# Strongly Coupled Pion Superfluid

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# Outline

- Motivation
- Nambu--Jona-Lasinio model & static potential
- Numerical Results
- Conclusion

# Motivation

- Lattice results at finite temperature, small $\eta / s$ in Ads/CFT and strong collective flow in heavy ion collision indicate the dense and hot quark matter is strongly coupled.
- Is the coupling the strongest at the phase transition of pion superfluid?

  Can the strongly coupled matter survive in the pion superfluid at high isospin density?
- Static potential can answer these two questions.

# Nambu-Jona-Lasinio model

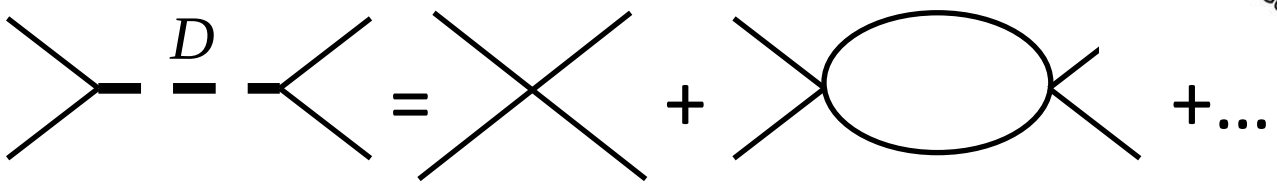
- NJL model (good model at finite $\mu_q$ and powerful for quark condensate)

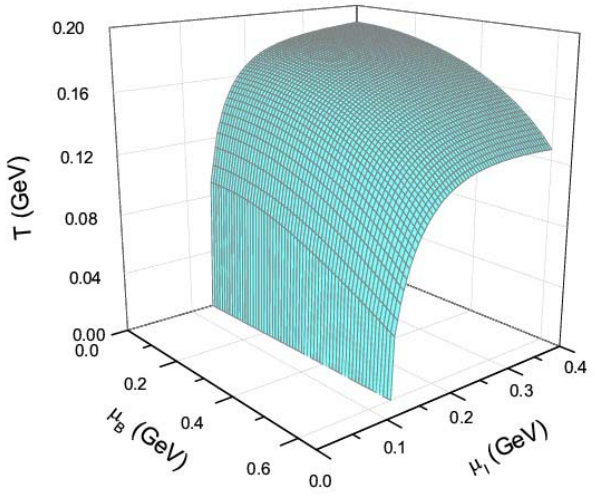
$$\mathcal{L} = \bar{\psi}\left(i\gamma^{\mu}\partial_{\mu} - m_0 + \mu\gamma_0\right)\psi + G\left[\left(\bar{\psi}\psi\right)^2 + \left(\bar{\psi}i\gamma_5\boldsymbol{\tau}\psi\right)^2\right]$$

- Mean field Approximation

$$\sigma = \langle\bar{\psi}\psi\rangle \qquad \pi = \sqrt{2}\langle\bar{\psi}i\gamma_5\tau_+\psi\rangle$$

- Quark interaction



# Static potential

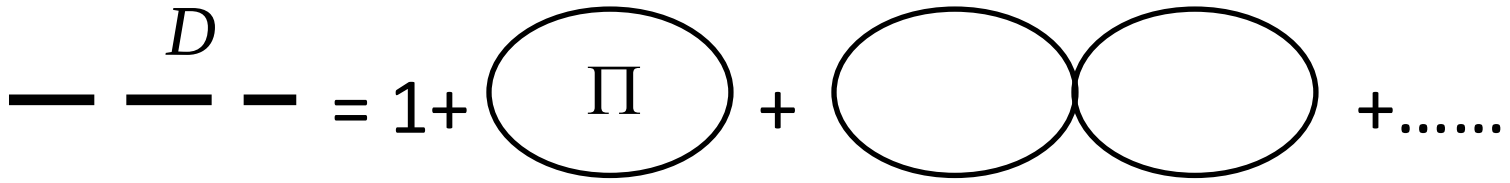
- Quark potential : Fourier transformation of D(k)

$$V(t,\vec{x}) = \mathcal{F}[D(k_0,\vec{k})]$$

- Static potential: set $k_0 = 0$

$$V(r) = \mathcal{F}[D(k_0 = 0,\vec{k})]$$

- To calculate D with RPA



# Static potential

- Pole approximation: diagonalize the $\Pi$ in flavor space, label the isospin eigenvectors with $\sigma, \pi_+, \pi_0, \pi_-$

$$V(r) = -\int \frac{d^3\mathbf{q}}{(2\pi)^3} \sum_j \frac{g^2_{jq\bar{q}}}{q^2 + M_j^2} e^{i\mathbf{q}\cdot\mathbf{r}} = -\sum_j \frac{g^2_{jq\bar{q}}}{4\pi r} e^{-M_j r}$$

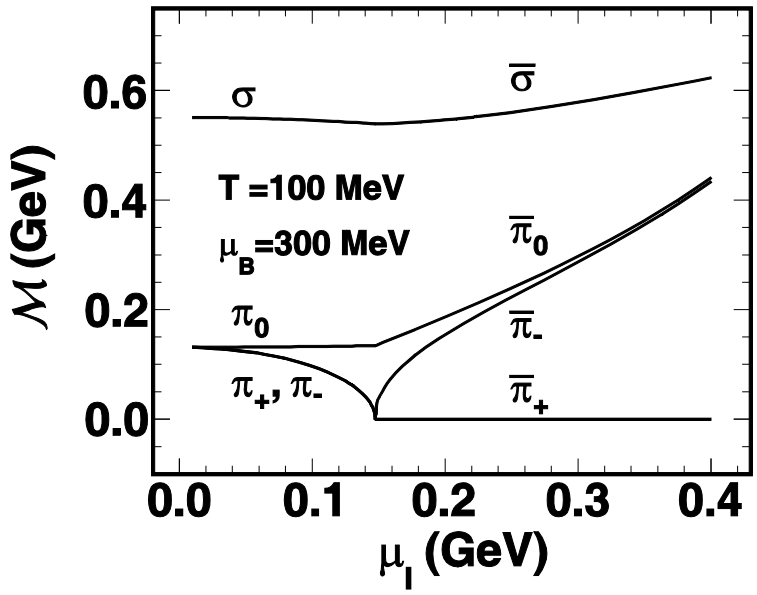
where $\det\left[1 - 2G\Pi\left(0, -\mathcal{M}_j\right)\right] = 0$

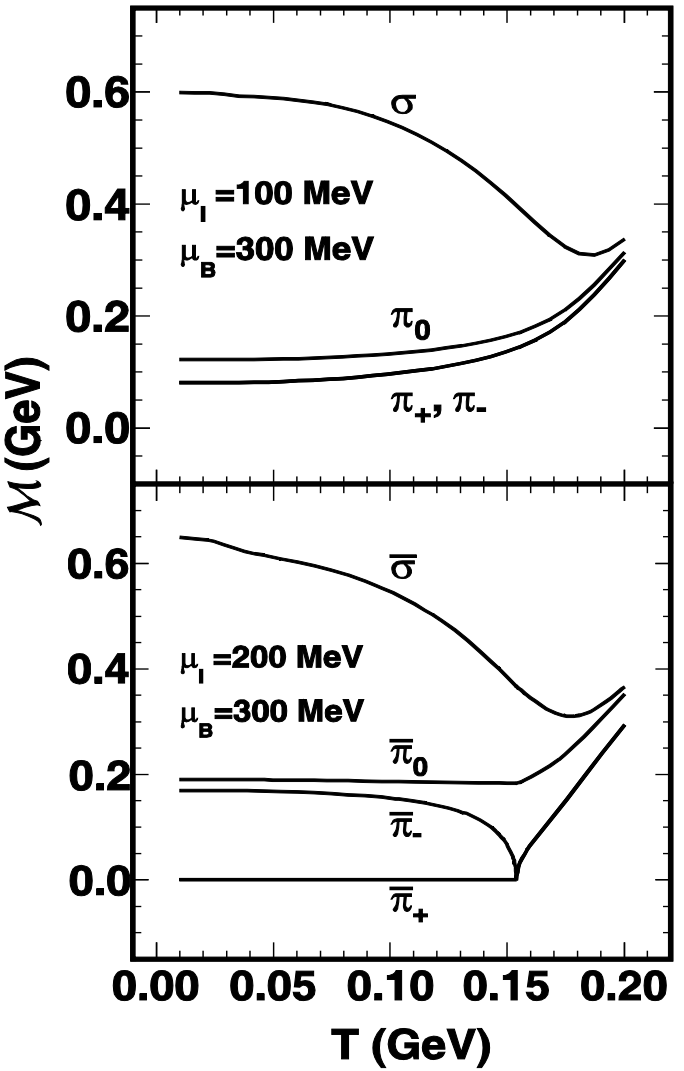
- Full calculation :

$$V(r) = -\frac{G}{\pi^2 r} \int dq \mathrm{Tr} \frac{q \sin(qr)}{1 - 2G\Pi(0, q^2)}$$

# Numerial Results(pole approach)

$$V(r) = -\sum_j \frac{g_{jq\bar{q}}^2}{4\pi r} e^{-M_j r}$$

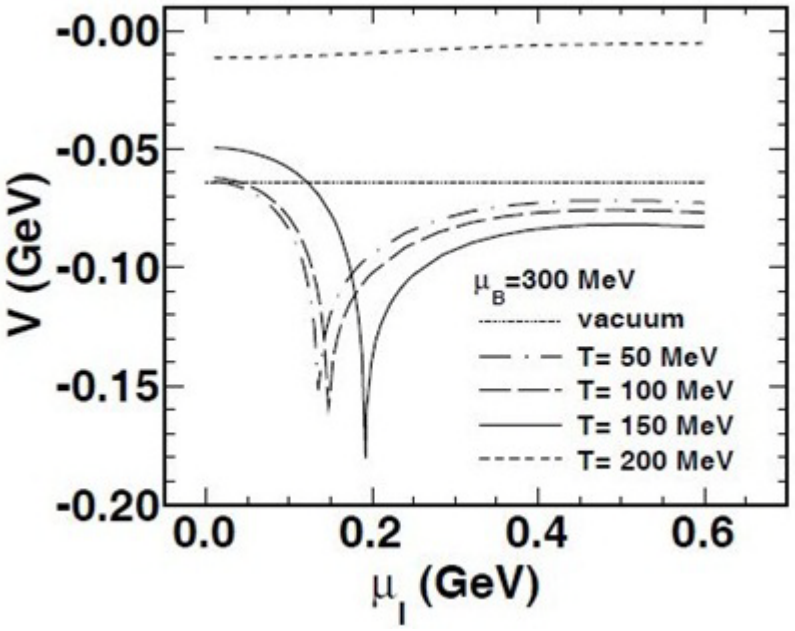


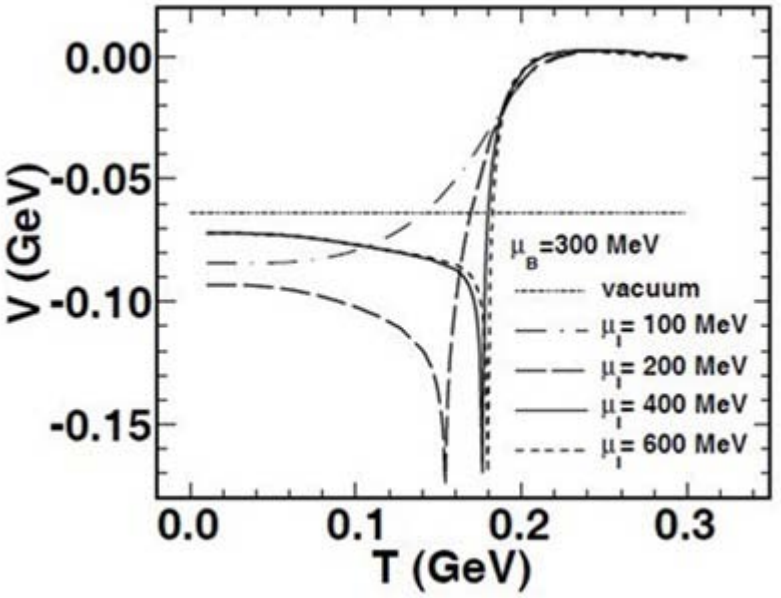




- Smaller M, deeper potential.
- Goldstone mode

->no screening->strongly coupled

# Numerical Results (Full results)

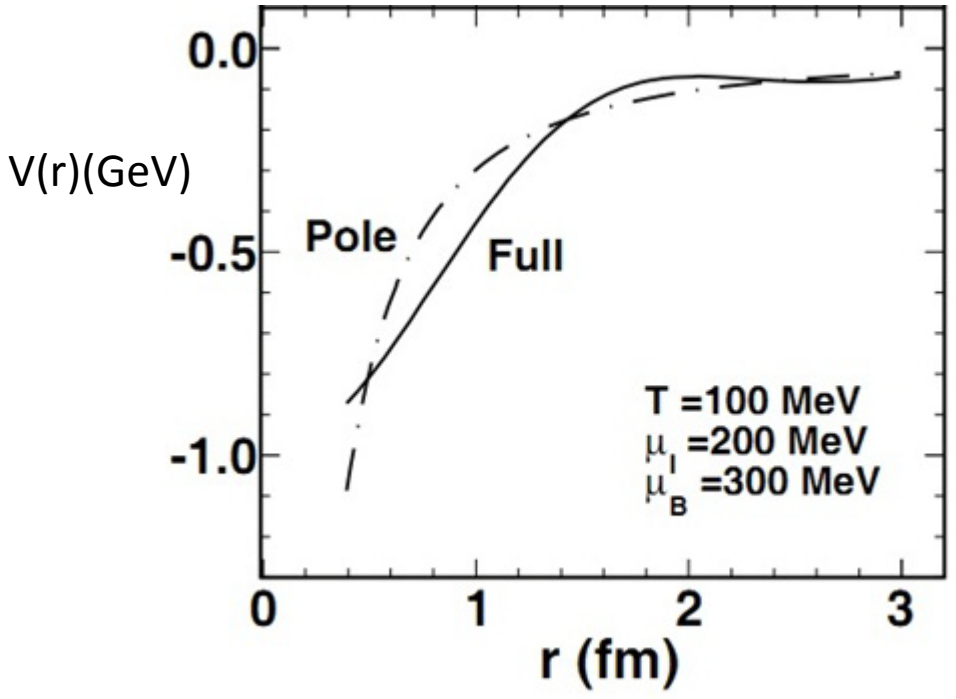



- Valley at phase transition points.
- Goldstone mode ->not be weakly coupled

# How good is the pole approximation

- Pole approximation agrees with full calculation.
- Numerical results can be explained by the behavior of screening masses.



# Conclusions

- Goldstone mode controls the pion superfluid.
- Quark matter is weakly coupled at high baryon chemical potential, but still strongly coupled at high isospin chemical potential.