Energy-Momentum Tensor Correlators in Hot Yang-Mills Theory

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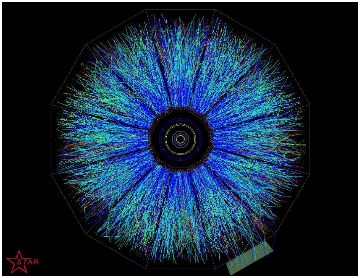
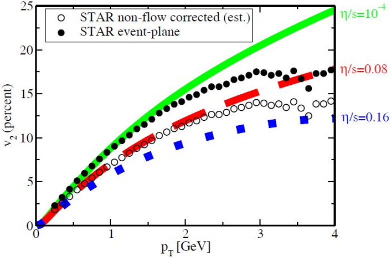
Puzzles from RHIC





RHIC observation: Hydrodynamics works well, but only if $\eta/s \lesssim 0.2$

- AdS/CFT: $\eta/s = \frac{1}{4\pi}$ in theories with gravity duals

Obvious questions: What are η, ζ,... in QCD? Is the plasma 'strongly coupled'? Is $\mathcal{N}=4$ SYM really a good model for QGP?

Ultimate answer only from non-perturbative calculations in QCD

Motivation I: Transport coefficients in hot QCD

Kubo formulas: Viscosities and other transport coeffs. obtainable from retarded Minkowski correlators of energy momentum tensor $T_{\mu\nu}$:

$$\eta = \lim_{\omega\to 0}\frac{1}{\omega}\,\mathrm{Im}\,D^{\mathrm{R}}_{12,12}(\omega,\mathbf{k}=0) \equiv \lim_{\omega\to 0}\frac{\rho_{12,12}(\omega,\mathbf{k}=0)}{\omega}$$

Problem: Lattice can only measure Euclidean correlators: Spectral density available only through inversion of

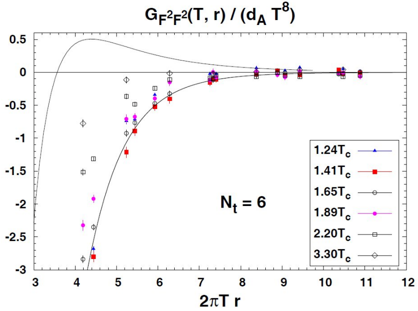
$$G(\tau) = \int_0^\infty \frac{\mathrm{d}\omega}{\pi}\rho(\omega)\frac{\cosh\frac{(\beta-2\tau)\omega}{2}}{\sinh\frac{\beta\omega}{2}}$$

$\therefore$ To extract the IR limit of ρ, need to understand its behavior also at $\omega \gtrsim \pi T$ — very non-trivial challenge for lattice QCD, requiring perturbative input

Motivation II: Correlators as measure of interaction

Spatial correlators measure screening in medium $\Rightarrow$ Comparison between lattice QCD, pQCD and AdS/CFT results offers insights into structure and properties of the QGP

Iqbal, Meyer (0909.0582): Lattice data for correlators of $\mathrm{Tr}\, F_{\mu\nu}^2$ in semi-quantitative agreement with strongly coupled $\mathcal{N} = 4$ SYM, while leading order pQCD result completely off. How about NLO?



Challenge for perturbation theory

Goal: Evaluate different Euclidean and Minkowskian correlators to high order in perturbation theory, and use the results to

1. Determine Operator Product Expansions at finite temperature
2. Evaluate the spectral densities and apply them to lattice calculations
3. Compare behavior of spatial correlators to lattice QCD and AdS/CFT

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Concretely: Specialize to scalar, pseudoscalar and shear operators

$$\theta \equiv c_\theta \, g_B^2 F^a_{\mu\nu} F^a_{\mu\nu} \,, \quad \chi \equiv c_\chi \, g_B^2 F^a_{\mu\nu} \widetilde{F}^a_{\mu\nu} \,, \quad \eta \equiv 2c_\eta T_{12} = -2c_\eta F^a_{1\mu} F^a_{2\mu}$$

and proceed from 1 to 3 working at NLO.

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When can perturbation theory be expected to converge?

$$\bar{\Lambda}_{x,T} \simeq \sqrt{\left(\bar{\Lambda}_x\right)^2 + \left(\bar{\Lambda}_T\right)^2} \sim \sqrt{\frac{1}{x^2} + (2\pi T)^2}$$

At least, if either $x \ll 1/\Lambda_{\rm QCD}$ ($\omega \gg \Lambda_{\rm QCD}$) or $T \gg \Lambda_{\rm QCD}$!

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Setting up the calculation

The plan: Work within finite-T SU(3) Yang-Mills theory

$$S_E = \int_0^\beta \mathrm{d}\tau \int \mathrm{d}^{3-2\epsilon}x \, \left\{ \frac{1}{4} F^a_{\mu\nu} F^a_{\mu\nu} \right\},$$

write down diagrammatic expansions for Euclidean correlators

$$G_\theta(x) \equiv \langle \theta(x)\theta(0) \rangle_c \, , \quad G_\chi(x) \equiv \langle \chi(x)\chi(0) \rangle \, , \quad G_\eta(x) \equiv \langle \eta(x)\eta(0) \rangle_c,$$

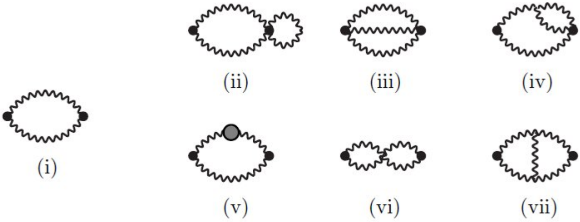
$$\widetilde{G}_\alpha(P) \equiv \int_x e^{-iP\cdot x} G_\alpha(x),$$

$$\rho_\alpha(\omega) \equiv \mathrm{Im}\, \widetilde{G}_\alpha(k_0 = -i(\omega + i\epsilon), \mathbf{k} = 0),$$

and evaluate the necessary integrals.

Setting up the calculation

End up with two-loop two-point diagrams in momentum space — doable by 'cutting' thermal lines and evaluating remaining 3*d* integrals



Summary of current NLO results

	OPEs	Spectral density	Coord. space
Scalar	[1]	[2]	[3]
Pseudoscalar	[1]	[2]	[3]
Shear	[4]	Underway [5]	Future

[1] Mikko Laine, Mikko Vepsäläinen, AV, 1008.3263
[2] Mikko Laine, AV, Yan Zhu, 1108.1259
[3] Mikko Laine, Mikko Vepsäläinen, AV, 1011.4439
[4] York Schröder, Mikko Vepsäläinen, AV, Yan Zhu, 1109.6548
[5] See talk by Yan Zhu

Note: Inclusion of fermions possible in all cases.

Results I: Wilson coefficients for OPE

In the UV, perform large momentum expansion of Euclidean correlators to obtain OPEs

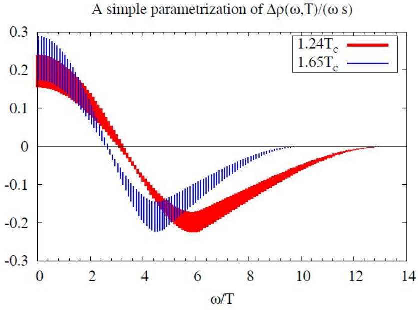
$$\frac{\Delta\widetilde{G}_\theta(P)}{4c_\theta^2 g^4} = \frac{3}{P^2}\left(\frac{p^2}{3}-p_n^2\right)\left[1+\frac{g^2N_c}{(4\pi)^2}\left(\frac{22}{3}\ln\frac{\bar{\mu}^2}{P^2}+\frac{203}{18}\right)\right](e+p)(T)$$
$$-\frac{2}{g^2b_0}\left[1+g^2b_0\ln\frac{\bar{\mu}^2}{\zeta_\theta P^2}\right](e-3p)(T)+\mathcal{O}\left(g^4,\frac{1}{P^2}\right)$$

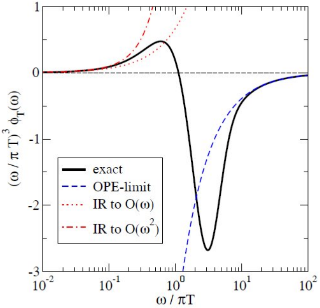
$$\frac{\Delta\widetilde{G}_\chi(P)}{-16c_\chi^2 g^4} = \frac{3}{P^2}\left(\frac{p^2}{3}-p_n^2\right)\left[1+\frac{g^2N_c}{(4\pi)^2}\left(\frac{22}{3}\ln\frac{\bar{\mu}^2}{P^2}+\frac{347}{18}\right)\right](e+p)(T)$$
$$+\frac{2}{g^2b_0}\left[1+g^2b_0\ln\frac{\bar{\mu}^2}{\zeta_\chi P^2}\right](e-3p)(T)+\mathcal{O}\left(g^4,\frac{1}{P^2}\right)$$

$$\frac{\Delta\widetilde{G}_\eta(P)}{4c_\eta^2} = -\left\{1+\frac{3}{P^2}\left(\frac{p^2}{3}-p_n^2\right)-\frac{1}{3}\frac{g^2N_c}{(4\pi)^2}\left[22+\frac{41}{P^2}\left(\frac{p^2}{3}-p_n^2\right)\right]\right\}(e+p)(T)$$
$$+\frac{4}{3g^2b_0}\left[1-g^2b_0\ln\zeta_\eta\right](e-3p)(T)+\mathcal{O}\left(g^4,\frac{1}{P^2}\right)$$

Note the absence of logs of $\bar{\mu}$ in the shear result.

Results II: Spectral densities

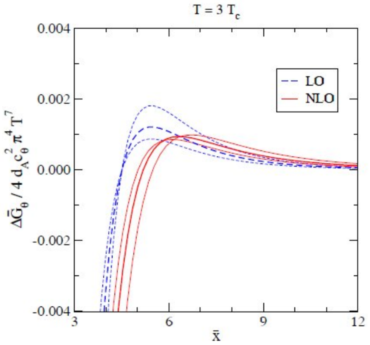




H. B. Meyer, 1002.3343

- Goal: To aid lattice determination of transport coefficients by providing non-trivial, dominant perturbative part of spectral density
- Promising results in bulk channel — technically more complicated shear calculation underway (see talks of Miao, Zhu)

Results III: Time averaged coord. sp. correlators



- Qualitatively, NLO results considerably closer to lattice than LO ones in bulk channel
- However: We computed time averaged correlator, not equal time
- AdS computation of same correlator in large-N_c YM underway

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Conclusions

- Information on correlation functions of the energy momentum tensor crucial for disentangling the properties of the QGP
  - Spectral densities needed in extracting transport coefficients from lattice QCD data
  - Spatial correlators a highly useful way test lattice, pQCD and holographic predictions
- NLO results in the bulk channel completed, shear channel underway
  - Results promising, but quantitative comparisons await
- If pure YM results useful, inclusion of fermions straightforward