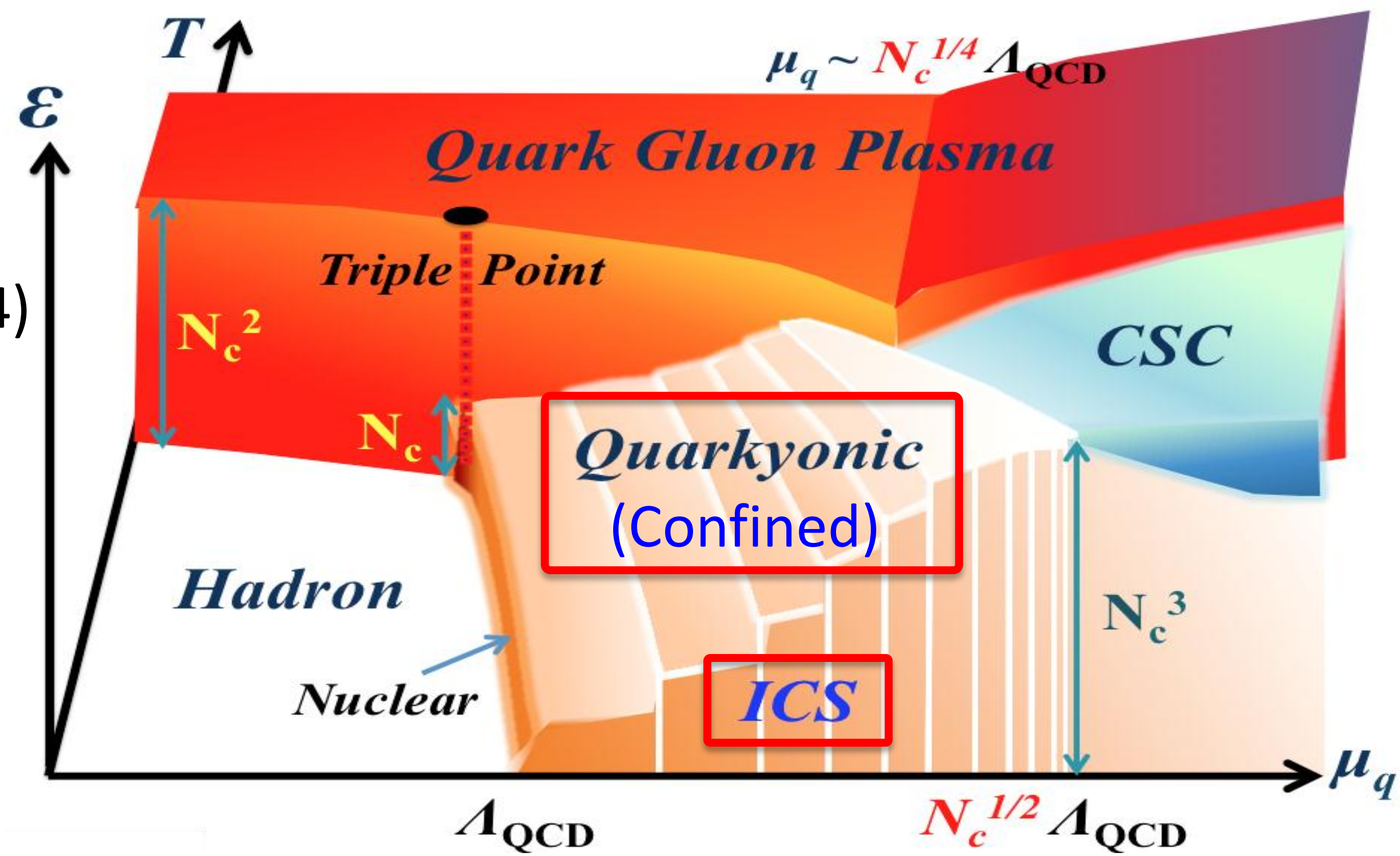
Interweaving Chiral Spirals

Toru Kojo (Bielefeld U.)

with: **K. Fukushima, Y. Hidaka, L. McLerran, R.D. Pisarski**

(arXiv: 1107.2124)



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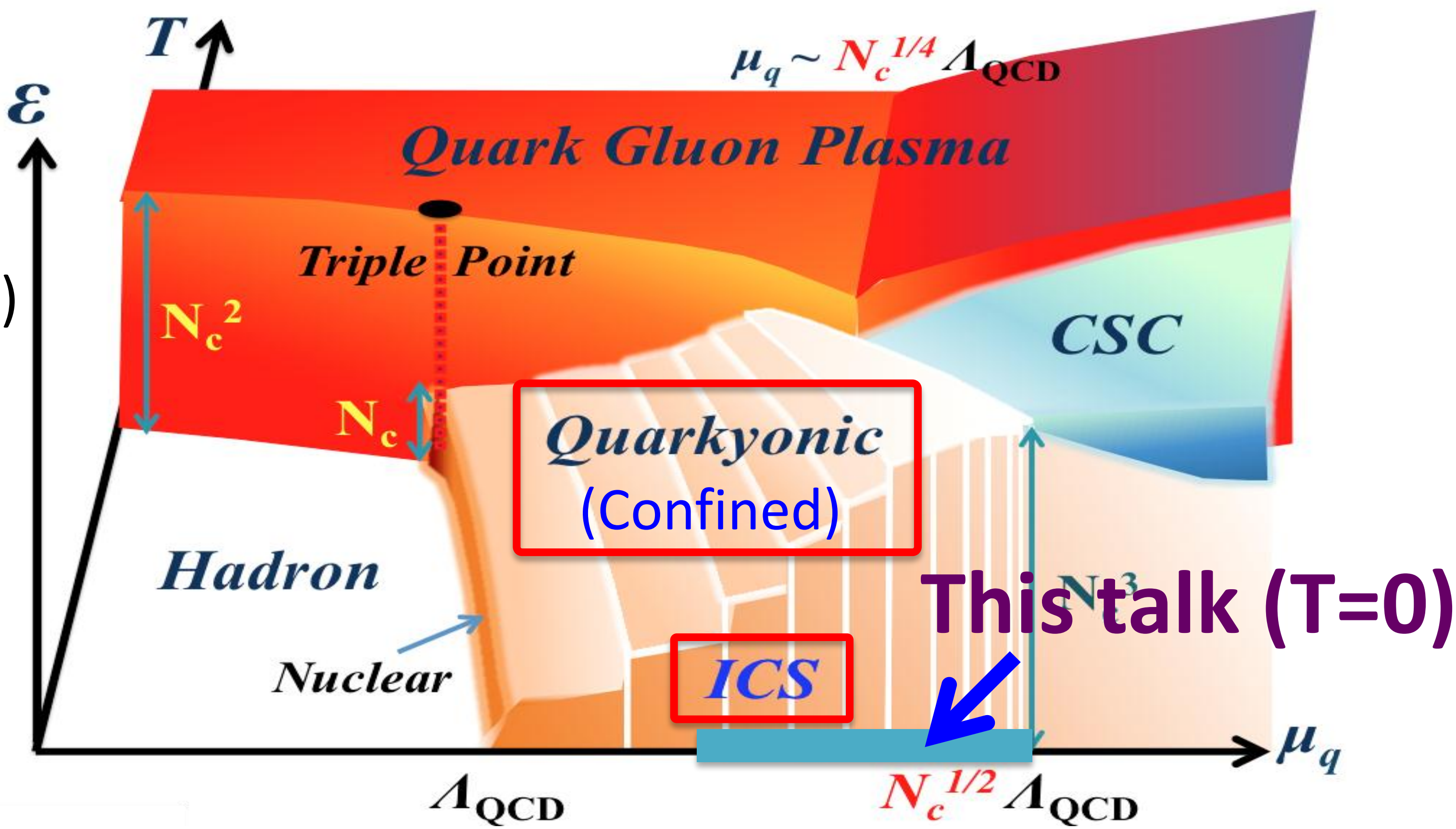
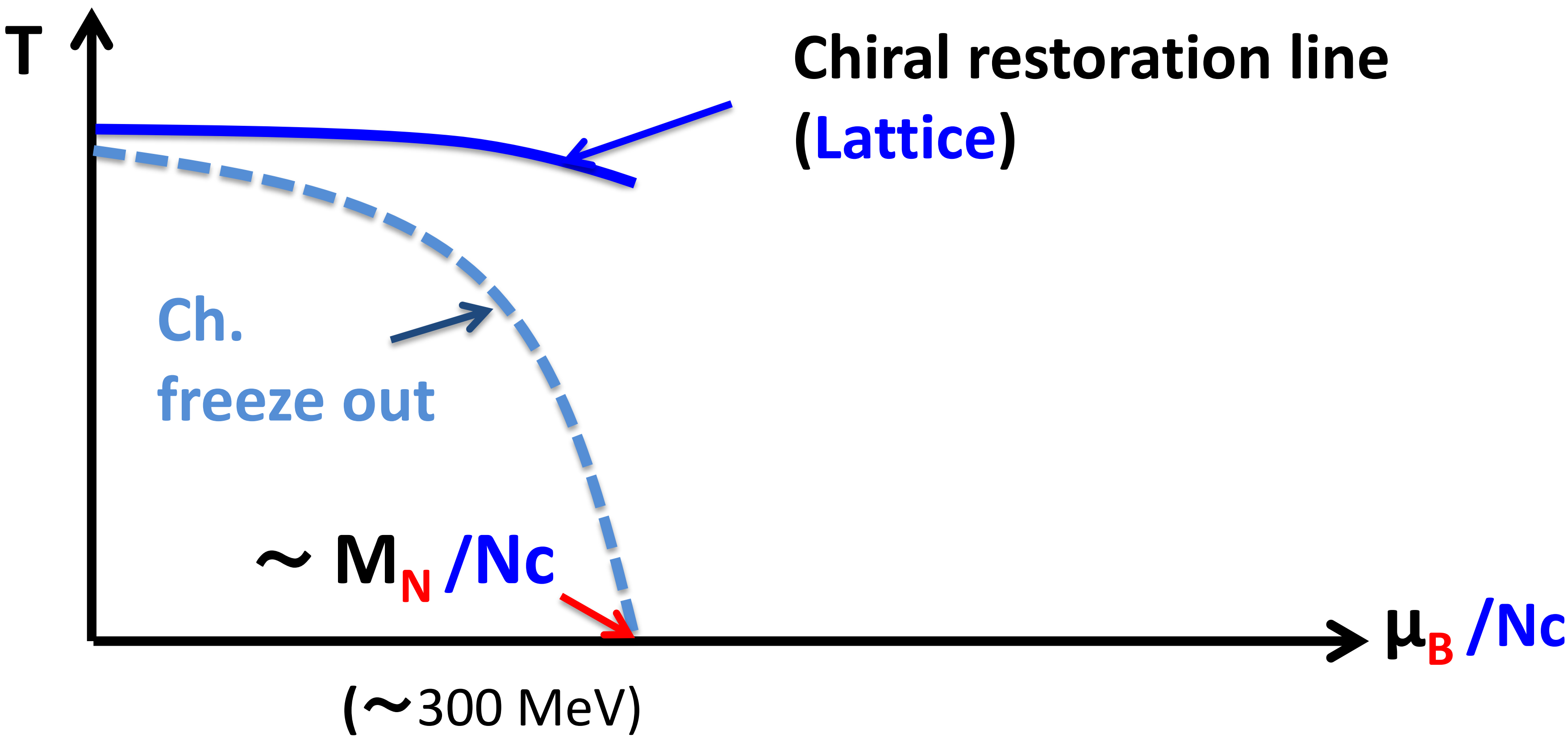
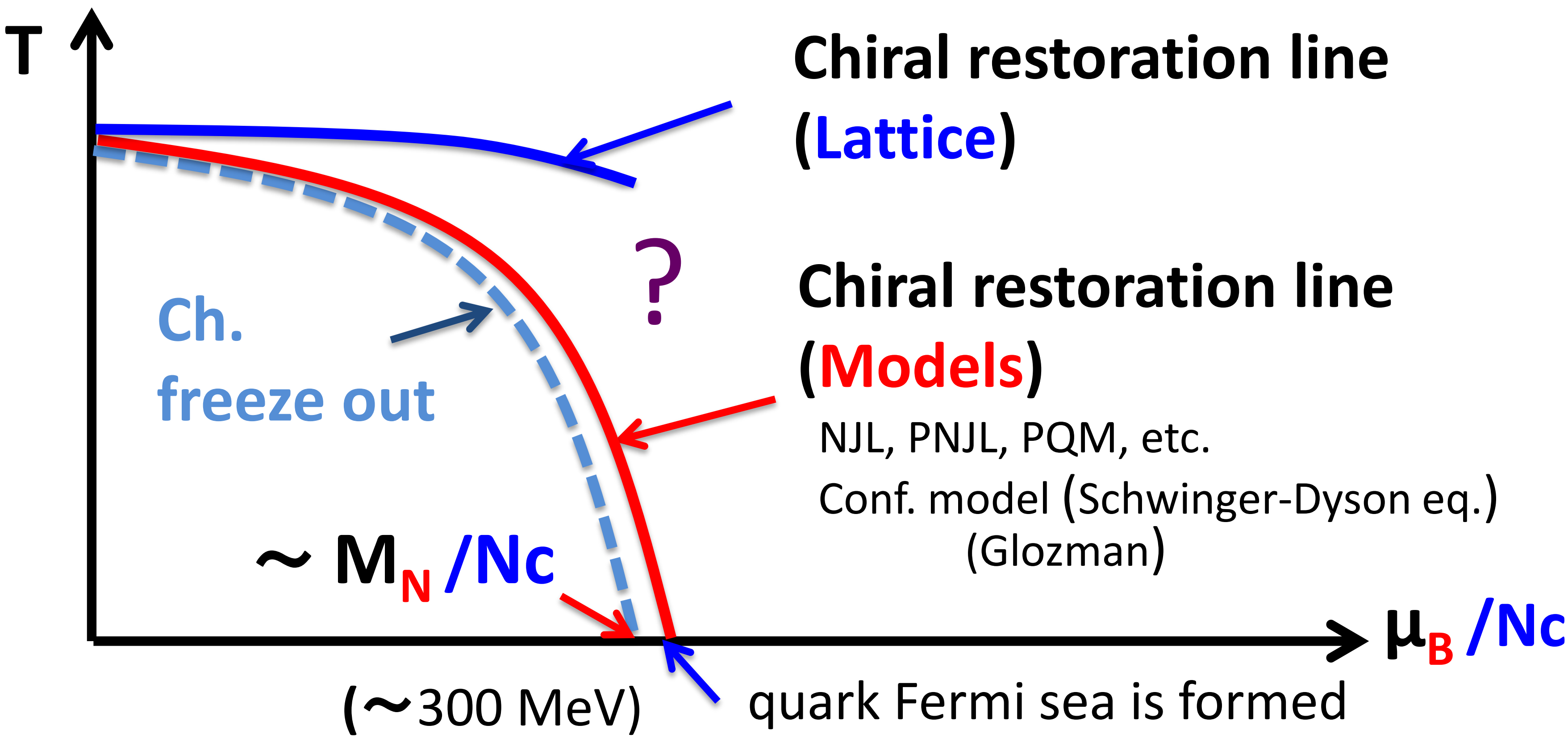


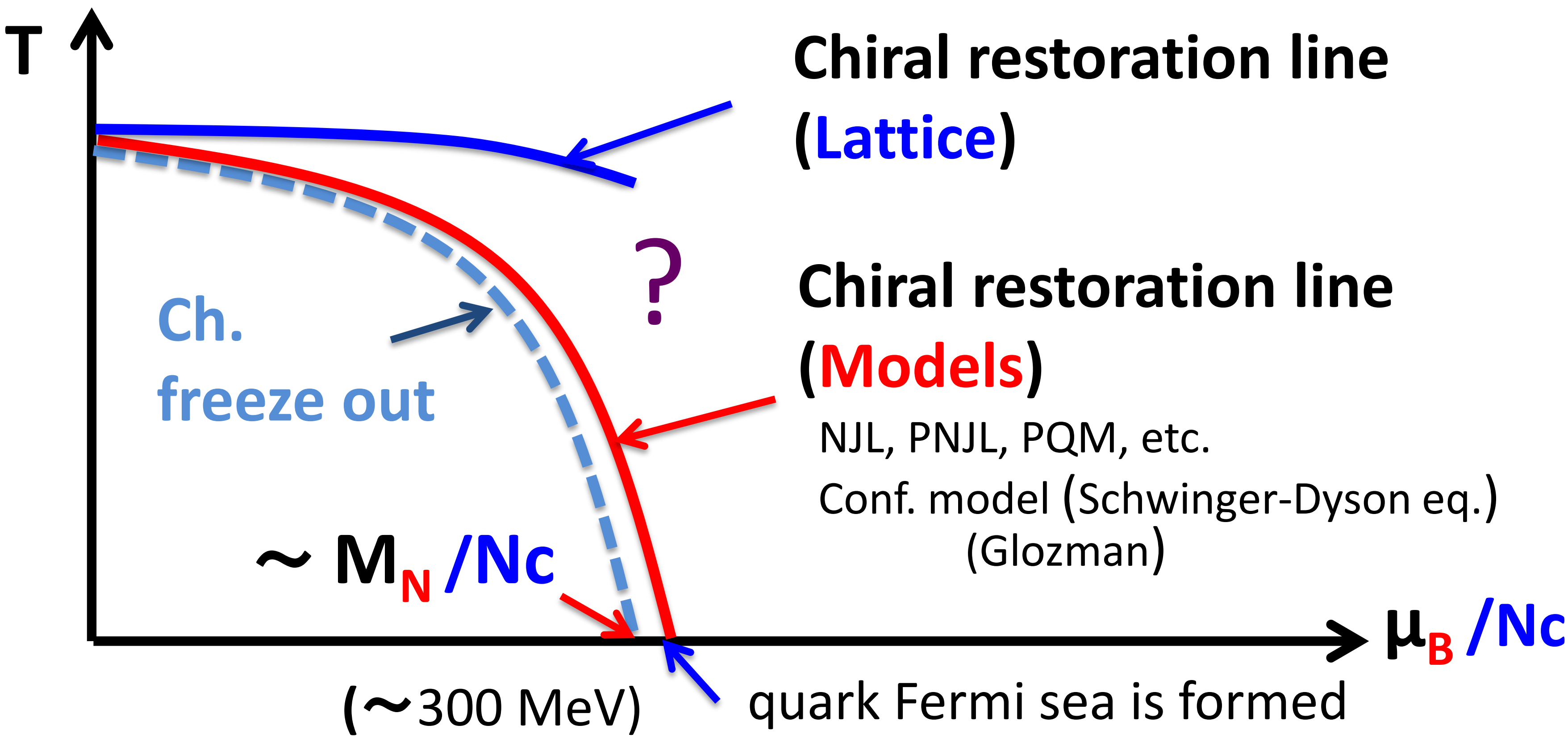
Chiral restoration



Chiral restoration



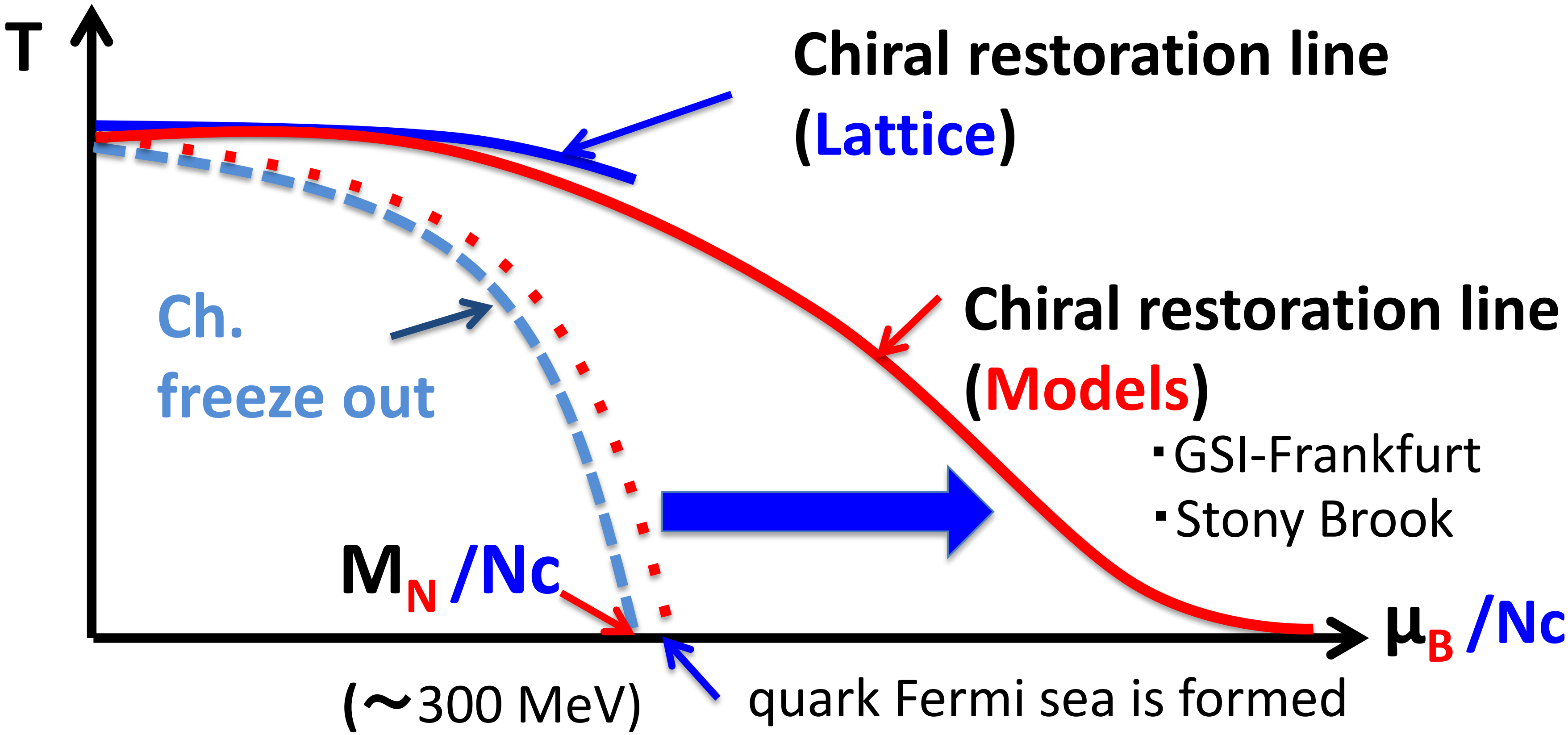
Chiral restoration



1, In conventional models, chiral restoration happens **quickly after the formation of the quark Fermi sea.**

2, Assumption: Chiral condensate is **const. everywhere.**

If we allow non-uniform condensates…



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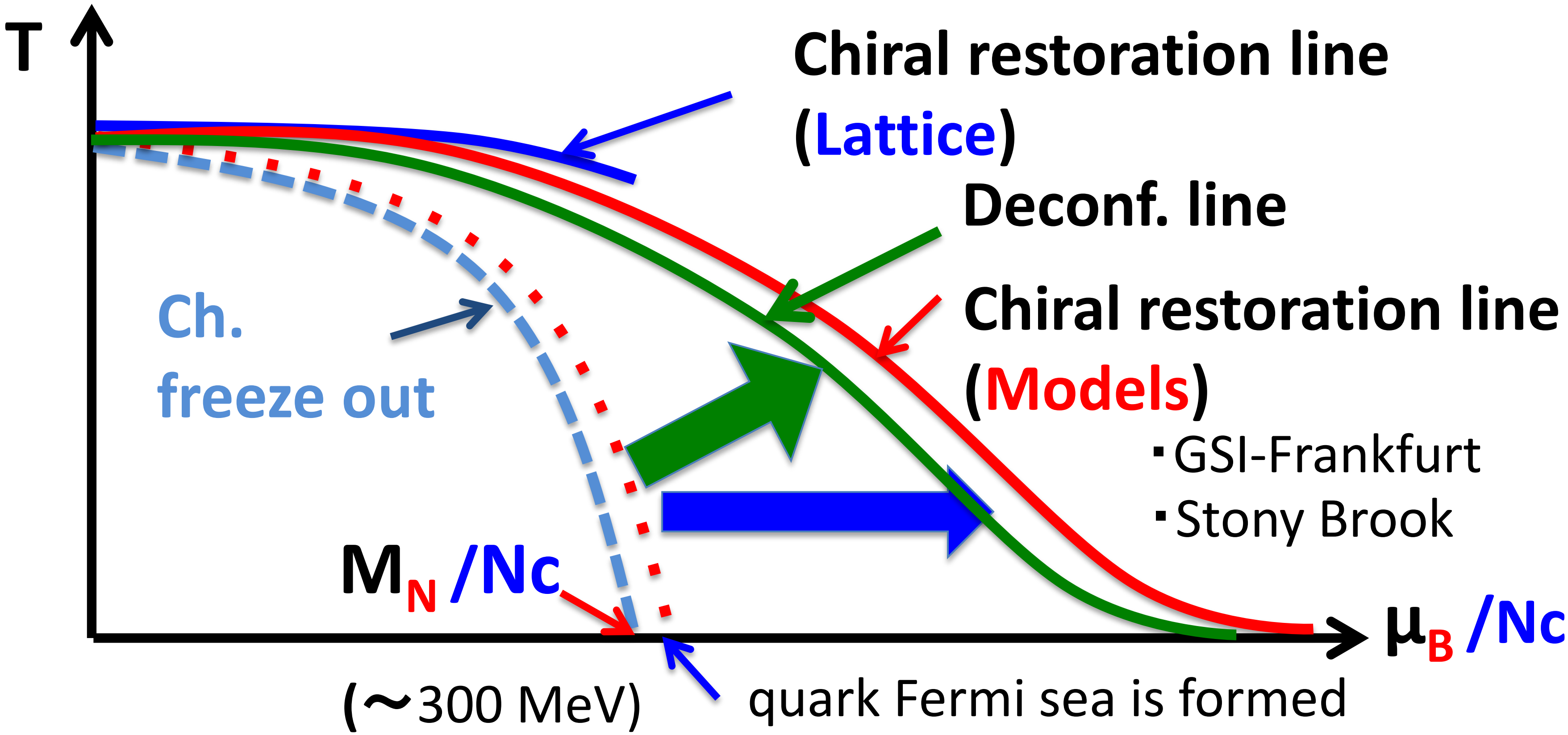
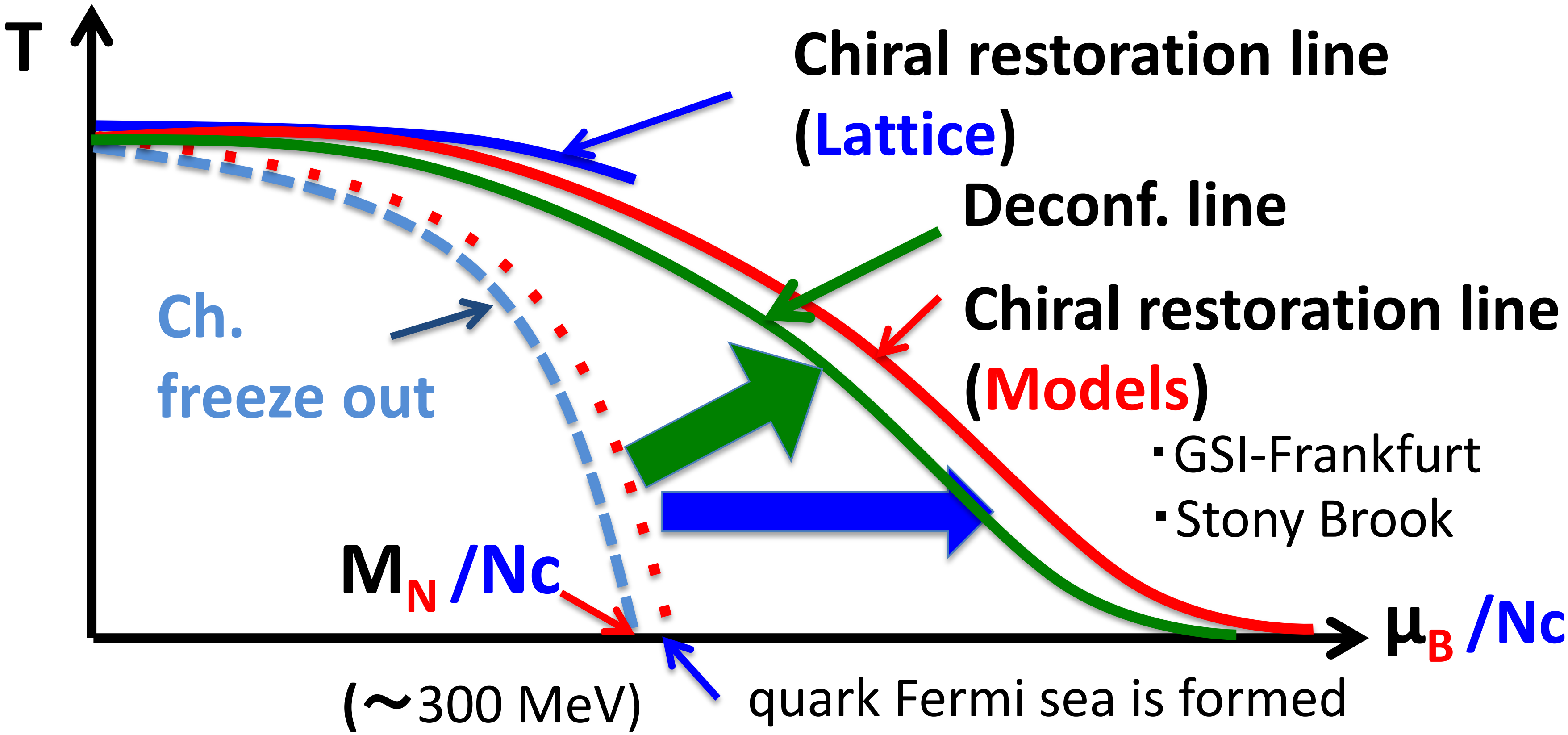


Deconfinment line would be also shifted because:

If we allow non-uniform condensates...



Deconfinment line would be also shifted because:

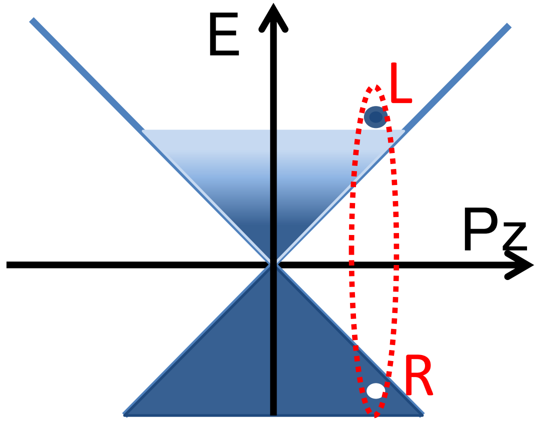
Non-uniform chiral condensate creates the **mass gap** of **quarks** near the Fermi surface.

→ The **pure glue** results are **less** affected by **massive** quarks.

Why restoration? (T=0)

- **Candidates of chiral pairing**

Dirac Type



P_{Tot}=0 (uniform)

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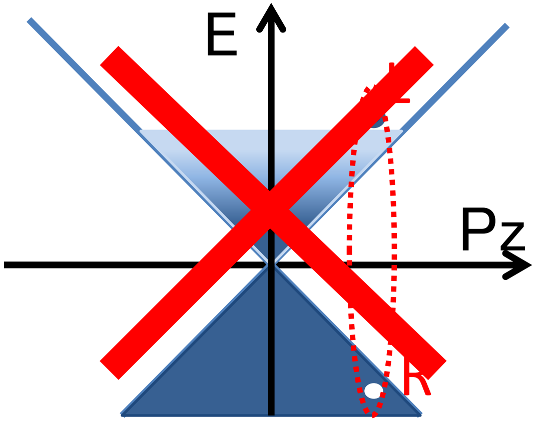
Dirac Type



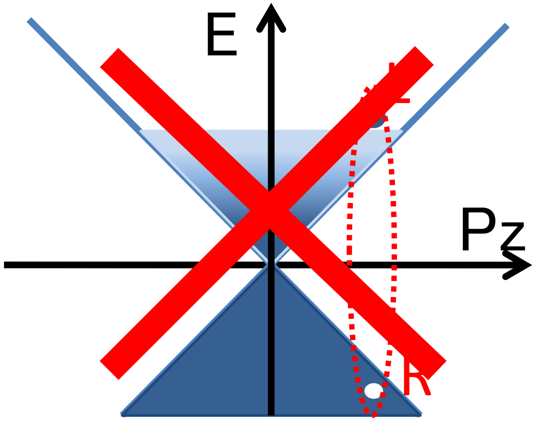
P_{Tot}=0 (uniform)

It costs large energy,
so does not occur spontaneously.

Why non-uniform? (T=0)

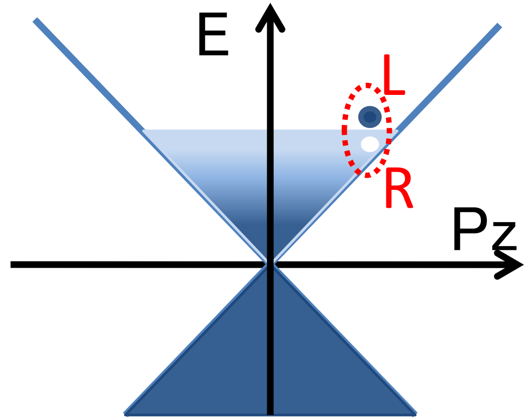
- **Candidates of chiral pairing**

Dirac Type



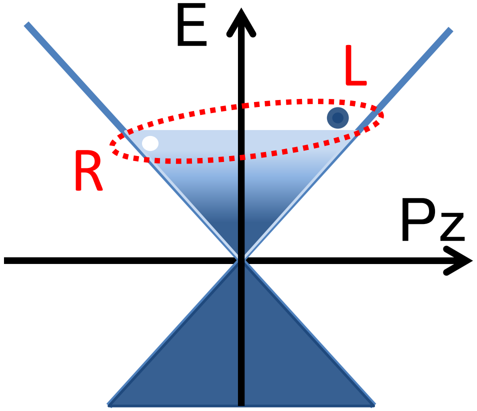
$P_{Tot}=0$ (uniform)

Exciton Type



$P_{Tot}=0$ (uniform)

Density wave

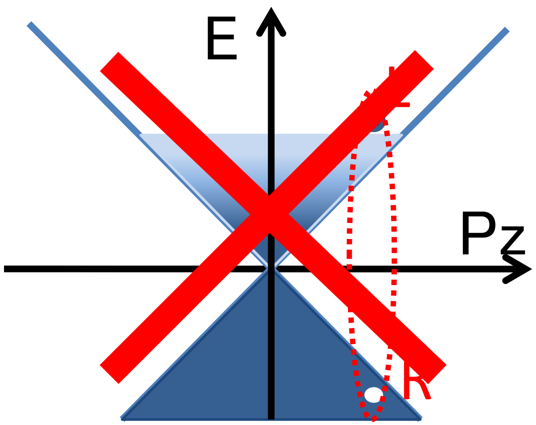


$P_{Tot}=2\mu$ (non-uniform)

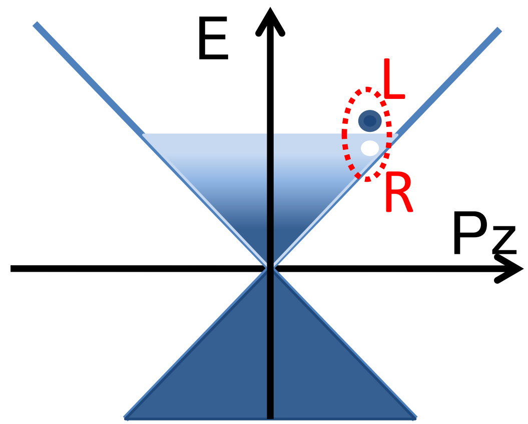
Why non-uniform? (T=0)

・Candidates of chiral pairing

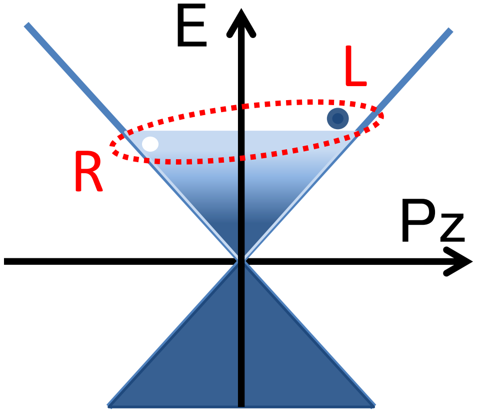
Dirac Type



Exciton Type

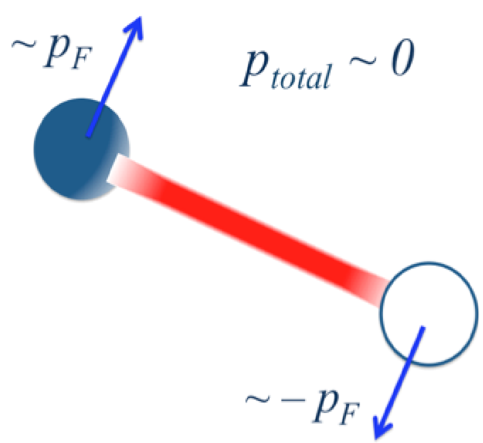


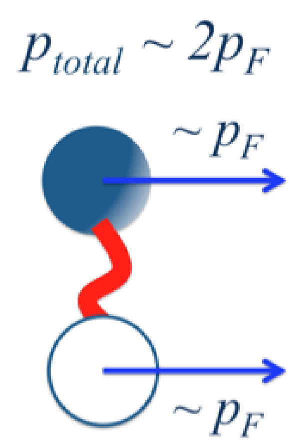
Density wave



$P_{Tot}=0$ (uniform) $P_{Tot}=0$ (uniform) $P_{Tot}=2\mu$ (non-uniform)

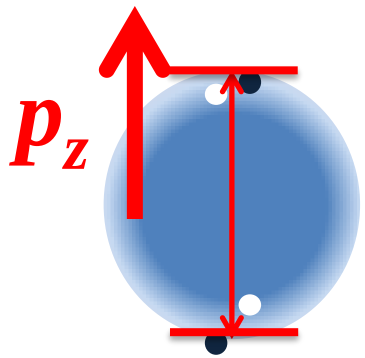
- ***Kinetic*** energy:
  comparable
- ***Potential*** energy:
  Big difference





Single Chiral Spiral

- **Choose one particular direction :**



Single Chiral Spiral

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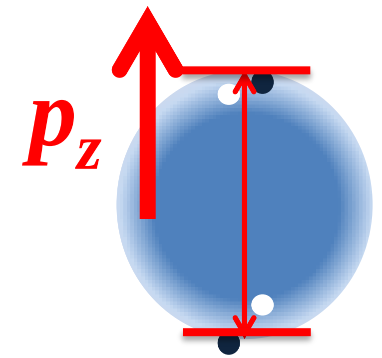
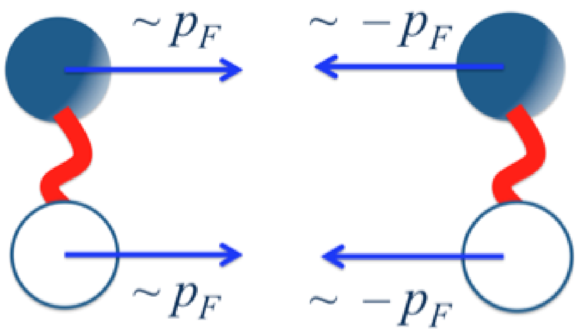


- Two kinds of condensates appear :

space-dep.



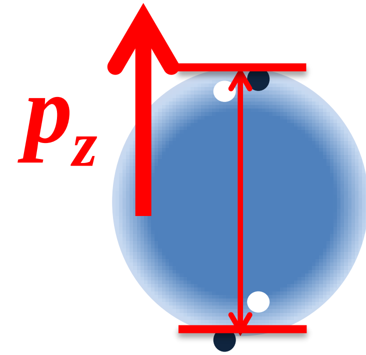
→ linear comb.

$$\langle \bar{\psi}\psi \rangle = \Delta \cos(2p_F z)$$

P-odd

$$\langle \bar{\psi} i\gamma_0 \underline{\gamma_z} \psi \rangle = \Delta \sin(2p_F z)$$

Single Chiral Spiral



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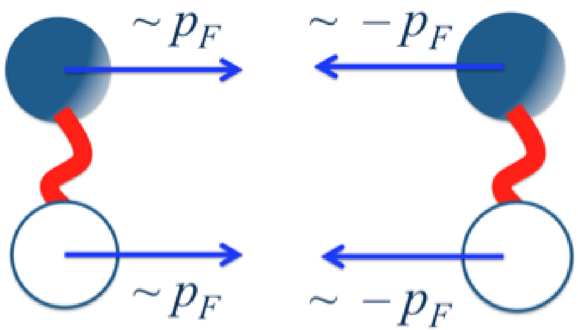

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$$\langle \bar\psi \psi \rangle = \Delta \cos(2 p_F z)$$

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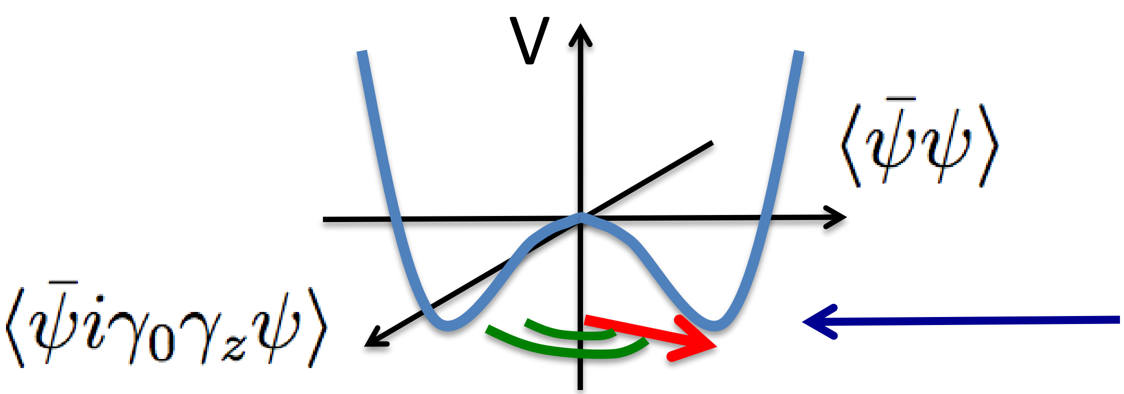
- Chiral rotation with fixed radius :



radius (for 1-pair)

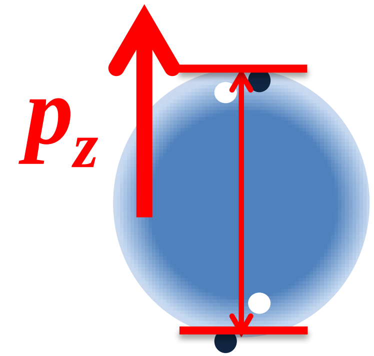
$\sim \Lambda_{QCD}^3$

period of rotation

$\Delta z \sim 1/2p_F$

Interweaving Chiral Spiral

So far we have considered only
the Chiral Spiral in **one direction**.



Is it possible to have CSs in multiple directions?

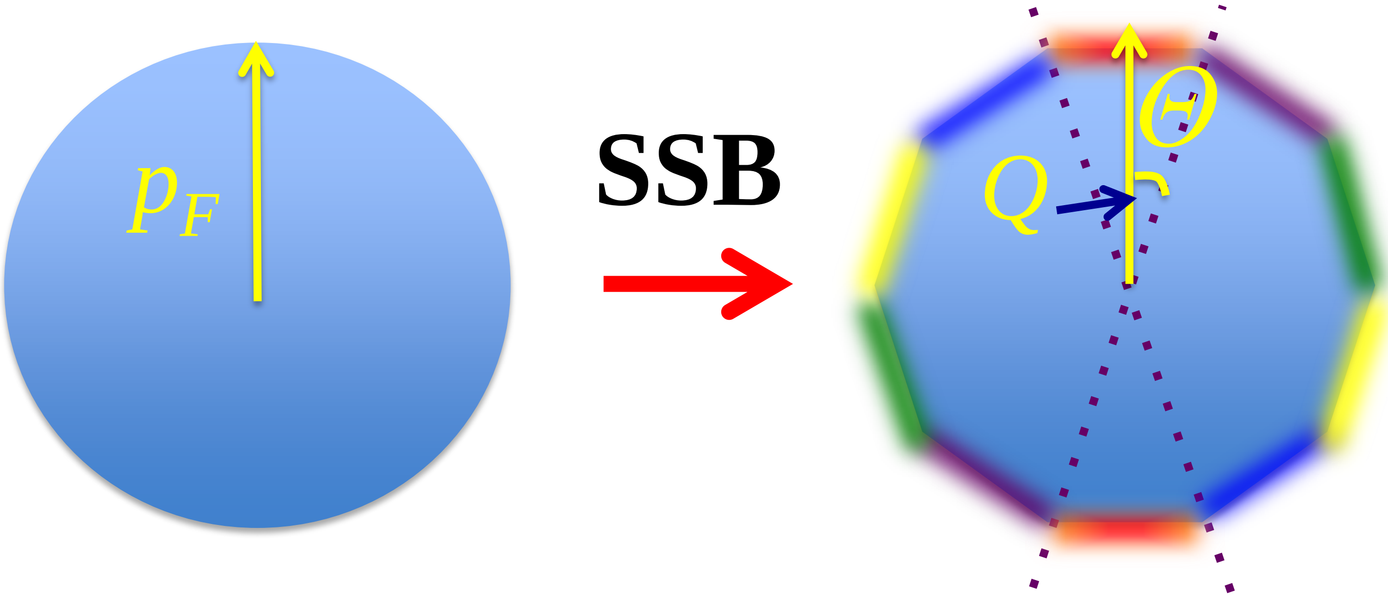
YES!

Pairs around the **entire Fermi surface** can condense.

Then, the free energy becomes comparable to the S-wave color super conductor.

(2+1) D Example

Rotational Sym. : $U(1) \longrightarrow Z_{2N_p}$ (N_p : Num. of patches)



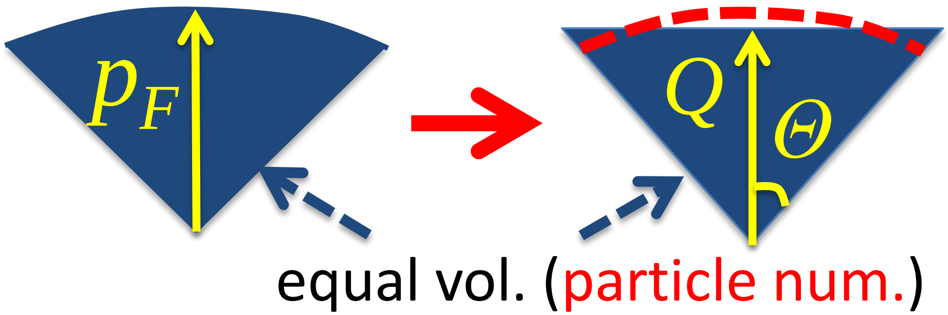
Variational parameter : angle $\Theta \sim 1/N_p$

We use canonical ensemble : $Q \rightarrow Q\,(\Theta, p_F)$

- **We will optimize the angle Θ**

Energetic gain v.s. cost

- **Cost : Deformation**
  (dominant for **large** Θ)



Energetic gain v.s. cost

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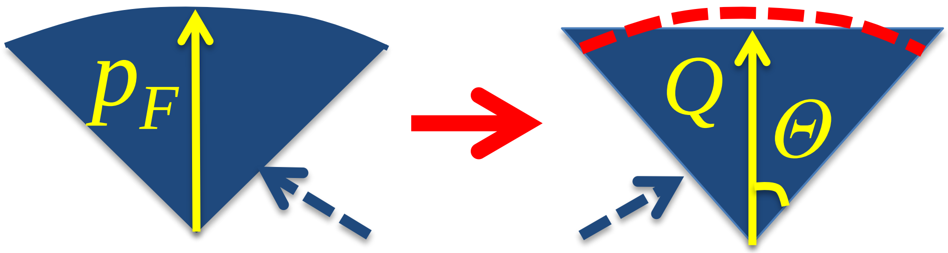
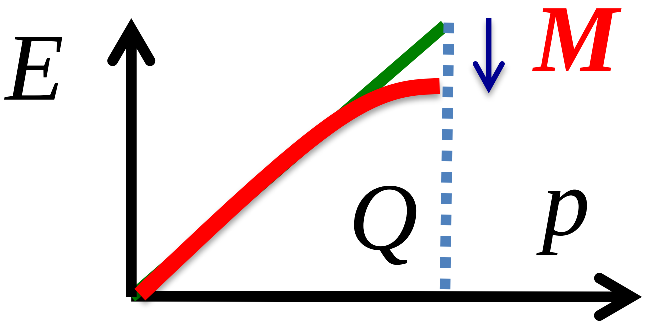


equal vol. (particle num.)

- **Gain : Mass gap origin**

Condensation effects



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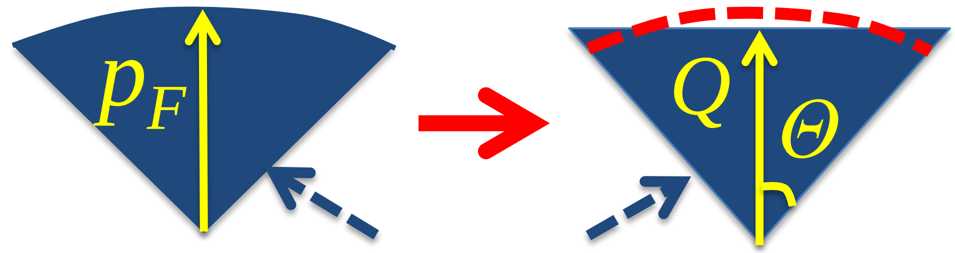

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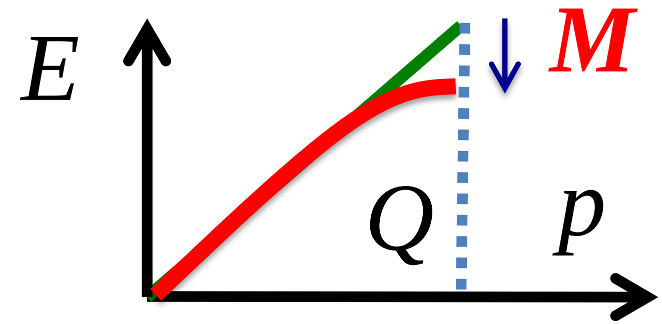
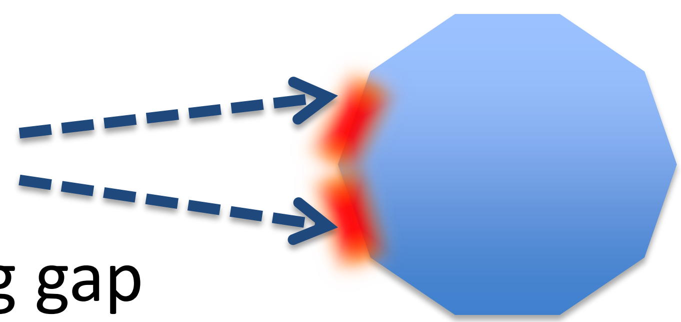


- **Cost : Interferences among CSs**
  (dominant for **small** Θ)

(**Model dep.** *!!*)

Condensate – Condensate int.



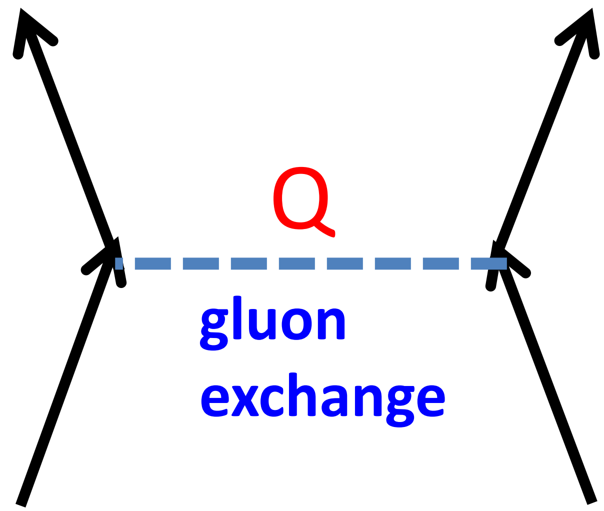
→ destroy one another, reducing gap

A schematic model

Strength of interactions is determined by

Momentum transfer, **NOT** by **quark momenta.**

→ **Even at high density**, **int.** **is strong** for **some processes**.



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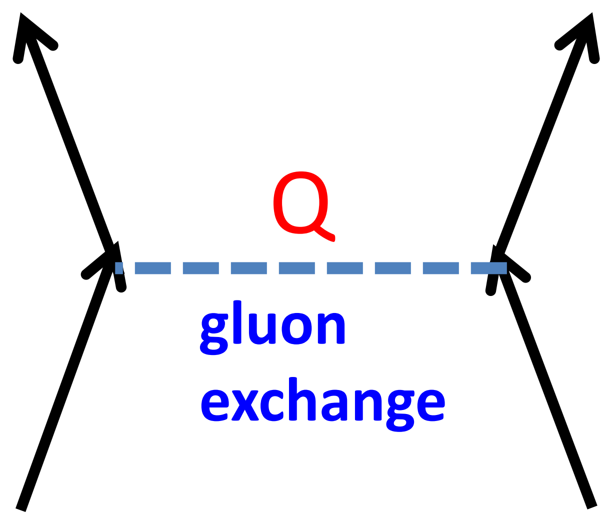
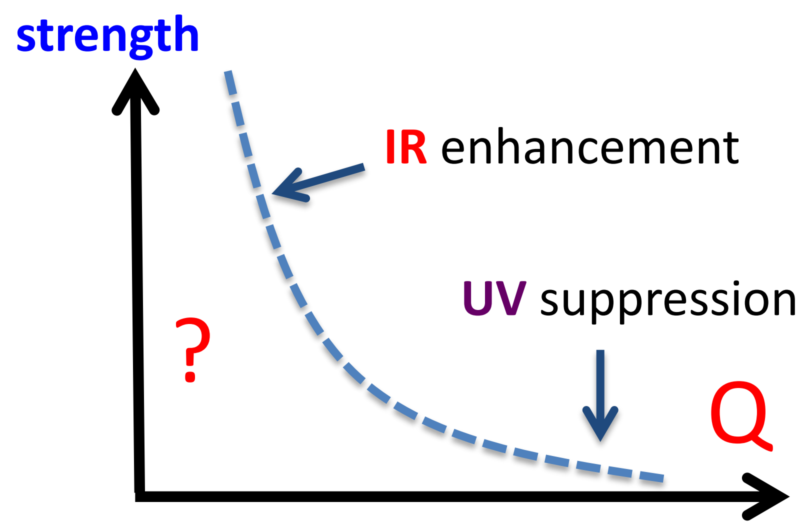
Therefore we use the int. with the following properties:





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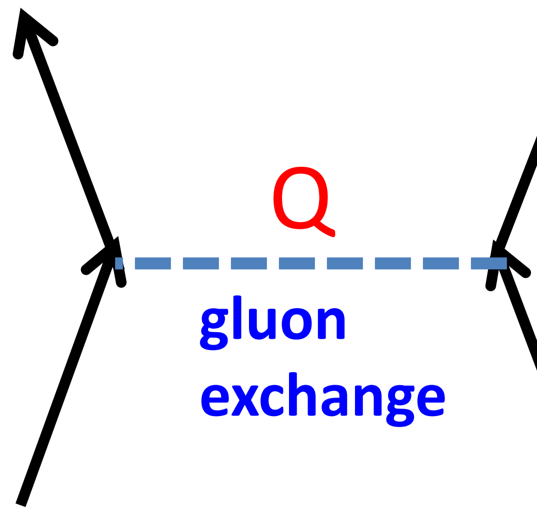

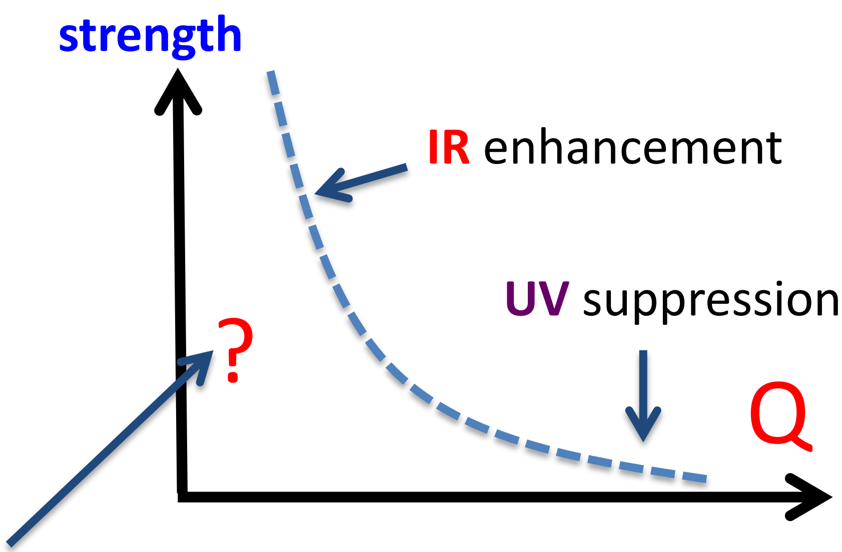

- The **detailed form in the IR region does not matter.**

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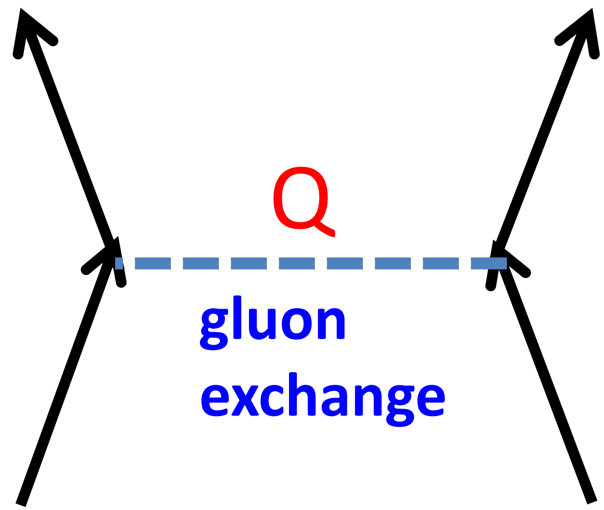

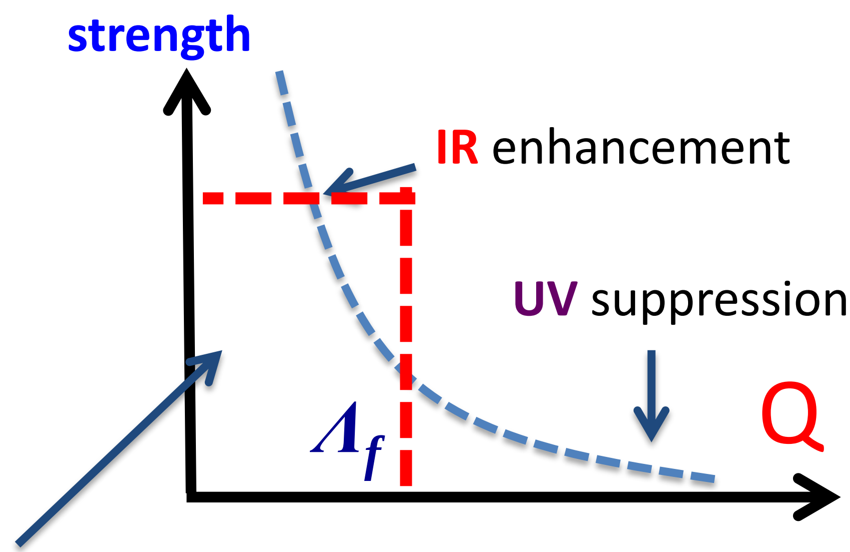

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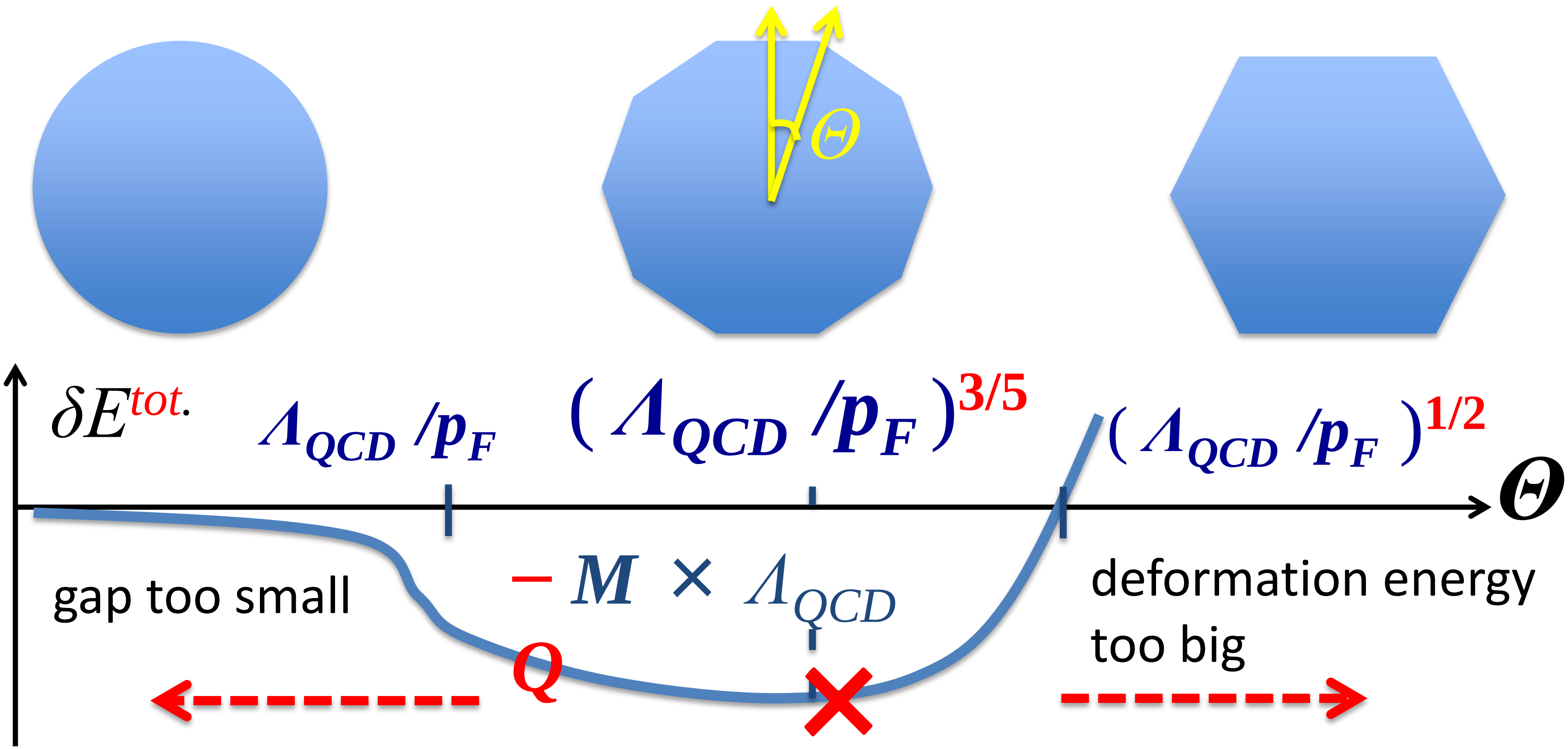
Energy Landscape (for fixed p_F)



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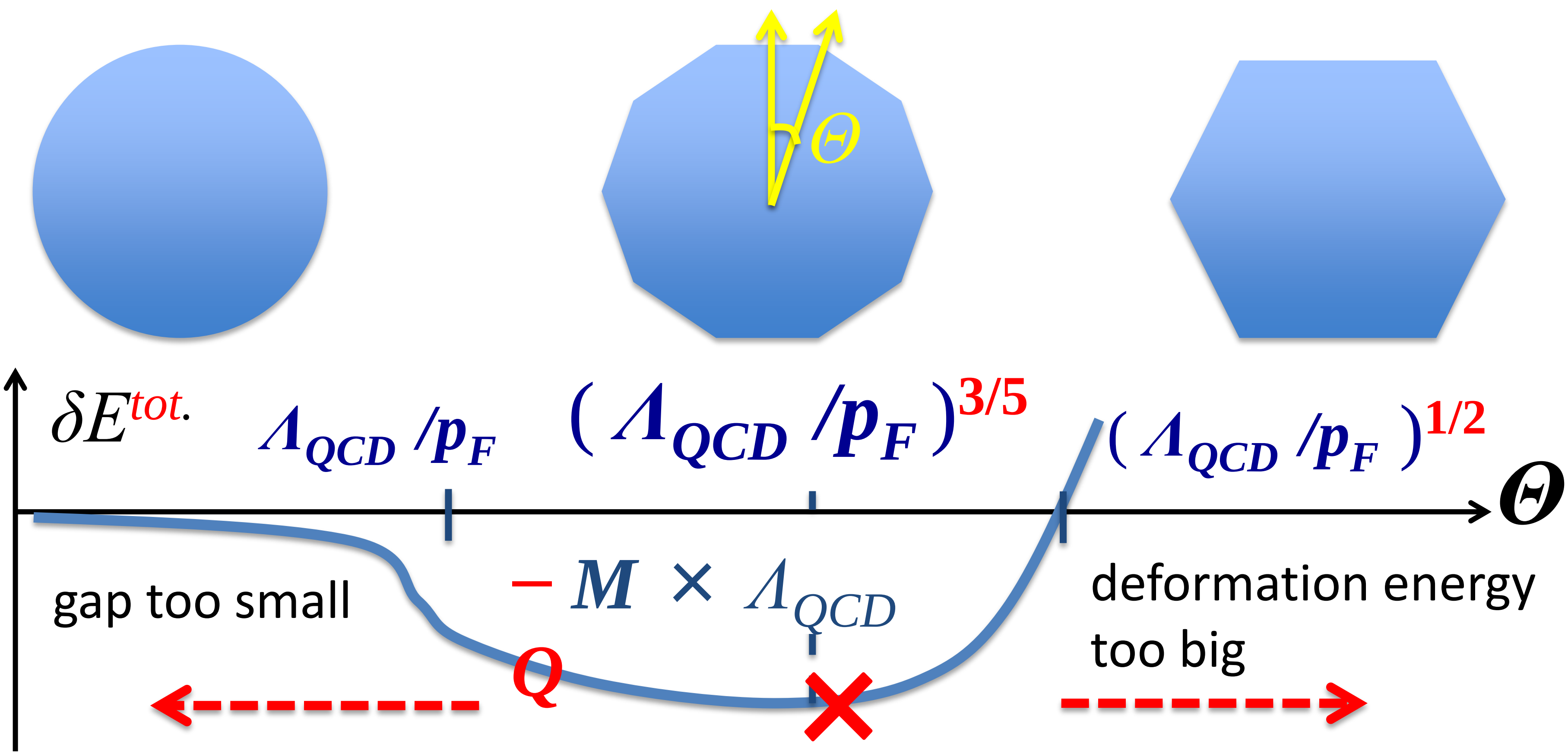
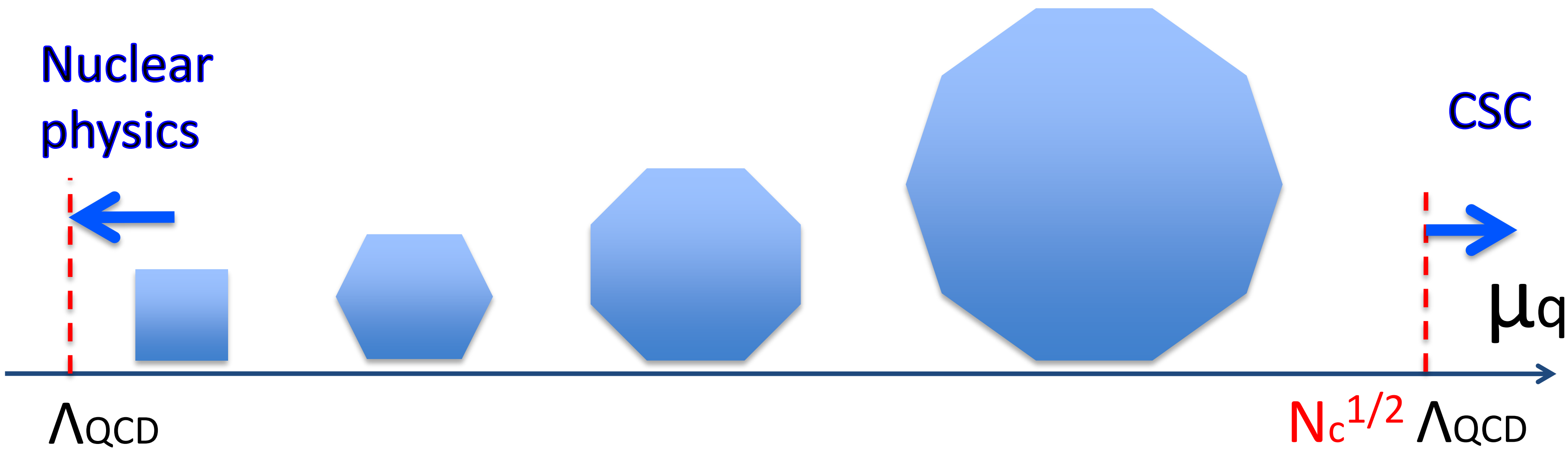


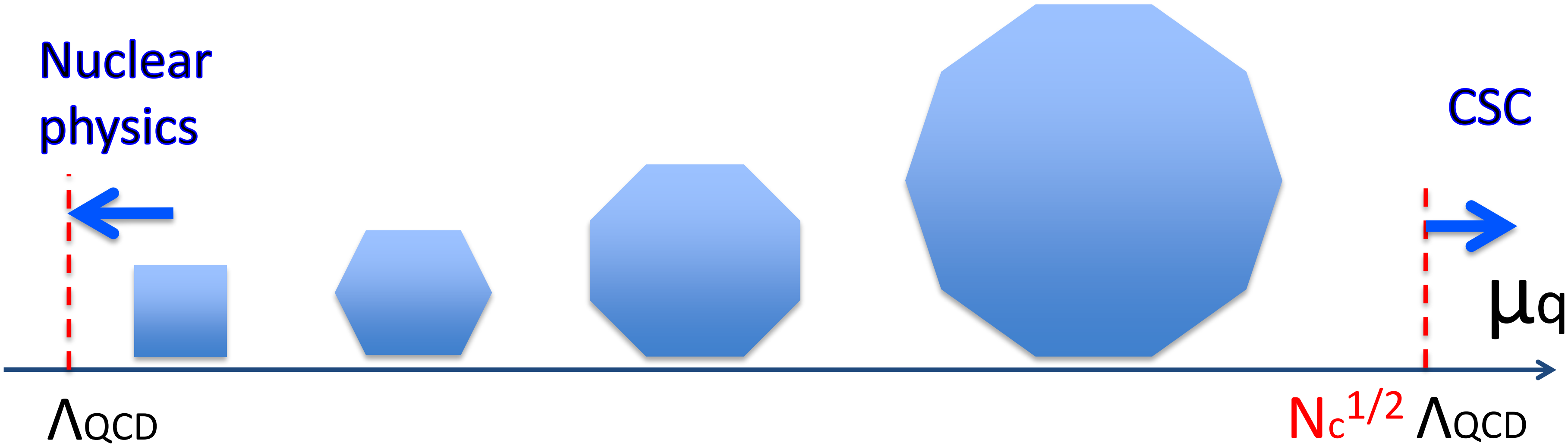
$$N_p \sim 1/\Theta \sim (p_F / \Lambda_{fQCD})^{3/5}$$

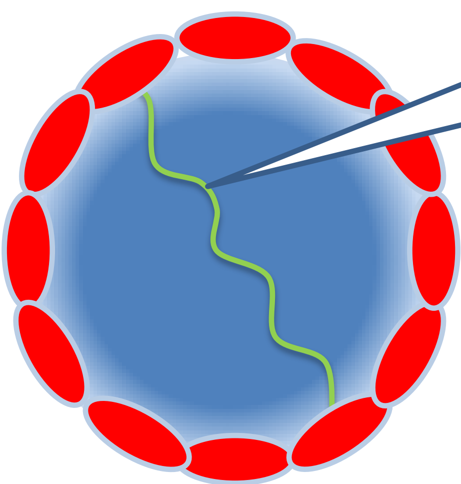
- **Patch num. depends upon density.**

(2+1) dim. ICS



(2+1) dim. ICS

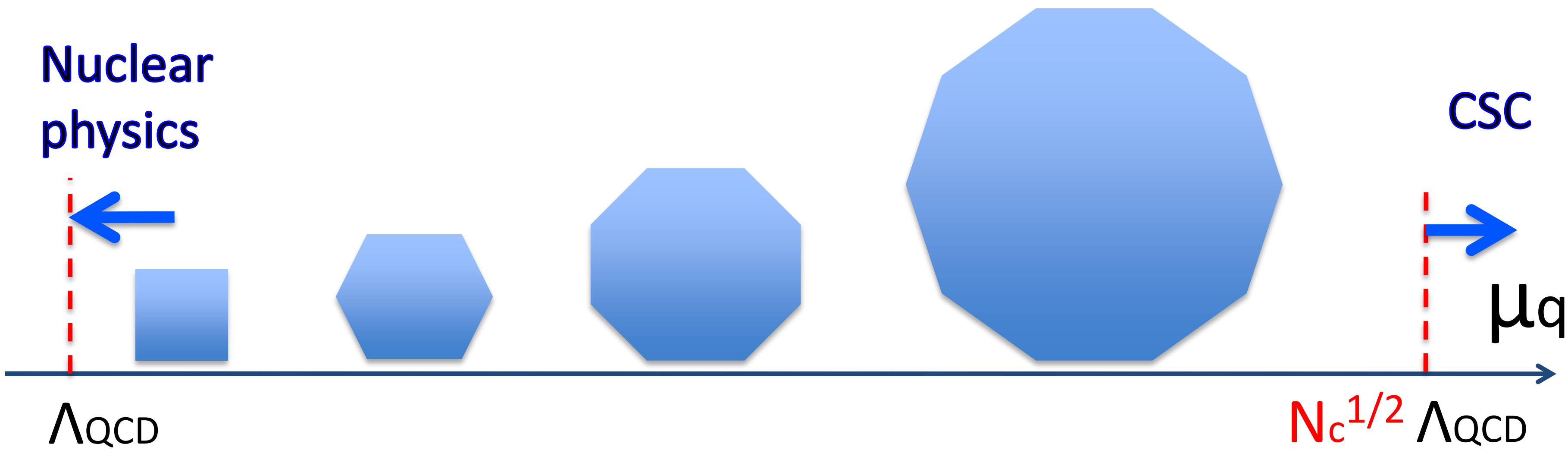


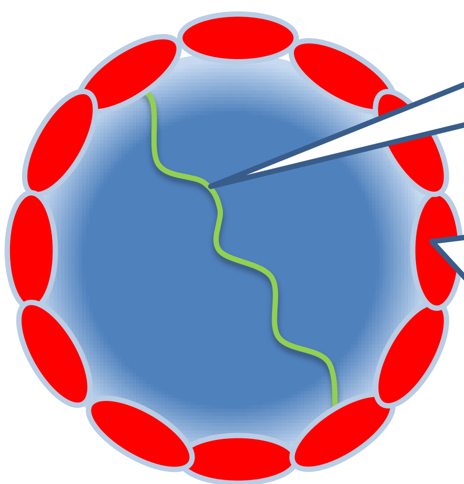


- Deeply inside: (perturbative quarks)

  Very likely Chiral sym. restored

(2+1) dim. ICS





- **Deeply inside:** (perturbative quarks)

  Very likely Chiral sym. restored

- **Near the Fermi surface:**

  Quarkyonic Chiral Spirals

  → Local violation of P & Chiral sym.

  → Quarks acquire the mass gap ,
  delaying the deconf. transition at finite density.

Summary & Outlook

The **ICS** has large impact for

chiral restoration & **deconfinement**.

- 1, The **low energy effective Lagrangian** → coming soon.
- 2, **Temperature effects** & **Transport properties**

(→ hopefully next CPOD)

Summary & Outlook

The **ICS** has large impact for

chiral restoration & **deconfinement**.

- 1, The **low energy effective Lagrangian** → coming soon.
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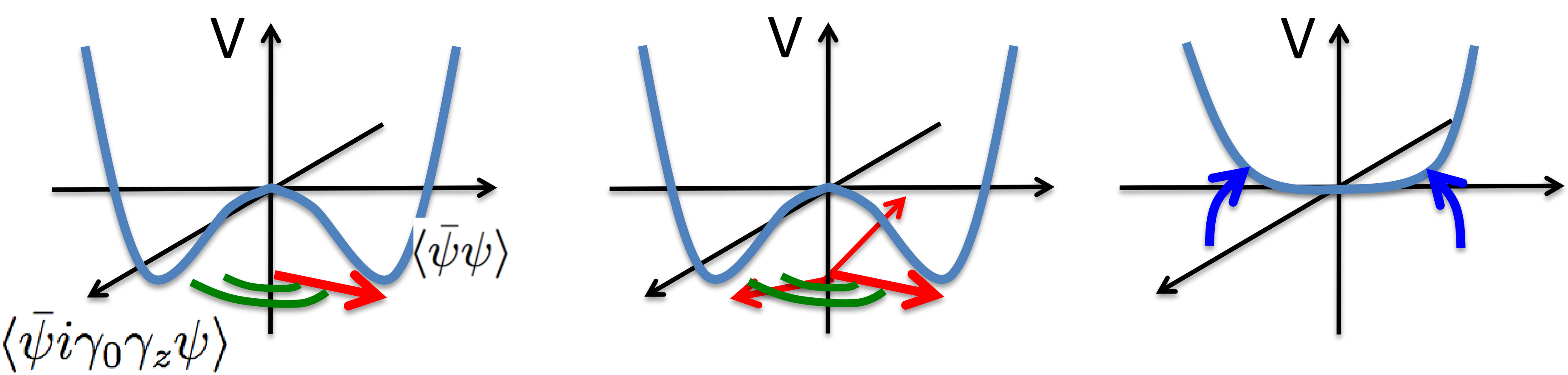
(→ hopefully next CPOD)

My guess :

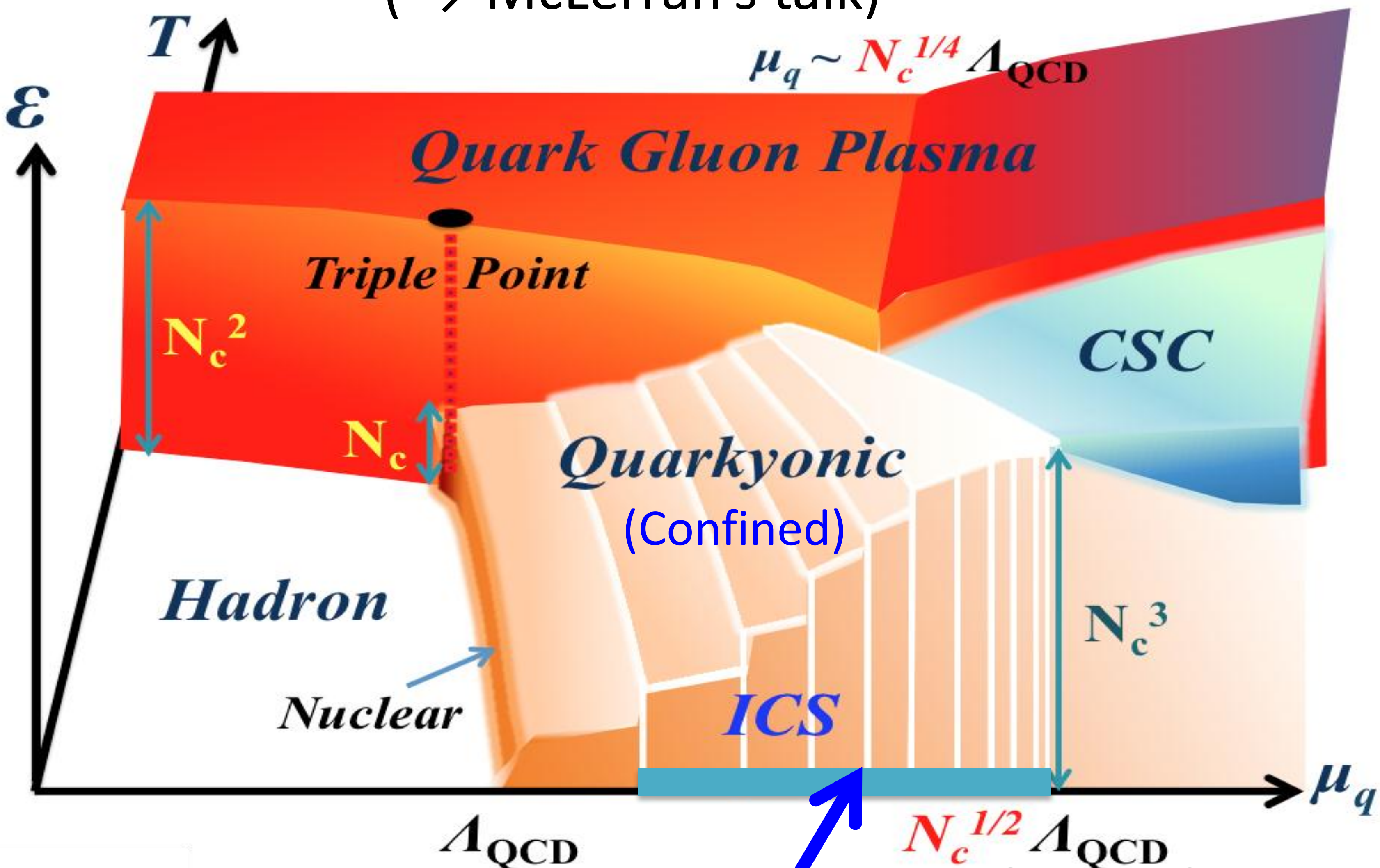
T=0	**0 << T < Tc**	**T ∼ Tc**
(inhomogeneous)	(homogeneous)	(linear realization)



Appendix

Large Nc phase diagram (2-flavor)

(→ McLerran's talk)

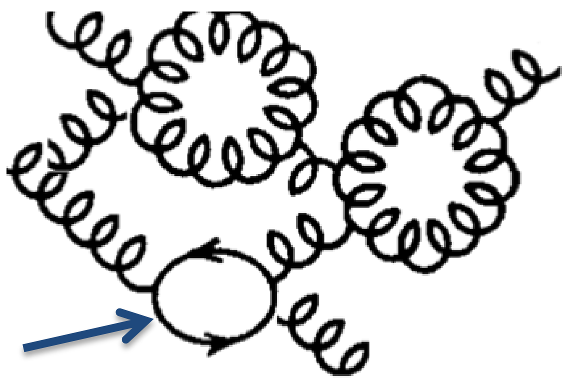


This talk (T=0)

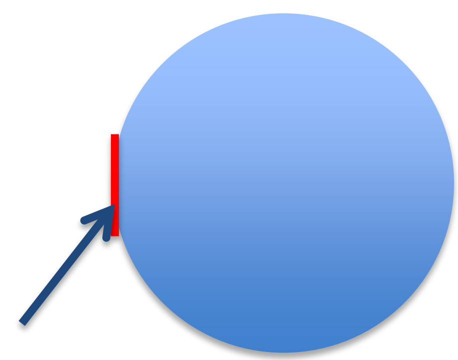
We will discuss

Consequences of **convolutional** effects

Nonpert. gluon dynamics



Fermi surface effects



emphasized by

Large Nc

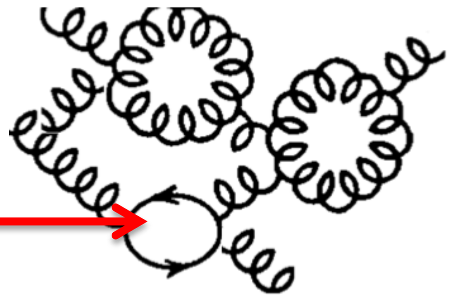
emphasized by

Large μ

How useful is such regime ?

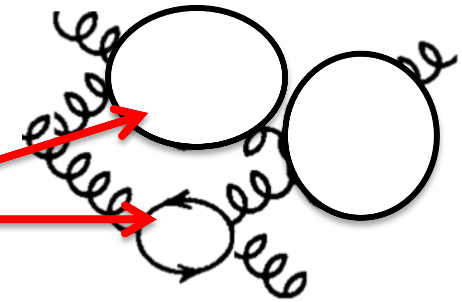
- **Two approximations compete :**

Vacuum:



small fraction

Large μ :



large fraction

So gluon sector will be eventually modified.

(Large Nc picture is no longer valid.)

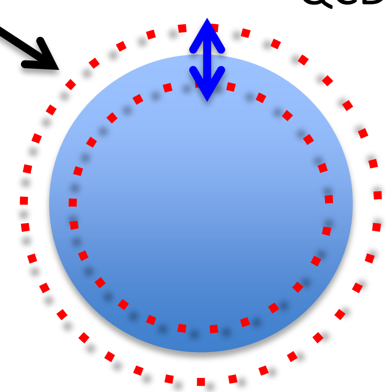
- **When modified? :**

gluon d.o.f: $N_c^2 \rightarrow N_c^2$

quark d.o.f: $N_c \rightarrow N_c \times (\mu/\Lambda_{QCD})^{d-1}$

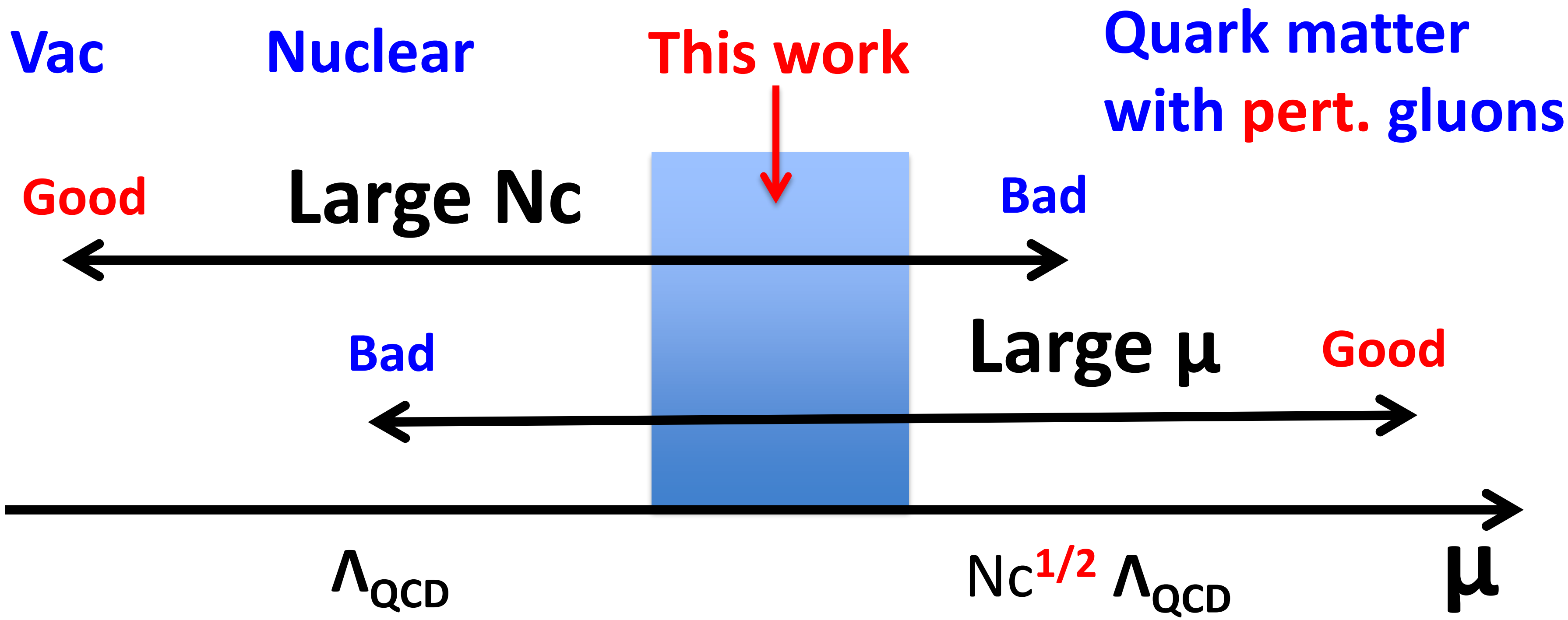
larger phase space

$\sim \Lambda_{QCD}$



For (**3**+1)D, $\mu \sim N_c^{1/2} \Lambda_{QCD}$.

Strategy



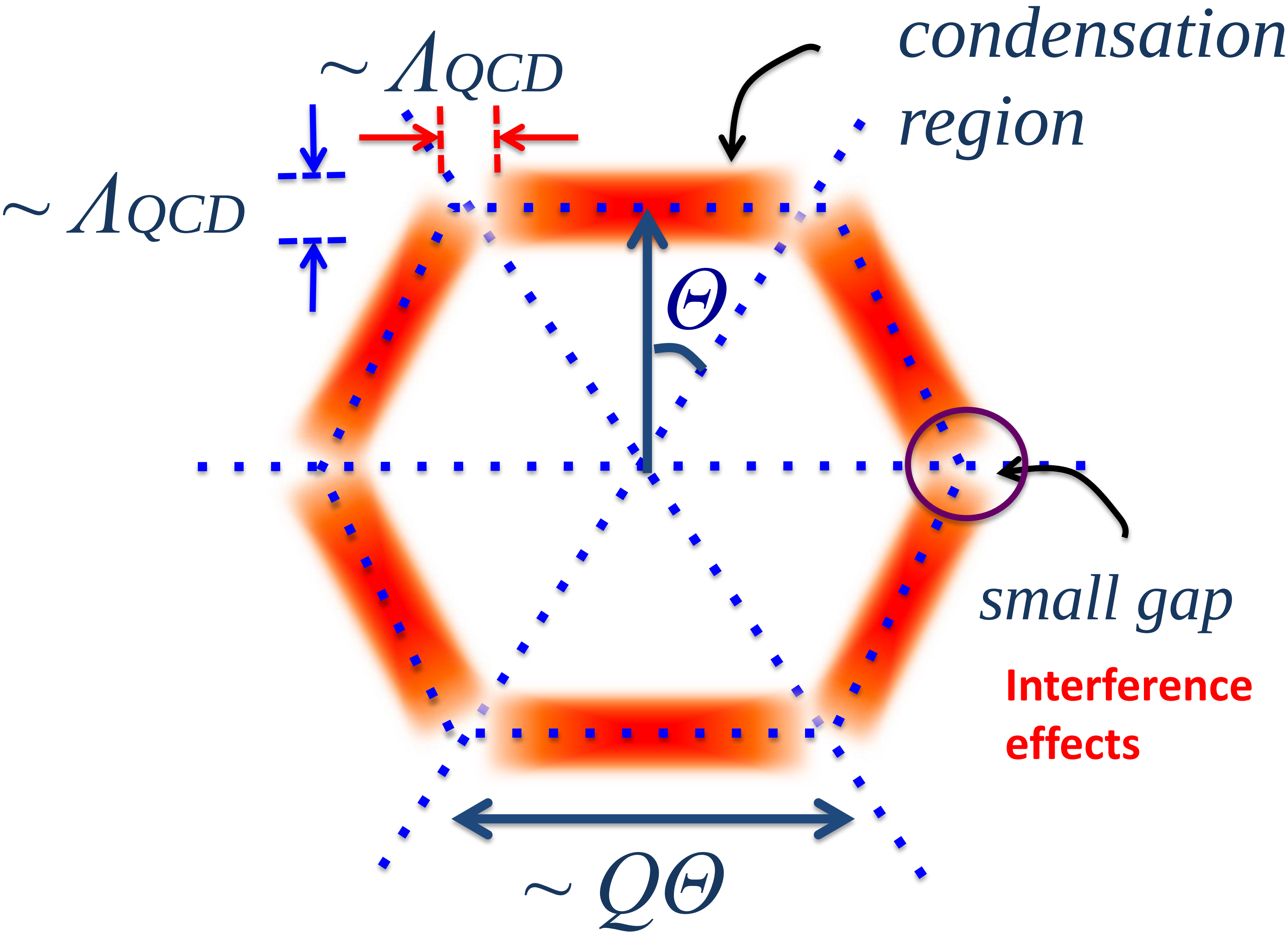
We will

1, Solve large Nc & μ, theoretically clean situation.

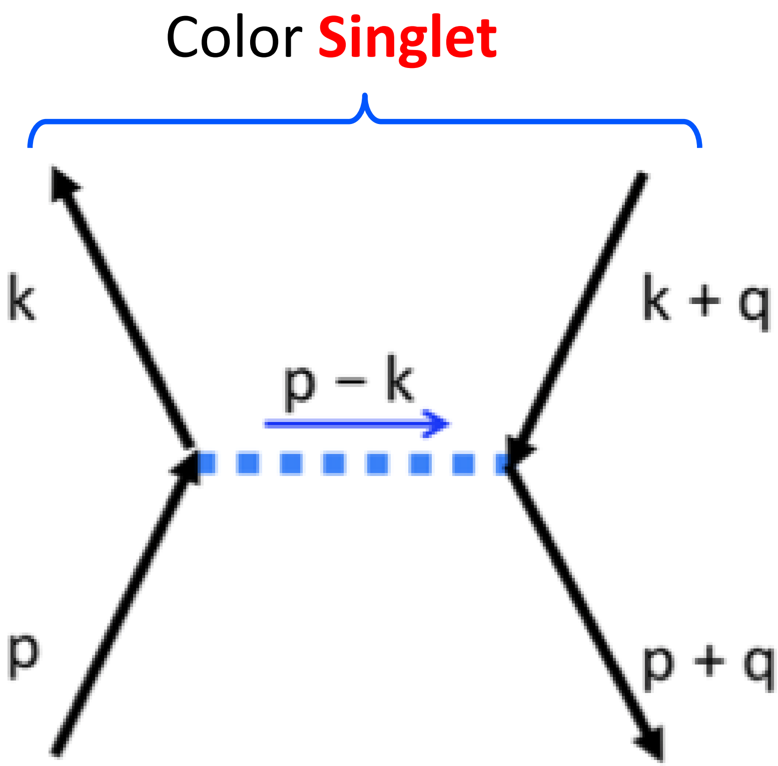
2, Construct the pert. theory of Λ_{QCD}/μ expansion.

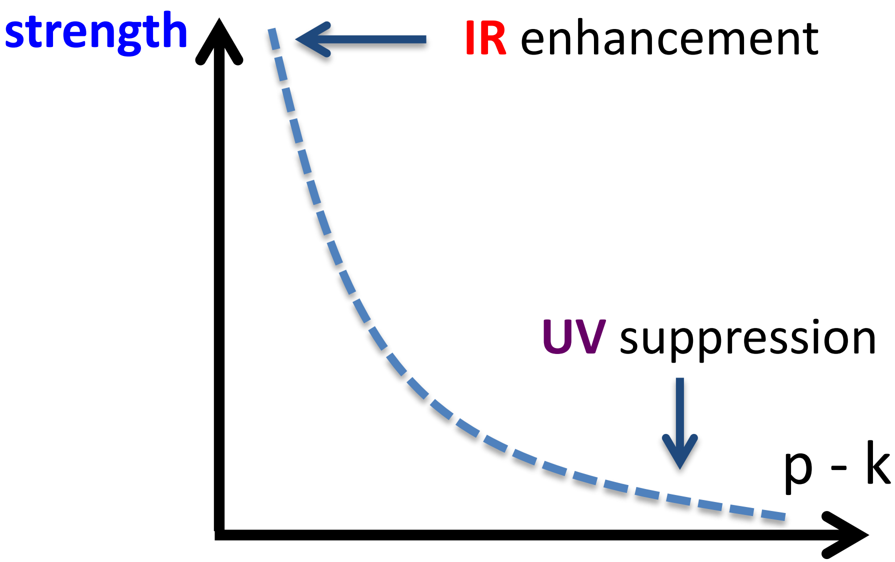
3, **Infer** what will happen in the **low** density region.

Gap distribution will be



A crude model with asymptotic freedom

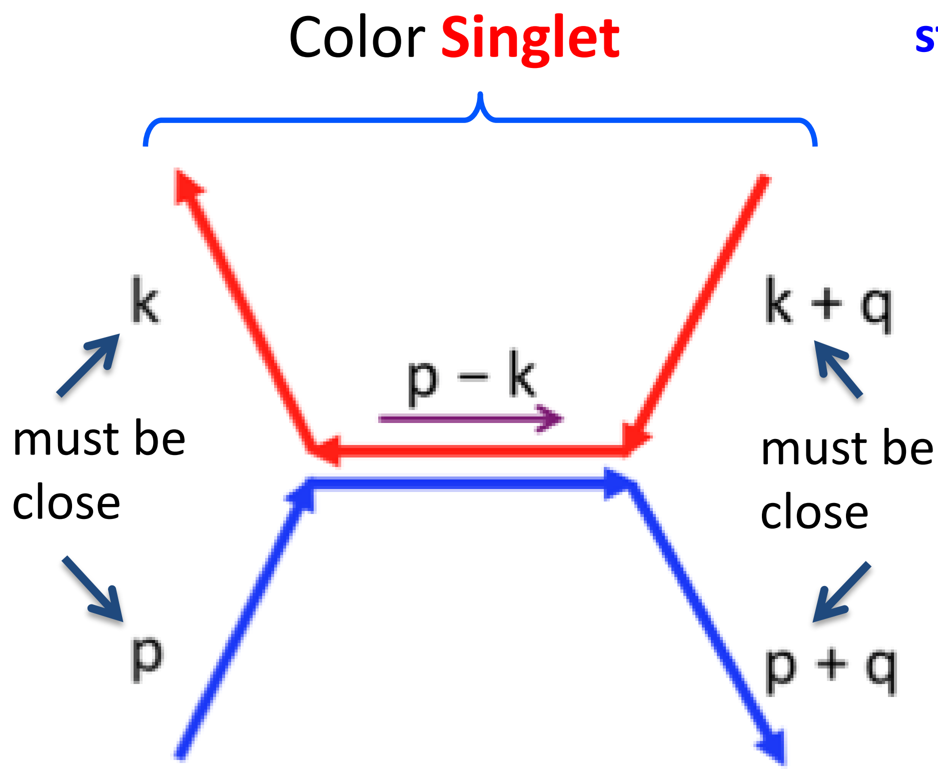


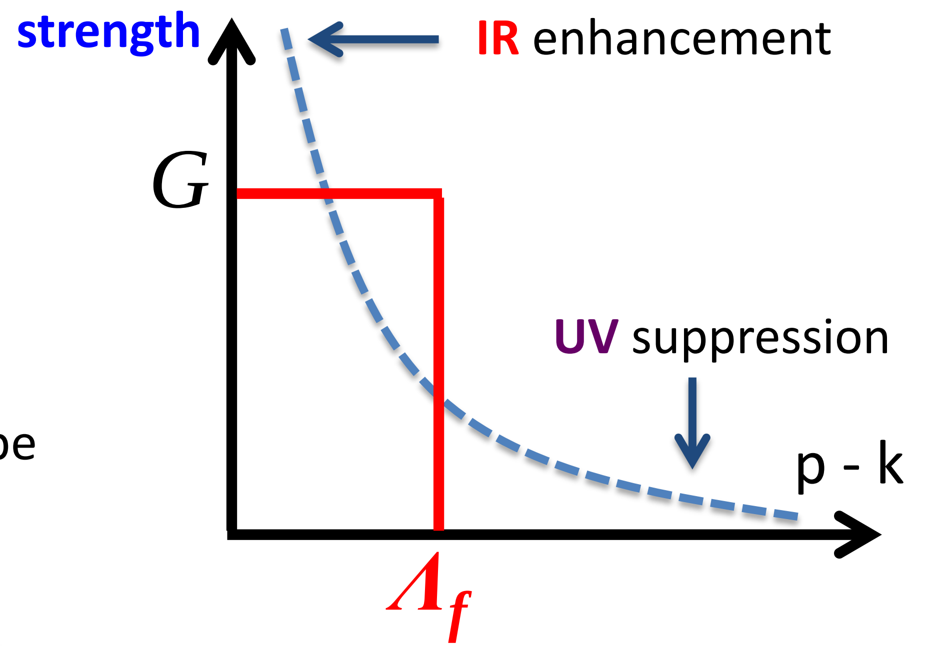


- ex) **Scalar** - **Scalar** channel

$$\frac{G}{N_c}\int dx^0 \int_{q,p,k} \left(\bar{\psi}(\vec{p}+\vec{q})\psi(\vec{p})\right)\left(\bar{\psi}(\vec{k})\psi(\vec{k}+\vec{q})\right)\theta_{p,k}$$

A crude model with asymptotic freedom





$$\theta_{p,k} \equiv \theta\left(\Lambda_f^2 - (\vec{p} - \vec{k})^2\right)$$

- ex) **Scalar** - **Scalar** channel

$$\frac{G}{N_c} \int dx^0 \int_{q,p,k} \left(\bar{\psi}(\vec{p}+\vec{q})\psi(\vec{p})\right) \left(\bar{\psi}(\vec{k})\psi(\vec{k}+\vec{q})\right) \theta_{p,k}$$

Comparison with other form factor models

	Typical model	**Ours**
function of :	**quark mom.**	**mom. transfer**
Strength at large μ :	weaken	unchange (at large Nc)

- As far as we estimate **overall** size of free energy,
  two pictures would not differ so much, because:

Hard quarks → ***Typical*** *int. : Hard*
(dominant in free energy)

- **However,** if we compare **energy difference b.t.w. phases**,
  typical part **largely** **cancel out**,
  so we **must** distinguish these two pictures.

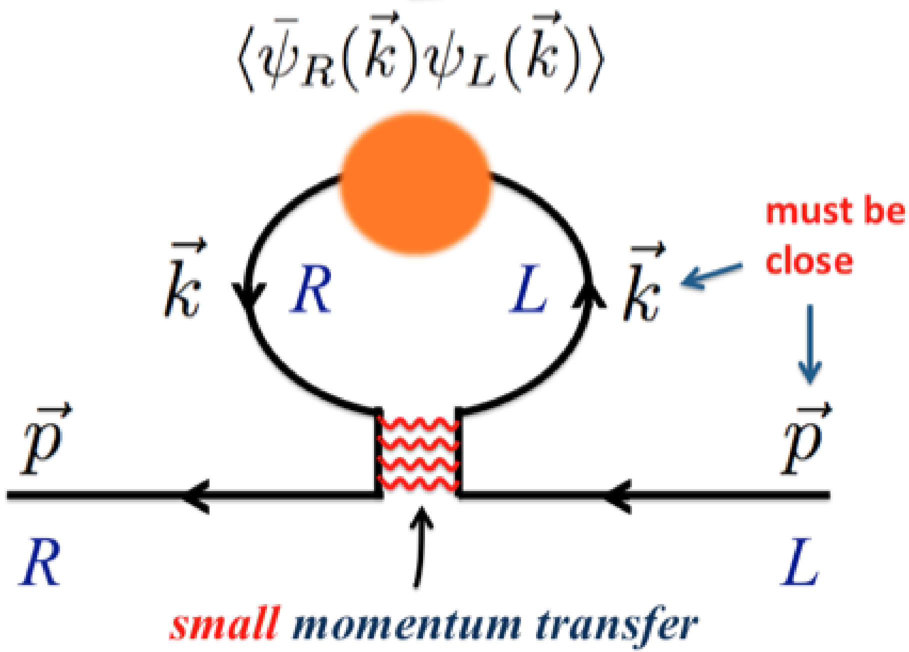
A key consequence of our form factor. 1

Quark Mass Self-energy (vacuum case)

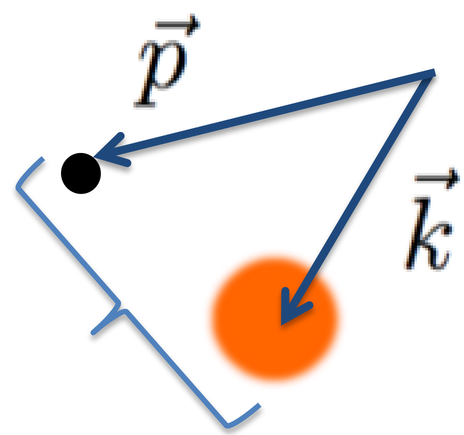
At Large Nc, largely comes from **Quark - Condensate int.**

(large amplitude ~ Nc)

(**Composite** objects with **internal** momenta)



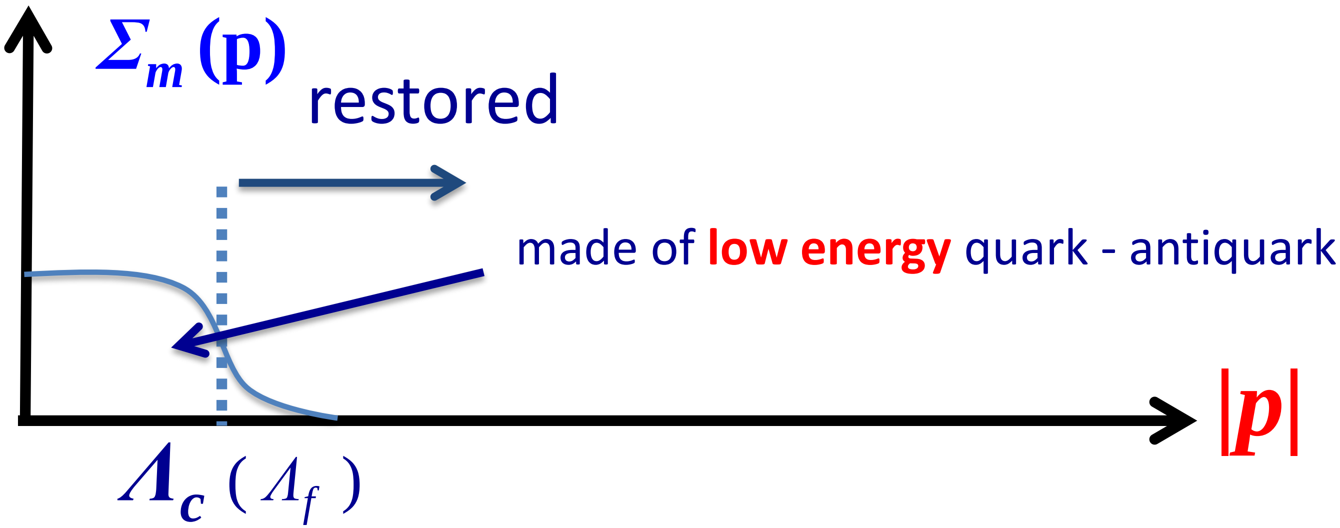
Mom. space



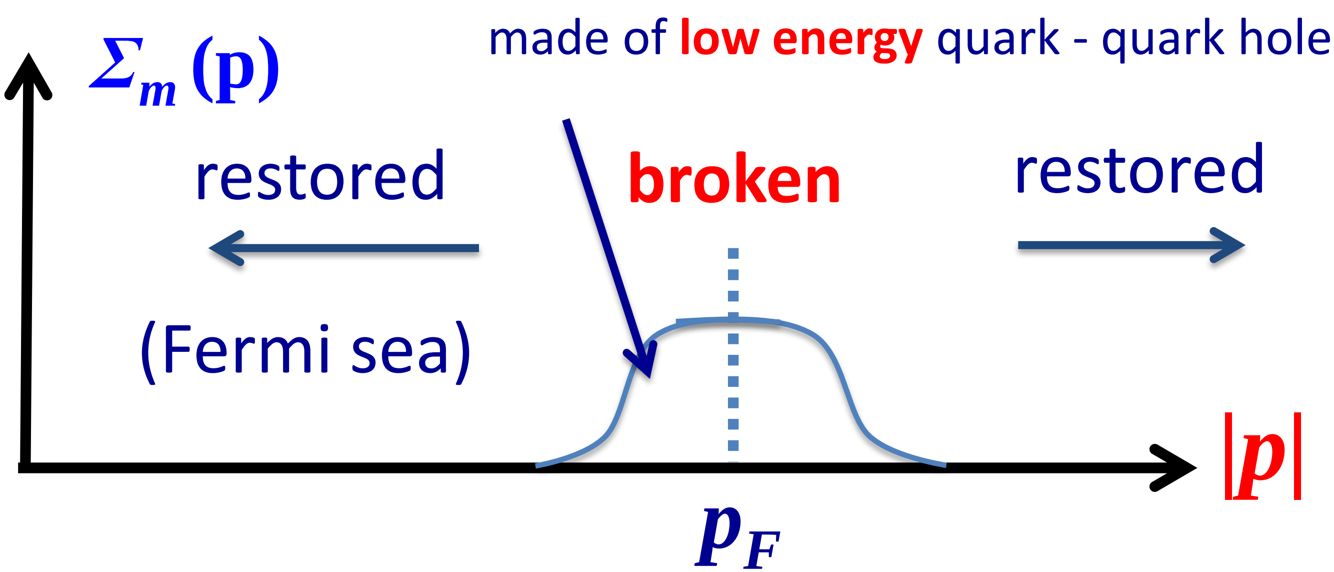
Decouple if p & k are very different

Relevant domain of Non-pert. effects

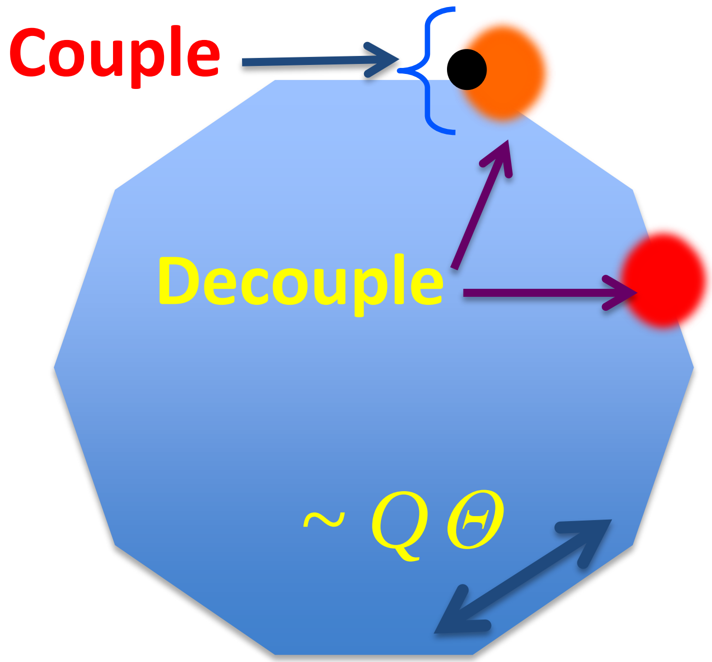
Vac.



Finite Density



3 Messages in this section



- 1, Condensates exist **only near the Fermi surface.**
- 2, Quark-Condensate int. **& Condensate-Condensate int.** are **local in mom. space.**

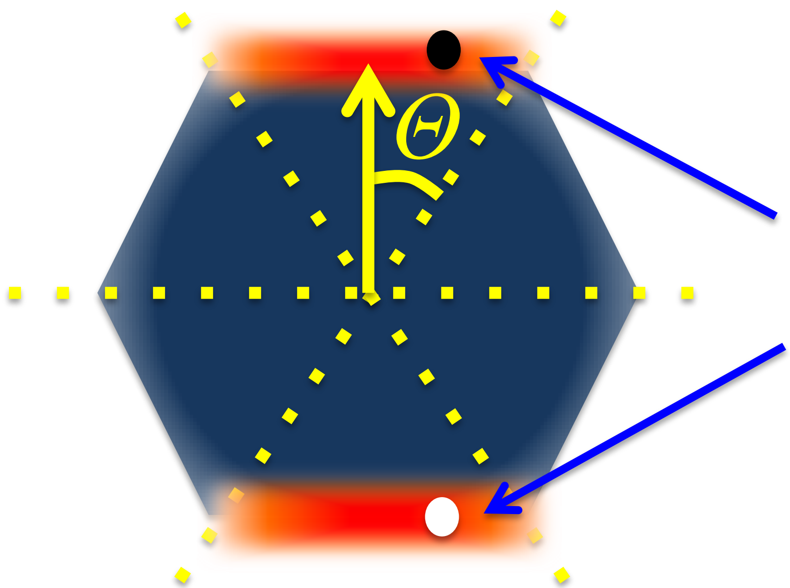
(Range $\sim \Lambda_f$)

- 3, **Interferences** among **differently oriented** CSs happens only **at the patch-patch boundaries**.

If $Q\,\Theta >> \Lambda_f$

Boundary int. is **rare process**, and can be treated as Pert.

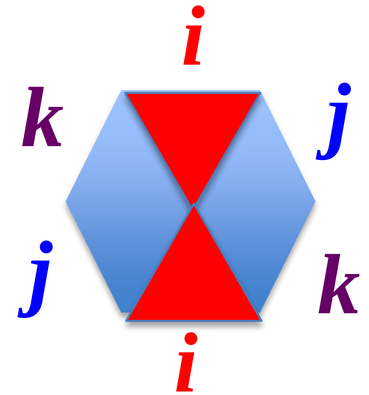
One Patch : Bases for Pert. Theory



Particle-hole combinations for one patch chiral spirals

Picking out one patch Lagrangian

ψ_i : momentum belonging to ***i*** -th patch



- **Kin. terms:** trivial to decompose

$$\mathcal{L}^{kin} \rightarrow \sum_i \bar{\psi}_i i \partial\!\!\!/ \psi_i \equiv \sum_i \mathcal{L}_i^{kin}$$

- **Int. terms:** Different patches can couple

$$\frac{G}{N_c} \sum_{i,j,k,l} \left((\bar{\psi}_i \psi_j)(\bar{\psi}_k \psi_l) + (\bar{\psi}_i i\gamma_5 \psi_j)(\bar{\psi}_k i\gamma_5 \psi_l) \right)$$

All fermions belong to the ***i*** -th patch

Patch - Patch int.

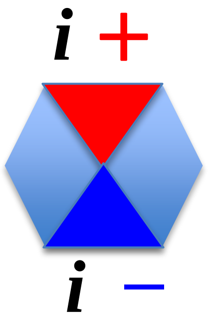
$$\mathcal{L} = \sum_i \mathcal{L}_i^{1patch} + \Delta\mathcal{L}$$

Dominant terms in One Patch, 1

“(1+1) D” “**chirality**” in ***i -th*** patch

$$\Gamma_{i5} \equiv \gamma_0 \gamma_{i||} \qquad \psi_{i\pm} \equiv \frac{1 \pm \Gamma_{i5}}{2} \psi_i$$

eigenvalue: **Moving direction**



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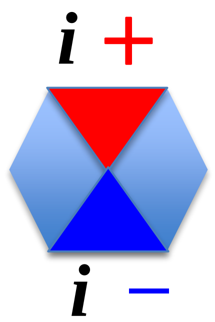

- **Fact :** "Chiral" **Non** - sym. terms → suppressed by ***1/Q***

Dominant terms in One Patch, 1

"(1+1) D" "**chirality**" in ***i -th*** patch

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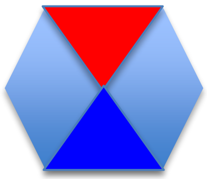


eigenvalue: **Moving direction**

- **Fact :** "Chiral" **Non** - sym. terms → suppressed by ***1/Q***

ex) free theory

- **Longitudinal Kin. (Sym.)**

$$\psi^\dagger_{\underline{i+}} i(\partial_0 - \partial_{i\|}) \psi_{\underline{i+}}$$

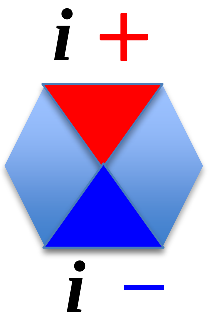
- **Transverse Kin. (Non-Sym.)**

$$\bar{\psi}_{\underline{i+}} i \not{\partial}_\perp \psi_{\underline{i-}}$$

Dominant terms in One Patch, 1

"(1+1) D" "**chirality**" in ***i -th*** patch

$$\Gamma_{i5} \equiv \gamma_0 \gamma_{i\parallel} \qquad \psi_{i\pm} \equiv \frac{1 \pm \Gamma_{i5}}{2} \psi_i$$



eigenvalue: **Moving direction**

- **Fact :** "Chiral" **Non** - sym. terms → suppressed by $1/Q$

ex) free theory

- **Longitudinal Kin. (Sym.)**

$$\psi^\dagger_{i+} i (\partial_0 - \partial_{i\parallel}) \psi_{i+}$$

- **Transverse Kin. (Non-Sym.)**

$$\bar{\psi}_{i+} i \partial\!\!\!/_\perp \psi_{i-}$$

excitation energy

$$\epsilon^{\text{free}}(\delta \vec{p}) = |\delta p_\parallel| + \frac{\delta p_\parallel^2 + p_\perp^2}{2Q} + \cdots$$

momentum **measured from Fermi surface**

Dominant terms in One Patch, 2

$$(\bar{\psi}\psi)^2$$

"Chiral" sym. part

$$\frac{1}{2}\Big((\bar{\psi}\psi)^2 + (\bar{\psi}i\Gamma_5\psi)^2\Big)$$

IR dominant

(must be **resummed** → **MF**)

Non - sym. part

$$\frac{1}{2}\Big((\bar{\psi}\psi)^2 - (\bar{\psi}i\Gamma_5\psi)^2\Big)$$

1/Q suppressed

(can be treated in **Pert**.)

Dominant terms in One Patch, 2

$$(\bar{\psi}\psi)^2$$

"Chiral" sym. part

$$\frac{1}{2}\Big((\bar{\psi}\psi)^2 + (\bar{\psi}i\Gamma_5\psi)^2\Big)$$

IR dominant

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Non - sym. part

$$\frac{1}{2}\Big((\bar{\psi}\psi)^2 - (\bar{\psi}i\Gamma_5\psi)^2\Big)$$

$1/Q$ suppressed

(can be treated in **Pert**.)

- **IR** dominant : **Unperturbed Lagrangian**

  Longitudinal Kin. + "Chiral" sym. 4-Fermi int.

  → Gap eq. can be reduced to (1+1) D

  (P_T - factorization)

- **IR** suppressed : **Perturbation**

  Transverse Kin. + Non - sym. 4-Fermi int.

Quick Summary of 1-Patch results

At leading order of Λ_{QCD}/μ

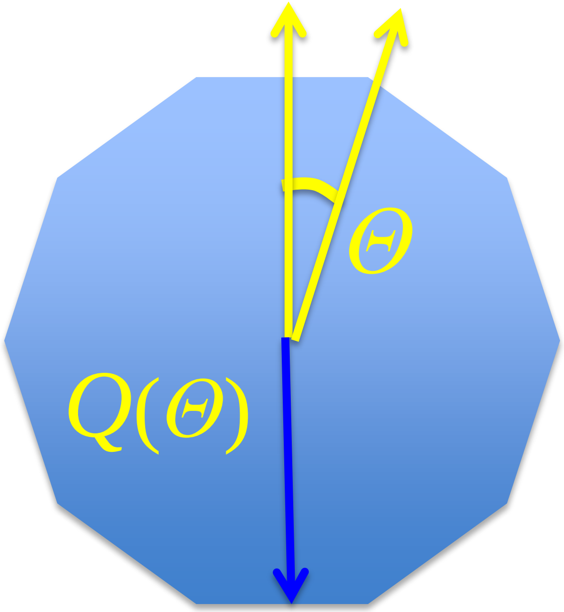
- **Integral** eqs. such as Schwinger-Dyson, Bethe-Salpeter, can be reduced from (2+1) D to (1+1) D. cf) *kT factorization*
- **Chiral Spirals** emerge, generating **large** quark mass gap. (even larger than vac. mass gap)
- **Quark num.** is **spatially uniform**. (in contrast to chiral density)

Pert. corrections

- **Quark num. oscillation.**
- **CSs : Plane wave → Solitonic**

} approach to **Baryonic Crystals**

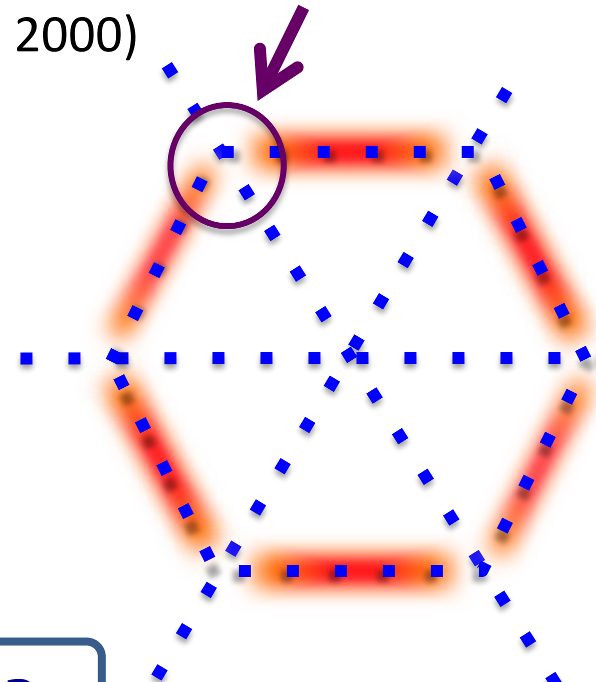
Multi-patches & Optimizing Θ



Multi-Patches: Boundary Effects

- Interferences among **differently oriented** CSs
  destroy one another, **reducing the mass gap**.
  (Checked by Pert. Numerical study by Rapp.et al 2000)

- Such effects arise
  only around patch boundaries.



reduction of gap : $\sim \Lambda_f$

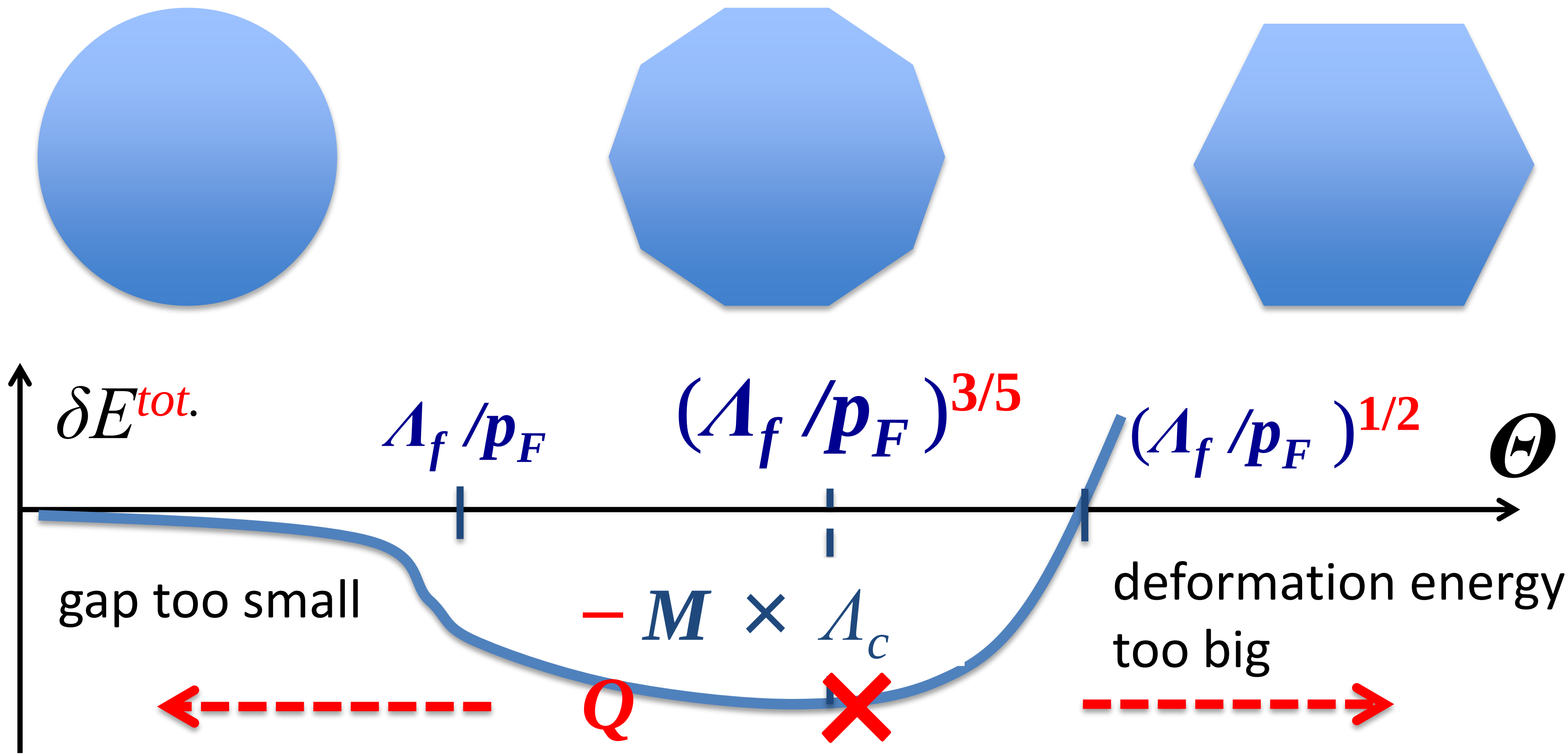
Phase space : $\sim N_p \times \Lambda_f^{\,2}$

Energetic Cost : $\sim N_p \times \Lambda_f^{\,3}$

$(N_p \sim 1/\Theta)$

(Remark: Such deconstruction effects are bigger
if CS'swave vectors take closer value.)

Energy Landscape



$$N_p \sim 1/\Theta \sim (p_F / \Lambda_f)^{3/5}$$

Chiral Spirals (CSs)

- One can find (1+1) D **solution** for the gap equation.
  (except boundaries of patches)

- The size of mass gap is $\sim \Lambda_f$, if we choose $G \sim 1/\Lambda_f$.

- The form of chiral condensates: **Spirals**

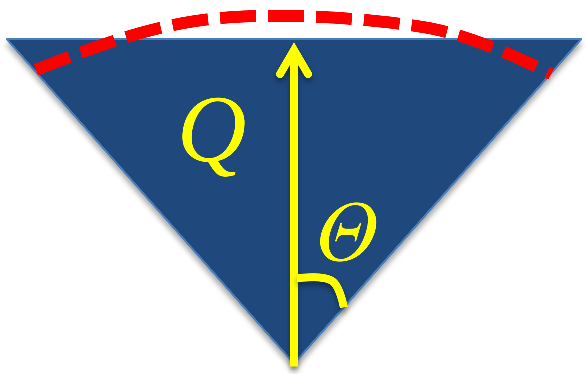
$$\langle \bar{\psi}_{i+} \psi_{i-} \rangle = \Delta \, \underline{e^{-2iQx_{i\parallel}}} \quad \& \quad \langle \bar{\psi}_{i-} \psi_{i+} \rangle = \Delta \, \underline{e^{2iQx_{i\parallel}}}$$

$$\Updownarrow$$

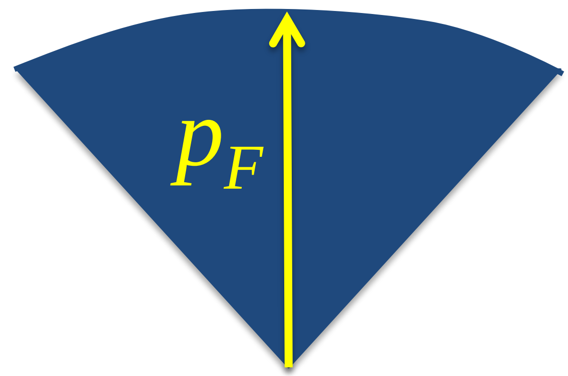
$$\langle \bar{\psi}_i \psi_i \rangle = \Delta \cos 2Qx_{i\parallel} \quad \& \quad \langle \bar{\psi}_i \underline{\gamma_0 \gamma_{i\parallel}} \psi_i \rangle = \Delta \sin 2Qx_{i\parallel}$$

- The subdominant terms can be computed
  systematically as $1/Q$ or Θ expansion .

Energetic cost of deformed Fermi sea



v.s.



- **Constraint:** **Canonical** ensemble $\rightarrow$ **Fermi vol. fixed**

$$\rightarrow \quad Q(\Theta) = p_F\left(1 - \frac{\Theta^2}{6} - \frac{\Theta^4}{40} + \cdots\right)$$

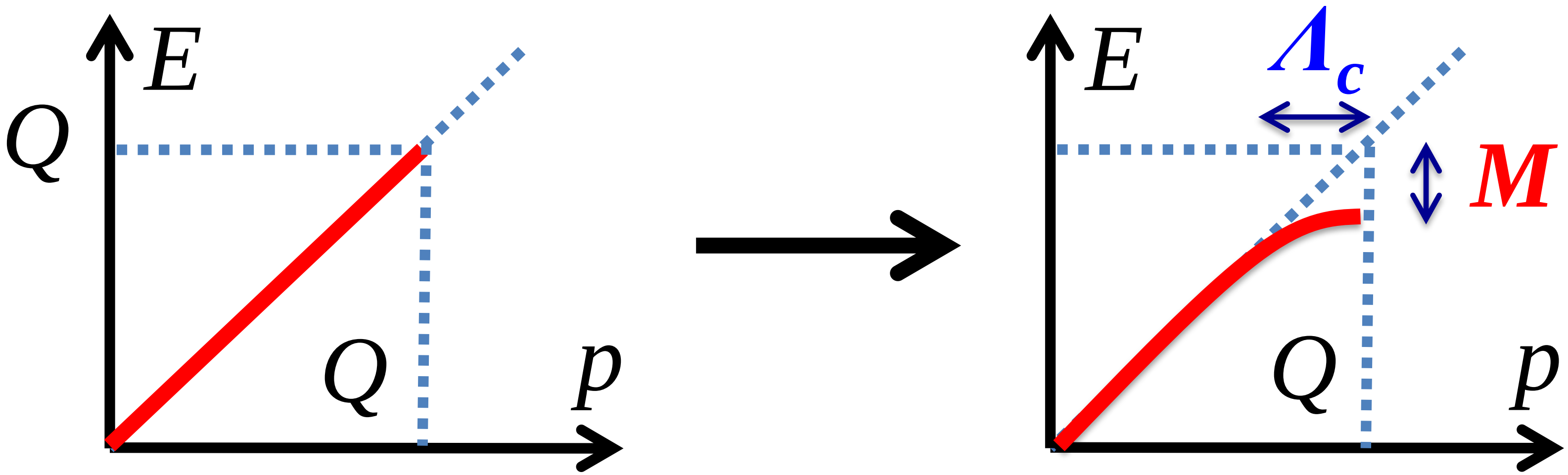
- **Energetic difference : (deformation energy)**

$$\delta E^{deform.} \sim N_p \times p_F{}^3 \times \Theta^5 \left(1 + O(\Theta^2)\right)$$

(This expression holds even if condensations occur.)

Condensation effects 1.

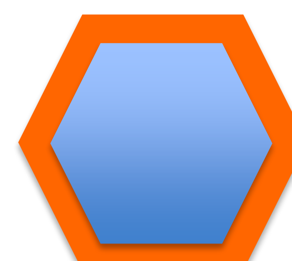
- **Gain** : **Less** single particle contributions
  (due to mass gap generated by condensates)



Fermions occupy energy levels only up to $Q - M$.

$$\delta E^{1particle} \sim -M \times \underbrace{\Lambda_c \times Q}_{\text{(phase space)}}$$

(after adding condensation energy)



Condensation effects 2.

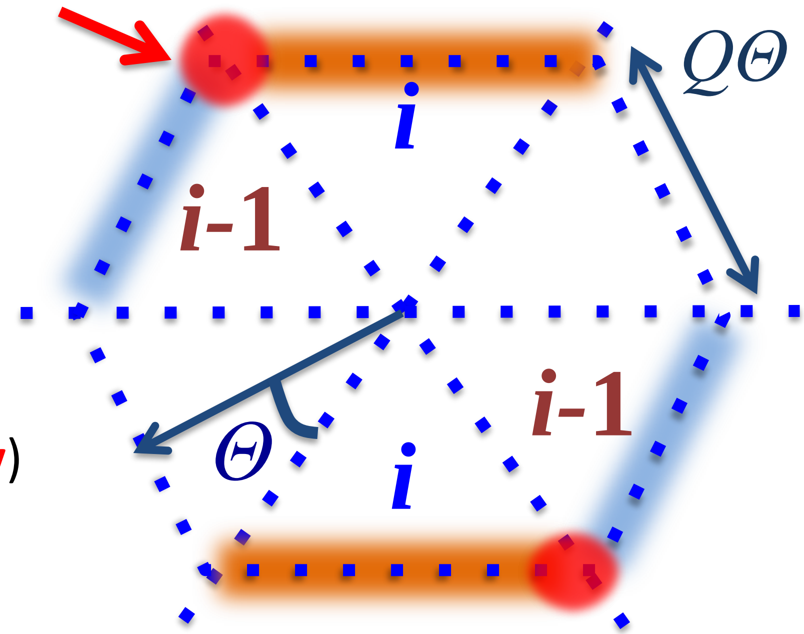
- **Cost** : **Induced interactions** b.t.w. CSs

Patch-Patch boundary

1, Int. between CSs happen **only**
within phase space, $\sim \Lambda_f^{\,2}$

2, The strength becomes smaller
with smaller size of M_B
(mass gap near the **boundary**)

3, The sign is **positive**.



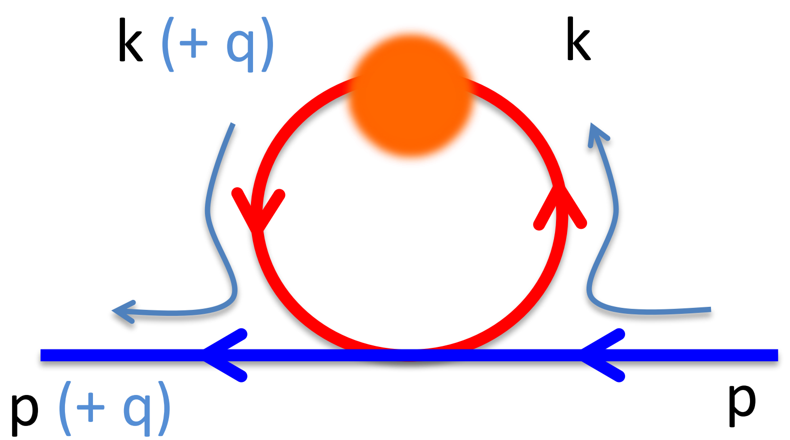
$$\delta E^{int.} \sim + N_p \times f_{int.}(M_B, \Theta)$$

(Num. of boundary points) ($f \rightarrow 0$ as $M_B \rightarrow 0$)

Consequences of form factor. 1

For **quarks – condensates int.** to happen,
their momentum domains **must be close** each other.

- **Schwinger-Dyson eq. for mass gap: (q=0 for vacuum)**



$$\Sigma_m(\vec{p}) = \int \frac{d\vec{k}}{(2\pi)^2} \frac{\Sigma_m(\vec{k})}{2\epsilon(\vec{k})} \theta_{p,k}$$

loop (condensate)

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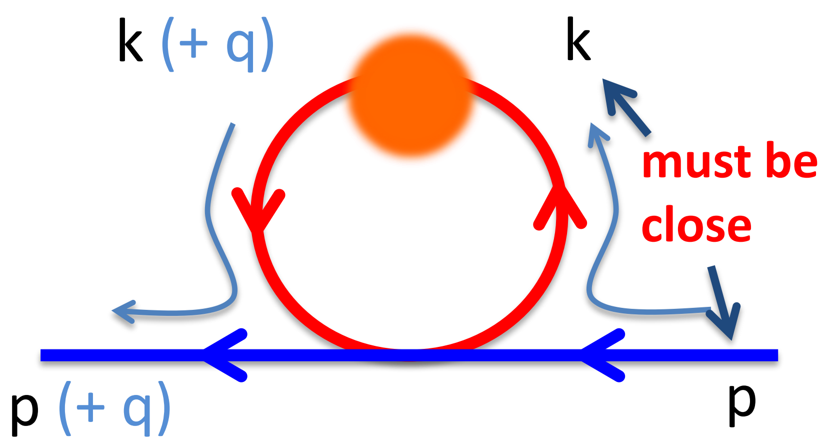

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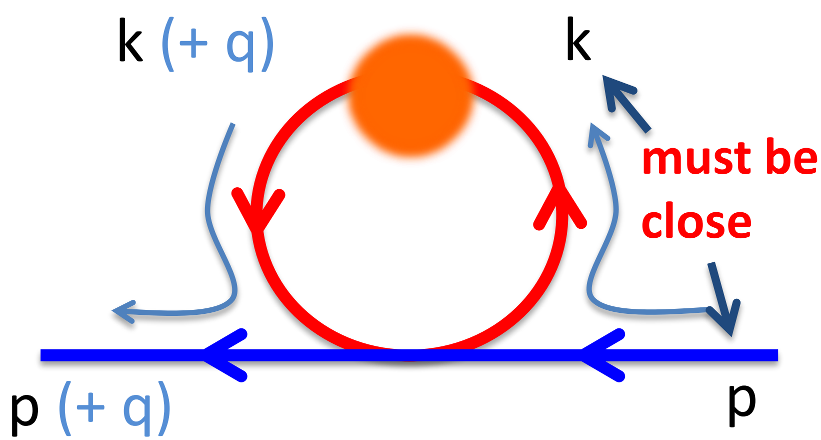

$$\Sigma_m(\vec{p}) = \int \frac{d\vec{k}}{(2\pi)^2} \frac{\Sigma_m(\vec{k})}{2\epsilon(\vec{k})} \theta_{p,k}$$

loop (condensate)

- Condensate created by fermions around momenta k
  can couple **only** to fermions with momenta $p \sim k$.
- UV cutoff for k is measured from p , **NOT from** 0 .

Consequences of form factor. 2

Dominant contributions to condensates : Low **energy** modes

(for vacuum)

$$\Sigma_m(\vec{p}) = \int \frac{d\vec{k}}{(2\pi)^2} \frac{\Sigma_m(\vec{k})}{2\epsilon(\vec{k})} \theta_{p,k}$$

When $p \longrightarrow \infty$:

- k must also go to ∞, so $\varepsilon(k) \longrightarrow \infty$.
- Phase space is **finite** : Nothing compensates denominator.

Consequences of form factor. 2

Dominant contributions to condensates : Low **energy** modes

(for vacuum)

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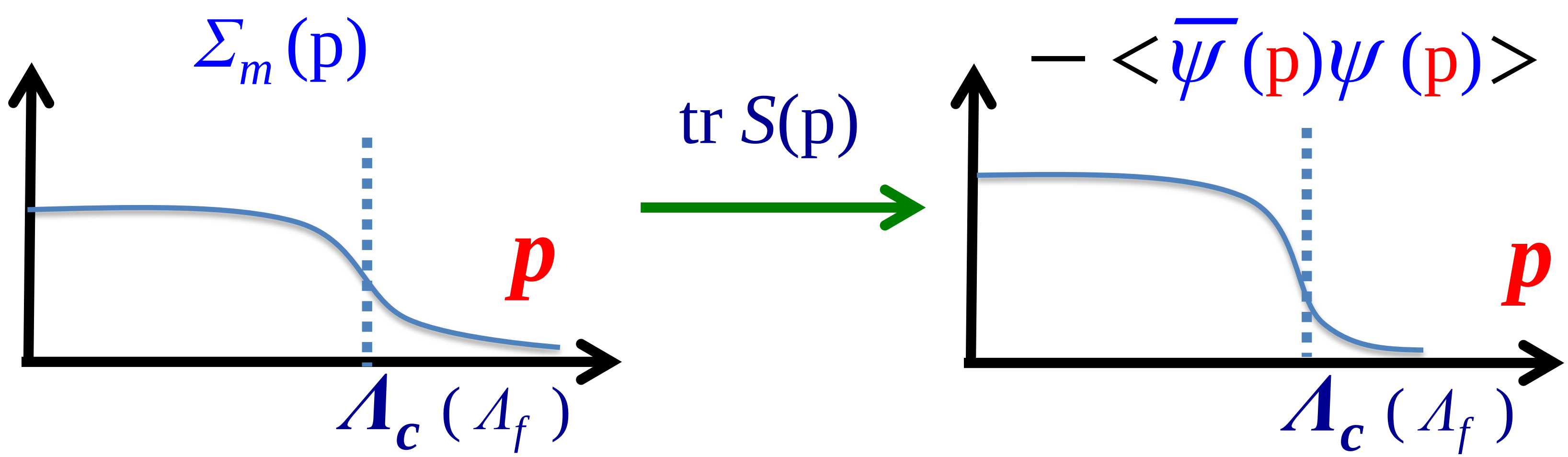
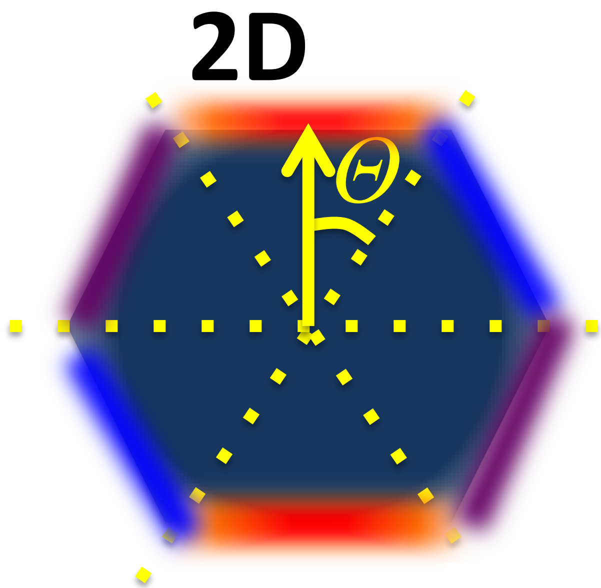


Remark)

- finite density: Low **energy** modes appear **near the Fermi surface**.



Our goal

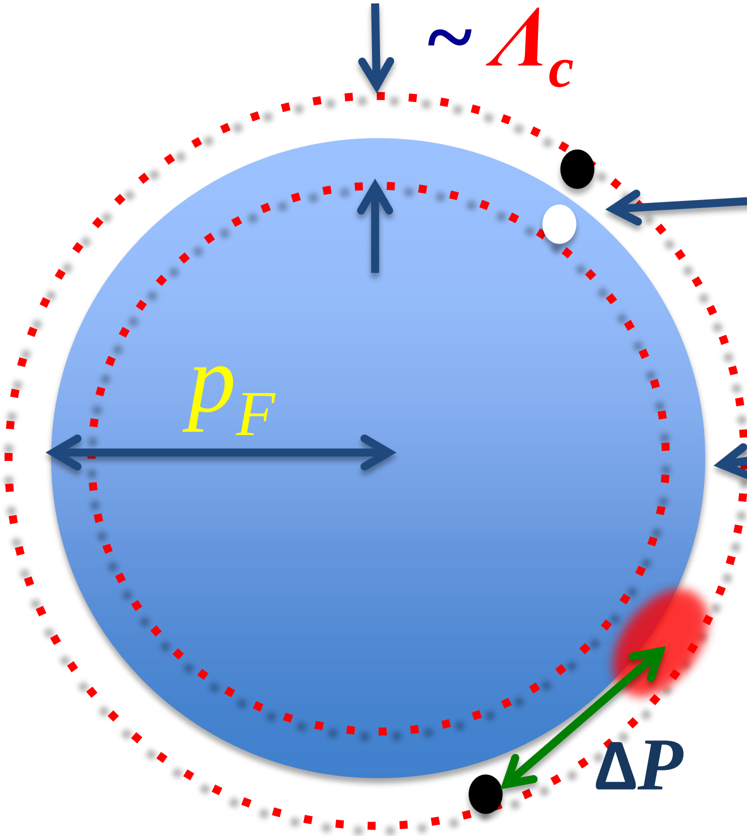
- To express the energy density as a function of theta, and to determine the best shape.

Approximations to be used

- **4-Fermi int. with a strong form factor**
- **Large Nc** (MF treatments)
- **Large density** (high density expansion, T=0)
- **(2+1) D** (simple **shape** of the Fermi surface)

→ **Simple analytic insights**

At very high density: $P_F \gg \Lambda_f$



- **Low energy modes:**

  particle - holes

  (near the Fermi surface)

- **Domain of condensations:**

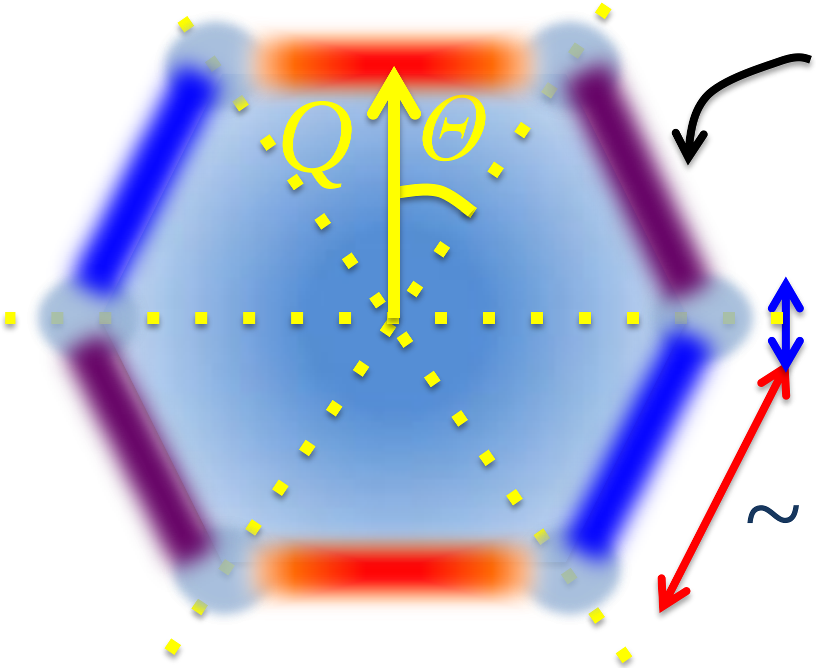
  limited to Fermi surface region

- **Decoupling:** **For $\Delta P \gg \Lambda_f$**

  Quarks **do not** couple to condensates in **very** different momentum domain.

Quark-condensate int. is **local** in **momentum space**.

Do we need to treat many CSs simultaneously ?



Domain of condensation

phase space

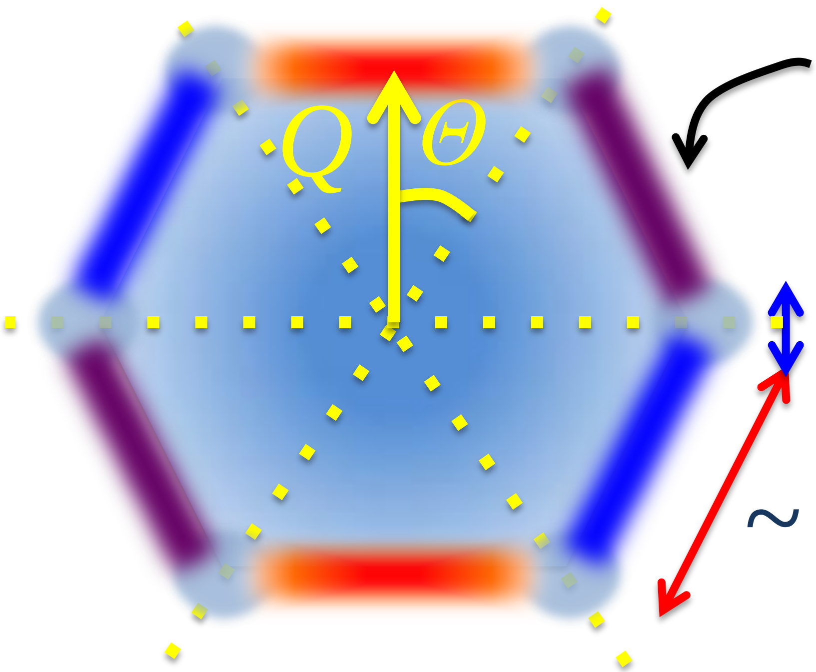
$\sim \Lambda_f \rightarrow \sim \Lambda_c \Lambda_f$

(for 1-boundary)

$\sim Q\,\Theta \rightarrow \sim \Lambda_c\, Q\,\Theta$

(for 1- patch)

Do we need to treat many CSs simultaneously ?



Domain of condensation

phase space

$\sim \Lambda_f \rightarrow \sim \Lambda_c \Lambda_f$

(for 1-boundary)

$\sim Q\,\Theta \rightarrow \sim \Lambda_c\, Q\,\Theta$

(for 1- patch)

We consider $\Lambda_f / p_F \ll \Theta \ll 1$ where

Phase space: 1- patch >> 1- boundary

Boundary effects
(Patch-Patch interactions) → **Small Perturbations** to the **1-patch problem**

Chiral Spirals (CSs)

- One can find (1+1) D **solution** for the gap equation.
  (**except** boundaries of patches)
- The size of mass gap is $\sim \Lambda_f$, if we choose $G \sim 1/\Lambda_f$.
- The form of chiral condensates: **Spirals**

$$\langle \bar{\psi}_{i+}\psi_{i-} \rangle = \Delta\, e^{-2iQx_{i\|}} \quad \& \quad \langle \bar{\psi}_{i-}\psi_{i+} \rangle = \Delta\, e^{2iQx_{i\|}}$$

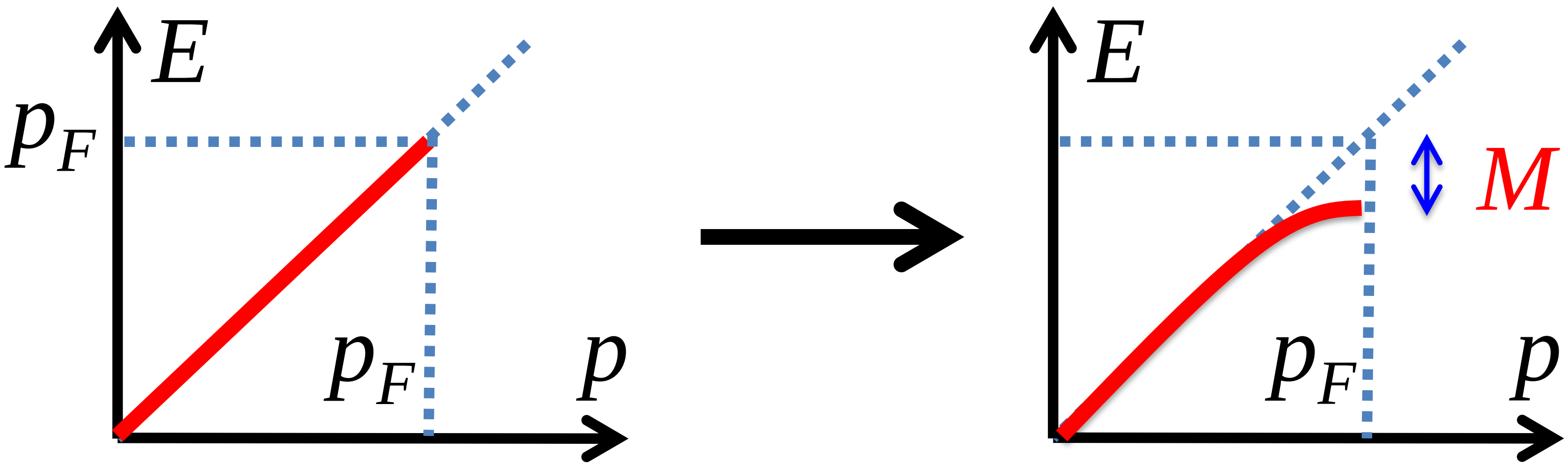
$\updownarrow$

$$\langle \bar{\psi}_{i}\psi_{i} \rangle = \Delta \cos 2Qx_{i\|} \quad \& \quad \langle \bar{\psi}_{i}\gamma_0\gamma_{i\|}\psi_{i} \rangle = \Delta \sin 2Qx_{i\|}$$

What is the best shape ?

- **Gain** : Less single particle contributions

(due to mass gap generated by condensates)



Fermions occupy levels only up to $\boldsymbol{p_F - M}$.

$$\delta E^{1\text{-}paticle} \sim -\boldsymbol{M} \times \boldsymbol{\Lambda} \times \boldsymbol{Q}$$