EOS is stiff.

⋇ MIT Bag model

⋇ Thermodynamics

$$f_q(|\vec{p}|;\mu_q) = \begin{cases} 1\,, & \vec{p}^{\,2} < p_{f,q}^2 \equiv \mu_q^2 - m_q^2 \\ 0\,, & \vec{p}^{\,2} > p_{f,q}^2 \end{cases}$$

with $m_{u,d} = 0$, $m_s = 150\ MeV$, bag constant

$$B = 90\ MeV\,fm^{-3}$$

or a density dependent bag constant (G.F.Burgio, et.al, PRC66, 025802 (2002))

$$B(\rho) = B_\infty + (B_0 - B_\infty)\exp\Big[-\beta\Big(\frac{\rho}{\rho_0}\Big)^2\Big]$$

with $\rho = \frac{1}{3}\sum n_q$, $\rho_0 = 0.17 fm^{-3}$, $B_\infty = 50\ MeV\,fm^{-3}$, $B_0 = 400\ MeV\,fm^{-3}$, $\beta = 0.17$,

$$\tilde{\mu}_q = \mu_q + \frac{dB(\rho)}{dn_q}$$

$$\tilde{P}_Q = P_Q + \sum_q n_q \frac{dB(\rho)}{dn_q}$$

⋇ Hadron phase: BBG theory

- ⋇ Bethe-Goldstone equation

$$G[\rho;\omega] = v + \sum_{k_a,k_b} v \frac{|k_a k_b\rangle Q \langle k_a k_b|}{\omega - e(k_a) - e(k_b)} G[\rho;\omega] \,, \qquad (2)$$

with BHF approximation

$$U(k;\rho) = \mathrm{Re} \sum_{k'<k_F} \left\langle kk' \middle| G[\rho; e(k) + e(k')] \middle| kk' \right\rangle_a \,, \qquad (3)$$

- ⋇ Nucleon-Nucleon (NN) two body potential:
  Bonn B (BOB); Argonne V18 (V18); Nijmegen 93 (N93)
- ⋇ Three body forces(TBF) - density dependent two body forces
  Urbana type(UIX); BOB; V18; N93
- ⋇ Hyperon: Σ^-, Λ...; Nijmegen soft core NY potential
- ⋇ EOS

$$\begin{aligned} \epsilon &= \epsilon_f + \frac{1}{2} \sum_{k<k_F} U(k,\rho) \\ \mu_i &= \partial\epsilon/\partial\rho_i \\ P &= \sum_i \rho_i \mu_i - \epsilon \end{aligned}$$

G. F.Burgio, et.al,PRC(2002), Z.H.Li and H.-J.Schulze, PRC (2008), ...

⋇ Beta stable stellar matter

⋇ Hadron matter

$$\epsilon(\rho_n, \rho_p, \rho_e, \rho_\mu, \ldots) \; = \; \epsilon_N + \epsilon_Y + \frac{(3\pi^2\rho_e)^{4/3}}{4\pi^2} + \rho_\mu m_\mu + \frac{1}{2m_\mu}\frac{(3\pi^2\rho_\mu)^{5/3}}{5\pi^2} \, ,$$

Beta equilibrium:

$$\begin{aligned} \mu_e &= \mu_\mu = \mu_n - \mu_p \\ \mu_{\Sigma^-} &= 2\mu_n - \mu_p \\ \mu_\Lambda &= \mu_n \end{aligned}$$

Charge neutrality:

$$0 \; = \; \rho_p - \rho_e - \rho_\mu - \rho_{\Sigma^-} \cdots$$

⋇ Quark matter

$$P(\mu_u, \mu_d, \mu_s, \mu_e, \mu_\mu) \; = \; P_Q + P_L$$

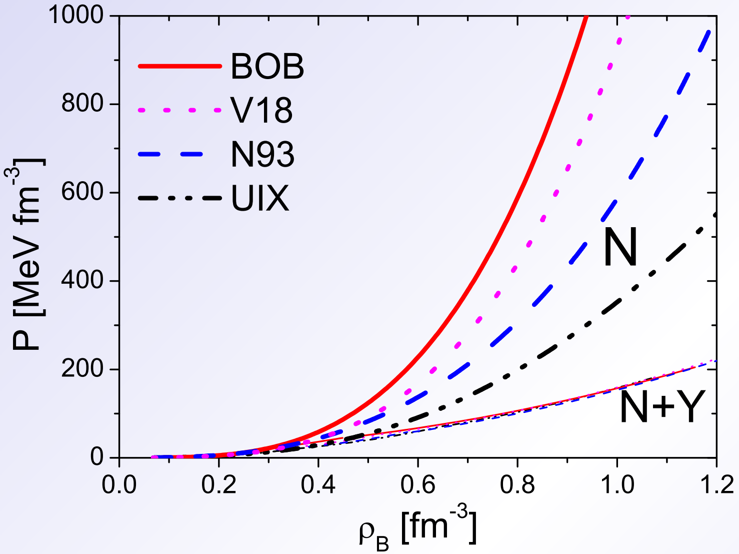
Beta equilibrium:

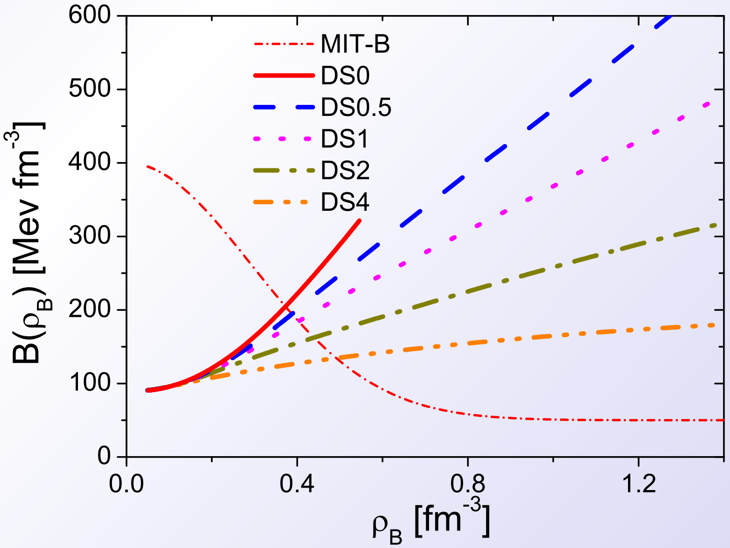
$$\mu_e \; = \; \mu_\mu = \mu_d - \mu_u = \mu_s - \mu_u$$

Charge neutrality:

$$0 \; = \; \frac{2\rho_u}{3} - \frac{\rho_d}{3} - \frac{\rho_s}{3} - \rho_e - \rho_\mu$$

⁕ Beta-stabled hadron matter (left) and effective bag constant (right)





$$\rho_B = (\rho_u + \rho_d + \rho_s)/3$$

$$B(\rho) \equiv \epsilon(\rho) - \epsilon_{free}(\rho)$$

⁂ Hadron-quark phase transition

- Maxwell construction

$$\mu_{B,H} = \mu_{B,Q} \, ,$$

$$P_H = P_Q \, ,$$

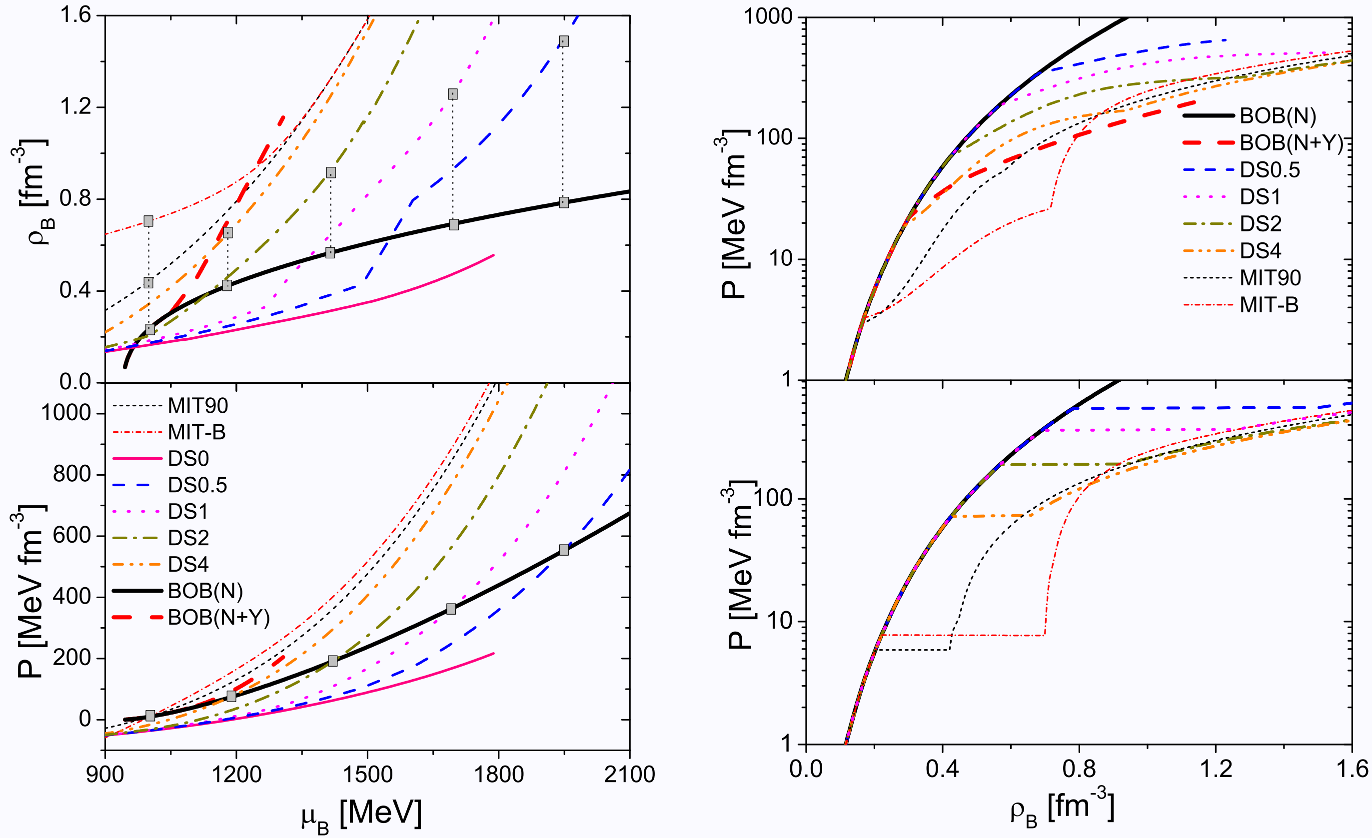
- Gibbs (Glendenning) construction - mixed phase

$$\begin{aligned}
\mu_{B,H} &= \mu_{B,Q} \, , \\
\mu_{e,H} &= \mu_{e,Q} \, , \\
P_H &= P_Q \, , \\
0 &= (1-x)(\rho_p - \rho_{\Sigma^-}) + x(\frac{2\rho_u}{3} - \frac{\rho_d}{3} - \frac{\rho_s}{3}) - \rho_e - \rho_\mu \, .
\end{aligned}$$

where 'x' is the fraction of quark phase in the mixed phase.

⋇ Hadron-quark phase transition:

Glendenning construction VS Maxwell construction



⋇ Interior Structure of Neutron Star

⋇ Tolman-Oppenheimer-Volkoff (TOV) equations

(spherically symmetric distribution of mass in hydrostatic equilibrium)

$$\frac{dP(r)}{dr} = -\frac{Gm(r)\epsilon(r)}{r^2} \frac{[1 + P(r)/\epsilon(r)]\left[1 + 4\pi r^3 P(r)/m(r)\right]}{1 - 2Gm(r)/r} ,$$

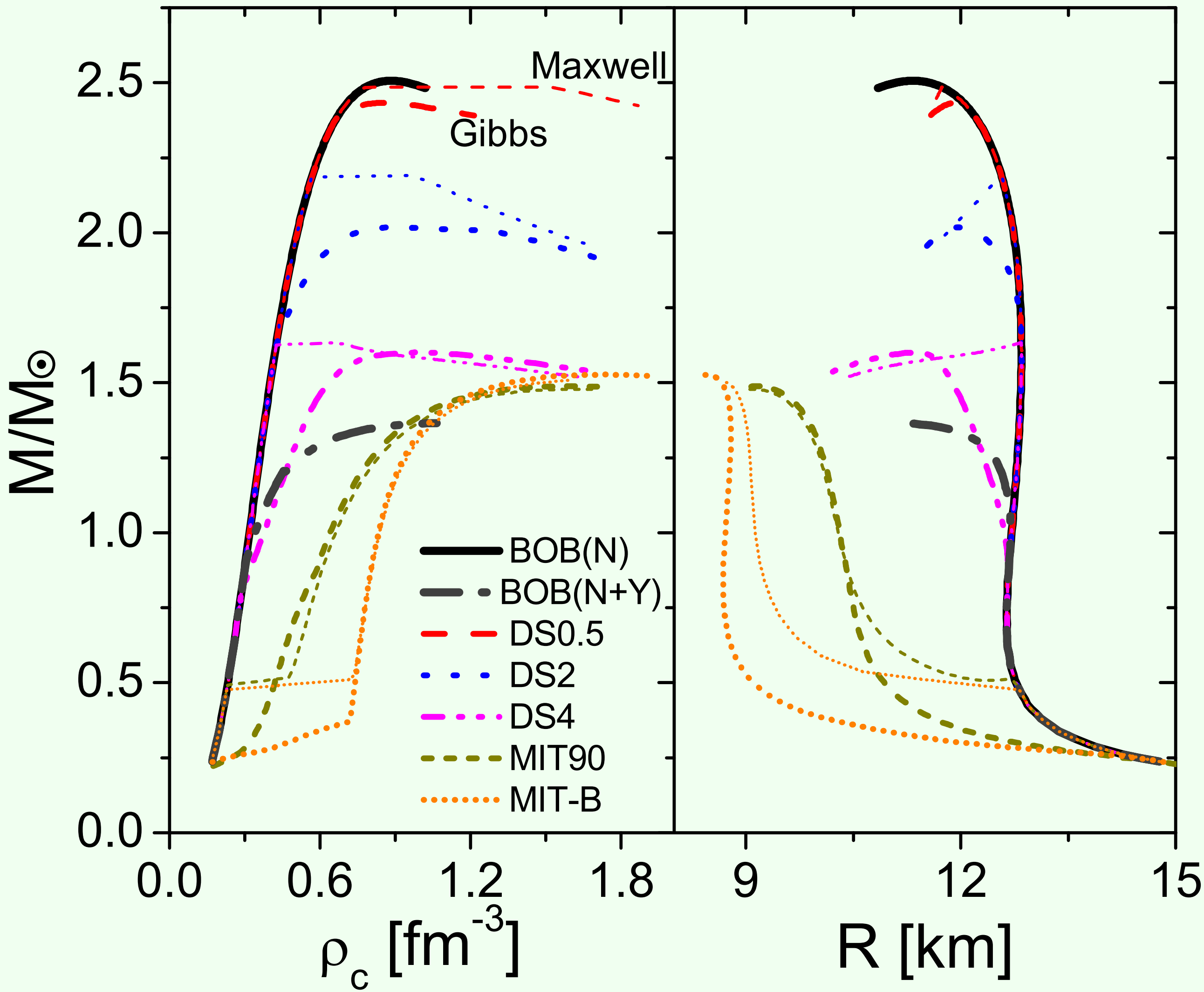
$$\frac{dm(r)}{dr} = 4\pi r^2 \epsilon(r) ,$$

Start with a central density ρ_c and corresponding $P(\rho_c), \epsilon(\rho_c)$ and integrate out until the ϵ at the surface equals the one of ion, the gravitational mass is

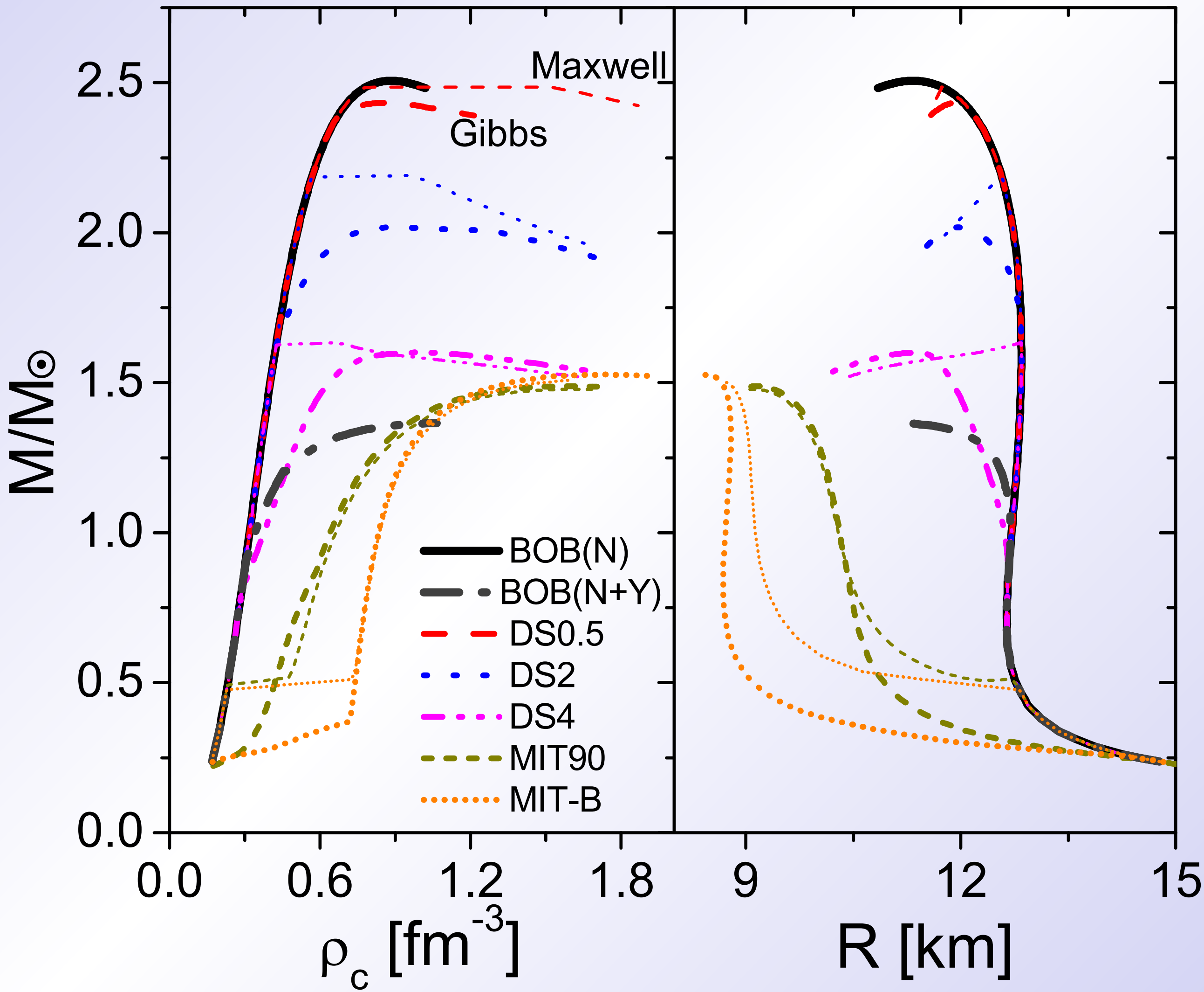
$$M \equiv m(R) = 4\pi \int_0^R dr\; r^2 \epsilon(r) .$$

For NS crust, use the EOS by Negele - Vautherin, Feynman - Metropolis-Teller and Baym - Pethick - Sutherland.

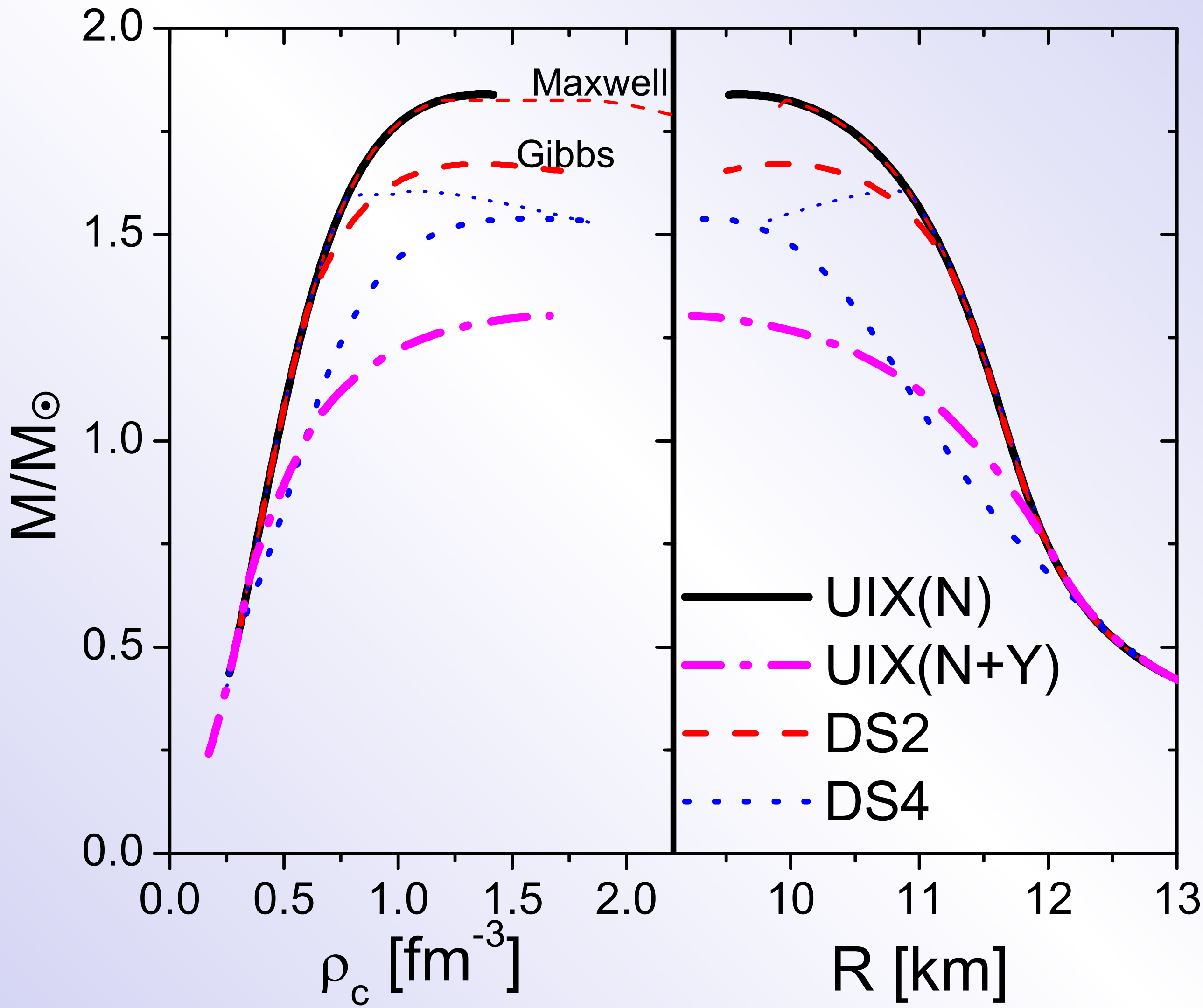
⋇ (Hybrid) Neutron Star: stiff hadronic EOS



⁎ (Hybrid) Neutron Star: stiff hadronic EOS



⋇ (Hybrid) Neutron Star: soft hadronic EOS



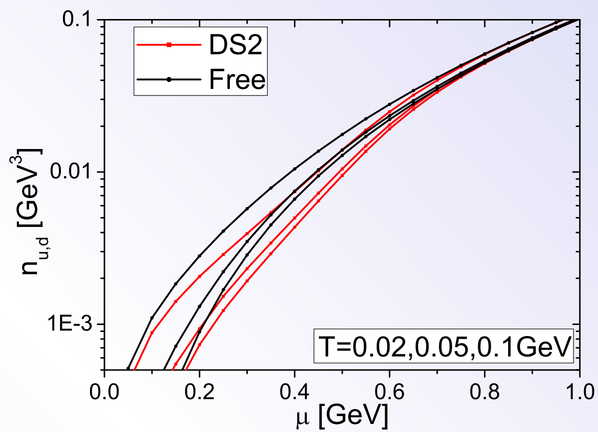
⋇ EOS at Finite T (preliminary)

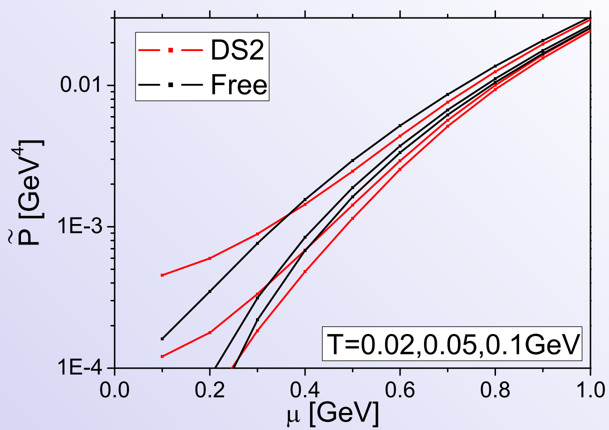
⋇ Pressure and entropy

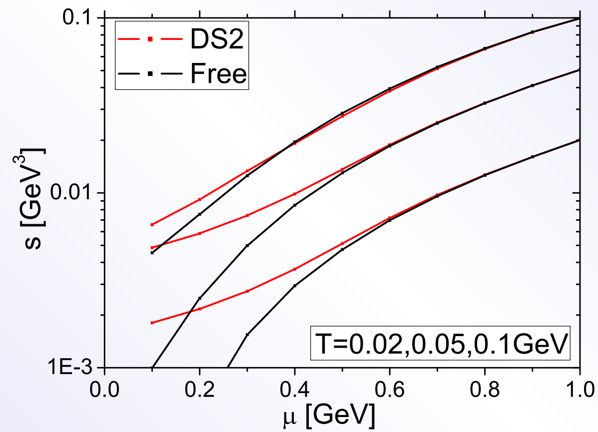
$$\tilde{P}_q(\mu_q, T) \equiv \int_{\mu_{q,0}}^{\mu_f} \mathrm{d}\mu\, n_q(\mu, T=0) + \int_0^T \mathrm{d}T\, s_{q,f}(\mu_f, T) + \int_{\mu_f}^{\mu_q} \mathrm{d}\mu\, n_q(\mu, T)\,,$$

$$s_q(\mu_q, T) \equiv \frac{\partial \tilde{P}_q(\mu_q, T)}{\partial T}$$

⋇ Num. results with $m_q = 0, \alpha = 2$







Summary

⁂ EOS of quark matter is stiff and phase transition at larger density, compare to bag model

⁂ Hybrid star with $2M_{\odot}$ requires stiff hadronic EOS and proper α

arXiv:1107.2497, appear on PRD soon

Outlook

⁂ EOS at $T > 0$: Proto-neutron stars and HIC

⁂ Improve ansatz for quark-gluon vertex and gluon propagator

Thank you!