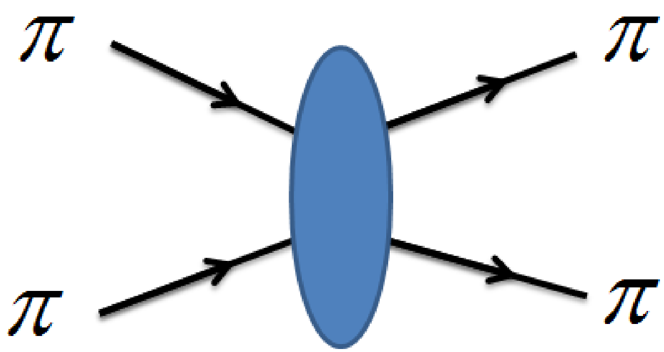
# π-π scattering and BEC-BCS crossover in pion superfluid





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# motivation

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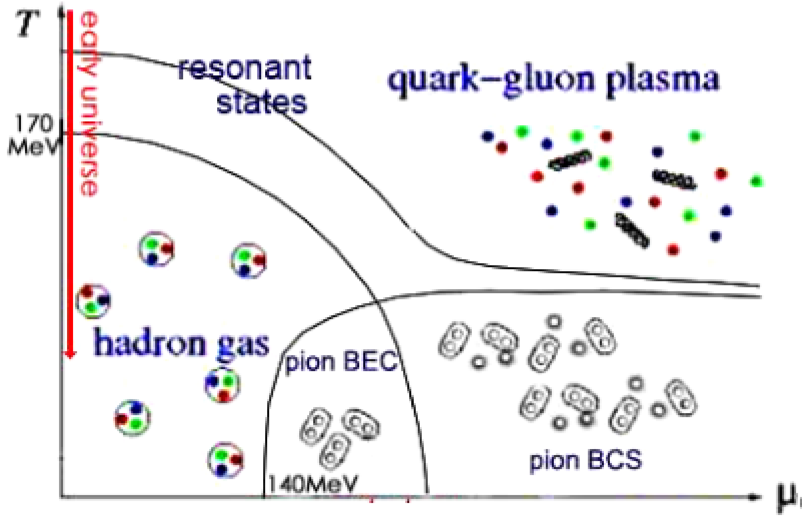
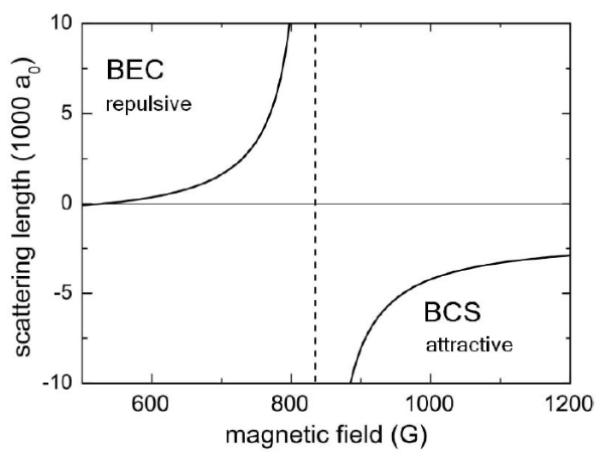



**asymmetric nuclear matter;**
**compact stars;**
**HIC at intermediate energy**

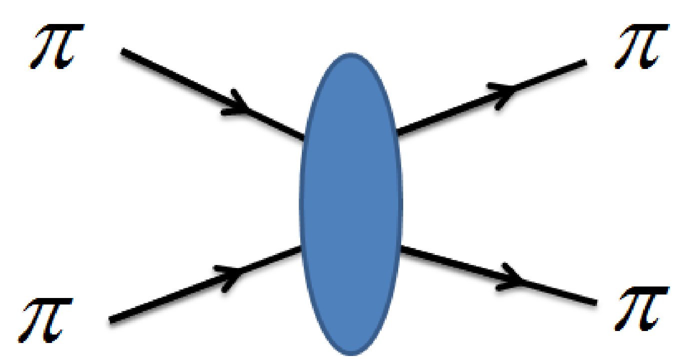
## cold atom:

**fermions scattering length**

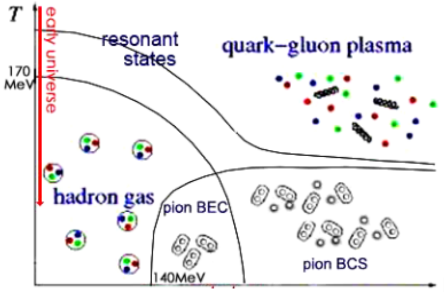



## QCD:

**pion scattering length $a_s$**



# Nambu-Jona-Lasinio model




**Temperature: vacuum excitation;**

**Density: vacuum condensate;**

**electron condensate → NJL model → quark system**

**NJL** ***with isospin symmetry breaking***

$$L_{NJL} = \bar{\psi}\left(i\gamma^{\mu}\partial_{\mu} - m_0 + \mu\gamma_0\right)\psi + G\left[\left(\bar{\psi}\psi\right)^2 + \left(\bar{\psi}i\tau_i\gamma_5\psi\right)^2\right]$$

***chiral and pion condensates***

$$\sigma = \left\langle\bar{\psi}\psi\right\rangle = \sigma_u + \sigma_d, \quad \sigma_u = \left\langle\bar{u}u\right\rangle, \quad \sigma_d = \left\langle\bar{d}d\right\rangle$$

$$\pi_+ = \sqrt{2}\left\langle\bar{u}i\gamma_5 d\right\rangle = \frac{\pi}{\sqrt{2}}e^{i\theta}, \quad \pi_- = \sqrt{2}\left\langle\bar{d}i\gamma_5 u\right\rangle = \frac{\pi}{\sqrt{2}}e^{i\theta}$$

***quark propagator in MF***

$$S^{-1}(p,\vec{q}) = \begin{pmatrix} \gamma^{\mu}p_{\mu} - \vec{\gamma}\cdot\vec{q} + \mu_u\gamma_0 - m & 2iG\pi\gamma_5 \\ 2iG\pi\gamma_5 & \gamma^{\mu}k_{\mu} + \vec{\gamma}\cdot\vec{q} + \mu_d\gamma_0 - m \end{pmatrix} \qquad m = m_0 - 2G\sigma$$

***thermodynamic potential and gap equations:***

$$\Omega = G(\sigma^2 + \pi^2) - \frac{T}{V}\mathrm{Tr}\,\mathrm{Ln}\,S^{-1}$$

$$\frac{\partial\Omega}{\partial\sigma} = 0, \quad \frac{\partial^2\Omega}{\partial\sigma^2} \ge 0, \quad \frac{\partial\Omega}{\partial\pi} = 0, \quad \frac{\partial^2\Omega}{\partial\pi^2} \ge 0$$

# mesons in RPA

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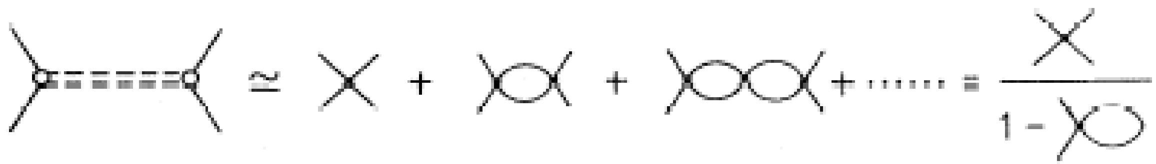
**meson propagator at RPA**



*considering all possible channels in the bubble summation*

$$U(k_0^2,\vec{k}^2)=\Gamma_i^*\left(\frac{2G}{1-2G\Pi(k_0^2,\vec{k}^2)}\right)_{ij}\Gamma_j$$

$$\Gamma_m=\begin{cases}1, & m=\sigma\\ i\tau_+\gamma_5, & m=\pi_+\\ i\tau_-\gamma_5, & m=\pi_-\\ i\tau_3\gamma_5, & m=\pi_0\end{cases};\quad \Gamma_m^*=\begin{cases}1, & m=\sigma\\ i\tau_-\gamma_5, & m=\pi_+\\ i\tau_+\gamma_5, & m=\pi_-\\ i\tau_3\gamma_5, & m=\pi_0\end{cases}$$

**meson polarization functions**

$$\Pi_{mn}(k)=i\int\frac{d^4p}{(2\pi)^4}\mathrm{Tr}\left(\Gamma_m^*S(p+k)\Gamma_nS(p)\right)$$

$$Det\left[1-2G\Pi(k_0^2,0)\right]=0$$

※ *pole of the propagator determines meson masses* $M_m$

※ *residue of the effective interaction at the pole determines quark-meson coupling constant* $g_{mq\bar{q}}$

**pion superfluid phase:**

**Goldstone** $M_{\bar{\pi}_+}=0$

meson eigen-modes $\bar{\sigma},\bar{\pi}_+,\bar{\pi}_-$ are linear combinations of $\sigma,\pi_+,\pi_-$

**resonant state** $M_{\bar{\sigma}}>2M_q \Longrightarrow g_{\bar{\sigma}q\bar{q}}=0$

# pi-pi scattering

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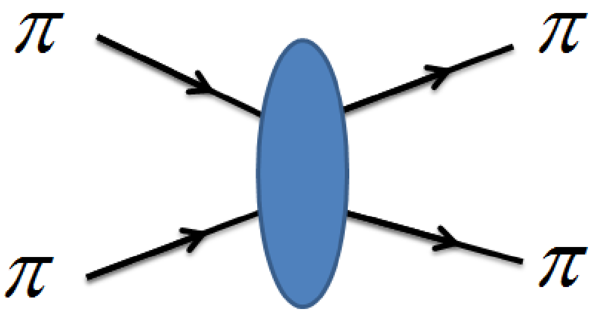



NJL: μ=0,T=0;
a0=0.157;
a2=-0.041;

$$\mathcal{T}_{ab,cd} = A(s,t,u)\delta_{ab}\delta_{cd} + B(s,t,u)\delta_{ac}\delta_{bd} + C(s,t,u)\delta_{ad}\delta_{bc}$$

$$s = (p_a + p_b)^2,\ t = (p_a - p_c)^2 \text{ and } u = (p_a - p_d)^2$$

**In the limit of scattering at threshold:**

scattering lengths $a$ (given in units of $m_\pi^{-1}$) → **dynamical meson mass**

$$a = \frac{1}{32\pi}\mathcal{T} \quad \left(s = 4M_\pi^2, t = u = 0\right)$$

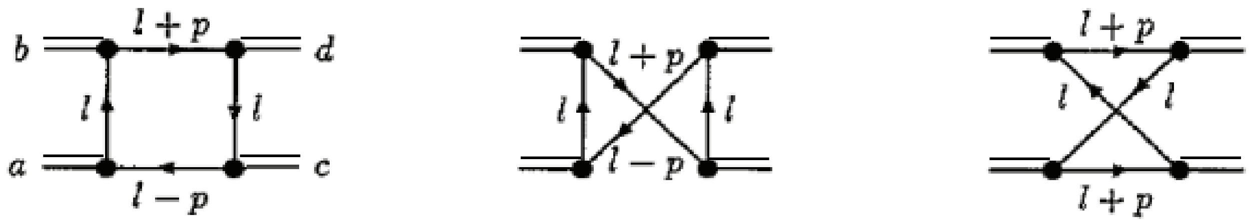
# pi-pi scattering



$T_{ab,cd}$ is calculated from the box and $\sigma$-propagation diagrams,

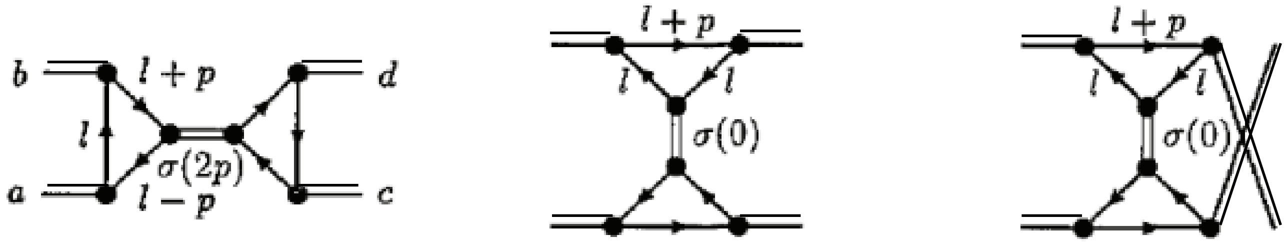
To the lowest order in $1/N_c$,

**box diagram:**




(a) (b) (c)

**σ-propagation diagram:**

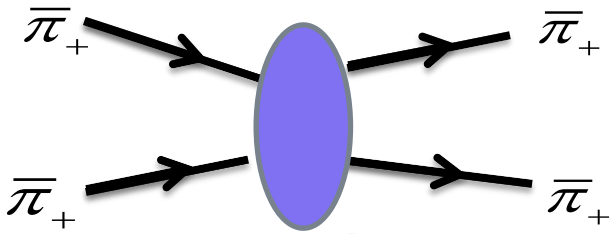



(d) (e) (f)

**NJL: (μ=0,T=0) a0=0.157;a2=-0.041;**
**consistent with experiment data and Weinberg limit.**

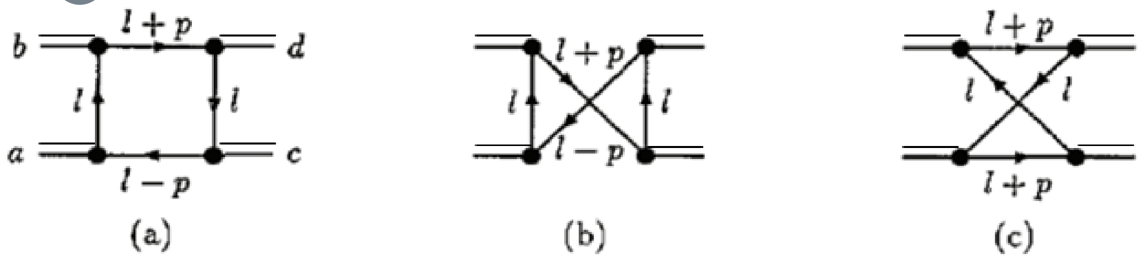
# pi-pi scattering in pion superfluid

**Goldstone mode dominates thermal and dynamic properties of pion superfluid**



$\overline{\pi}_+$ **is Goldstone mode with** $M_{\overline{\pi}_+} = 0$

**box diagram:**




$$iT = -2\int \frac{d^4 l}{(2\pi)^4}\, Tr\left[\gamma_5\tau_2(ig_+)(iS_1)\gamma_5\tau_2(ig_+)(iS_2)\gamma_5\tau_2(ig_+)(iS_3)\gamma_5\tau_2(ig_+)(iS_4)\right]$$

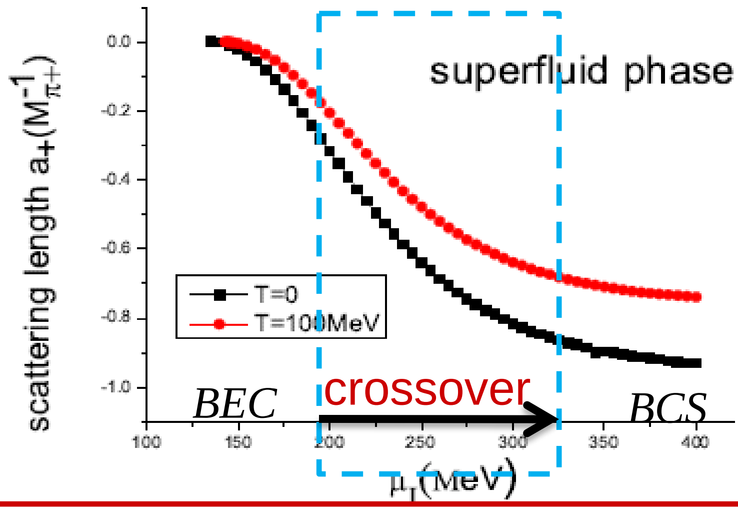
**threshold condition:** $p_\nu = (M_{\overline{\pi}^+} = 0, \vec{p} = \vec{0})$

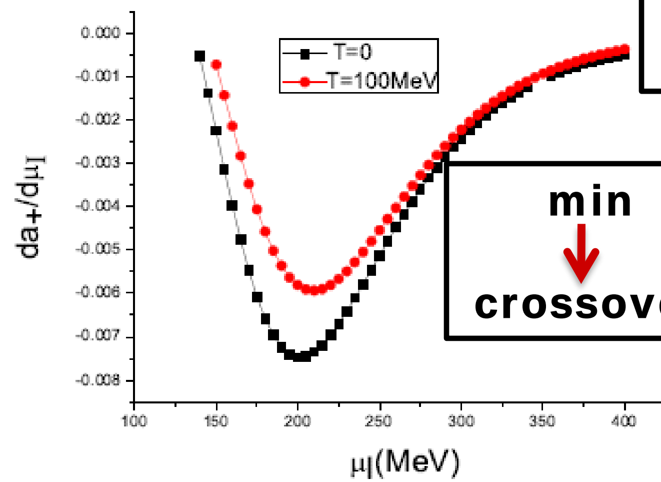
$$T_+ = 3T_a \qquad a = \frac{1}{32\pi} T$$

$$iT_a = -8ig_+^4 N_c \int \frac{d^3 l}{(2\pi)^3}\left[\frac{f'(E_-^+) + f'(-E_+^+)}{(2E^+)^2} - 2\frac{f(E_-^+) - f(-E_+^+)}{(2E^+)^3} + \frac{f'(E_-^-) + f'(-E_+^-)}{(2E^-)^2} - 2\frac{f(E_-^-) - f(-E_+^-)}{(2E^-)^3}\right]$$

# numerical results (I)

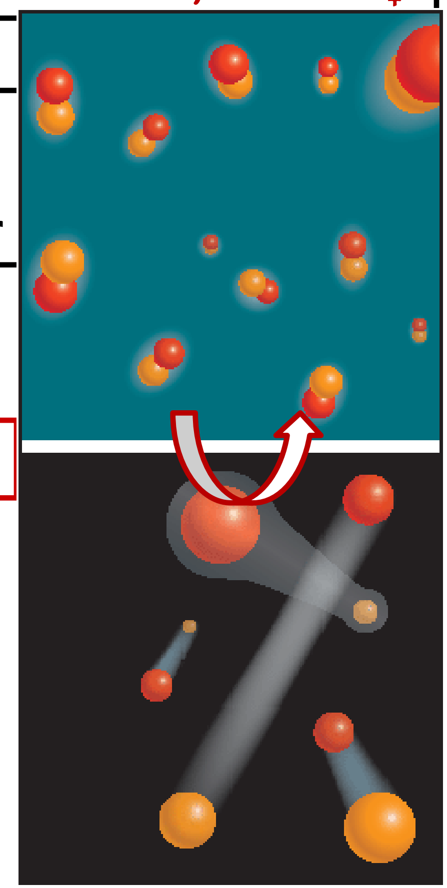
## BEC to BCS crossover in pion superfluid



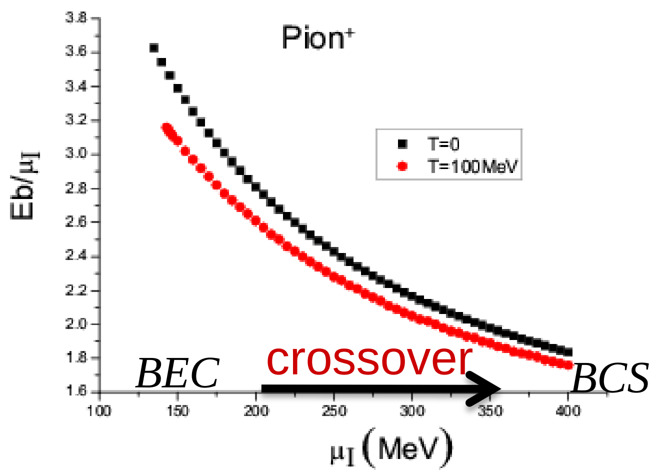



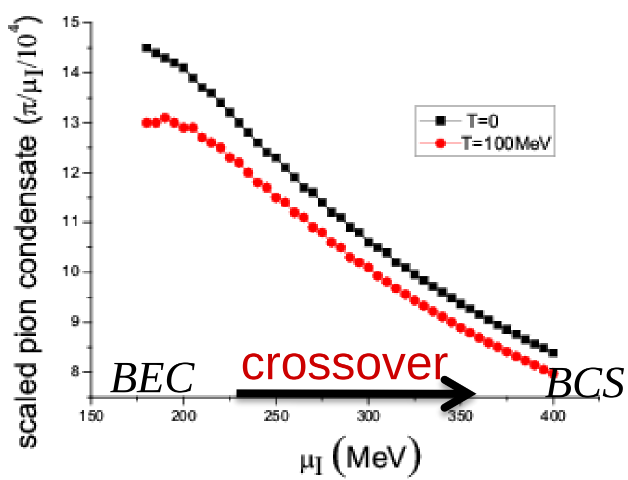

min → crossover

**BEC: separate molecular， small $a_+$**



**$a_+$ is controlled by the competition of coupling and density.**



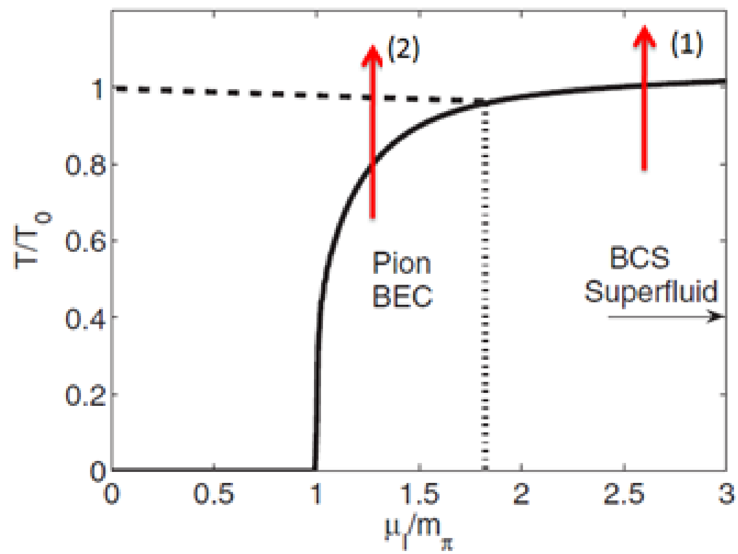




**BCS: overlapped molecular， large $a_+$**

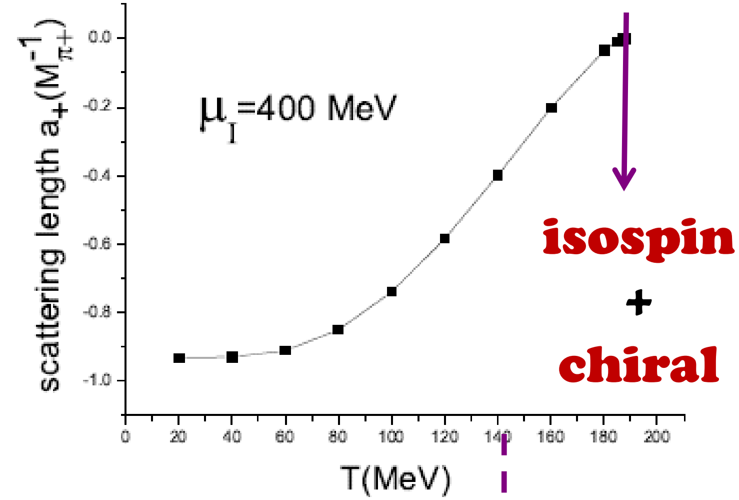
# numerical results (II)



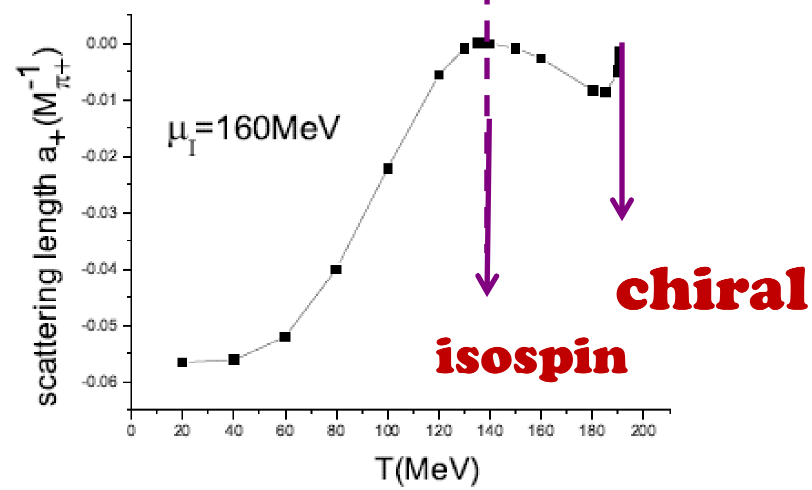
## Where is the strongest coupling between quarks?




(1)




(2)




**Quarks are most strongly coupled at phase transition temperature!!!**

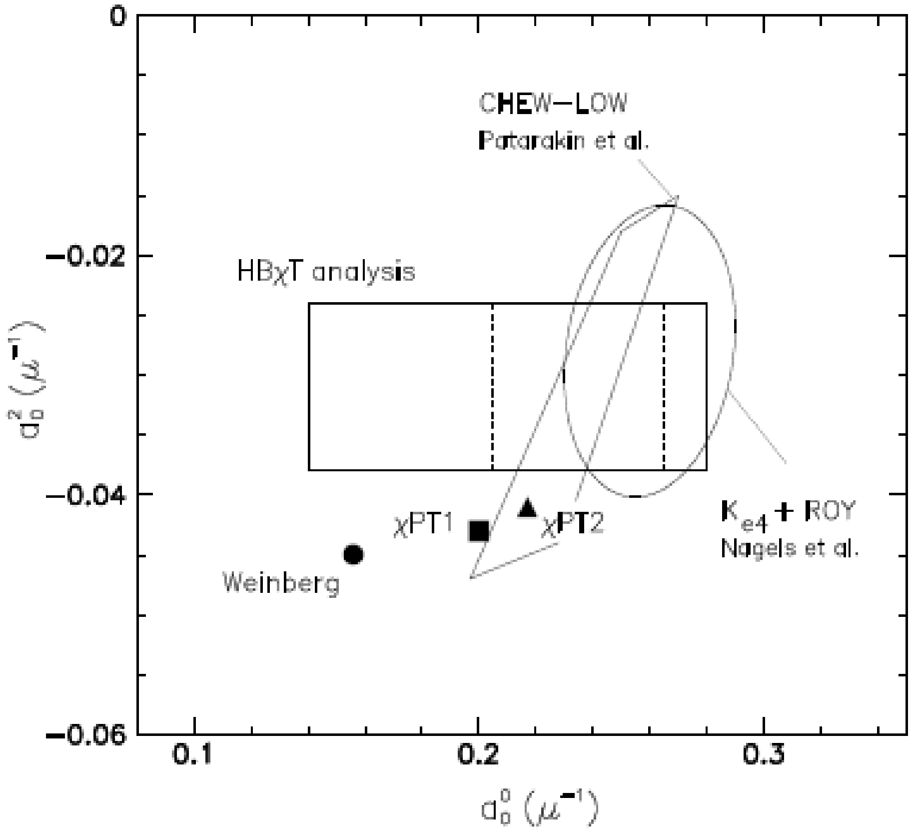
# Conclusion

1. **We calculate the $\pi$-$\pi$ scattering in pion superfluid in frame of SU(2) NJL model.**

2. $\pi$-$\pi$ **scattering provides a direct signal for BEC to BCS crossover in pion superfluid, which is large in BCS and small in BEC.**

3. **At the phase transition line, quarks are most strongly coupled with zero scattering length.**

$\bar{\sigma}$ **is resonant state with** $g_{\bar{\sigma}q\bar{q}} = 0$

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