Critical behaviors of higher order cumulant ratios in O(1,2,4) models

Xue Pan[1], Lizhu Chen[1], X. S. Chen[2] and Yuanfang Wu[1]
[1]Central China Normal University
[2]Institute of Theoretical Physics, Chinese Academy of Sciences

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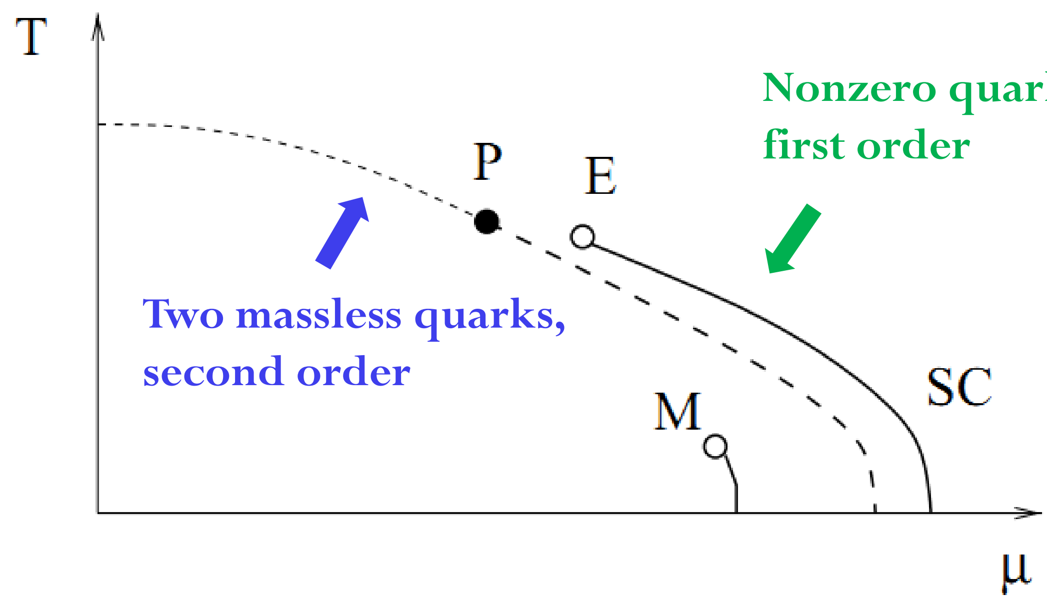
Outline

- Motivation
- O(N) models and the cumulant ratios
- Behaviors of the cumulant ratios
- Summary and outlook

QCD universality class



M. Stephanov, K.Rajagopal, and E. Shuryak , PRL81, 4816(1998)

Chiral phase transition ⟹ { **3d O(4) model** / **3d O(2) model** }

Critical end point $\xrightarrow{\textbf{O(1)}}$ **3d Ising model**

Robert D.Pisarski and Frank Wilczek, PRD29(1984)338

J. Engels, S.Holtmann, T.Mendes, T.Schulze, PLB514(2001)299

Higher order cumulants

- Higher order cumulants are more sensitive to the critical end point.

  1. More sensitive to the correlation length

$$\left\langle (\delta N)^3 \right\rangle \sim \xi^{4.5}, \left\langle (\delta N)^4 \right\rangle - 3\left\langle (\delta N)^2 \right\rangle^2 \sim \xi^7$$

M.A.Stephanov,PRL,102(2009)032301

  2. Sign changes

  Sign change of skewness and negative kurtosis.

M.Asakawa, S.Ejiri, M.Kitazawa, PRL103(2009)262301

M.A.Stephanov, PRL107(2011)052301

- Sixth order cumulant : the first divergent cumulant in the chiral limit.

B. Friman, F. Karsch, K. Redlich and V. Skokov, arXiv: 1103.3511

- It is helpful to study the critical behaviors of the cumulants directly by O(1,2,4) models.

O(N) models and the cumulant ratios

- Definition of O(N) models : $\beta\mathscr{H} = -J \sum_{\langle i,j \rangle} \mathbf{S}_i \cdot \mathbf{S}_j - \mathbf{H} \cdot \sum_i \mathbf{S}_i$

  $\mathbf{S}_i$ is an N-component unit vector at site i.

- In the chiral limit : $m_q = 0 \xrightarrow{\text{O(4)}} \vec{H} = 0$

- Order parameter : $m = \left|\vec{M}\right| = \left|\sum_{i=1}^{L^3} \vec{s}_i / L^3\right|$

  A.L. Talapov and H. W. J. Blote
  arXiv:cond-mat/9603013

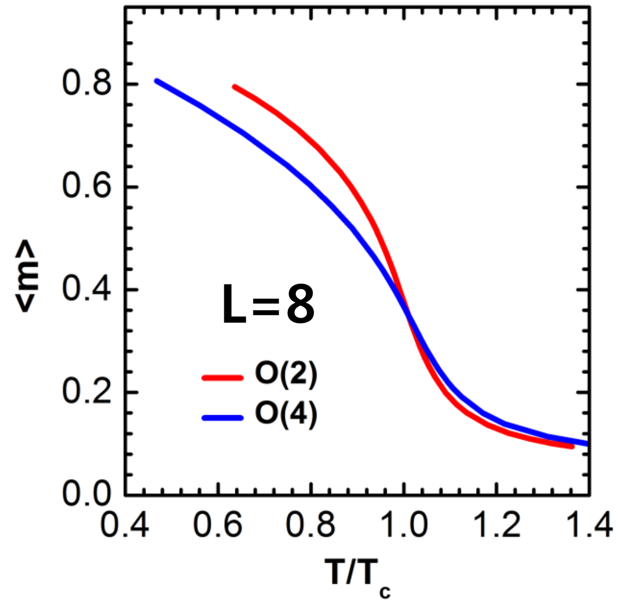
- The cumulant ratios:

$$\delta m = m - \langle m \rangle \qquad K_2 = \langle \delta m^2 \rangle \qquad R_{3,2} = \frac{\langle \delta m^3 \rangle}{\langle \delta m^2 \rangle}$$

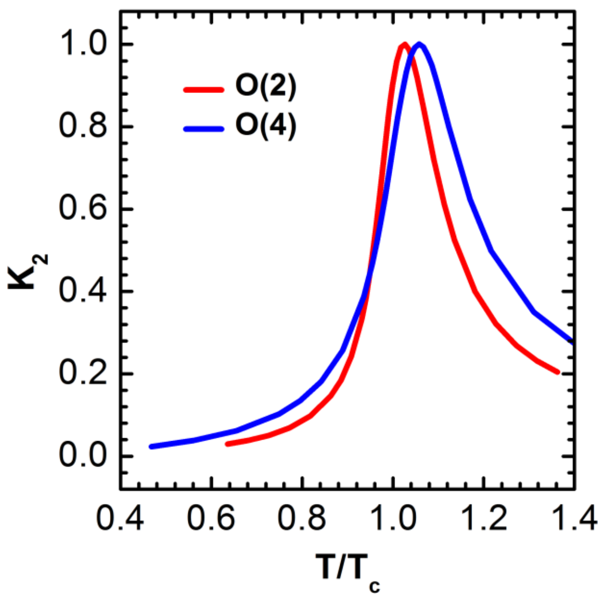
$$R_{4,2} = \frac{\langle \delta m^4 \rangle}{\langle \delta m^2 \rangle} - 3\langle \delta m^2 \rangle \qquad R_{6,2} = \frac{\langle \delta m^6 \rangle - 15\langle \delta m^4 \rangle\langle \delta m^2 \rangle - 10\langle \delta m^3 \rangle^2 + 30\langle \delta m^2 \rangle^3}{\langle \delta m^2 \rangle}$$

Behavior of $\langle m \rangle$ and K_2

- The order parameter decreases with the increase of T.



- There is a cusp in K_2 near T_c .

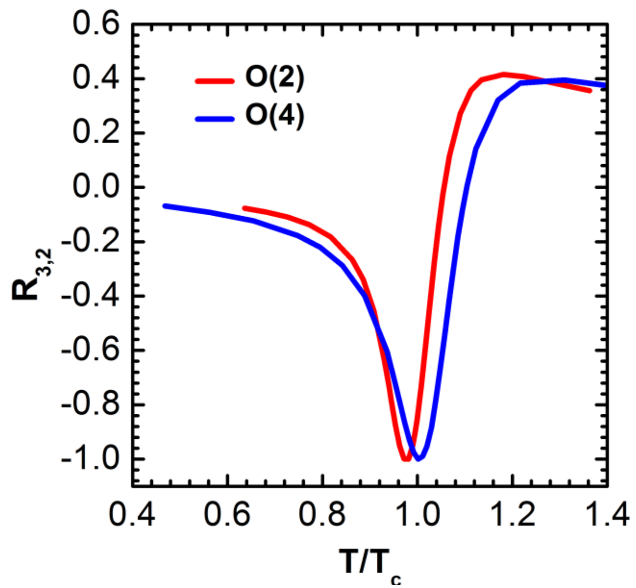


- The trends are qualitatively similar in O(2) and O(4) models.

Behaviors of $R_{3,2}$ and $R_{4,2}$

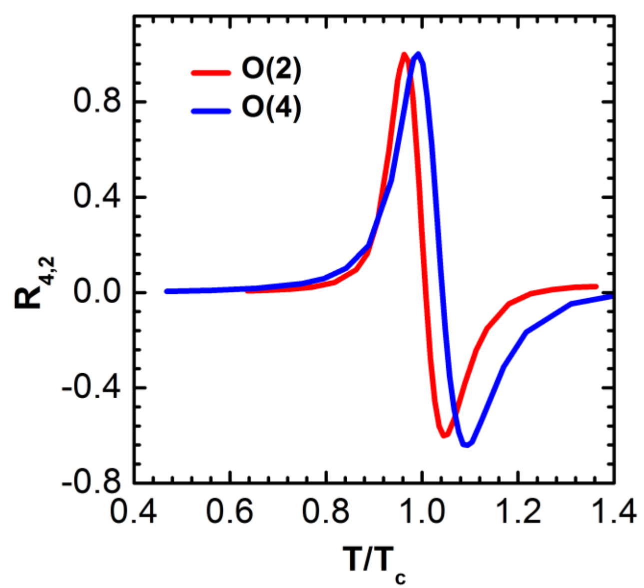
- There is a valley near T_c .
- $R_{3,2}$ changes sign near T_c .

M.Asakawa, S.Ejiri, M.Kitazawa, PRL103(2009)262301

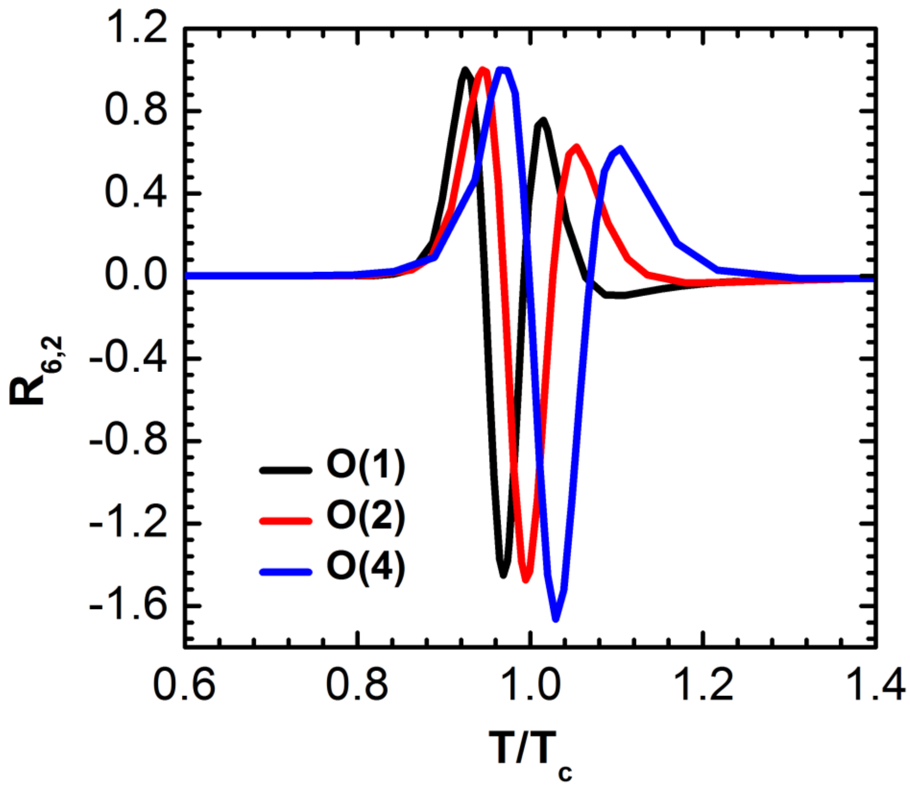


- $R_{4,2}$ oscillates when passing T_c .
- $R_{4,2}$ changes to negative at $T > T_c$.

M.A.Stephanov, PRL107 (2011) 052301
R.V. Gavai, Sourendu Gupta, Phys Lett B 696 (2011) 459
Lizhu Chen, Xue Pan, Xiaosong Chen, Yuanfang Wu, arXiv: 1010.1166



Behavior of $R_{6,2}$



- $R_{6,2}$ has a maximum close to the transition region before they drop sharply.
- $R_{6,2}$ shows pronounced minima with $R_{6,2} < 0$ in the vicinity of the critical temperature.

Summary

- Cumulant ratios in 3d O(1), O(2) and O(4) models without magnetic field are calculated.
- The cumulant ratios all change sharply and become negative in the transition region.
- The qualitative behaviors of cumulant ratios are similar in O(1), O(2) and O(4) models.

Outlook

- The cumulant ratios in 3d O(2) and O(4) models with magnetic field ($m_q \neq 0$) are ongoing.

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THANKS !