A novel method for computing correlators and spectral functions in perturbation theory

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Outline

- Introduction & Motivation (Miao and Vuorinen's talk)
- Setup
- Correlators
- Spectral functions
- Summary and outlook

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Setup

- Energy-Momentum Tensor $T_{\mu\nu} = \frac{1}{4}\delta_{\mu\nu}F^a_{\alpha\beta}F^a_{\alpha\beta} - F^a_{\mu\alpha}F^a_{\nu\alpha}\,,$
- Operators: $\theta \equiv c_\theta\, g_{\mathrm{B}}^2 F^a_{\mu\nu}F^a_{\mu\nu}\,, \quad \chi \equiv c_\chi\, \epsilon_{\mu\nu\rho\sigma} g_{\mathrm{B}}^2 F^a_{\mu\nu}F^a_{\rho\sigma}\,,$
- Define
  - $G_\theta(x) \equiv \langle \theta(x)\theta(0)\rangle_{\mathrm{c}}\,,$
  - $G_\chi(x) \equiv \langle \chi(x)\chi(0)\rangle\,,$
  - $G_\eta(x) \equiv 2c_\eta^2 X_{\mu\nu,\alpha\beta}(x)\, \langle T_{\mu\nu}(x)\, T_{\alpha\beta}(0)\rangle_c\,,$

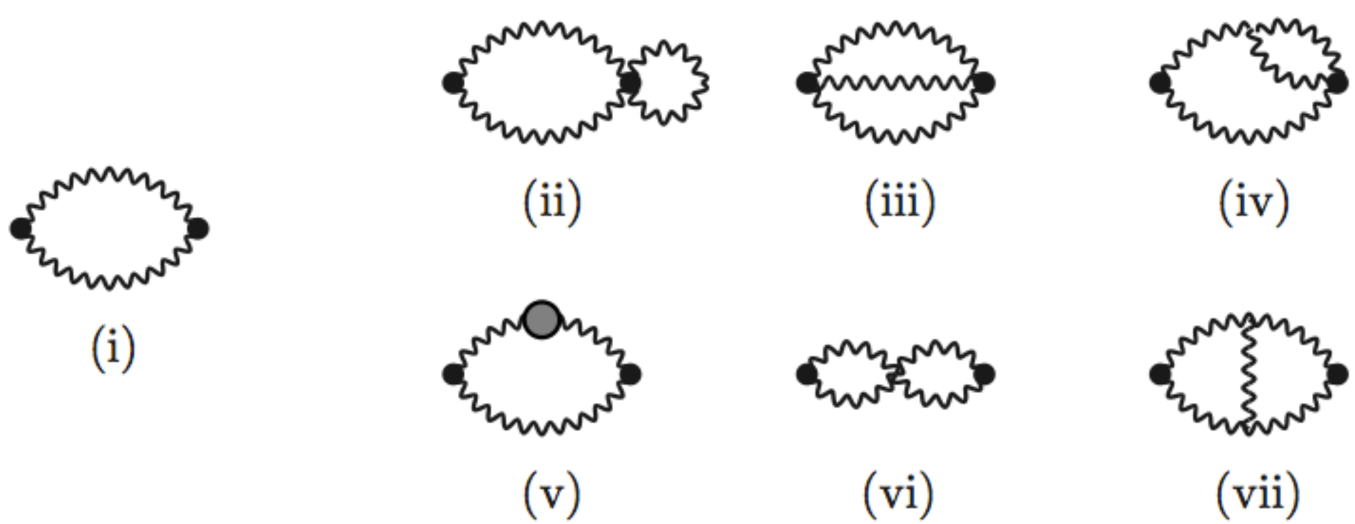
  where $X_{\mu\nu,\alpha\beta} \equiv P^T_{\mu\nu}P^T_{\alpha\beta} - \frac{D-2}{2}(P^T_{\mu\alpha}P^T_{\nu\beta} + P^T_{\mu\beta}P^T_{\nu\alpha})\,,$

  $G_\eta(x) = -16c_\eta^2\, \langle T_{12}(x)\, T_{12}(0)\rangle_c\,.$

- We use $D = 4 - 2\epsilon$ to regularize our calculation.

Correlators

The LO and NLO Feynman graphs contributing to the correlators



- Wick contraction .
- Simplify the results to a minimal number of independent “master” sum-integrals.
- Carry out Matsubara Sums and expand them in terms of large P to get the results in UV limit (short distance and/or large frequency).

Results: Mikko Laine, Mikko Vepsäläinen, Aleksi Vuorinen, 1011.4439
York Schröder, Mikko Vepsäläinen, Aleksi Vuorinen, Yan Zhu, 1109.6548

Spectral Functions

$$\rho(\omega) = \mathrm{Im}\left[\tilde{G}(P)\right]_{P\to(-i[\omega+i0^+],\mathbf{0})} .$$

- After Matsubara Sums, the imaginary part can be extracted with

$$\mathrm{Im}\left[\frac{1}{\omega \pm i0^+}\right] = \mp\pi\delta(\omega) .$$

- Example:

$$\mathcal{I}_{\mathrm{j}}(P) \equiv \sum\!\!\!\!\!\!\!\int_{Q,R} \frac{P^6}{Q^2R^2[(Q-R)^2+\lambda^2](Q-P)^2(R-P)^2} .$$

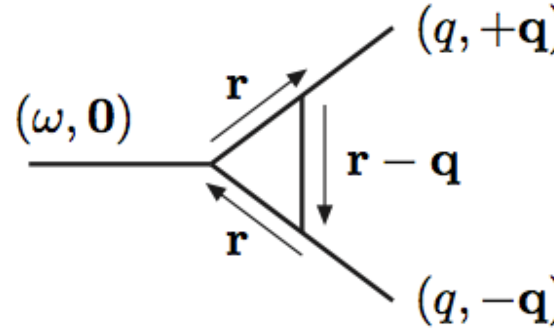
Denoting $E_q \equiv q\,, \quad E_r \equiv r\,, \quad E_{qr} \equiv \sqrt{(\mathbf{q}-\mathbf{r})^2+\lambda^2}\,,$

$\rho_{\mathcal{I}_j}(\omega)$

$$\rho_{\mathcal{I}_j}(\omega) = \int_{\mathbf{q},\mathbf{r}} \frac{\omega^6 \pi}{4qrE_{qr}} \Bigg\{$$

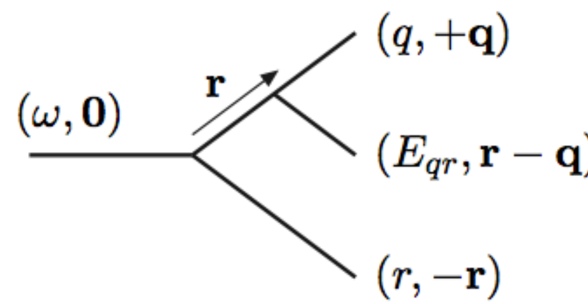
$$\frac{1}{8q^2}\Big[\delta(\omega-2q)-\delta(\omega+2q)\Big]\times$$
$$\times\Bigg[\left(\frac{1}{(q+r-E_{qr})(q+r)}-\frac{1}{(q-r+E_{qr})(q-r)}\right)(1+2n_q)(n_{qr}-n_r)$$
$$+\left(\frac{1}{(q+r+E_{qr})(q+r)}-\frac{1}{(q-r-E_{qr})(q-r)}\right)(1+2n_q)(1+n_{qr}+n_r)\Bigg]$$
$$+\frac{1}{8r^2}\Big[\delta(\omega-2r)-\delta(\omega+2r)\Big]\times$$
$$\times\Bigg[\left(\frac{1}{(q+r-E_{qr})(q+r)}-\frac{1}{(q-r-E_{qr})(q-r)}\right)(1+2n_r)(n_{qr}-n_q)$$
$$+\left(\frac{1}{(q+r+E_{qr})(q+r)}-\frac{1}{(q-r+E_{qr})(q-r)}\right)(1+2n_r)(1+n_{qr}+n_q)\Bigg]$$

Factorized int./ Virtual correction



$$+\Big[\delta(\omega-q-r-E_{qr})-\delta(\omega+q+r+E_{qr})\Big]\frac{(1+n_{qr})(1+n_q+n_r)+n_q n_r}{(q+r+E_{qr})^2(q-r+E_{qr})(q-r-E_{qr})}$$
$$+\Big[\delta(\omega-q-r+E_{qr})-\delta(\omega+q+r-E_{qr})\Big]\frac{n_{qr}(1+n_q+n_r)-n_q n_r}{(q+r-E_{qr})^2(q-r+E_{qr})(q-r-E_{qr})}$$
$$+\Big[\delta(\omega-q+r-E_{qr})-\delta(\omega+q-r+E_{qr})\Big]\frac{n_r(1+n_q+n_{qr})-n_q n_{qr}}{(q-r+E_{qr})^2(q+r+E_{qr})(q+r-E_{qr})}$$
$$+\Big[\delta(\omega+q-r-E_{qr})-\delta(\omega-q+r+E_{qr})\Big]\frac{n_q(1+n_r+n_{qr})-n_r n_{qr}}{(q-r-E_{qr})^2(q+r+E_{qr})(q+r-E_{qr})}\Bigg\}$$

Phase space int./ Real correction



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Virtual Correction

$$\rho^{(\mathrm{fz})}_{\mathcal{I}_\mathrm{j}}(\omega) = \int_{\mathbf{q},\mathbf{r}} \frac{\omega^6 \pi}{4qrE_{qr}} \times$$
$$\times \frac{1}{8r^2}\Big[\delta(\omega - 2r) - \delta(\omega + 2r)\Big] \times$$
$$\times \Bigg[\left(\frac{1}{(q+r-E_{qr})(q+r)} - \frac{1}{(q-r-E_{qr})(q-r)}\right)(1+2n_r)(n_{qr}-n_q)$$
$$+\left(\frac{1}{(q+r+E_{qr})(q+r)} - \frac{1}{(q-r+E_{qr})(q-r)}\right)(1+2n_r)(1+n_{qr}+n_q)$$

- $$\int_{\mathbf{r}} \pi\delta(\omega - 2r) = \frac{\omega^2}{16\pi} \,,$$
- $$\int_{\mathbf{q}} \frac{1}{2qE_{qr}} = \frac{1}{4\pi^2\omega}\int_0^\infty \mathrm{d}q \int_{E^-_{qr}}^{E^+_{qr}} \mathrm{d}E_{qr} \,, \quad E^\pm_{qr} \equiv \sqrt{\left(q \pm \frac{\omega}{2}\right)^2 + \lambda^2} \,.$$
- $$\int_0^\infty \mathrm{d}q \int_{\sqrt{(q-\frac{\omega}{2})^2+\lambda^2}}^{\sqrt{(q+\frac{\omega}{2})^2+\lambda^2}} \mathrm{d}E_{qr} = \int_\lambda^\infty \mathrm{d}E_{qr} \int_{|\frac{\omega}{2}-\sqrt{E^2_{qr}-\lambda^2}|}^{\frac{\omega}{2}+\sqrt{E^2_{qr}-\lambda^2}} \mathrm{d}q \,.$$

Virtual Correction

$$\rho^{(\mathrm{fz})}_{\mathcal{I}_\mathrm{j}}(\omega) = \int_{\mathbf{q},\mathbf{r}} \frac{\omega^6 \pi}{4qrE_{qr}} \times$$

$$\times \frac{1}{8r^2}\Big[\delta(\omega - 2r) - \delta(\omega + 2r)\Big] \times \quad \omega > 0$$

$$\times \Bigg[\bigg(\frac{1}{(q+r-E_{qr})(q+r)} - \frac{1}{(q-r-E_{qr})(q-r)}\bigg)(1+2n_r)(n_{qr}-n_q)$$

$$+ \bigg(\frac{1}{(q+r+E_{qr})(q+r)} - \frac{1}{(q-r+E_{qr})(q-r)}\bigg)(1+2n_r)(1+n_{qr}+n_q)$$

- $$\int_\mathbf{r} \pi\delta(\omega - 2r) = \frac{\omega^2}{16\pi} \, ,$$
- $$\int_\mathbf{q} \frac{1}{2qE_{qr}} = \frac{1}{4\pi^2\omega}\int_0^\infty \mathrm{d}q \int_{E^-_{qr}}^{E^+_{qr}} \mathrm{d}E_{qr} \, , \quad E^\pm_{qr} \equiv \sqrt{\Big(q \pm \frac{\omega}{2}\Big)^2 + \lambda^2} \, .$$
- $$\int_0^\infty \mathrm{d}q \int_{\sqrt{(q-\frac{\omega}{2})^2+\lambda^2}}^{\sqrt{(q+\frac{\omega}{2})^2+\lambda^2}} \mathrm{d}E_{qr} = \int_\lambda^\infty \mathrm{d}E_{qr} \int_{|\frac{\omega}{2}-\sqrt{E^2_{qr}-\lambda^2}|}^{\frac{\omega}{2}+\sqrt{E^2_{qr}-\lambda^2}} \mathrm{d}q \, .$$

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Virtual Correction

$$\rho^{(\mathrm{fz})}_{\mathcal{I}_{\mathrm{j}}}(\omega) = \int_{\mathbf{q},\mathbf{r}} \frac{\omega^6 \pi}{4qrE_{qr}} \times$$
$$\times \frac{1}{8r^2}\Big[\delta(\omega - 2r) - \delta(\omega + 2r)\Big] \times \quad \omega > 0$$
$$\times \Bigg[\bigg(\frac{1}{(q+r-E_{qr})(q+r)} - \frac{1}{(q-r-E_{qr})(q-r)}\bigg)(1+2n_r)(n_{qr}-n_q)$$
$$+ \bigg(\frac{1}{(q+r+E_{qr})(q+r)} - \frac{1}{(q-r+E_{qr})(q-r)}\bigg)(1+2n_r)(1+n_{qr}+n_q)$$

- $$\int_{\mathbf{r}} \pi\delta(\omega - 2r) = \frac{\omega^2}{16\pi} \,,$$
- $$\int_{\mathbf{q}} \frac{1}{2qE_{qr}} = \frac{1}{4\pi^2\omega}\int_0^\infty \mathrm{d}q \int_{E^-_{qr}}^{E^+_{qr}} \mathrm{d}E_{qr} \,, \quad E^\pm_{qr} \equiv \sqrt{\Big(q \pm \frac{\omega}{2}\Big)^2 + \lambda^2} \,.$$
- $$\int_0^\infty \mathrm{d}q \int_{\sqrt{(q-\frac{\omega}{2})^2+\lambda^2}}^{\sqrt{(q+\frac{\omega}{2})^2+\lambda^2}} \mathrm{d}E_{qr} = \int_\lambda^\infty \mathrm{d}E_{qr} \int_{|\frac{\omega}{2}-\sqrt{E^2_{qr}-\lambda^2}|}^{\frac{\omega}{2}+\sqrt{E^2_{qr}-\lambda^2}} \mathrm{d}q \,.$$

Virtual Correction

$$\rho^{(\mathrm{fz})}_{\mathcal{I}_\mathrm{j}}(\omega) = \int_{\mathbf{q},\mathbf{r}} \frac{\omega^6 \pi}{4qrE_{qr}} \times$$

$$\times \frac{1}{8r^2}\Big[\delta(\omega - 2r) - \delta(\omega + 2r)\Big] \times \quad \omega > 0$$

$$\times \Bigg[\left(\frac{1}{(q+r-E_{qr})(q+r)} - \frac{1}{(q-r-E_{qr})(q-r)}\right)(1+2n_r)(n_{qr}-n_q)$$

$$+\left(\frac{1}{(q+r+E_{qr})(q+r)} - \frac{1}{(q-r+E_{qr})(q-r)}\right)(1+2n_r)(1+n_{qr}+n_q)$$

- $\rho^{(\mathrm{fz,p})}_{\mathcal{I}_\mathrm{j}}(\omega) \approx \frac{\omega^4}{(4\pi)^3}(1+2n_{\frac{\omega}{2}}) \int_\lambda^{\frac{\omega}{2}} \frac{\mathrm{d}q}{q} \ln\left|\frac{q+\sqrt{q^2-\lambda^2}}{q-\sqrt{q^2-\lambda^2}}\right| .$
- $\int_\mathbf{q} \frac{1}{2qE_{qr}} = \frac{1}{4\pi^2\omega}\int_0^\infty \mathrm{d}q \int_{E^-_{qr}}^{E^+_{qr}} \mathrm{d}E_{qr} \,, \quad E^\pm_{qr} \equiv \sqrt{\left(q \pm \frac{\omega}{2}\right)^2 + \lambda^2} .$
- $\int_0^\infty \mathrm{d}q \int_{\sqrt{(q-\frac{\omega}{2})^2+\lambda^2}}^{\sqrt{(q+\frac{\omega}{2})^2+\lambda^2}} \mathrm{d}E_{qr} = \int_\lambda^\infty \mathrm{d}E_{qr} \int_{|\frac{\omega}{2}-\sqrt{E^2_{qr}-\lambda^2}|}^{\frac{\omega}{2}+\sqrt{E^2_{qr}-\lambda^2}} \mathrm{d}q \,.$

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Virtual Correction

$$\rho^{(\mathrm{fz})}_{\mathcal{I}_\mathrm{j}}(\omega) = \int_{\mathbf{q},\mathbf{r}} \frac{\omega^6 \pi}{4qrE_{qr}} \times$$

$$\times \frac{1}{8r^2}\Big[\delta(\omega - 2r) - \delta(\omega + 2r)\Big] \times \quad \omega > 0$$

$$\times \Bigg[\left(\frac{1}{(q+r-E_{qr})(q+r)} - \frac{1}{(q-r-E_{qr})(q-r)}\right)(1+2n_r)(n_{qr}-n_q)$$

$$+\left(\frac{1}{(q+r+E_{qr})(q+r)} - \frac{1}{(q-r+E_{qr})(q-r)}\right)(1+2n_r)(1+n_{qr}+n_q)$$

- $$\rho^{(\mathrm{fz,p})}_{\mathcal{I}_\mathrm{j}}(\omega) \approx \frac{\omega^4}{(4\pi)^3}(1+2n_{\frac{\omega}{2}}) \int_\lambda^{\frac{\omega}{2}} \frac{\mathrm{d}q}{q} \ln\left|\frac{q+\sqrt{q^2-\lambda^2}}{q-\sqrt{q^2-\lambda^2}}\right| .$$

- $$\rho^{(\mathrm{fz,e})}_{\mathcal{I}_\mathrm{j}}(\omega) = \frac{\omega^4}{(4\pi)^3}(1+2n_{\frac{\omega}{2}})\Bigg\{$$

- $$\int_0^\infty \mathrm{d}q\, n_q\, \mathbb{P}\left[\frac{1}{q+\frac{\omega}{2}} \ln\left|\frac{\lambda^2}{2q\omega-\lambda^2}\right| + \frac{1}{q-\frac{\omega}{2}} \ln\left|\frac{\lambda^2}{2q\omega+\lambda^2}\right|\right]$$

$$+ \int_\lambda^\infty \mathrm{d}q\, n_q \left[\frac{1}{q}\ln\left|\frac{q+\frac{\lambda^2}{\omega}+\sqrt{q^2-\lambda^2}}{q+\frac{\lambda^2}{\omega}-\sqrt{q^2-\lambda^2}}\right| + \frac{1}{q}\ln\left|\frac{q-\frac{\lambda^2}{\omega}+\sqrt{q^2-\lambda^2}}{q-\frac{\lambda^2}{\omega}-\sqrt{q^2-\lambda^2}}\right|\right]\Bigg\} .$$

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Real Correction

$$\rho^{(\mathrm{ps})}_{\mathcal{I}_j}(\omega) \equiv \int_{\mathbf{q},\mathbf{r}} \frac{\omega^6 \pi}{4qrE_{qr}} \Big\{$$

$$+ \Big[\delta(\omega - q - r - E_{qr}) - \delta(\omega + q + r + E_{qr})\Big] \frac{(1+n_{qr})(1+n_q+n_r) + n_q n_r}{(q+r+E_{qr})^2 (q-r+E_{qr})(q-r-E_{qr})}$$

$$+ \Big[\delta(\omega - q - r + E_{qr}) - \delta(\omega + q + r - E_{qr})\Big] \frac{n_{qr}(1+n_q+n_r) - n_q n_r}{(q+r-E_{qr})^2 (q-r+E_{qr})(q-r-E_{qr})}$$

$$+ \Big[\delta(\omega - q + r - E_{qr}) - \delta(\omega + q - r + E_{qr})\Big] \frac{n_r(1+n_q+n_{qr}) - n_q n_{qr}}{(q-r+E_{qr})^2 (q+r+E_{qr})(q+r-E_{qr})}$$

$$+ \Big[\delta(\omega + q - r - E_{qr}) - \delta(\omega - q + r + E_{qr})\Big] \frac{n_q(1+n_r+n_{qr}) - n_r n_{qr}}{(q-r-E_{qr})^2 (q+r+E_{qr})(q+r-E_{qr})} \Big\} .$$

- $$\int_{\mathbf{q},\mathbf{r}} \frac{\pi}{4qrE_{qr}} = \frac{2}{(4\pi)^3} \int_0^\infty \mathrm{d}q \int_0^\infty \mathrm{d}r \int_{E_{qr}^-}^{E_{qr}^+} \mathrm{d}E_{qr} \,, \quad E_{qr}^{\pm} \equiv \sqrt{(q \pm r)^2 + \lambda^2} \,.$$

- $$\begin{aligned} (1+n_{qr})(1+n_q+n_r) + n_q n_r &= n_q n_r n_{qr} \left(e^{q+r+E_{qr}} - 1\right) , \\ n_{qr}(1+n_q+n_r) - n_q n_r &= n_q n_r n_{qr} \left(e^{q+r} - e^{E_{qr}}\right) , \\ n_r(1+n_q+n_{qr}) - n_q n_{qr} &= n_q n_r n_{qr} \left(e^{q+E_{qr}} - e^{r}\right) , \\ n_q(1+n_r+n_{qr}) - n_r n_{qr} &= n_q n_r n_{qr} \left(e^{r+E_{qr}} - e^{q}\right) , \end{aligned}$$

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Real Correction

$$\rho^{(\mathrm{ps})}_{\mathcal{I}_j}(\omega) \equiv \int_{\mathbf{q},\mathbf{r}} \frac{\omega^6 \pi}{4qrE_{qr}} \Big\{$$

$0 < \lambda < \omega$

$$+ \Big[\delta(\omega - q - r - E_{qr}) - \delta(\omega + q + r + E_{qr})\Big] \frac{(1+n_{qr})(1+n_q+n_r)+n_q n_r}{(q+r+E_{qr})^2(q-r+E_{qr})(q-r-E_{qr})}$$

$$+ \Big[\delta(\omega - q - r + E_{qr}) - \delta(\omega + q + r - E_{qr})\Big] \frac{n_{qr}(1+n_q+n_r)-n_q n_r}{(q+r-E_{qr})^2(q-r+E_{qr})(q-r-E_{qr})}$$

$$+ \Big[\delta(\omega - q + r - E_{qr}) - \delta(\omega + q - r + E_{qr})\Big] \frac{n_r(1+n_q+n_{qr})-n_q n_{qr}}{(q-r+E_{qr})^2(q+r+E_{qr})(q+r-E_{qr})}$$

$$+ \Big[\delta(\omega + q - r - E_{qr}) - \delta(\omega - q + r + E_{qr})\Big] \frac{n_q(1+n_r+n_{qr})-n_r n_{qr}}{(q-r-E_{qr})^2(q+r+E_{qr})(q+r-E_{qr})} \Big\} .$$

- $$\int_{\mathbf{q},\mathbf{r}} \frac{\pi}{4qrE_{qr}} = \frac{2}{(4\pi)^3} \int_0^\infty \mathrm{d}q \int_0^\infty \mathrm{d}r \int_{E_{qr}^-}^{E_{qr}^+} \mathrm{d}E_{qr} , \quad E_{qr}^\pm \equiv \sqrt{(q\pm r)^2 + \lambda^2} .$$

- $$\begin{aligned} (1+n_{qr})(1+n_q+n_r)+n_q n_r &= n_q n_r n_{qr}\left(e^{q+r+E_{qr}} - 1\right) , \\ n_{qr}(1+n_q+n_r)-n_q n_r &= n_q n_r n_{qr}\left(e^{q+r} - e^{E_{qr}}\right) , \\ n_r(1+n_q+n_{qr})-n_q n_{qr} &= n_q n_r n_{qr}\left(e^{q+E_{qr}} - e^{r}\right) , \\ n_q(1+n_r+n_{qr})-n_r n_{qr} &= n_q n_r n_{qr}\left(e^{r+E_{qr}} - e^{q}\right) , \end{aligned}$$

Real Correction

$0 < \lambda < \omega$

$$\begin{aligned}
\rho^{(\mathrm{ps})}_{\mathcal{I}_j}(\omega) \equiv \int_{\mathbf{q},\mathbf{r}} \frac{\omega^6\pi}{4qrE_{qr}}\Big\{ \\
&+ \Big[\delta(\omega - q - r - E_{qr}) - \delta(\omega + q + r + E_{qr})\Big] \frac{(1+n_{qr})(1+n_q+n_r)+n_q n_r}{(q+r+E_{qr})^2(q-r+E_{qr})(q-r-E_{qr})} \\
&+ \Big[\delta(\omega - q - r + E_{qr}) - \delta(\omega + q + r - E_{qr})\Big] \frac{n_{qr}(1+n_q+n_r)-n_q n_r}{(q+r-E_{qr})^2(q-r+E_{qr})(q-r-E_{qr})} \\
&+ \Big[\delta(\omega - q + r - E_{qr}) - \delta(\omega + q - r + E_{qr})\Big] \frac{n_r(1+n_q+n_{qr})-n_q n_{qr}}{(q-r+E_{qr})^2(q+r+E_{qr})(q+r-E_{qr})} \\
&+ \Big[\delta(\omega + q - r - E_{qr}) - \delta(\omega - q + r + E_{qr})\Big] \frac{n_q(1+n_r+n_{qr})-n_r n_{qr}}{(q-r-E_{qr})^2(q+r+E_{qr})(q+r-E_{qr})}\Big\} .
\end{aligned}$$

- $$\int_{\mathbf{q},\mathbf{r}} \frac{\pi}{4qrE_{qr}} = \frac{2}{(4\pi)^3}\int_0^\infty \mathrm{d}q \int_0^\infty \mathrm{d}r \int_{E^-_{qr}}^{E^+_{qr}} \mathrm{d}E_{qr} \,, \quad E^\pm_{qr} \equiv \sqrt{(q\pm r)^2+\lambda^2} \,.$$

- $$\begin{aligned}
(1+n_{qr})(1+n_q+n_r)+n_q n_r &= n_q n_r n_{qr}\big(e^{q+r+E_{qr}}-1\big) \,, \\
n_{qr}(1+n_q+n_r)-n_q n_r &= n_q n_r n_{qr}\big(e^{q+r}-e^{E_{qr}}\big) \,, \\
n_r(1+n_q+n_{qr})-n_q n_{qr} &= n_q n_r n_{qr}\big(e^{q+E_{qr}}-e^{r}\big) \,, \\
n_q(1+n_r+n_{qr})-n_r n_{qr} &= n_q n_r n_{qr}\big(e^{r+E_{qr}}-e^{q}\big) \,,
\end{aligned}$$

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Real Correction

$$\rho^{(\mathrm{ps})}_{\mathcal{I}_j}(\omega) \equiv \frac{2\omega^4}{(4\pi)^3}\int_0^\infty \mathrm{d}q \int_0^\infty \mathrm{d}r \int_{E_{qr}^-}^{E_{qr}^+} \mathrm{d}E_{qr}\, n_q n_r n_{qr} \Bigg\{$$

$$\text{(i)} \quad \frac{\delta(\omega - q - r - E_{qr})}{(2r-\omega)(2q-\omega)}\Big(1 - e^{q+r+E_{qr}}\Big)$$

$$\text{(ii)} \quad + \frac{\delta(\omega - q - r + E_{qr})}{(2r-\omega)(2q-\omega)}\Big(e^{E_{qr}} - e^{q+r}\Big)$$

$$\text{(iii)} \quad + \frac{\delta(\omega + q - r - E_{qr})}{(2r-\omega)(2q+\omega)}\Big(e^{r+E_{qr}} - e^{q}\Big)$$

$$\text{(iv)} \quad + \frac{\delta(\omega - q + r - E_{qr})}{(2r+\omega)(2q-\omega)}\Big(e^{q+E_{qr}} - e^{r}\Big)\Bigg\}\,.$$

$$\text{(i)} \quad \int_0^\infty \mathrm{d}q \int_0^\infty \mathrm{d}r \int_{E_{qr}^-}^{E_{qr}^+} \mathrm{d}E_{qr}\, \delta(\omega - q - r - E_{qr})\,\phi(q,r,E_{qr})$$

$$= \int_0^{\frac{\omega^2-\lambda^2}{2\omega}} \mathrm{d}q \int_{\frac{\omega(\omega-2q)-\lambda^2}{2\omega}}^{\frac{\omega(\omega-2q)-\lambda^2}{2(\omega-2q)}} \mathrm{d}r\,\phi(q,r,\omega-q-r)\,,$$

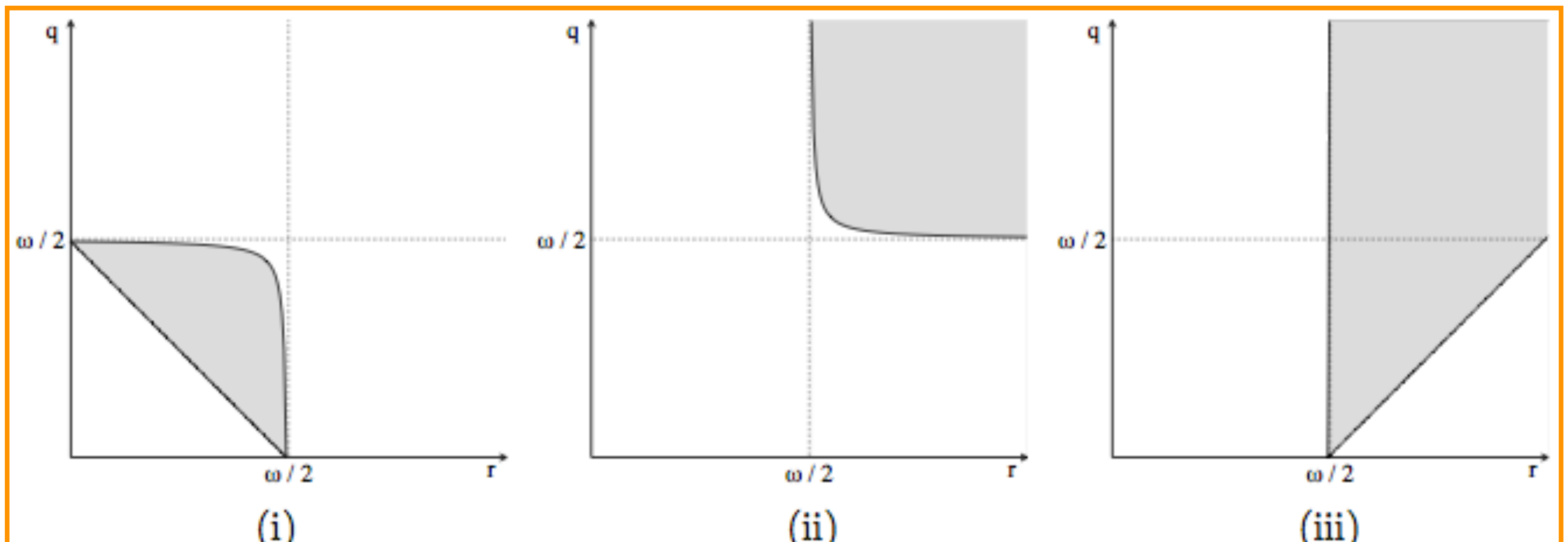
$$\text{(ii)} \quad \int_0^\infty \mathrm{d}q \int_0^\infty \mathrm{d}r \int_{E_{qr}^-}^{E_{qr}^+} \mathrm{d}E_{qr}\, \delta(\omega - q - r + E_{qr})\,\phi(q,r,E_{qr})$$

$$= \int_{\frac{\omega}{2}}^\infty \mathrm{d}q \int_{\frac{\omega(2q-\omega)+\lambda^2}{2(2q-\omega)}}^\infty \mathrm{d}r\,\phi(q,r,-\omega+q+r)\,,$$

$$\text{(iii)} \quad \int_0^\infty \mathrm{d}q \int_0^\infty \mathrm{d}r \int_{E_{qr}^-}^{E_{qr}^+} \mathrm{d}E_{qr}\, \delta(\omega + q - r - E_{qr})\,\phi(q,r,E_{qr})$$

$$= \int_0^\infty \mathrm{d}q \int_{\frac{\omega(\omega+2q)-\lambda^2}{2(\omega+2q)}}^{\frac{\omega(\omega+2q)-\lambda^2}{2\omega}} \mathrm{d}r\,\phi(q,r,\omega+q-r)\,,$$

Original integration ranges for $\lambda = \omega/10$.



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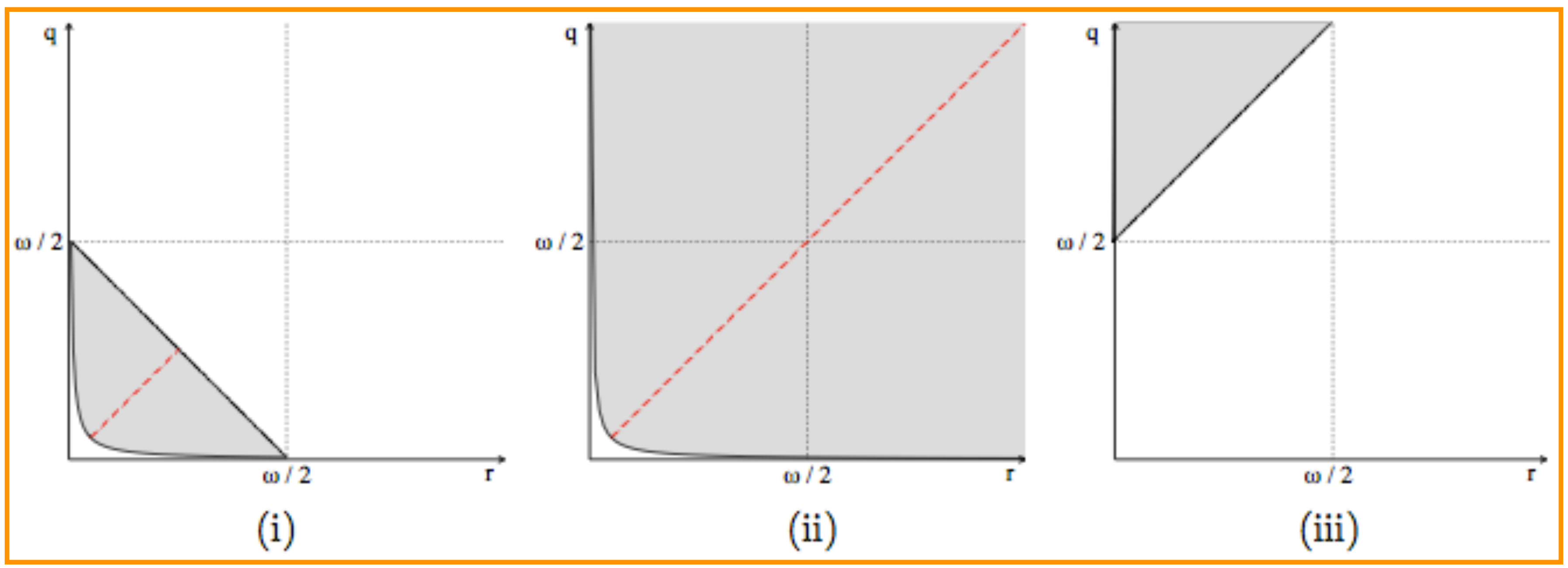
Real Correction

For turning all denominators into 1/(4qr):

$$
\begin{aligned}
\text{(i)}: &\quad q \to \frac{\omega}{2} - q\,, \quad r \to \frac{\omega}{2} - r\,, \\
\text{(ii)}: &\quad q \to \frac{\omega}{2} + q\,, \quad r \to \frac{\omega}{2} + r\,, \\
\text{(iii)}: &\quad q \to -\frac{\omega}{2} + q\,, \quad r \to \frac{\omega}{2} + r\,,
\end{aligned}
$$



Integration ranges after the shifts, for $\lambda = \omega/10$.

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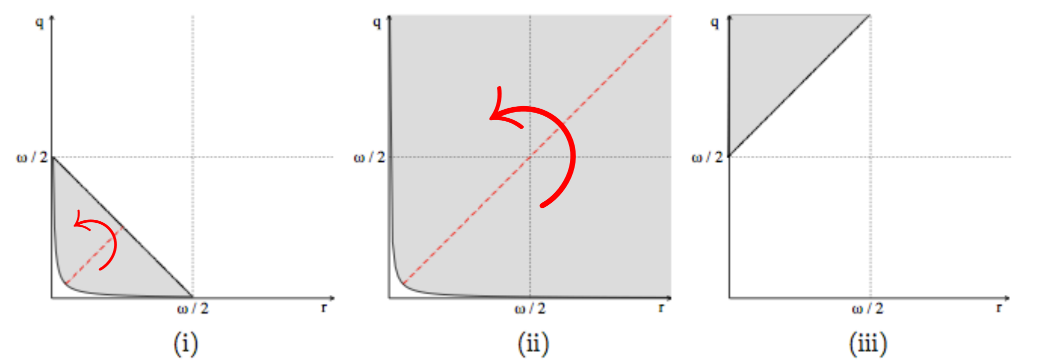
Real Correction

$$
\begin{aligned}
\rho^{(\mathrm{ps})}_{\mathcal{I}_j}(\omega) \;=\; & \frac{\omega^4}{2(4\pi)^3}(e^{\omega}-1)\Bigg\{ \\
\text{(i)} \quad & -\int_{\frac{\lambda^2}{2\omega}}^{\frac{\omega}{2}} \mathrm{d}q \int_{\frac{\lambda^2}{4q}}^{\frac{\omega(\omega-2q)+\lambda^2}{2\omega}} \mathrm{d}r\, \mathbb{P}\left(\frac{1}{qr}\right) n_{\frac{\omega}{2}-q}\, n_{\frac{\omega}{2}-r}\, n_{q+r} \\
\text{(ii)} \quad & -\int_{0}^{\infty} \mathrm{d}q \int_{\frac{\lambda^2}{4q}}^{\infty} \mathrm{d}r\, \mathbb{P}\left(\frac{1}{qr}\right) n_{\frac{\omega}{2}+q}\, n_{\frac{\omega}{2}+r}\, n_{q+r}\, e^{q+r} \\
\text{(iii)} \quad & +\int_{\frac{\omega}{2}}^{\infty} \mathrm{d}q \int_{-\frac{\lambda^2}{4q}}^{\frac{\omega(2q-\omega)-\lambda^2}{2\omega}} \mathrm{d}r\, \mathbb{P}\left(\frac{2}{qr}\right) n_{q-\frac{\omega}{2}}\, n_{\frac{\omega}{2}+r}\, n_{q-r}\, e^{q-\frac{\omega}{2}} \Bigg\}\,.
\end{aligned}
$$

Divergent!



$$\text{(i)} \quad n_{\frac{\omega}{2}-q}\, n_{\frac{\omega}{2}-r}\, n_{q+r}(1-e^{\omega}) = -(1+2n_{\frac{\omega}{2}})\left[1+n_{q+r}+n_{\frac{\omega}{2}-q}+(1+n_{\frac{\omega}{2}-r})\frac{n_{q+r}n_{\frac{\omega}{2}-q}}{n_r^2}\right].$$

$$\text{(ii)} \quad n_{\frac{\omega}{2}+q}\, n_{\frac{\omega}{2}+r}\, n_{q+r}\, e^{q+r}(1-e^{\omega}) = (1+2n_{\frac{\omega}{2}})\left[-n_{q+r}+n_{q+\frac{\omega}{2}}-(1+n_{q+\frac{\omega}{2}})\frac{n_{q+r}n_{r+\frac{\omega}{2}}}{n_r^2}\right].$$

$$\text{(iii)} \quad n_{q-\frac{\omega}{2}}\, n_{\frac{\omega}{2}+r}\, n_{q-r}\, e^{q-\frac{\omega}{2}}(e^{\omega}-1) = (1+2n_{\frac{\omega}{2}})\left[n_{q-\frac{\omega}{2}}-n_q-n_{q-\frac{\omega}{2}}\frac{(1+n_{q-r})(n_q-n_{r+\frac{\omega}{2}})}{n_r n_{-\frac{\omega}{2}}}\right].$$

For integrals:

- Depend on q+r only, we can integrate them by integrating (x,y)=(q+r,q-r),
- Independent on r, one can integrate r first.

$\rho_{\mathcal{I}_j}(\omega)$

- Collect every part together and simplify them with $\lambda \ll \omega$,
- All the divergent terms cancel each other, we can set $\lambda \to 0$ in the end.

$$
\begin{aligned}
&\frac{(4\pi)^3 \rho_{\mathcal{I}_j}(\omega)}{\omega^4(1+2n_{\frac{\omega}{2}})} = \\
&\quad \int_0^{\frac{\omega}{4}} \mathrm{d}q\, n_q \left[\left(\frac{1}{q-\frac{\omega}{2}} - \frac{1}{q}\right)\ln\left(1-\frac{2q}{\omega}\right) - \frac{\frac{\omega}{2}}{q(q+\frac{\omega}{2})}\ln\left(1+\frac{2q}{\omega}\right)\right] \\
&+ \int_{\frac{\omega}{4}}^{\frac{\omega}{2}} \mathrm{d}q\, n_q \left[\left(\frac{2}{q-\frac{\omega}{2}} - \frac{1}{q}\right)\ln\left(1-\frac{2q}{\omega}\right) - \frac{\frac{\omega}{2}}{q(q+\frac{\omega}{2})}\ln\left(1+\frac{2q}{\omega}\right) - \frac{1}{q-\frac{\omega}{2}}\ln\left(\frac{2q}{\omega}\right)\right] \\
&+ \int_{\frac{\omega}{2}}^{\infty} \mathrm{d}q\, n_q \left[\left(\frac{2}{q-\frac{\omega}{2}} - \frac{2}{q}\right)\ln\left(\frac{2q}{\omega}-1\right) - \frac{\frac{\omega}{2}}{q(q+\frac{\omega}{2})}\ln\left(1+\frac{2q}{\omega}\right) + \left(\frac{1}{q} - \frac{1}{q-\frac{\omega}{2}}\right)\ln\left(\frac{2q}{\omega}\right)\right] \\
&+ \int_0^{\frac{\omega}{2}} \mathrm{d}q \int_0^{\frac{\omega}{4}-|q-\frac{\omega}{4}|} \mathrm{d}r \left(-\frac{1}{qr}\right) \frac{n_{\frac{\omega}{2}-q}\, n_{q+r}(1+n_{\frac{\omega}{2}-r})}{n_r^2} \\
&+ \int_{\frac{\omega}{2}}^{\infty} \mathrm{d}q \int_0^{q-\frac{\omega}{2}} \mathrm{d}r \left(-\frac{1}{qr}\right) \frac{n_{q-\frac{\omega}{2}}(1+n_{q-r})(n_q - n_{r+\frac{\omega}{2}})}{n_r n_{-\frac{\omega}{2}}} \\
&+ \int_0^{\infty} \mathrm{d}q \int_0^{q} \mathrm{d}r \left(-\frac{1}{qr}\right) \frac{(1+n_{q+\frac{\omega}{2}})n_{q+r}n_{r+\frac{\omega}{2}}}{n_r^2} + \mathcal{O}(\lambda \ln \lambda)\,.
\end{aligned}
$$

Numerical Calculation

Complex integral function

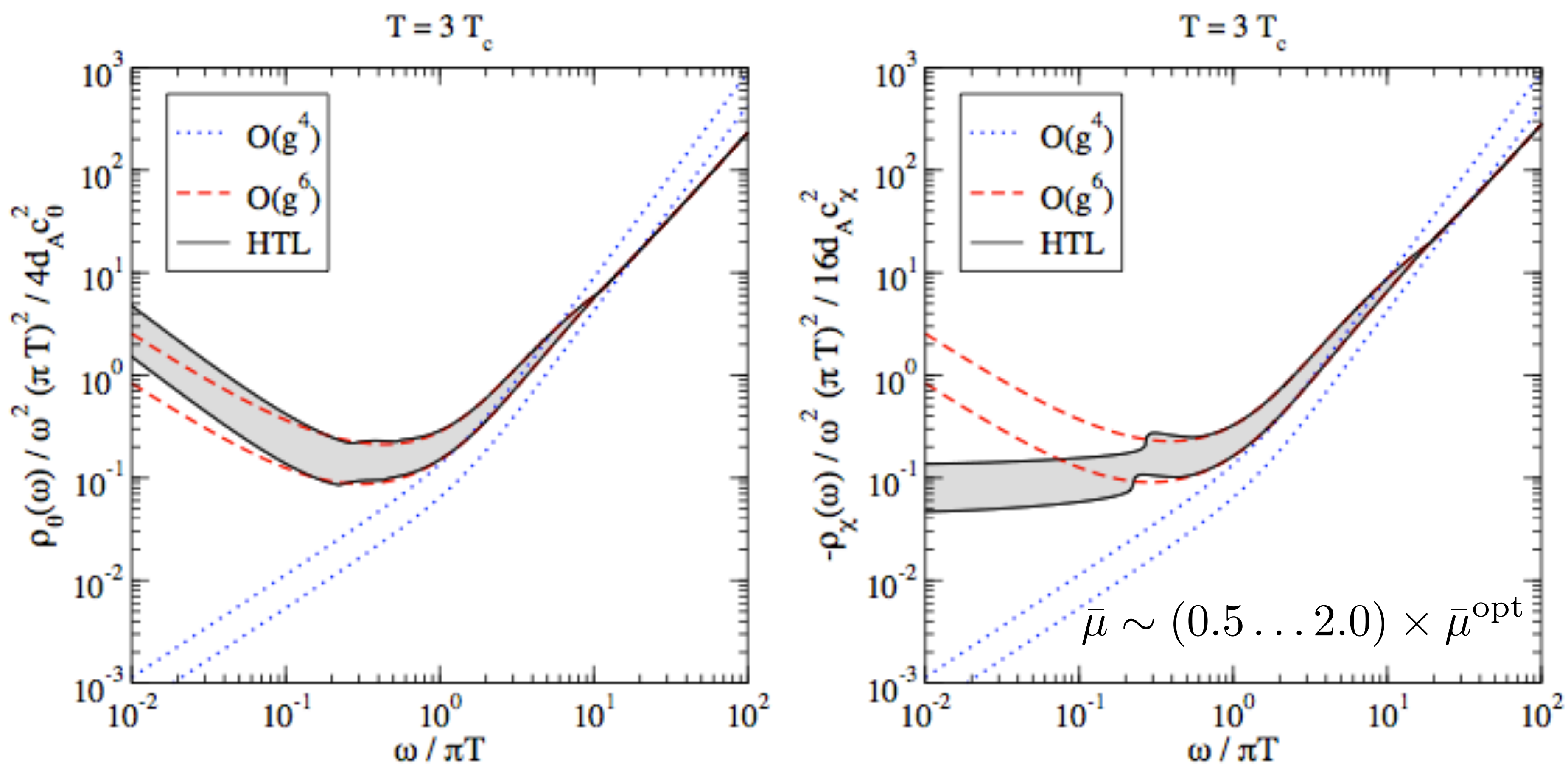
$$\frac{\rho_\theta(\omega)}{4d_A c_\theta^2} = \frac{\pi\omega^4}{(4\pi)^2}\left(1+2n_{\frac{\omega}{2}}\right)\left\{g^4 + \frac{g^6 N_c}{(4\pi)^2}\left[\frac{22}{3}\ln\frac{\bar\mu^2}{\omega^2} + \frac{73}{3} + 8\,\phi_T(\omega)\right]\right\} + \mathcal{O}(g^8)$$

$$\frac{-\rho_\chi(\omega)}{16 d_A c_\chi^2} = \frac{\pi\omega^4}{(4\pi)^2}\left(1+2n_{\frac{\omega}{2}}\right)\left\{g^4 + \frac{g^6 N_c}{(4\pi)^2}\left[\frac{22}{3}\ln\frac{\bar\mu^2}{\omega^2} + \frac{97}{3} + 8\,\phi_T(\omega)\right]\right\} + \mathcal{O}(g^8)$$

In this results, the terms with the structure $\omega^n\delta(\omega)$ have been omitted!

- In the regime $\omega \gg \pi T$, “Fast apparent convergence”: $\ln(\bar\mu_\theta^{\mathrm{opt}(\omega)}) \equiv \ln(\omega) - \frac{73}{44}$, $\ln(\bar\mu_\chi^{\mathrm{opt}(\omega)}) \equiv \ln(\omega) - \frac{97}{44}$.
- When $\omega \ll \pi T$, EQCD: $\ln(\bar\mu_{\theta,\chi}^{\mathrm{opt}(T)}) \equiv \ln(4\pi T) - \gamma_E - \frac{1}{22}$.
- The scale parameter $\Lambda_{\overline{\mathrm{MS}}} \equiv \lim_{\bar\mu\to\infty} \bar\mu\,[b_0 g^2]^{-b_1/2b_0^2} \exp[-\frac{1}{2b_0 g^2}]$, $T_c = 1.25\Lambda_{\overline{\mathrm{MS}}}$,

Spectral Functions



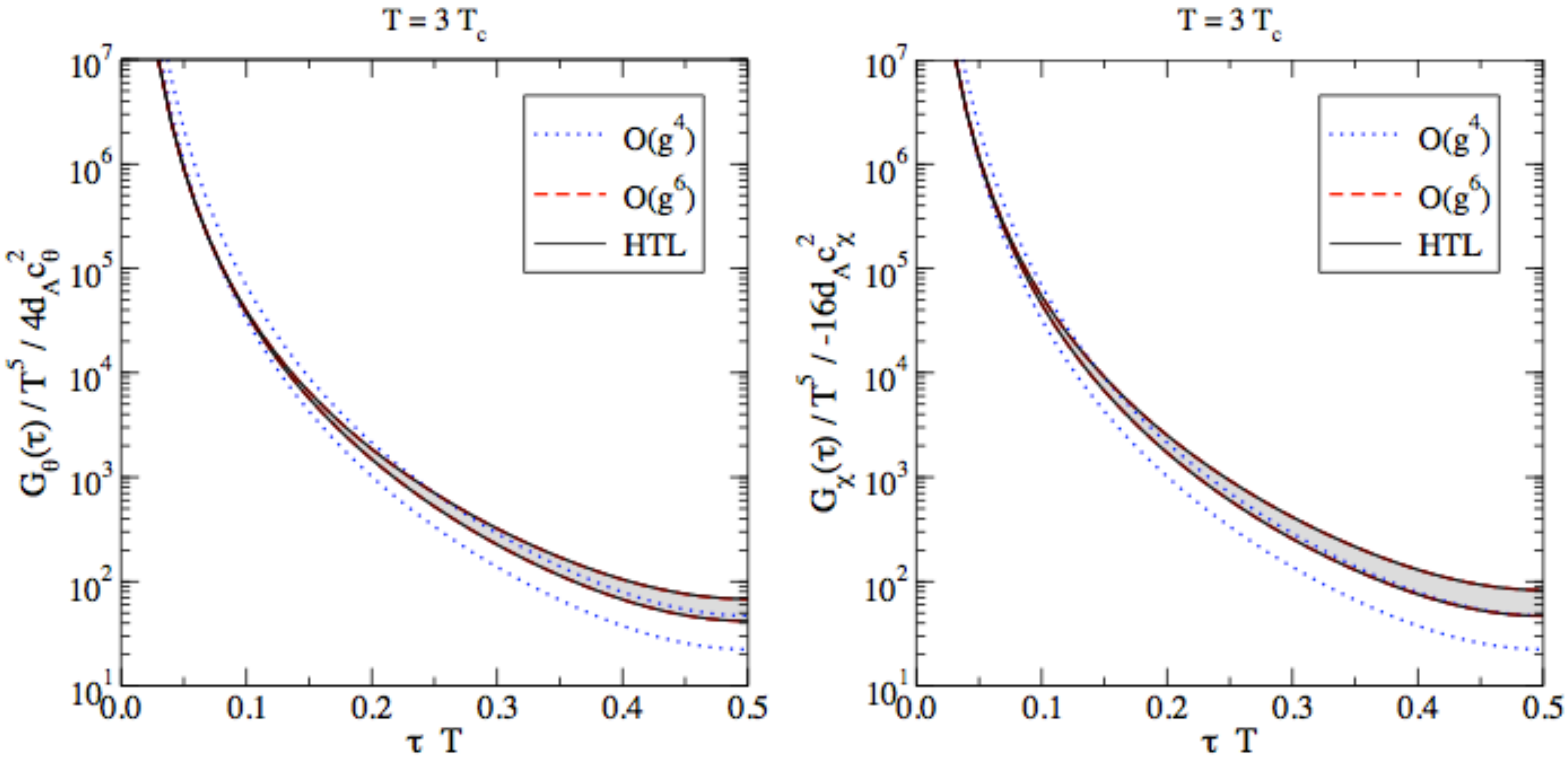
$$\frac{\rho_\theta(\omega)}{4d_A c_\theta^2} = \frac{\pi\omega^4}{(4\pi)^2}\left(1+2n_{\frac{\omega}{2}}\right)\left\{g^4 + \frac{g^6 N_c}{(4\pi)^2}\left[\frac{22}{3}\ln\frac{\bar{\mu}^2}{\omega^2} + \frac{73}{3} + 8\,\phi_T(\omega)\right]\right\} + \mathcal{O}(g^8)$$

$$\frac{-\rho_\chi(\omega)}{16d_A c_\chi^2} = \frac{\pi\omega^4}{(4\pi)^2}\left(1+2n_{\frac{\omega}{2}}\right)\left\{g^4 + \frac{g^6 N_c}{(4\pi)^2}\left[\frac{22}{3}\ln\frac{\bar{\mu}^2}{\omega^2} + \frac{97}{3} + 8\,\phi_T(\omega)\right]\right\} + \mathcal{O}(g^8)$$

$$\rho^{\rm QCD}_{\rm resummed} = \rho^{\rm QCD}_{\rm resummed} - \rho^{\rm HTL}_{\rm resummed} + \rho^{\rm HTL}_{\rm resummed} \approx \rho^{\rm QCD}_{\rm naive} - \rho^{\rm HTL}_{\rm naive} + \rho^{\rm HTL}_{\rm resummed}\,.$$

Imaginary-time Correlators

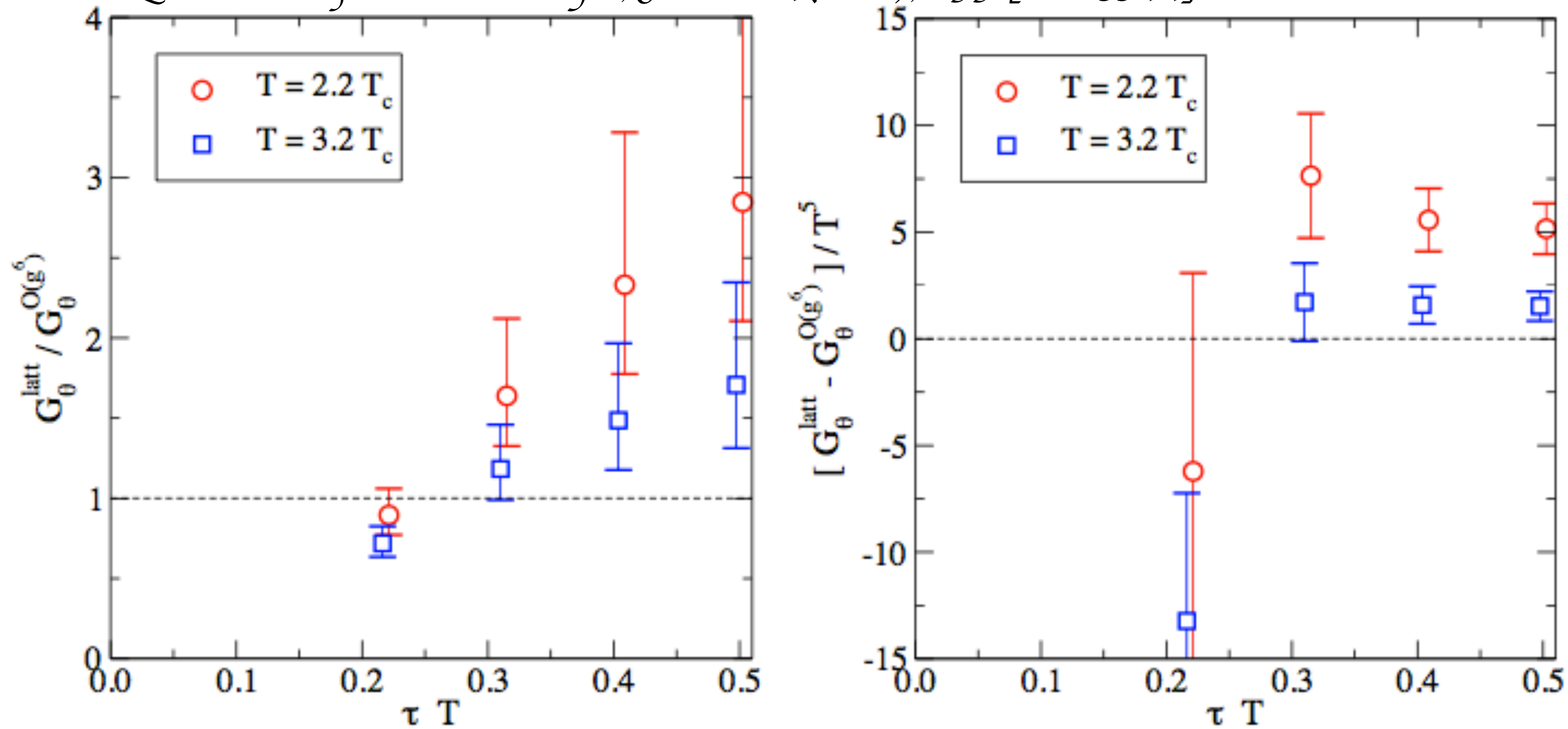
$$G(\tau) = \int_0^\infty \frac{d\omega}{\pi} \rho(\omega, \mathbf{0}) \frac{\cosh\left(\frac{\beta}{2} - \tau\right)\omega}{\sinh\frac{\beta\omega}{2}} \,.$$



- Considerable difference between LO and NLO in spectral function leads to a small correction to the imaginary-time correlators.

Comparing with LQCD

Lattice data from H.B. Meyer, JHEP 04(2010), 099 [10023344]



- The ratio shows good agreement at short distance.
- The difference no longer shows the short distance divergence.
  A model independent analytic continuation could be attempted.

Summary and Outlook

- A novel method has been evaluated to calculate the correlators and spectral functions to NLO in pure Yang-Mills theory.
- Spectral functions in bulk channel has been computed with this method, which is very helpful to constrain the corresponding correlator and determine the thermal coefficient.
- The application to shear channel is underway.
- Production rate of axion and dilation in cosmology, as well as the spectral function of electromagnetic current in QCD,..., can be evaluated with this method.

Thanks for your attention!