Baryon Number Cumulants and Proton Number Cumulants in Relativistic Heavy Ion Collisions

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S. Ejiri, M. Kitazawa, and M. A., PRL 103 (2009) 262301
M. Kitazawa and M.A., (2011) to be published

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Two Meanings of Fluctuation Observables

- Long History of Fluctuation Observations (including intermittency) in HI Collisions

  - Distinction of Phases (conserved charge fluctuation)

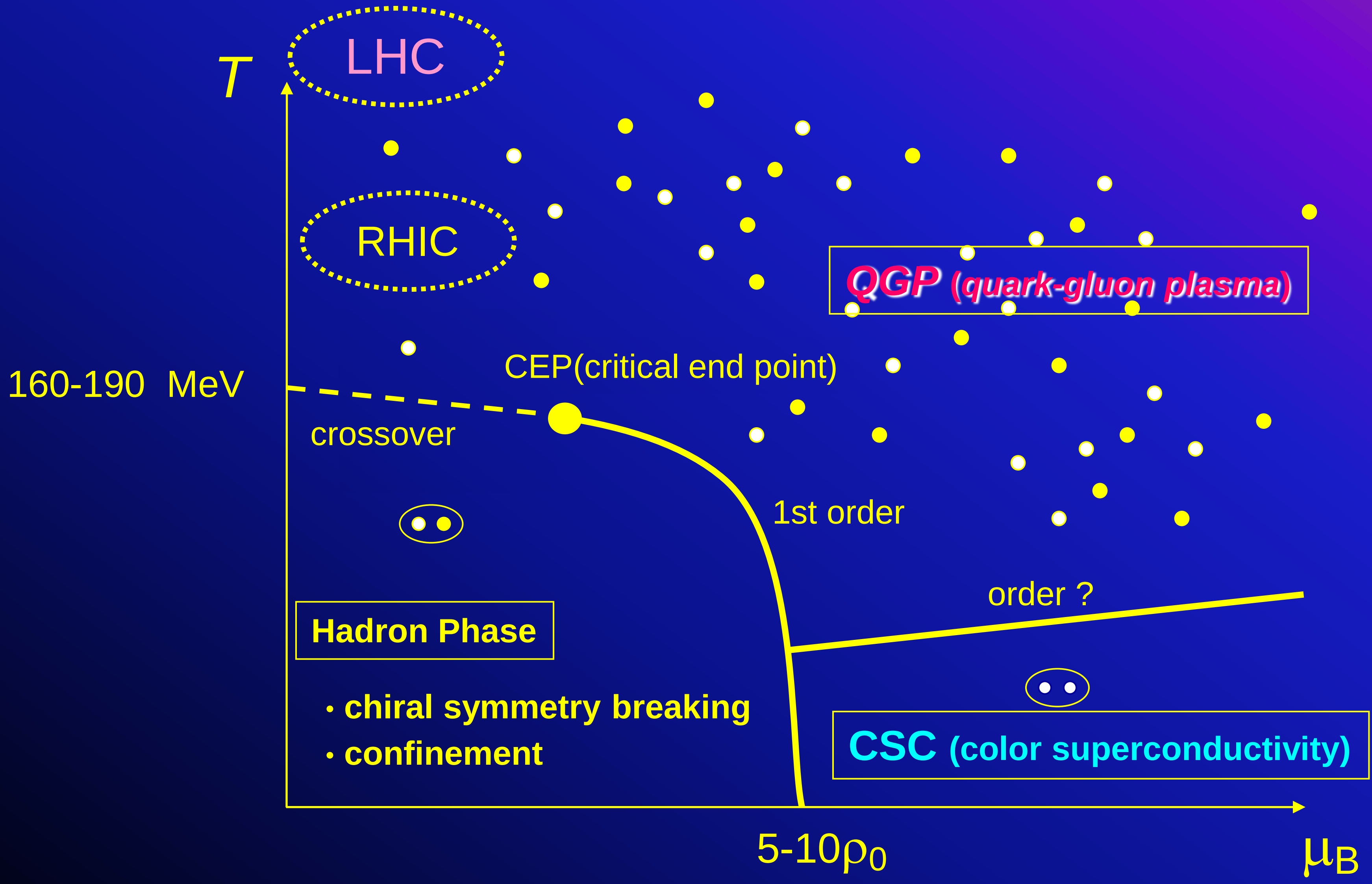
    Asakawa, Heinz, Müller, Jeon, Koch, Ejiri, Kitazawa, …

  - Detection of Increase of Fluctuation (or Correlation Length) at 2nd order Phase Transition

    Critical End Point = 2nd order phase transition point

    ⇨ Phase structure of QCD

QCD Phase Diagram



M. Asakawa (Osaka University)

Higher Moments

- Recently higher moments have attracted quite a lot of attention
  - (Roughly) Two Reasons
    - Larger critical exponents around CEP (=Aξ^z) Stephanov (2008)
    - Sign change across the phase transition (crossover) line

Asakawa, Ejiri, Kitazawa (2009)

⇨ Next Slide

Odd Power Fluctuation Moments

- Fluctuation of Conserved Charges: not subject to final state interactions
  - Usually even power fluctuations such as $\left\langle (\delta Q)^2 \right\rangle$ have been considered
  - Usual Fluctuations such as $\left\langle (\delta Q)^2 \right\rangle$: positive definite

⇒ Absolute values carry information of states (D-measure)

Asakawa, Heinz, Müller, Jeon, Koch

On the other hand,

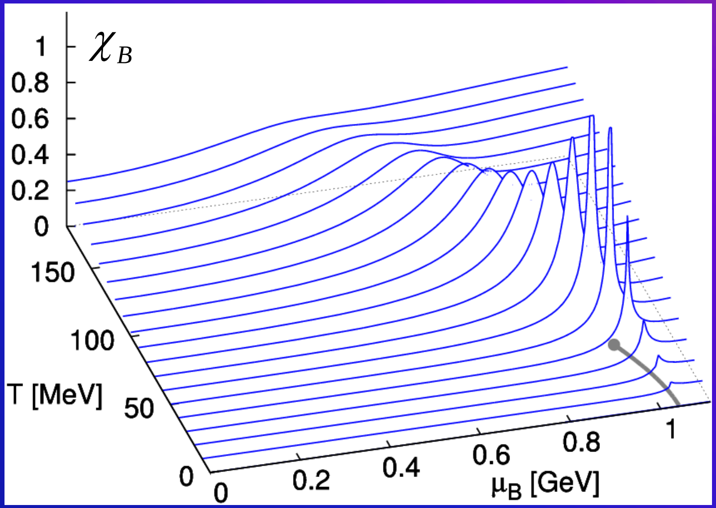
✓ Odd power fluctuations : NOT positive definite

- In general, do not vanish (exception, $\langle \delta A \rangle$)
- *Sign also carry information of states*

Physical Meaning of 3rd Fluc. Moment

χ_B : Baryon number susceptibility

in general, has a peak along phase transition



⇨ $\frac{\partial \chi_B}{\partial \mu_B}$ changes the sign at QCD phase boundary !

- In the Language of fluctuation moments:

$$\chi_B = -\frac{1}{V}\frac{\partial^2 \Omega}{\partial \mu_B^2} = \frac{\langle (\delta N_B)^2 \rangle}{VT}$$

$$\frac{\partial \chi_B}{\partial \mu_B} = -\frac{1}{V}\frac{\partial^3 \Omega}{\partial \mu_B^3} = \frac{\langle (\delta N_B)^3 \rangle}{VT^2} \equiv m_3(\mathrm{BBB})$$

more information than usual fluctuation

(Hopefully) More Easily Measured Moments

- Third Moment of Electric Charge Fluctuation

$$m_3(\mathrm{QQQ}) \equiv \frac{\langle (\delta N_Q)^3 \rangle}{VT^2} = -\frac{1}{V}\frac{\partial^3 \Omega}{\partial \mu_Q^3}$$

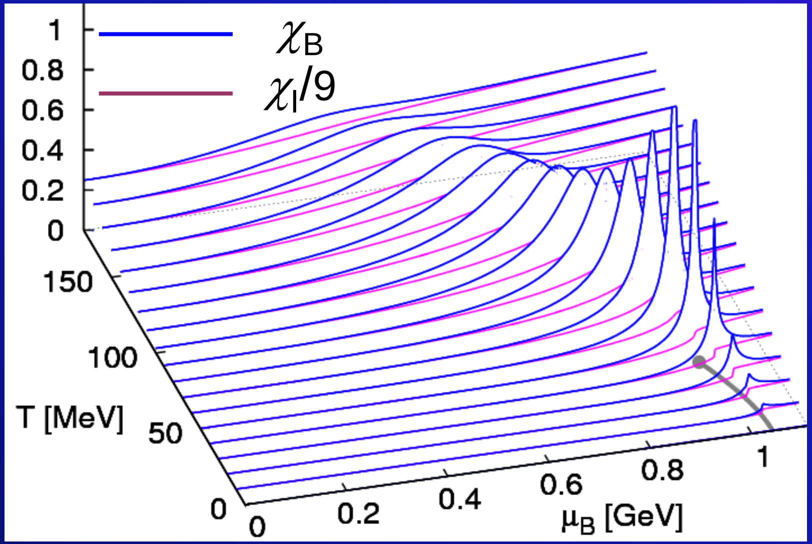
$$N_Q = \frac{2}{3}N_u - \frac{1}{3}N_d = \frac{1}{2}N_B + \frac{1}{2}N_I$$

$$m_3(\mathrm{QQQ}) \equiv \frac{1}{8}\frac{\partial}{\partial \mu_B}\left(\frac{1}{27}\chi_B + \chi_I\right)$$

singular @CEP (for $\frac{1}{27}\chi_B$)

iso-vector susceptibility (for χ_I)
nonsingular when Isospin-symm.

Hatta and Stephanov 2002

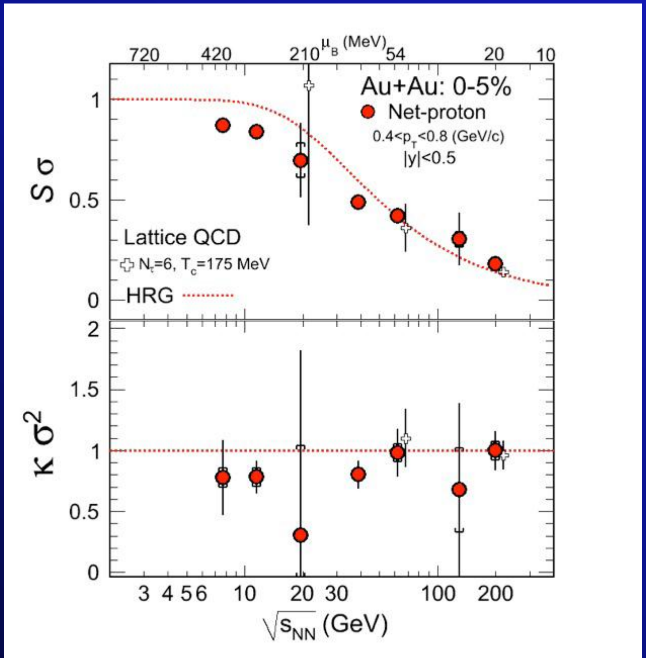


Recent Progress: Proton Cumulants

- Proton Fluctuation has been attracting a lot of interest because it can be observed experimentally
- Proton Fluctuation diverges at CEP

Hatta and Stephanov, 2003

- Comparisons of experimental results and lattice predictions have been done （e.g. Gupta et al., Science 2011)



STAR, QM2011

$$\chi_B^{(n)}\left(\frac{T}{T_c},\frac{\mu_B}{T}\right)=\frac{1}{T^n}\frac{\partial^n}{\partial(\mu_B/T)^n}P\left(\frac{T}{T_c},\frac{\mu_B}{T}\right)\bigg|_{T/T_c}$$

$$S\sigma=\frac{T\chi_B^{(3)}}{\chi_B^{(2)}}$$

$$\kappa\sigma^2=\frac{T^2\chi_B^{(4)}}{\chi_B^{(2)}}$$

Experiment: Net Proton
Theory: Net Baryon

Is this harmless?

Protons and Baryons

The question here is how these are related to each other:

$$\left\langle \left(\delta N_p\right)^n \right\rangle_c \Longleftrightarrow \left\langle \left(\delta N_{\mathrm{B}}\right)^n \right\rangle_c$$

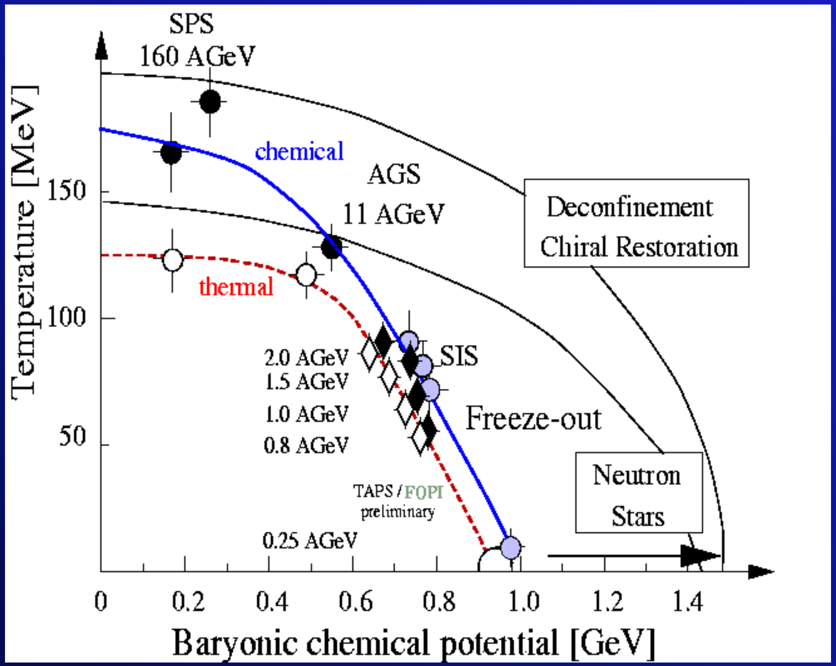
In free nucleon gas in equilibrium,

$$\left\langle \left(\delta N_{\mathrm{B}}\right)^n \right\rangle_c = 2\left\langle \left(\delta N_p\right)^n \right\rangle_c$$

Otherwise, in general,

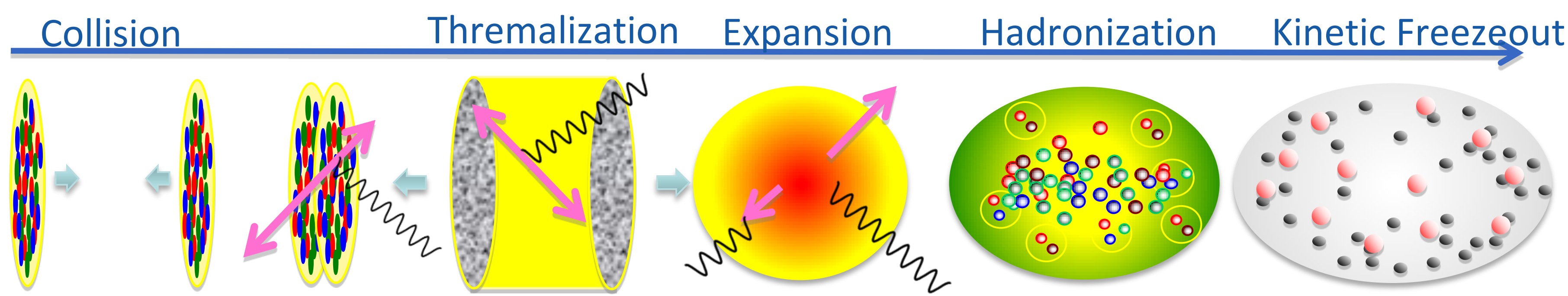
$$\left\langle \left(\delta N_{\mathrm{B}}\right)^n \right\rangle_c \neq 2\left\langle \left(\delta N_p\right)^n \right\rangle_c$$

Freezeouts



- Net proton may be considered as an alternative of net baryon
- Chemical freezeout is close to the crossover, and (anti)proton number is expected to be fixed early (?)

➢ But Not all particle numbers and fluctuations are fixed at chemical freezeout



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Exception

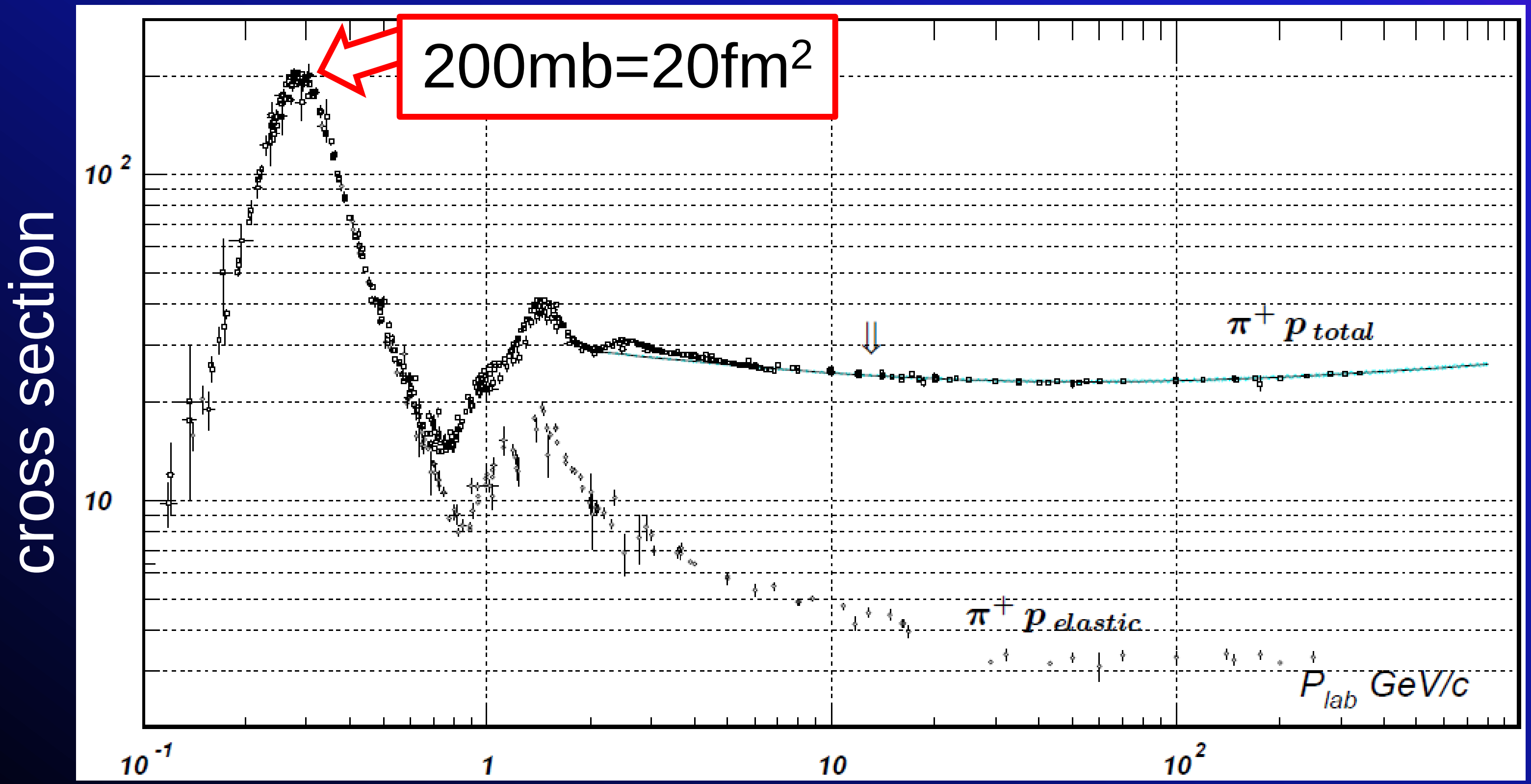
If there are low mass resonances, exception happens

In our case at hand, Δ resonances

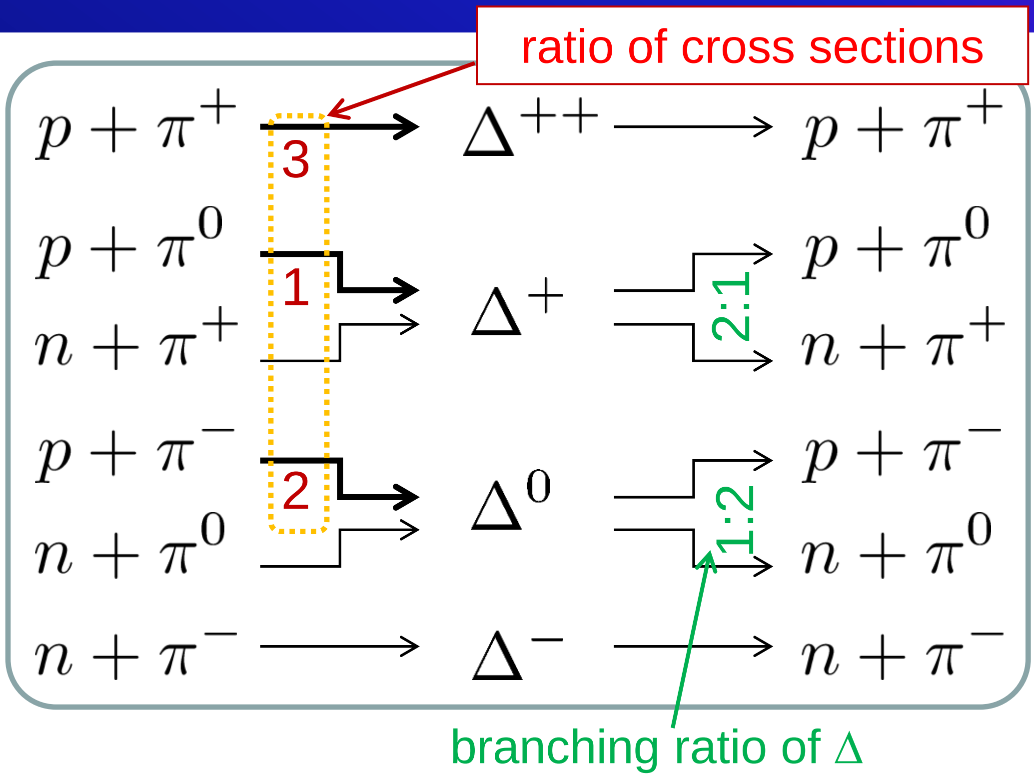
$$p, n \searrow \quad \Delta(1232) \quad \nearrow p, n$$
$$\pi \nearrow \qquad\qquad\quad \searrow \pi$$

$$I = 3/2$$

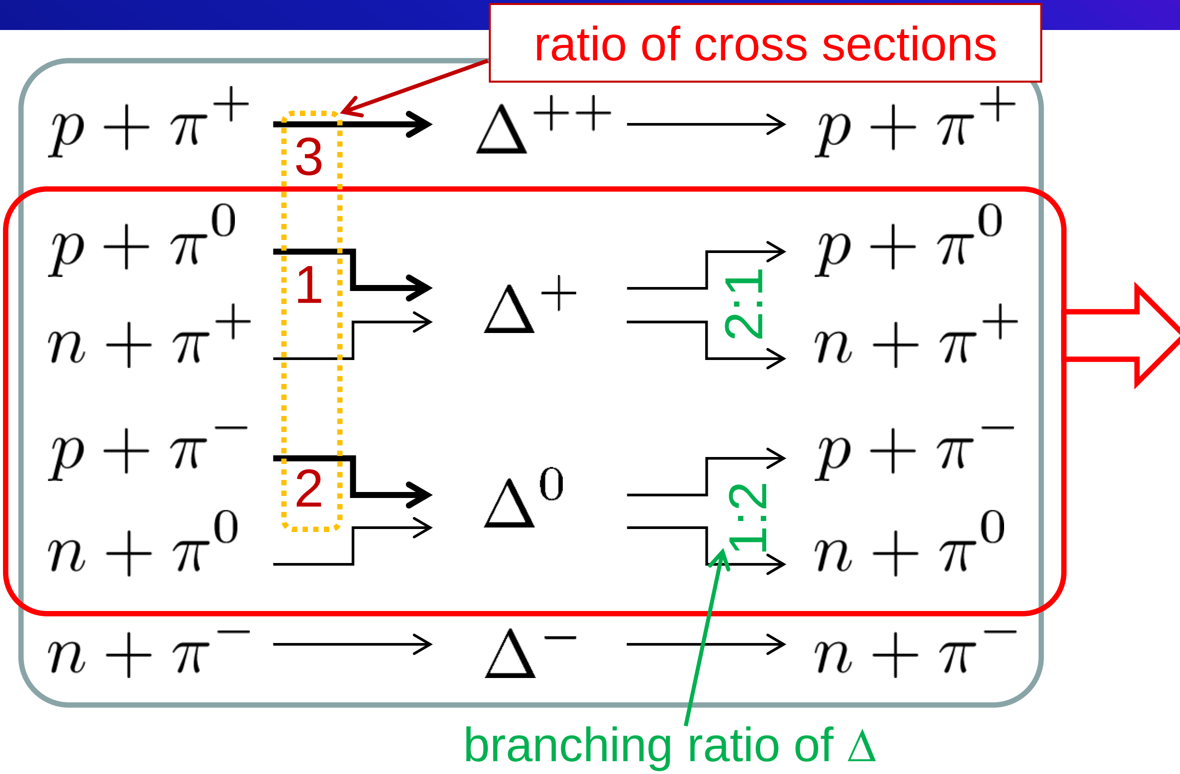
$$\Gamma \simeq 1.8 \text{ [fm]}$$



Effect of Δ


ratio of cross sections
$p+\pi^+$
$p+\pi^0$
$n+\pi^+$
$p+\pi^-$
$n+\pi^0$
$n+\pi^-$
3
1
2
Δ^{++}
Δ^+
Δ^0
Δ^-
2:1
1:2
$p+\pi^+$
$p+\pi^0$
$n+\pi^+$
$p+\pi^-$
$n+\pi^0$
$n+\pi^-$
branching ratio of Δ

Effect of Δ



$$p+\pi \to \Delta^{+,0}$$

$$\to p : n$$

$$= 5 : 4$$

How long is the mean free time?

ratio of cross sections

$$p+\pi^+ \longrightarrow \Delta^{++} \longrightarrow p+\pi^+$$

$$p+\pi^0,\ n+\pi^+ \longrightarrow \Delta^{+} \longrightarrow p+\pi^0,\ n+\pi^+$$

$$p+\pi^-,\ n+\pi^0 \longrightarrow \Delta^{0} \longrightarrow p+\pi^-,\ n+\pi^0$$

$$n+\pi^- \longrightarrow \Delta^{-} \longrightarrow n+\pi^-$$

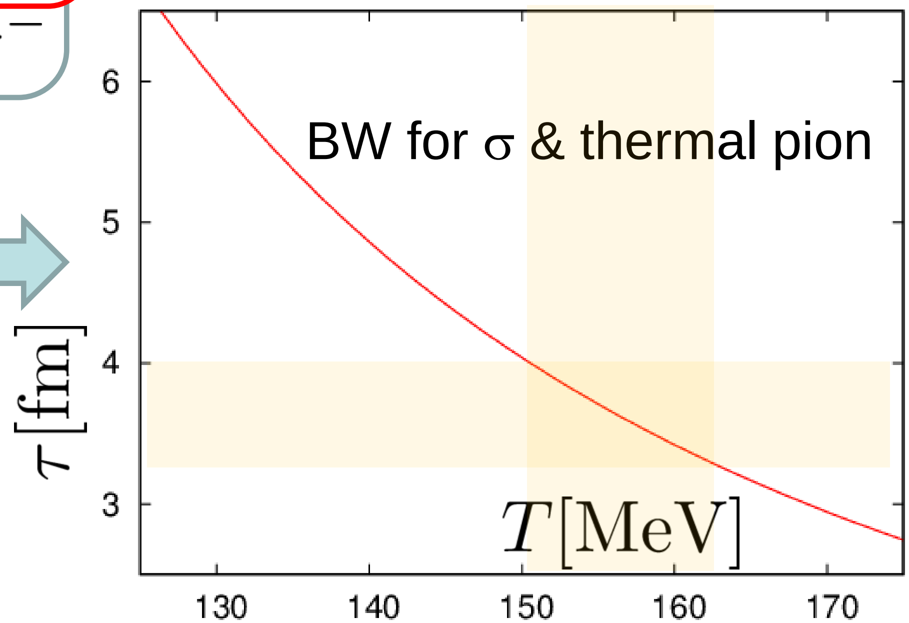
Ratio of cross sections: 3 (Δ^{++}), 1 (Δ^{+}), 2 (Δ^{0})

branching ratio of Δ: 2:1 (Δ^{+}), 1:2 (Δ^{0})

$$p+\pi \to \Delta^{+,0} \to p:n = 5:4$$

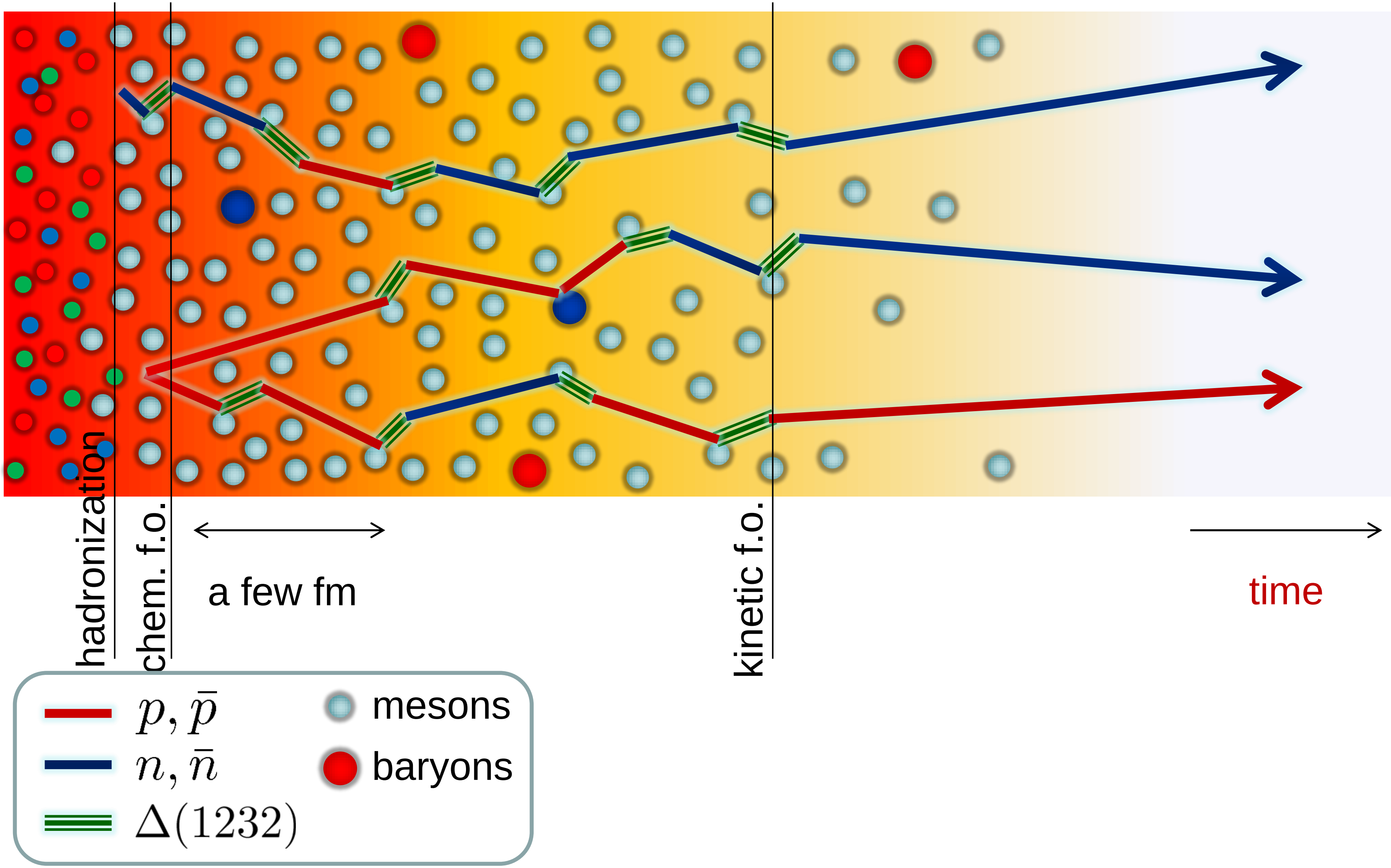
Meantime to create Δ^+ or Δ^0

$$\tau^{-1} = \int \frac{d^3k_\pi}{(2\pi)^3}\sigma(E_{\rm cm})v_\pi n(E_\pi)$$

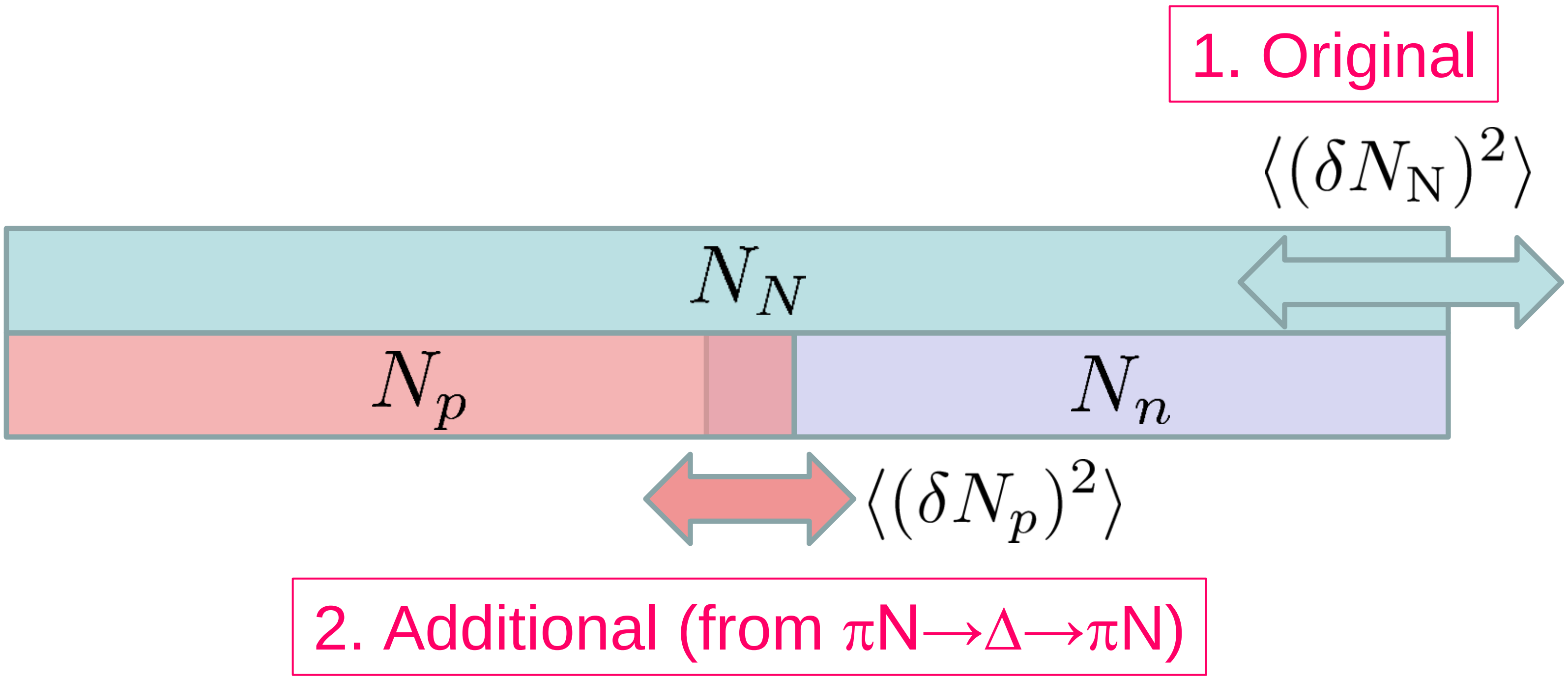


$\tau \leq$ a few fm

Nucleon Isospin Randomization in Pion Gas

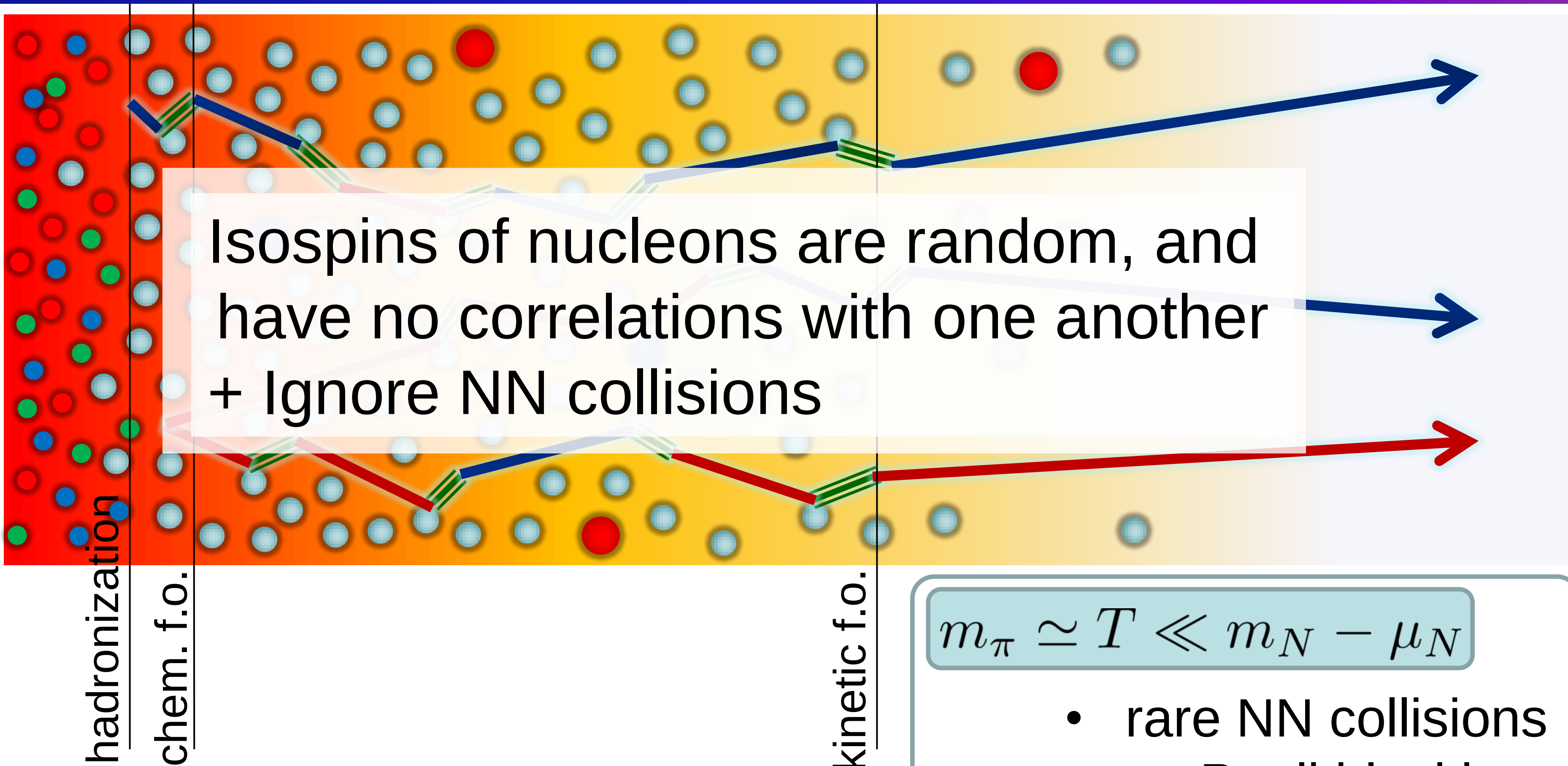


Production of Additional Fluctuation



- In, general, fluctuations of N_N and N_p are different
- Additional N_p fluctuations are created by (thermal) pions

Dilute Nucleon Approximation



Isospins of nucleons are random, and have no correlations with one another
+ Ignore NN collisions

- $p, \bar{p}$
- $n, \bar{n}$
- $\Delta(1232)$
- mesons
- baryons

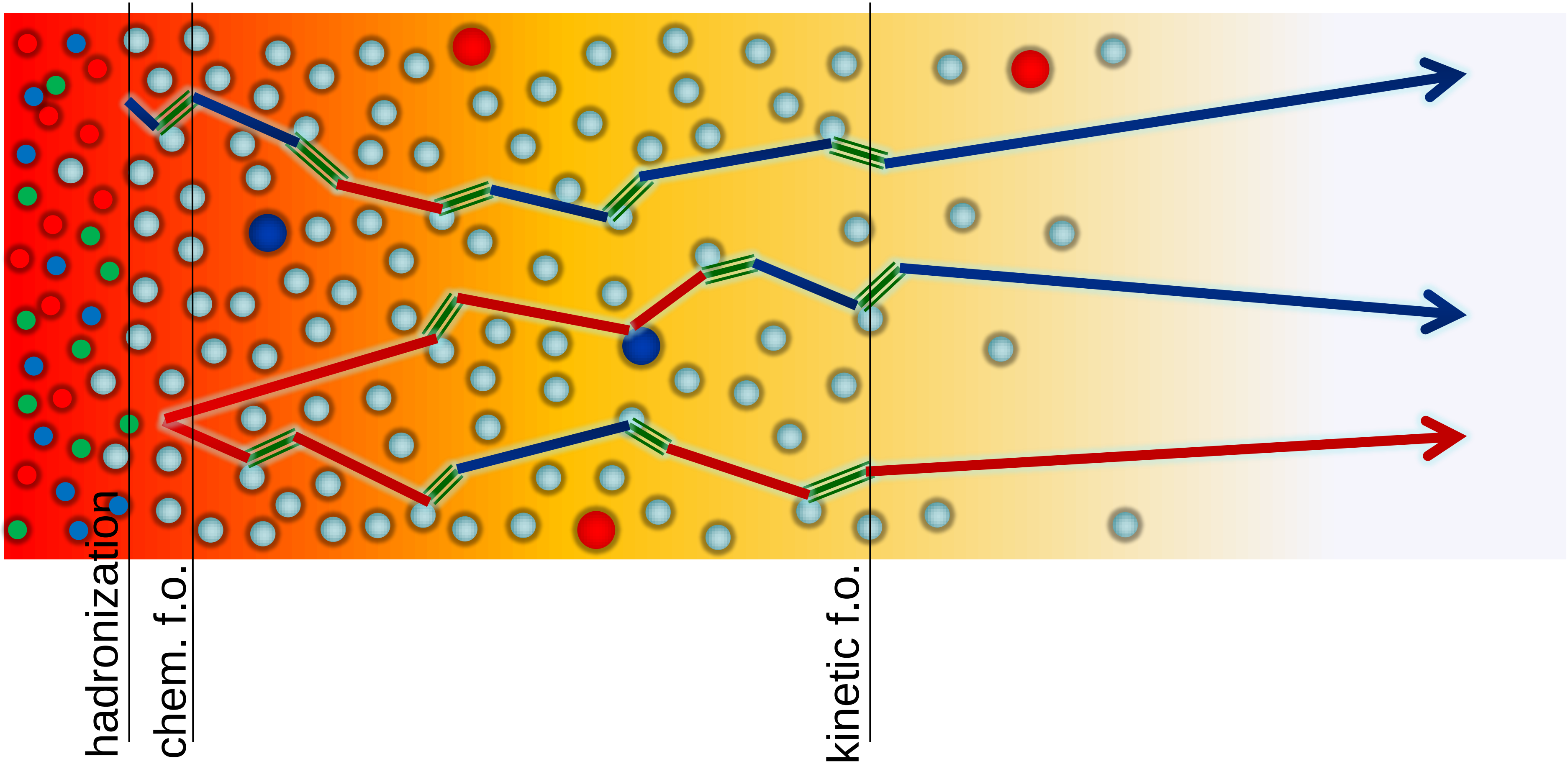
$$m_\pi \simeq T \ll m_N - \mu_N$$

- rare NN collisions
- no Pauli blocking

$$n_N \ll n_\pi$$

- many pions

Probability Distribution



$$P_i(N_p, N_n, N_{\bar{p}}, N_{\bar{n}}) \Rightarrow P_f(N_p, N_n, N_{\bar{p}}, N_{\bar{n}})$$

Probability Distribution

$$P_i(N_p, N_n, N_{\bar{p}}, N_{\bar{n}}) = P(N_N^{(\mathrm{net})}, N_N^{(\mathrm{tot})}, N_p, N_{\bar{p}})$$

$$N_{\mathrm{N}}^{(\mathrm{net})} = N_p + N_n - N_{\bar{p}} - N_{\bar{n}}$$

$$N_{\mathrm{N}}^{(\mathrm{tot})} = N_p + N_n + N_{\bar{p}} + N_{\bar{n}}$$

In the dilute approximation, $N_N^{(\mathrm{net})}$ and $N_N^{(\mathrm{tot})}$ are conserved, i.e.
$N_p + N_n$ and $N_{\bar{p}} + N_{\bar{n}}$ are conserved separately

When $N_p + N_n \equiv N_N$ and $N_{\bar{p}} + N_{\bar{n}} \equiv N_{\bar{N}}$ are fixed
and hadron phase is long enough compared to the mean free time of (anti)nucleons, the final state (anti)proton distribution is given by the binomial distribution

$$B(N_p; N_N)\left(B(N_{\bar{p}}; N_{\bar{N}})\right)$$

Probability Distribution

- As a result, the final state distribution is factorized as follows:

$$P_f(N_p, N_n, N_{\bar{p}}, N_{\bar{n}}) = F(N_N^{(\mathrm{net})}, N_N^{(\mathrm{tot})}) B(N_N; N_p) B(N_{\bar{N}}; N_{\bar{p}})$$

$$F(N_N^{(\mathrm{net})}, N_N^{(\mathrm{tot})}) = \sum_{N_p, N_{\bar{p}}} P(N_N^{(\mathrm{net})}, N_N^{(\mathrm{tot})}, N_p, N_{\bar{p}})$$

$$P_i(N_p, N_n, N_{\bar{p}}, N_{\bar{n}}) = P(N_N^{(\mathrm{net})}, N_N^{(\mathrm{tot})}, N_p, N_{\bar{p}})$$

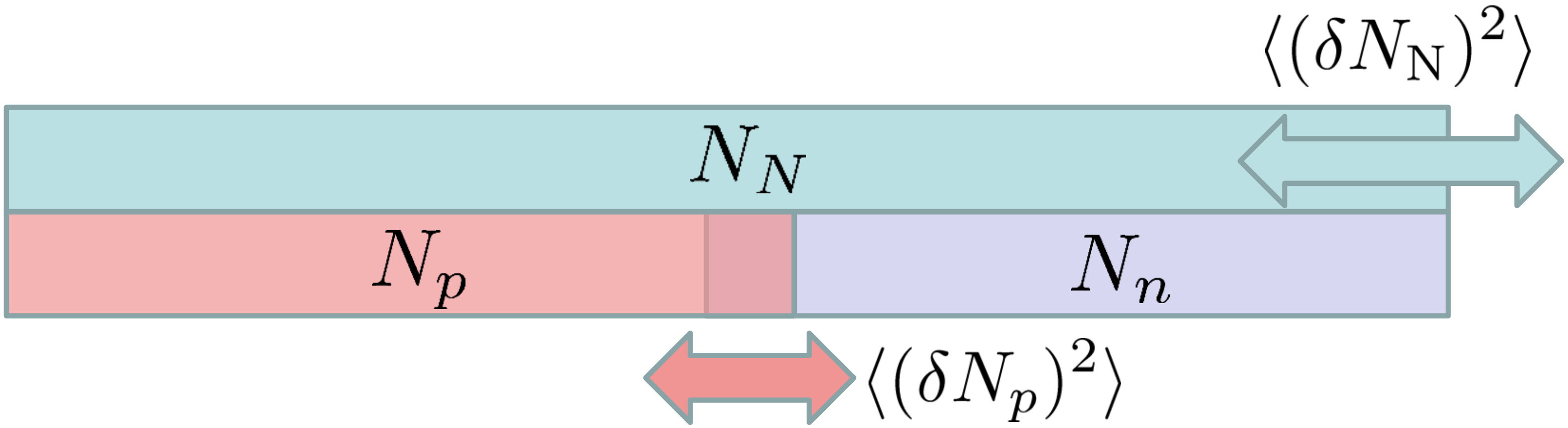
$$= P'(N_N, N_{\bar{N}}, N_p, N_{\bar{p}})$$

The two variable function "F" includes the initial information (correlation)

This form of P_f enables to relate proton moments and nucleon moments

Proton and Nucleon Moments

1. Original



2. Additional (from πN→Δ→πN)

$$\left\langle \left(\delta N_p^{(\mathrm{net})} \right)^2 \right\rangle = \frac{1}{4} \left\langle \left(\delta N_N^{(\mathrm{net})} \right)^2 \right\rangle + \frac{1}{4} \left\langle N_N^{(\mathrm{tot})} \right\rangle$$

$$\left\langle \left(\delta N_N^{(\mathrm{net})} \right)^2 \right\rangle = 4 \left\langle \left(\delta N_p^{(\mathrm{net})} \right)^2 \right\rangle - 2 \left\langle N_p^{(\mathrm{tot})} \right\rangle$$

for isospin symmetric matter

- For free nucleon gas

$$\left\langle \left(\delta N_p^{(\mathrm{net})} \right)^2 \right\rangle = \frac{1}{2} \left\langle \left(\delta N_N^{(\mathrm{net})} \right)^2 \right\rangle$$

Proton and Nucleon Moments

Similarly,

$$N_{\mathrm{B}} \longrightarrow N_p$$

$$\left\langle \left(\delta N_p^{(\mathrm{net})}\right)^3 \right\rangle = \frac{1}{8}\left\langle \left(\delta N_{\mathrm{B}}^{(\mathrm{net})}\right)^3 \right\rangle + \frac{3}{8}\left\langle \delta N_{\mathrm{B}}^{(\mathrm{net})} \delta N_{\mathrm{B}}^{(\mathrm{tot})} \right\rangle$$

$$\left\langle \left(\delta N_p^{(\mathrm{net})}\right)^4 \right\rangle_c = \frac{1}{16}\left\langle \left(\delta N_{\mathrm{B}}^{(\mathrm{net})}\right)^4 \right\rangle_c + \frac{3}{8}\left\langle \left(\delta N_{\mathrm{B}}^{(\mathrm{net})}\right)^2 \delta N_{\mathrm{B}}^{(\mathrm{tot})} \right\rangle + \frac{3}{16}\left\langle \left(\delta N_{\mathrm{B}}^{(\mathrm{net})}\right)^2 \right\rangle - \frac{1}{8}\left\langle N_B^{(\mathrm{tot})} \right\rangle$$

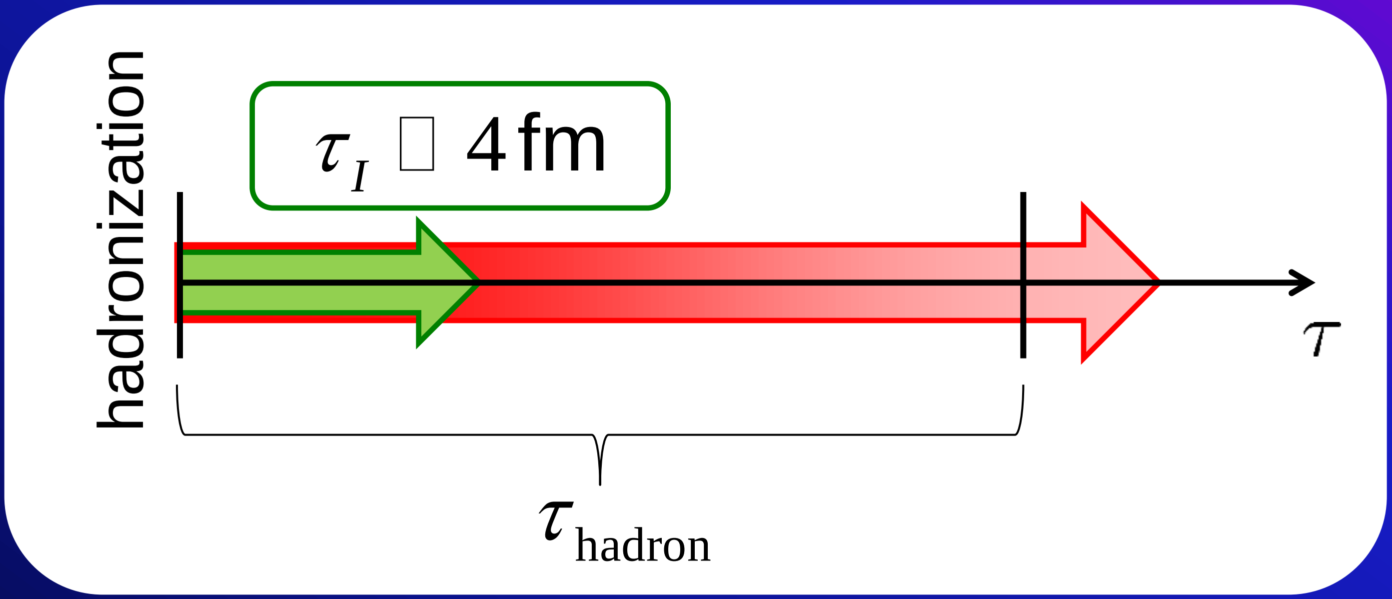
$$N_p \longrightarrow N_{\mathrm{B}}$$

$$\left\langle \left(\delta N_{\mathrm{B}}^{(\mathrm{net})}\right)^3 \right\rangle = 8\left\langle \left(\delta N_p^{(\mathrm{net})}\right)^3 \right\rangle - 12\left\langle \delta N_p^{(\mathrm{net})} \delta N_p^{(\mathrm{tot})} \right\rangle + 6\left\langle N_p^{(\mathrm{net})} \right\rangle$$

$$\left\langle \left(\delta N_{\mathrm{B}}^{(\mathrm{net})}\right)^4 \right\rangle_c = 16\left\langle \left(\delta N_p^{(\mathrm{net})}\right)^4 \right\rangle_c - 48\left\langle \left(\delta N_p^{(\mathrm{net})}\right)^2 \delta N_p^{(\mathrm{tot})} \right\rangle + 48\left\langle \left(\delta N_p^{(\mathrm{net})}\right)^2 \right\rangle + 12\left\langle \left(\delta N_p^{(\mathrm{tot})}\right)^2 \right\rangle - 26\left\langle N_p^{(\mathrm{tot})} \right\rangle$$

$$\left\langle \delta N^4 \right\rangle_c = \left\langle (\delta N)^4 \right\rangle - 3\left\langle (\delta N)^2 \right\rangle^2$$

Time Scales



- τ_I : time scale to realize isospin randomization
- τ_{hadron} : time scale of hadron phase duration

τ_{hadron} ⇐ result of state-of-art hydro + cascade calculation

Result of Hydro+Cascade Calculation

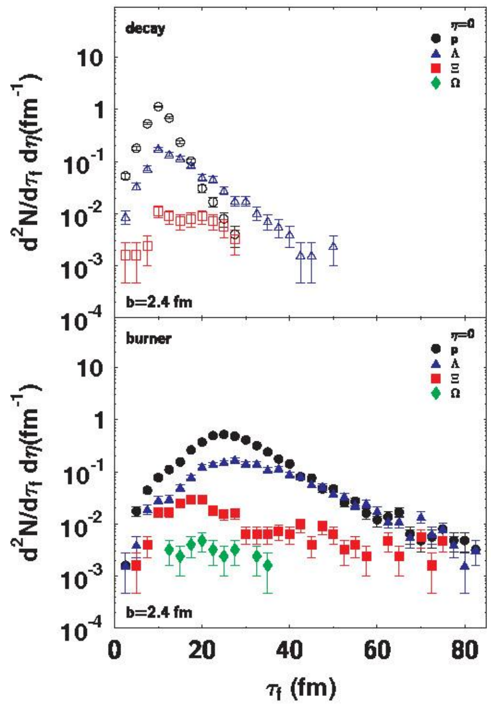


FIG. 22. (Color online) Freeze-out time distribution of baryons for hydro+decay (open symbols, above) and hydro+UrQMD (solid symbols, below) at midrapidity.

providing us with an estimate on the lifetime of the hadronic phase around 10–20 fm/c. Note that this estimate is subject to the same systematic uncertainties discussed previously in the context of the overall lifetime of the system.

Nonaka and Bass, PRC 2007

Freezeout time distribution

without after-burner

with after-burner

τ_{hadron} : 10 ~ 20 fm

$\tau_I \ll \tau_{hadron}$ isospin: randomized

N_N or N_B: Strange Baryon Contribution

Branching Ratios:

Λ $\quad m_\Lambda \simeq 1116[\mathrm{MeV}]$

$$\Rightarrow p : n \simeq 1.6 : 1$$

Σ $\quad m_\Sigma \simeq 1190[\mathrm{MeV}]$

$$\Rightarrow p : n \simeq 1 : 1.8$$

Decay modes:

$$\Lambda \to p + \pi^- \quad 64\%$$
$$\Lambda \to n + \pi^0 \quad 36\%$$

$$\Sigma^+ \to p + \pi^0 \quad 52\%$$
$$\Sigma^+ \to n + \pi^+ \quad 48\%$$

$$\Sigma^0 \to \Lambda \to p + \pi^- \quad 64\%$$
$$\Sigma^0 \to \Lambda \to n + \pi^0 \quad 36\%$$

$$\Sigma^- \longrightarrow n + \pi^-$$

Considering hyperon contribution is small, regard these ratios even,
and incorporate (anti)protons from these decays into the binomial distribution,
then, $N_N \rightarrow N_B$

Summary

■ Conserved Charges and Higher Moments:

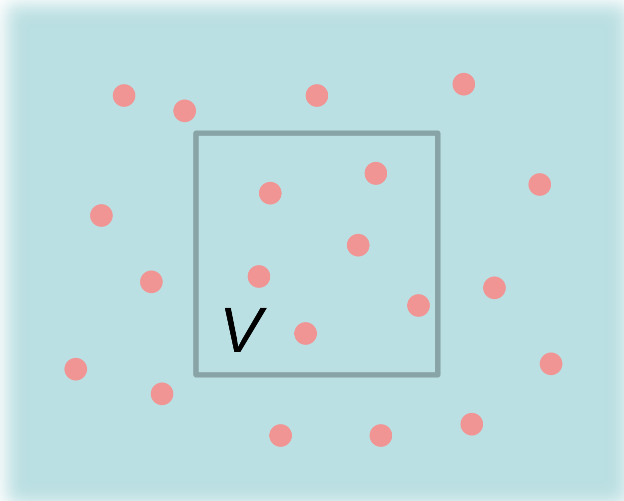
- Third Fluctuation Moments of Conserved Charges take negative values in regions on the FAR SIDE of Phase Transition (more information!)

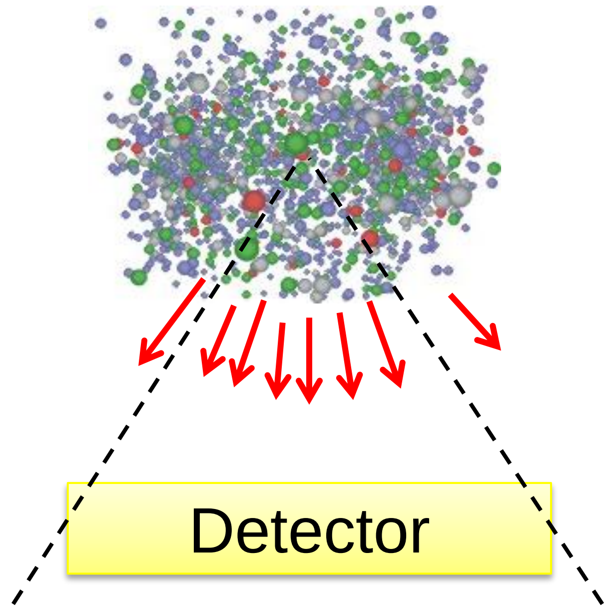
■ Proton Number Cumulants and Baryon Number Ones:

- Proton Number Cumulants are not frozen at chemical freezeout
- Mean Free Time of Protons in hadron phase is very short owing to Δ formation and isospin is randomized
- Final p, $\bar{p}$, n, $\bar{n}$ distributions are factorized (NOT an assumption!)
- This makes it possible to relate the initial baryon number cumulants and final proton number cumulants, and vice versa
- Extension to isospin nonsymmetric case is straightforward, and in progress

Fluctuations in Heavy Ion Collisions

- Observables in equilibrium is fluctuating

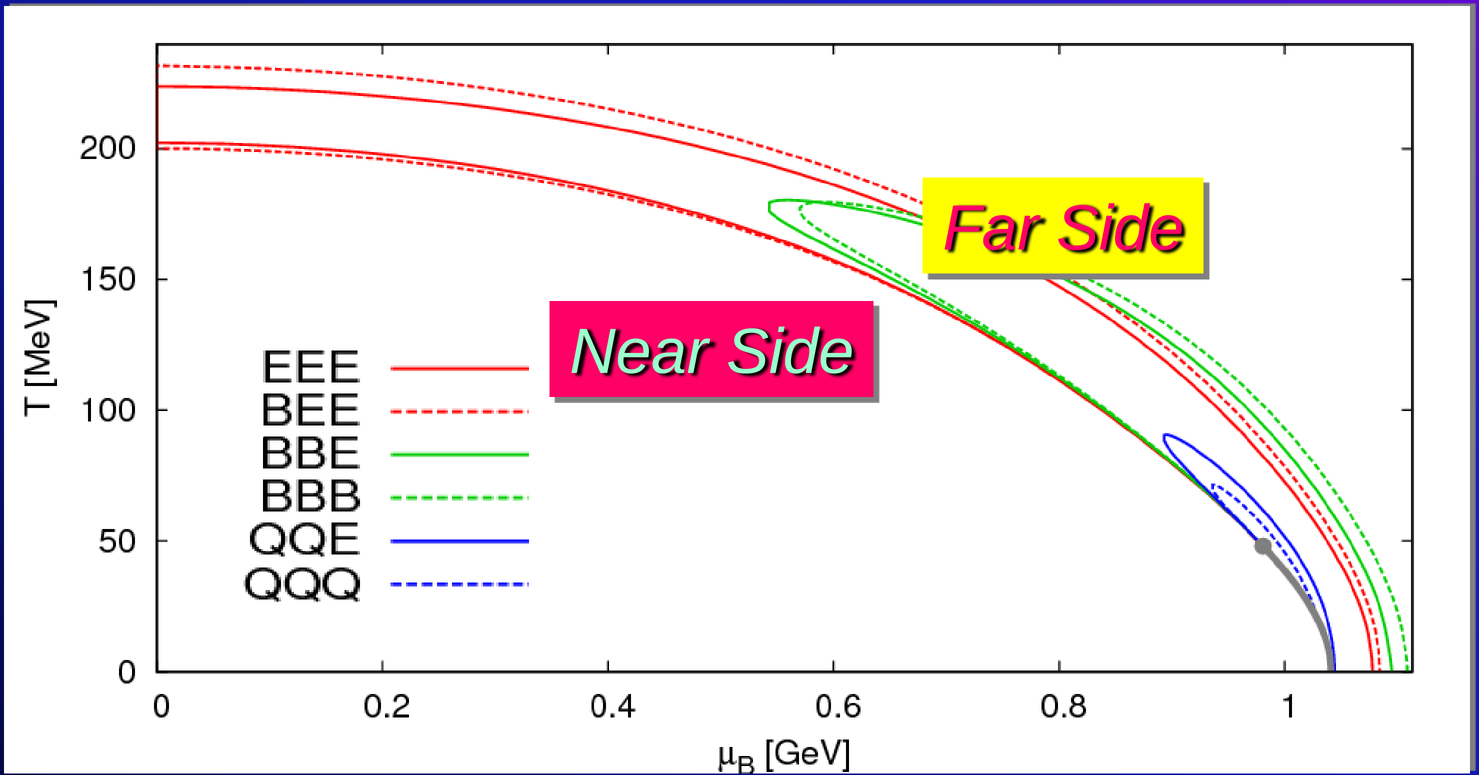




- Momentum and Space correlation is essential

- Fluctuations reflect properties of matter.
  - Enhancement near the critical end point
    Stephanov, Rajagopal, Shuryak (98); Hatta, Stephanov (02); Stephanov (09)
  - Ratios between cumulants of conserved charges
    Asakawa, Heinz, Müller (00); Jeon, Koch (00); Ejiri, Karsch, Redlich(06)
  - Signs of higher order cumulants
    Asakawa, Ejiri, Kitazawa (09); Friman et al. (11); Stephanov (11)

Comparison of Various Moments



2-flavor NJL with standard parameters

G=5.5GeV^{-2}
m_q=5.5MeV
Λ=631MeV

- Different moments have different regions with negative moments

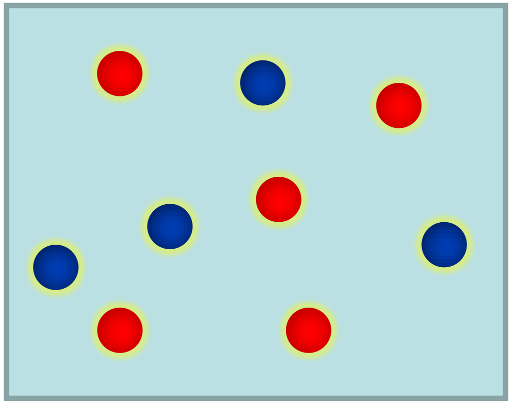
→ By comparing the signs of various moments, possible to pin down the origin of moments

- Negative m_3(EEE) region extends to T-axis (in this particular model)
- Sign of m_3(EEE) may be used to estimate heat conductivity

Free Nucleon Gas Case

$T, \mu_{\rm B} \ll m_{\rm N}$ ⇒ Poisson distribution $P_\lambda(N)$

$$\mathcal{P}(N_p, N_n) = P_\lambda(N_p) P_\lambda(N_n)$$
$$= P_{2\lambda}(N_p + N_n) B_{1/2}(N_p; N_p + N_n)$$



- The factorization is satisfied in free nucleon gas.

$$\mathcal{P}_{\rm free}(N_p, N_n, N_{\bar p}, N_{\bar n}) = \underbrace{P_{\bar N_{\rm N}}(N_{\rm N}) P_{\bar N_{\bar{\rm N}}}(N_{\bar{\rm N}})} B(N_p; N_{\rm N}) B(N_{\bar p}; N_{\bar{\rm N}})$$

$$F(N_{\rm N}, N_{\bar{\rm N}}) = P_{\bar N_{\rm N}}(N_{\rm N}) P_{\bar N_{\bar{\rm N}}}(N_{\bar{\rm N}})$$