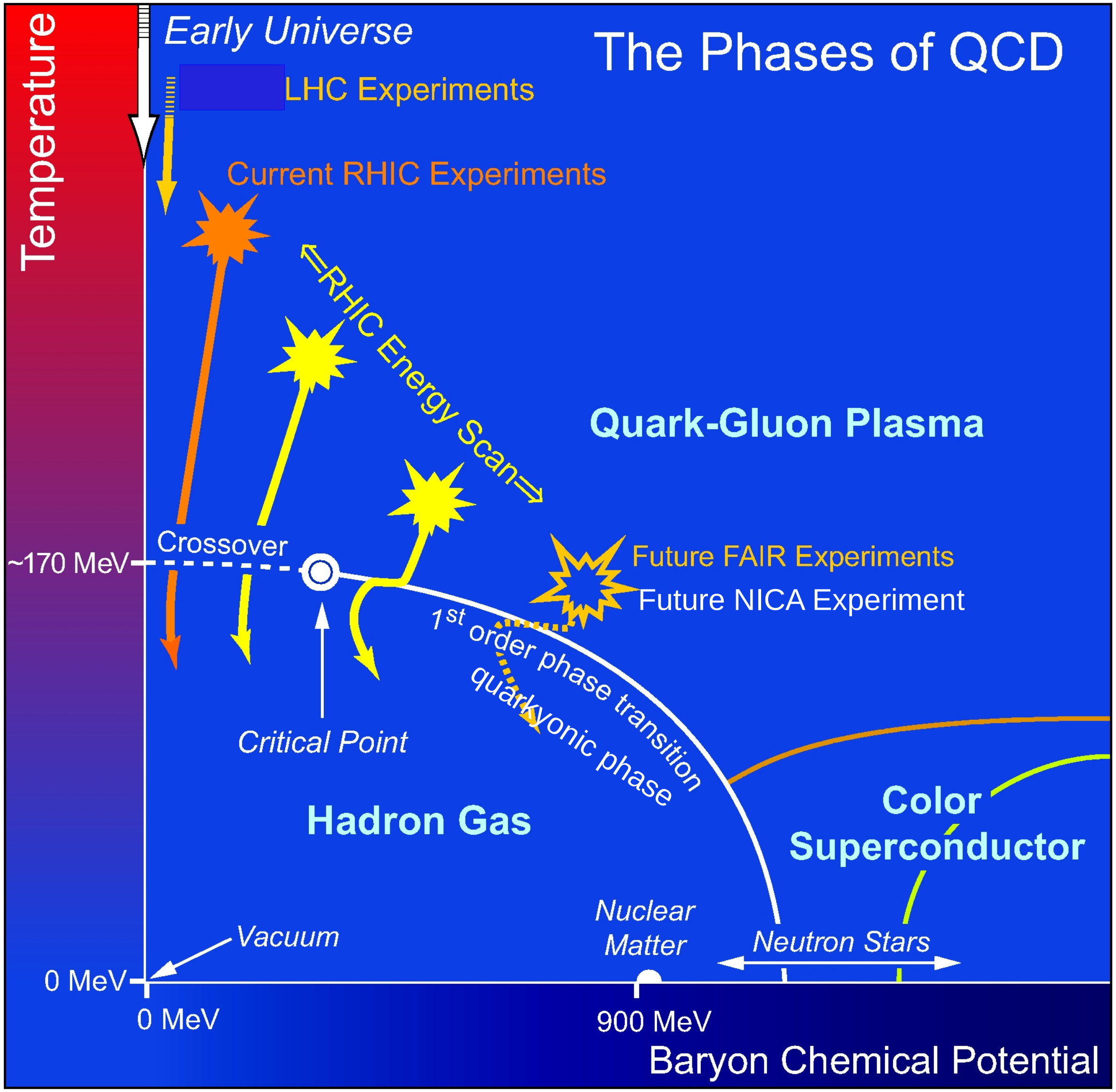
Determination of freeze-out conditions from lattice QCD

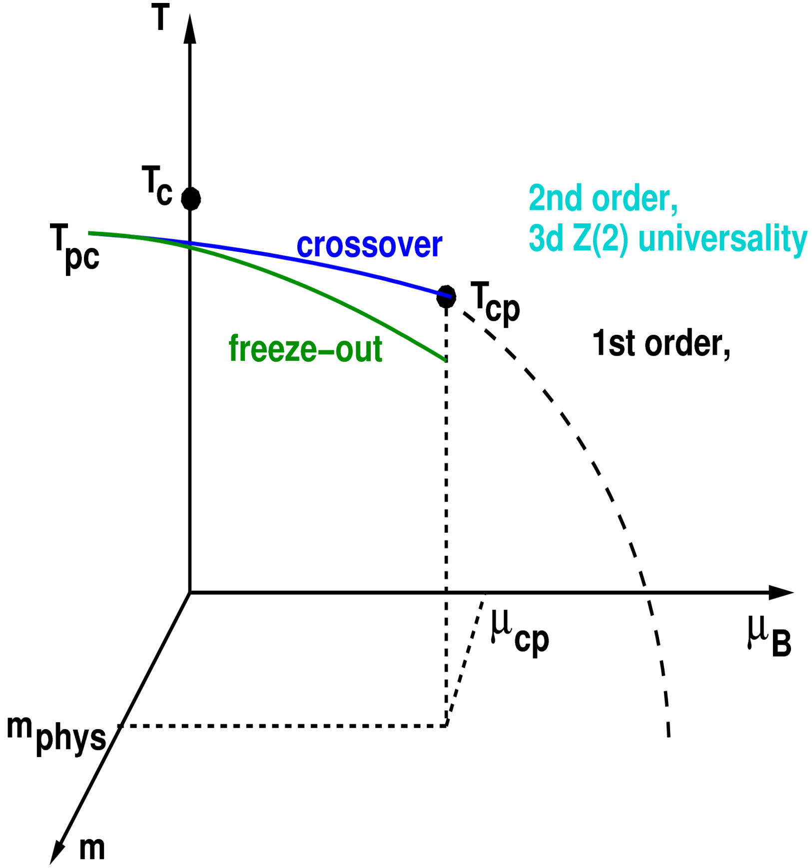
Frithjof Karsch, BNL&Bielefeld



OUTLINE:

- QCD phase diagram close to the chiral limit
- cumulants of charge fluctuations as probe for the proximity to criticality
- freeze-out conditions without using model input

Phase diagram for $\mu_B \geq 0,\ m_q > 0$



- crossover line

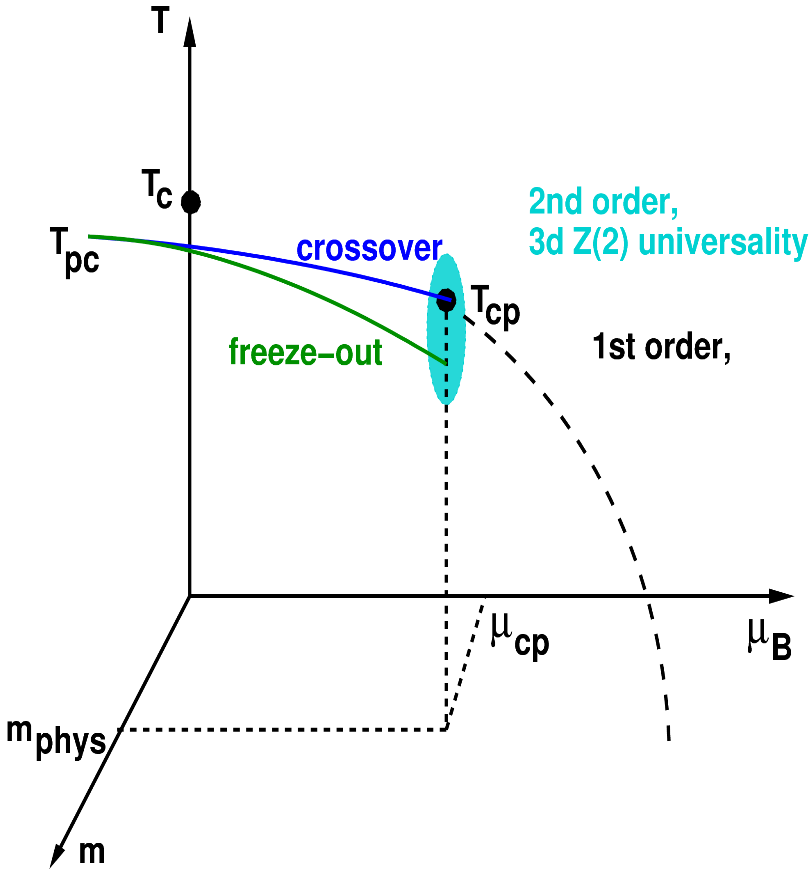
$$\frac{T_c(\mu_B)}{T_c} = 1 - \kappa_{B,m} \left(\frac{\mu_B}{T}\right)^2 - \mathcal{O}(\mu_q^4)$$

- freeze-out line

$$T(\mu_B) = a - b\mu_B^2 - c\mu_B^4$$

Phase diagram for $\mu_B \geq 0,\ m_q > 0$

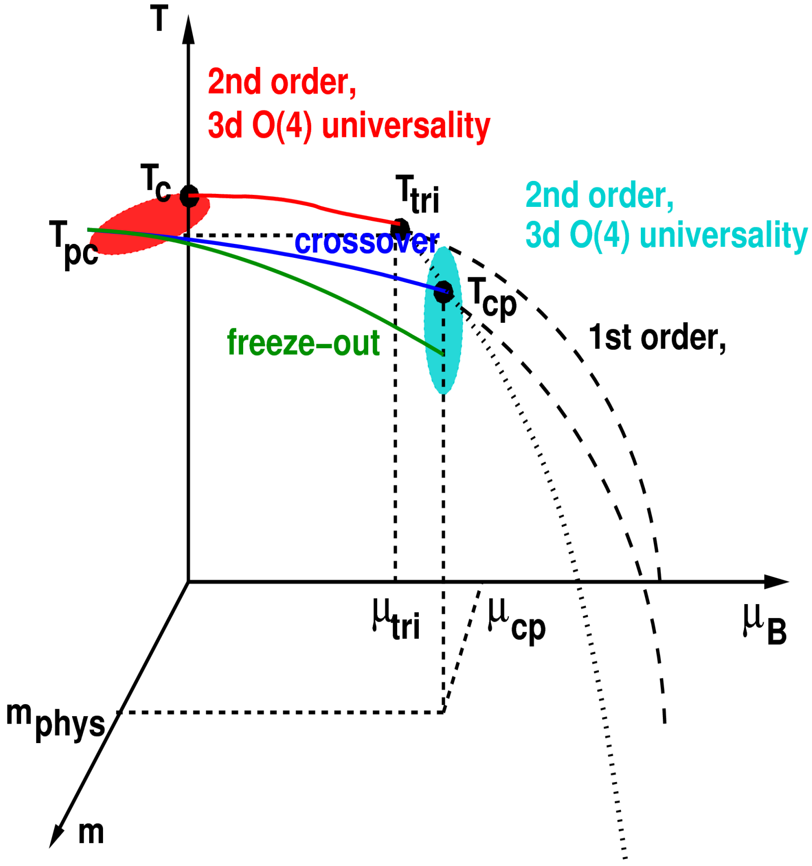
Does freeze-out occur close to a critical point?



- crossover line
  physics on crossover line controlled by universal scaling relations ?
- freeze-out line
  Is the crossover line related to the experimentally determined freeze-out curve?

Phase diagram for $\mu_B \geq 0,\ m_q > 0$

Does freeze-out occur close to a critical point?



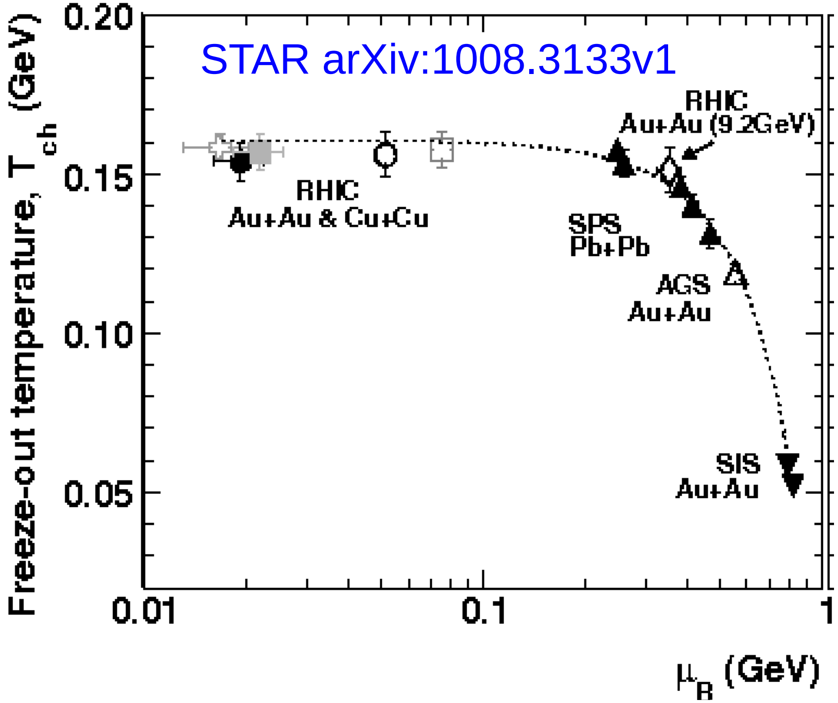
- critical line at $m_q=0$

$$\frac{T_c(\mu_B)}{T_c} = 1 - \kappa_{B,0}\left(\frac{\mu_B}{T}\right)^2 - \mathcal{O}(\mu_B^4)$$

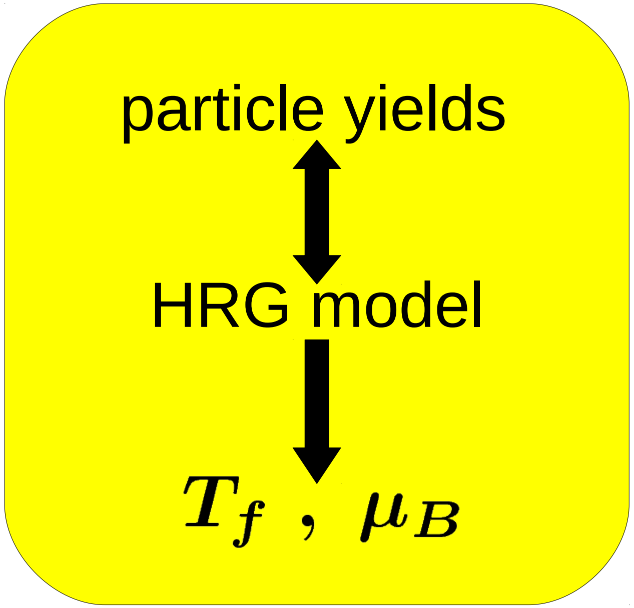
- crossover line
  physics on crossover line controlled by universal scaling relations ?

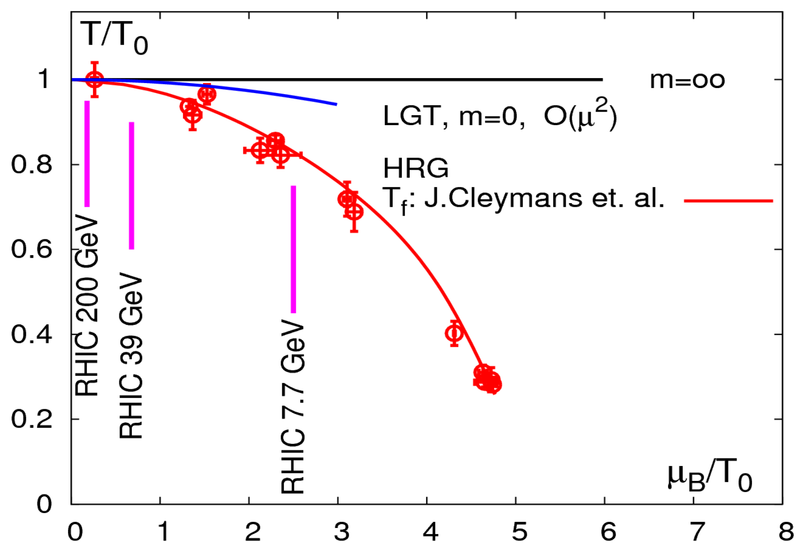
- freeze-out line
  Is the crossover line related to the experimentally determined freeze-out curve?

Freeze-out conditions at RHIC



freeze-out parameter determined in HIC by comparing ratios of hadron yields to HRG model calculations





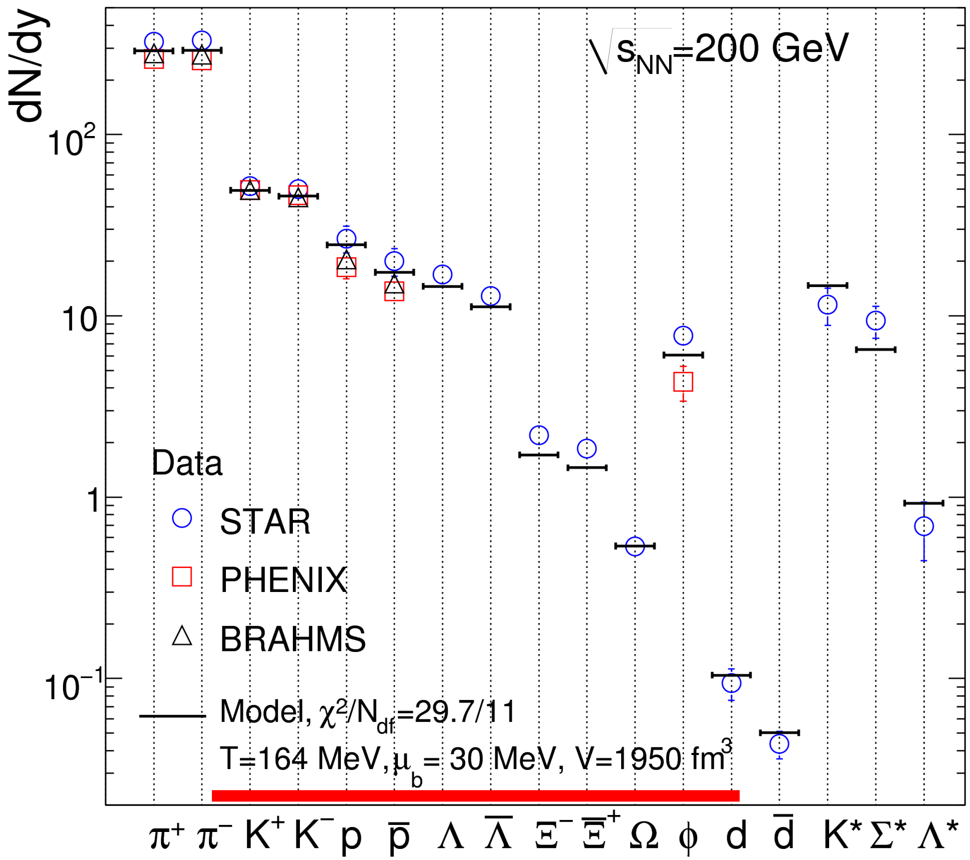
freeze-out line and chiral (phase) transition seem to have different slopes for small μ_B

J. Cleymans et al, PRC73, 034905 (2006)

BNL-Bielefeld-GSI, arXiv:1011.3130

G. Endrodi et al., arXiv:1102.1356

Recall: Determining T_f and μ_B using the HRG model



A. Andronic et al., arXiv:1106.6321[nucl-th]

ratios of hadron yields determine

T_f and μ_B (μ_Q, μ_S) e.g.,

$$\frac{N_B}{N_{\bar{B}}} = e^{2\mu_B/T_f} \Rightarrow \frac{\mu_B}{T_f}$$

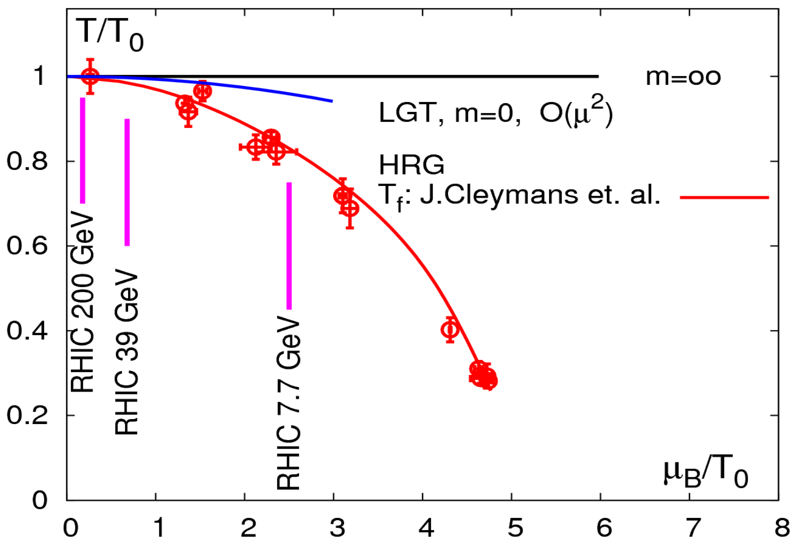
absolute yields (mean values) determine VT^3

$$\frac{\mu_B}{T_f} = 0.183$$

HRG model:

$$\ln \mathcal{Z}^{M/B}_{m_i} = \frac{VT^3}{2\pi^2} d_i \left(\frac{m_i}{T}\right)^2 \sum_{k=1}^{\infty} (\pm 1)^{k+1} \frac{1}{k^2} K_2(km_i/T) \cosh(k\mu_B B_i + ..)$$

The RHIC low energy runs



measure cumulants of charge fluctuations

susceptibilities = cumulants of charge fluctuations

$$\delta N_X = N_X - \langle N_X \rangle$$

$$VT^3 \chi^{(2)}_{X,\mu} = \langle (\delta N_X)^2 \rangle$$

$$VT^3 \chi^{(4)}_{X,\mu} = \langle (\delta N_X)^4 \rangle - 3\langle (\delta N_X)^2 \rangle^2$$

....

generalized susceptibilities in QCD = Taylor expansion coefficients of finite density partition function

$$\frac{p}{T^4} = \sum_{n=0}^{\infty} \frac{1}{n!} \chi^{(n)}_{B,0}(T) \left(\frac{\mu_B}{T}\right)^n$$

$$\chi^{(n)}_{X,0} = \frac{1}{VT^3} \frac{\partial^n \ln Z}{\partial (\mu_X/T)^n}\bigg|_{\mu_X=0} \quad , \; X = B,\ Q,\ S$$

similar: for mixed susceptibilities and expansion for $\mu_S,\ \mu_Q \neq 0$

Consistency check: Baryon number susceptibilities in the HRG

- the pressure in the HRG model

$$\frac{P}{T^4} = \frac{1}{\pi^2}\sum_i d_i \left(\frac{m_i}{T}\right)^2 K_2\left(\frac{m_i}{T}\right) \cosh[(B_i\mu_B + S_i\mu_S + Q_i\mu_Q)/T]$$

$$\mu_S = \mu_Q = 0 :$$

$$\kappa_B\sigma_B^2 = 1$$

$$S_B\sigma_B = \tanh(\mu_B/T)$$

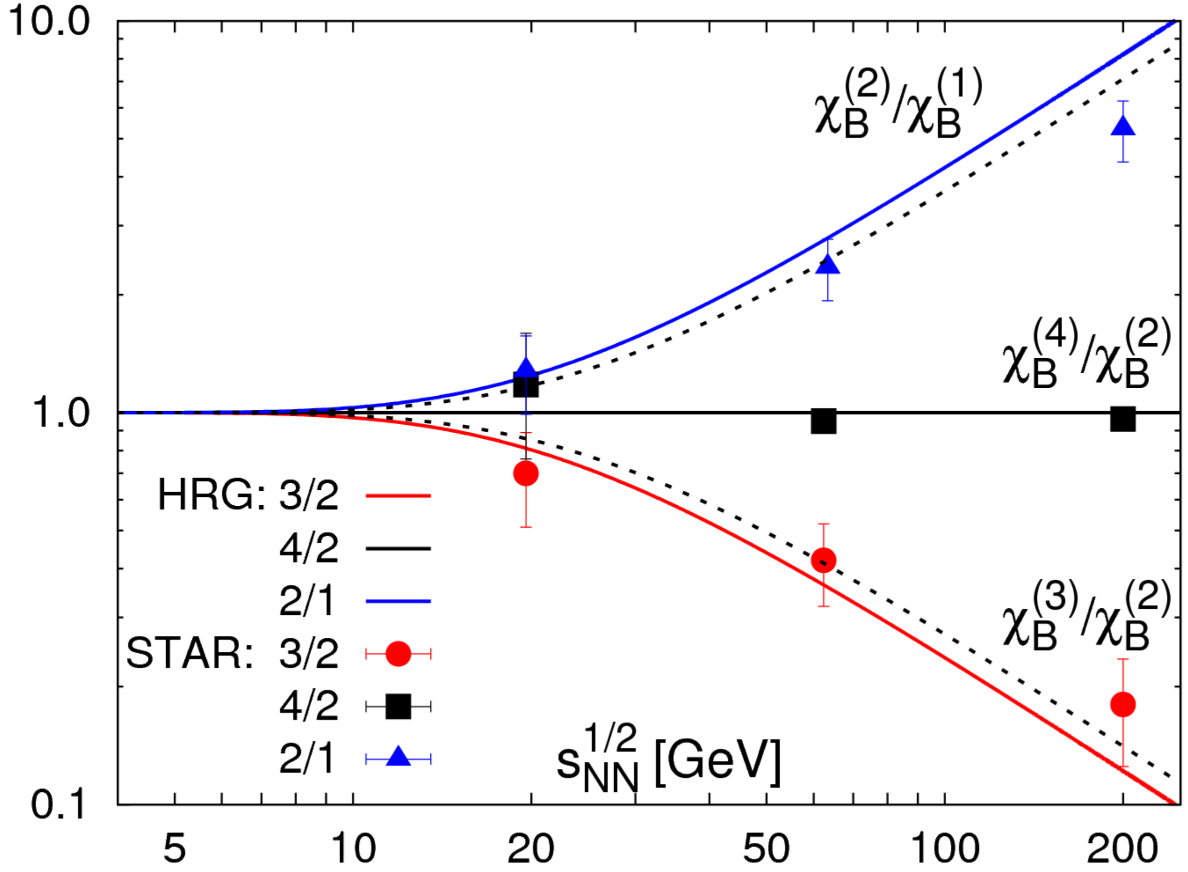
$$\frac{\sigma_B^2}{M_B} = \frac{1}{S_B\sigma_B}$$

FK, K. Redlich, arXiv:1007.2581

other HRG consequence:

$$\frac{S_B\sigma_B^3}{M_B} \equiv \frac{\chi_B^{(3)}}{\chi_B^{(1)}} = 1\,, \qquad \frac{\kappa_B\sigma_B}{S_B} \equiv \frac{\chi_B^{(4)}}{\chi_B^{(3)}} = \coth(\mu_B/T)$$

Cumulants of net-baryon fluctuations



dashed:

$$\mu_Q = \mu_S = 0$$

solid:

$$\mu_Q(\mu_B),\ \mu_S(\mu_B)$$

on freeze-out curve

$\chi_B^{(1)}$ ~ input for V

FK, K Redlich, arXiv:1007.2581
data from STAR: arXiv:1004.4959

charge fluctuations at freeze-out agree 'almost' with HRG model predictions

significant deviations between HRG model and data for the variance ($\chi_B^{(2)}$)?

(talks by K. Redlich, V. Skokov)

Freeze-out parameters from QCD

While hadron yields are "not easy" to obtain from lattice QCD calculations, fluctuations of conserved charges and their higher order cumulants are directly accessible in lattice QCD calculations (in particular for small or vanishing chemical potential)

eventually we should get rid of model calculations and should be able to determine the freeze-out parameters directly by comparing experimental results with (lattice) QCD calculations

assume that you have never heard about the HRG model;
try to determine freeze-out conditions by comparing experiment with QCD

Mean, variance, skewness & kurtosis

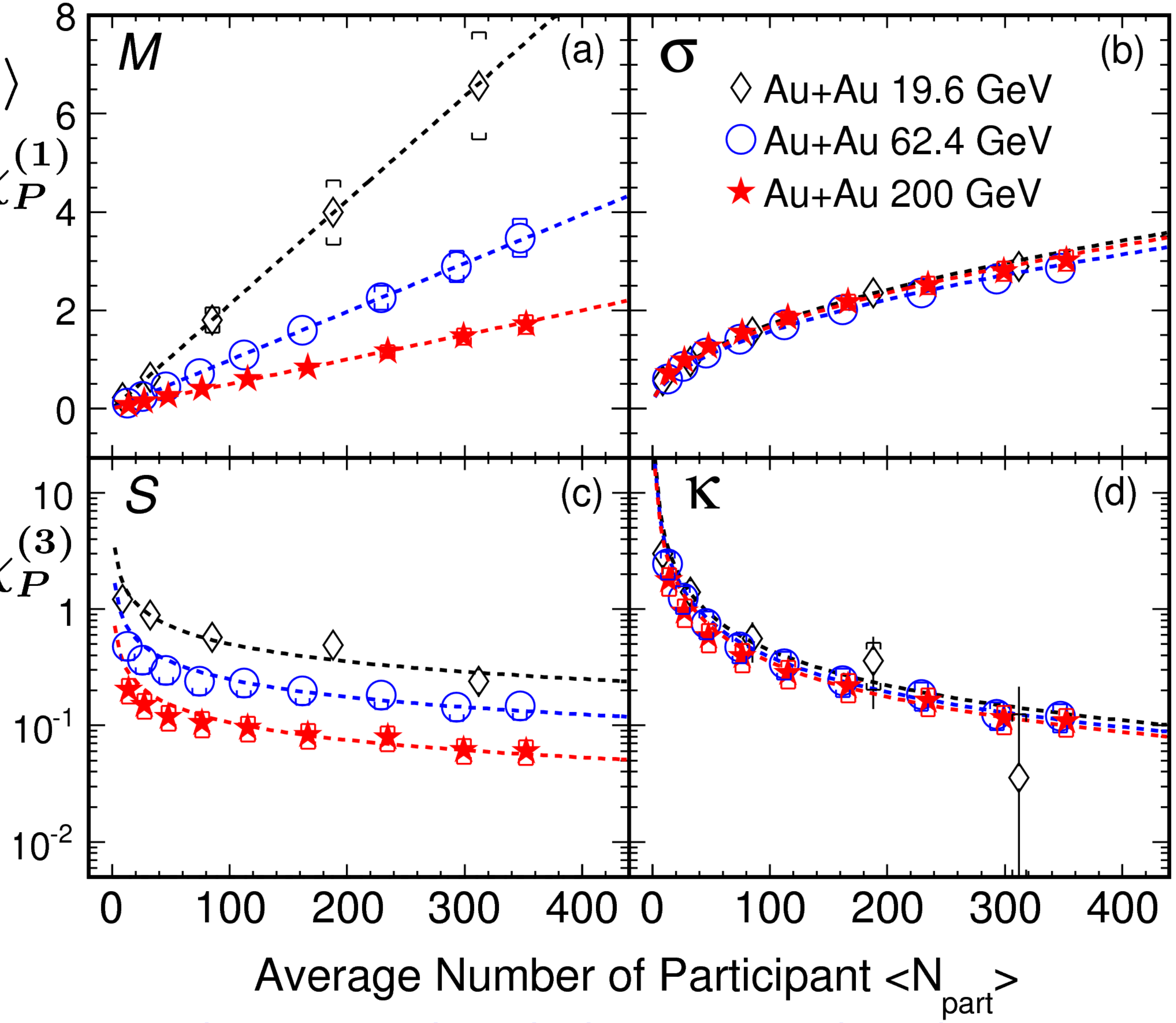
STAR results from RHIC low energy run

$$M = \langle \delta N_P \rangle = VT^3 \chi_P^{(1)}$$

$$\sigma^2 = \langle (\delta N_P)^2 \rangle = VT^3 \chi_P^{(2)}$$

$$S\sigma^3 = VT^3 \chi_P^{(3)}$$

$$\kappa\sigma^4 = VT^3 \chi_P^{(4)}$$



ratios fix $T_f, \mu_B, \ldots$ while absolute values determine VT^3

M. M. Aggarwal et al. (STAR Collaboration), PRL, 105 (2010) 22302

Mean, variance, skewness & kurtosis

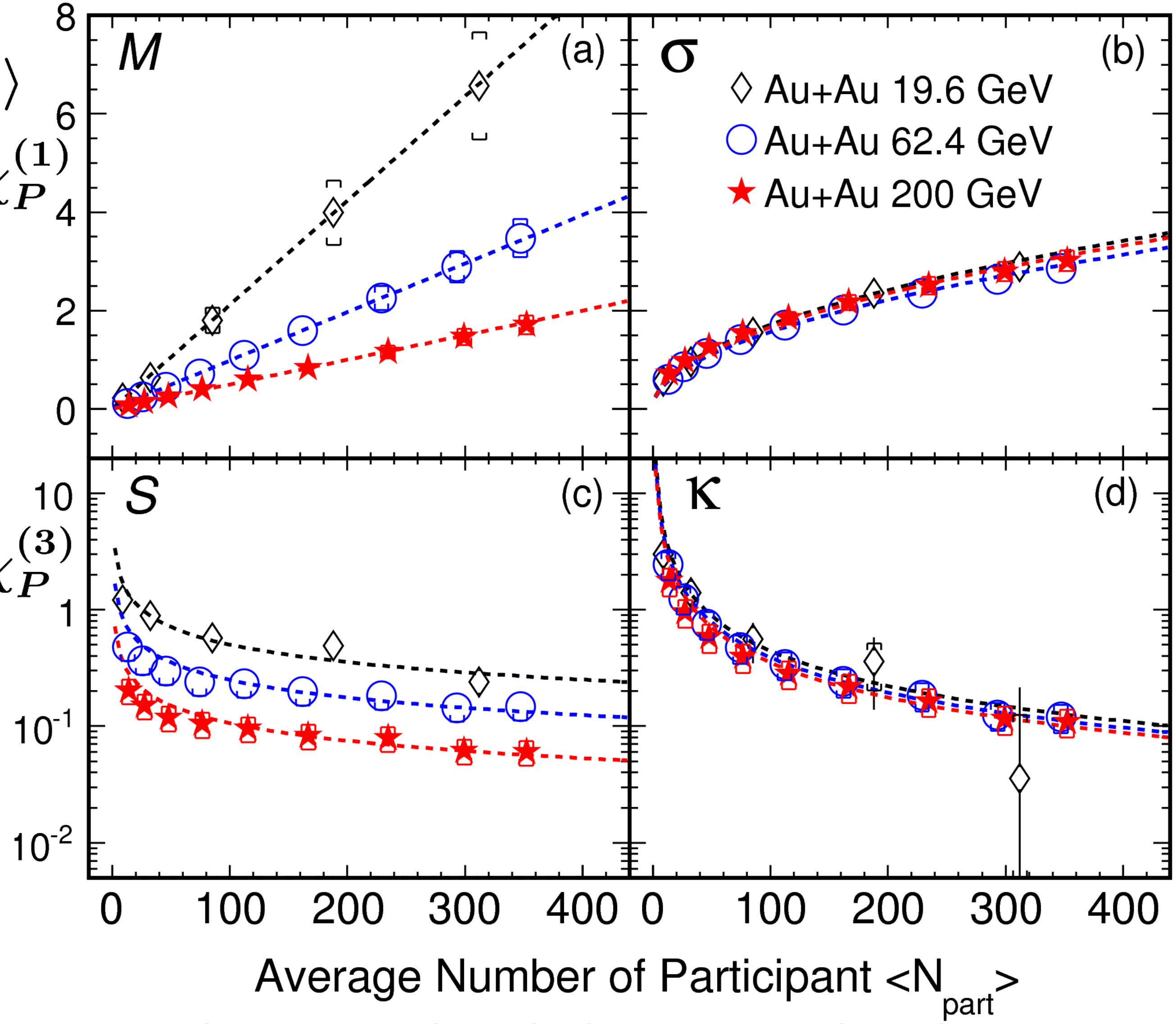
STAR results from RHIC low energy run

$$M = \langle \delta N_P \rangle = VT^3 \chi_P^{(1)}$$

$$\sigma^2 = \langle (\delta N_P)^2 \rangle = VT^3 \chi_P^{(2)}$$

$$S\sigma^3 = VT^3 \chi_P^{(3)}$$

$$\kappa\sigma^4 = VT^3 \chi_P^{(4)}$$



Caution:

$$P \neq B$$

ignore this for a while;
come back to it later

M. M. Aggarwal et al. (STAR Collaboration), PRL, 105 (2010) 22302

Taylor expansions of baryon number susceptibilities

$$\chi_{B,\mu}^{(n)} = \sum_{k=0}^{\infty} \frac{1}{k!} \chi_{B,0}^{(k+n)}(T) \left(\frac{\mu_B}{T}\right)^k$$

mean: $$M_B = VT^3 \chi_{B,\mu}^{(1)} = VT^3 \left(\frac{\mu_B}{T}\chi_{B,0}^{(2)} + \ldots\right)$$

variance: $$\sigma_B^2 = VT^3 \chi_{B,\mu}^{(2)} = VT^3 \left(\chi_{B,0}^{(2)} + \frac{1}{2}\left(\frac{\mu_B}{T}\right)^2 \chi_{B,0}^{(4)} + \ldots\right)$$

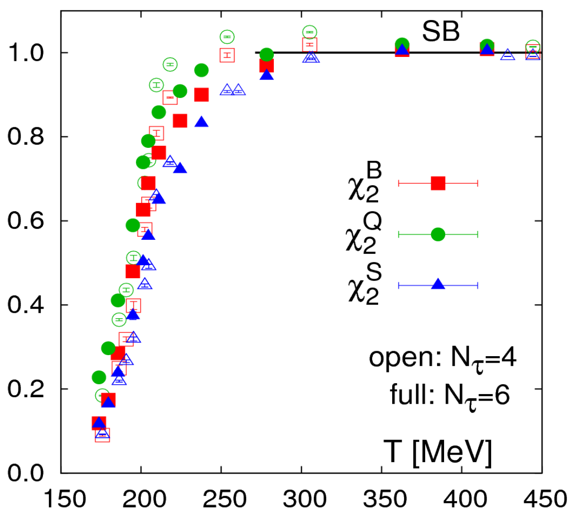
skewness and kurtosis and volume independent ratios of susceptibilities

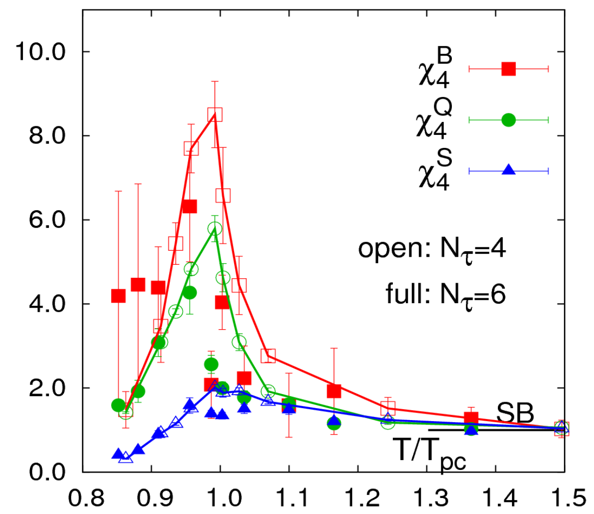
$$S_B \equiv \frac{\langle(\delta N_B)^3\rangle}{\sigma_B^3} \; , \quad \kappa_B \equiv \frac{\langle(\delta N_B)^4\rangle}{\sigma_B^4} - 3$$

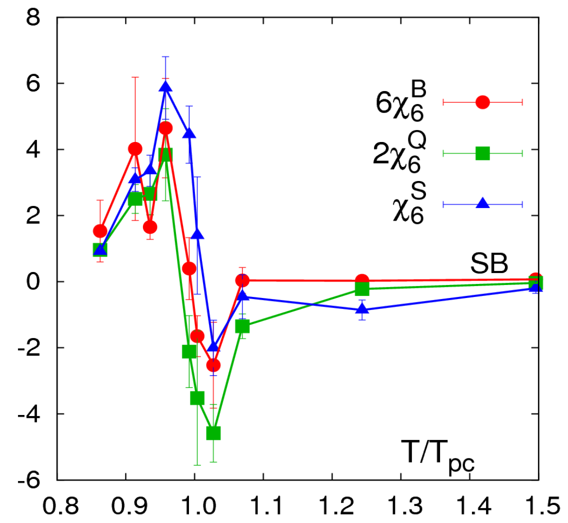
$$\frac{\sigma_B^2}{M_B} = \frac{\chi_B^{(2)}}{\chi_B^{(1)}}, \qquad S_B\sigma_B = \frac{\chi_B^{(3)}}{\chi_B^{(2)}}, \qquad \kappa_B\sigma_B^2 = \frac{\chi_B^{(4)}}{\chi_B^{(2)}}$$

Cumulants of net-charge susceptibilities in Lattice QCD

p4-action: $16^3 \times 4,\ 24^3 \times 6$







M. Cheng et al, Phys. Rev. D79 (2009) 074505

- results from an O(a^2) improved action (p4) on coarse lattices
- currently repeated with highly improved action (HISQ) on fine lattices

(talk by Swagato Mukherjee)

generic features: larger deviations from HRG model for higher order cumulants

$\chi_2^X, \chi_4^X \geq 0$ for all $T > 0$

$\chi_6^X > 0$ for all $T \geq T_0$

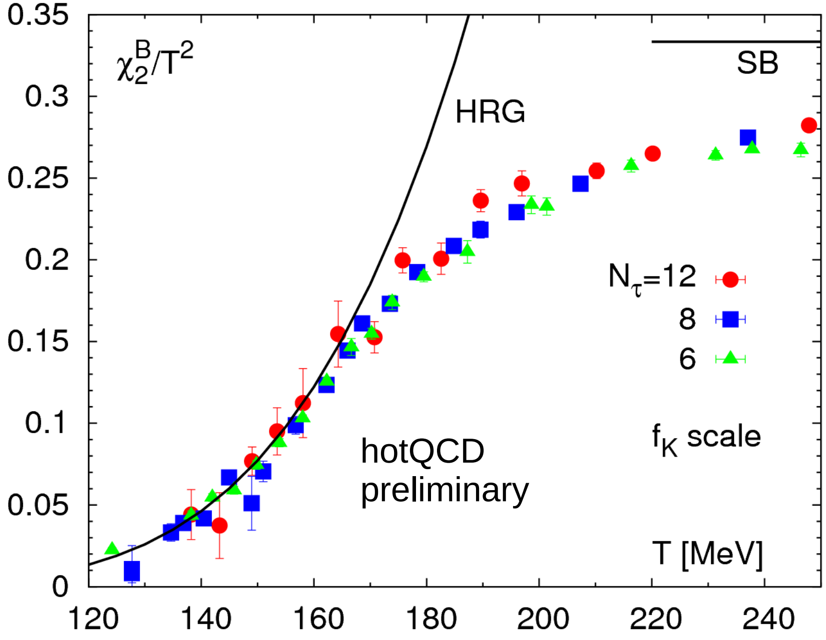
structure consistent with O(4) universality/ crossover transition

Quadratic charge fluctuations

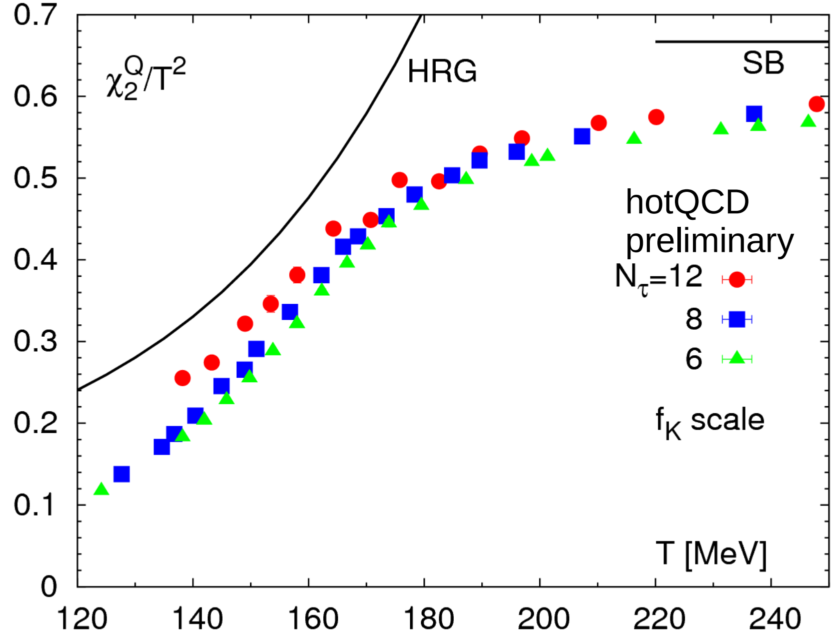
preliminary results from hotQCD; using the HISQ action at three values of the lattice cut-off

comparison of baryon number and electric charge fluctuations with the corresponding HRG model calculations

net baryon number:
deviations start for $T \gtrsim 170$ MeV



generic lattice problems larger in the electric charge sector

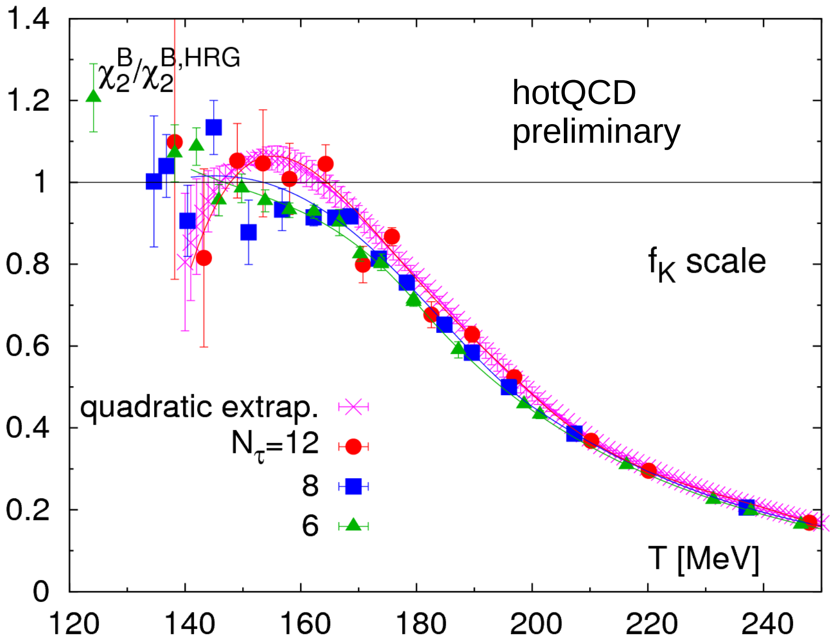


Quadratic charge fluctuations

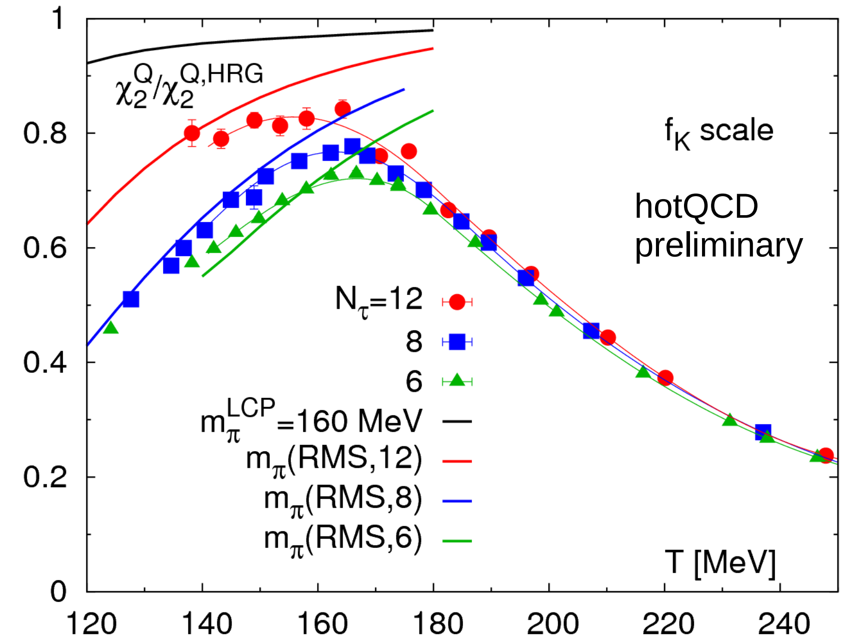
preliminary results from hotQCD; using the HISQ action at three values of the lattice cut-off

comparison of baryon number and electric charge fluctuations with the corresponding HRG model calculations

net baryon number:
deviations start for $T \gtrsim 170$ MeV



generic lattice problems larger in the electric charge sector, BUT:



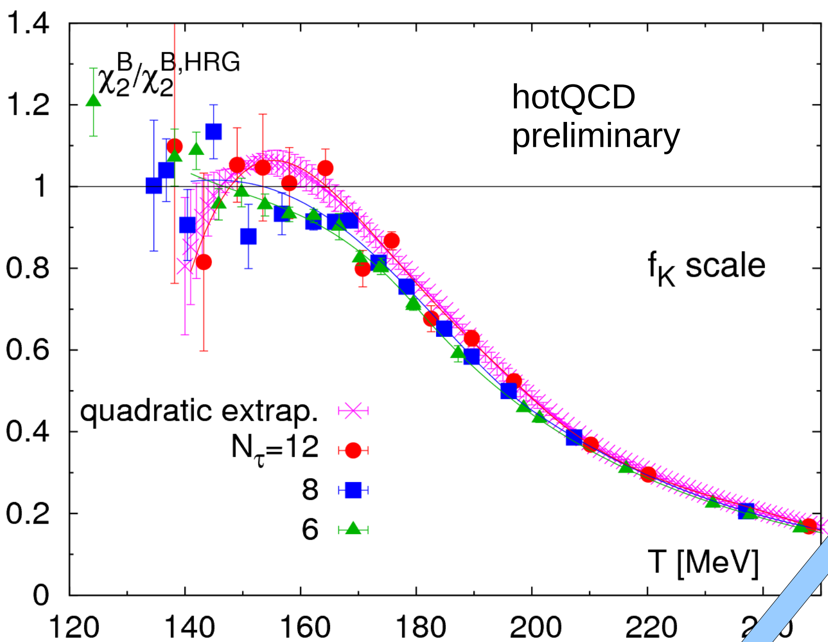
deviations from HRG result seem to be significant already for $T \gtrsim 160$ MeV

Quadratic charge fluctuations

preliminary results from hotQCD; using the HISQ action at three values of the lattice cut-off

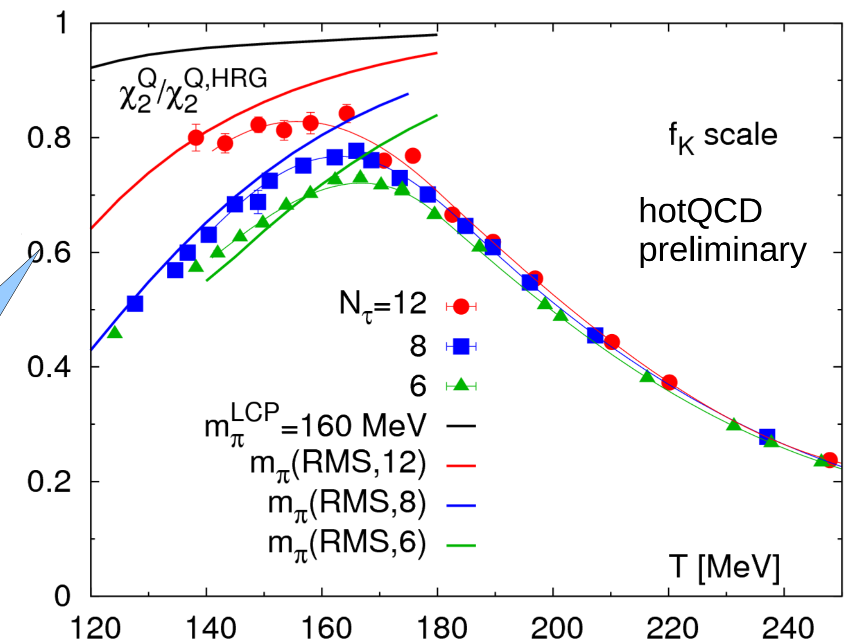
comparison of baryon number and electric charge fluctuations with the corresponding HRG model calculations

net baryon number:
deviations start for $T \gtrsim 170$ MeV



suffers from taste symmetry violations inherent to the staggered fermion scheme

generic lattice problems larger in the electric charge sector, BUT:



deviations from HRG result seem to be significant already for $T \gtrsim 160$ MeV

BARYO-meter

LGT: Even-odd ratios of cumulants are BARYO-meters

(Taylor expansion)

e.g.: $$\frac{\chi_{B,\mu}^{(2)}}{\chi_{B,\mu}^{(1)}} = \frac{\chi_{B,0}^{(2)} + \frac{1}{2}\chi_{B,0}^{(4)}(\mu_B/T)^2 + \ldots}{\chi_{B,0}^{(2)}(\mu_B/T) + \frac{1}{6}\chi_{B,0}^{(4)}(\mu_B/T)^3 + \ldots}$$

$$= \frac{T}{\mu_B}\left[\frac{1 + \frac{1}{2}\frac{\chi_{B,0}^{(4)}}{\chi_{B,0}^{(2)}}(\mu_B/T)^2 + \ldots}{1 + \frac{1}{6}\frac{\chi_{B,0}^{(4)}}{\chi_{B,0}^{(2)}}(\mu_B/T)^2 + \ldots}\right]$$

similar: $\chi_{B,\mu}^{(4)}/\chi_{B,\mu}^{(3)}$

→ at 200 GeV: $\frac{\chi_B^{(2)}}{\chi_B^{(1)}} = 5.3(9)$ (STAR)

→ $\frac{\mu_B}{T_f} = 0.19(3)$

LO LGT;
NLO contributes less than 2%

BARYO-meter

LGT: Even-odd ratios of cumulants are BARYO-meters

(Taylor expansion)

e.g.: $$\frac{\chi_{B,\mu}^{(2)}}{\chi_{B,\mu}^{(1)}} = \frac{\chi_{B,0}^{(2)} + \frac{1}{2}\chi_{B,0}^{(4)}(\mu_B/T)^2 + \dots}{\chi_{B,0}^{(2)}(\mu_B/T) + \frac{1}{6}\chi_{B,0}^{(4)}(\mu_B/T)^3 + \dots}$$

$$= \frac{T}{\mu_B}\left[\frac{1 + \frac{1}{2}\frac{\chi_{B,0}^{(4)}}{\chi_{B,0}^{(2)}}(\mu_B/T)^2 + \dots}{1 + \frac{1}{6}\frac{\chi_{B,0}^{(4)}}{\chi_{B,0}^{(2)}}(\mu_B/T)^2 + \dots}\right]$$

similar:

$$\chi_{B,\mu}^{(4)}/\chi_{B,\mu}^{(3)}$$

	STAR	QCD	HRG
$\sqrt{s_{NN}}$	$\chi_P^{(2)}/\chi_P^{(1)}$	μ_B/T_f	μ_B/T_f
200	5.3(9)	0.190(30)(4)	0.183
63.4	2.35(42)	0.43(8)(3)	0.43

we assume here: $\chi_P^{(2)}/\chi_P^{(1)} = \chi_B^{(2)}/\chi_B^{(1)}$

(second error is systematics from NLO; at 39 GeV corrections are about 20%)

THERMO-meter

LGT: Even-even ratios of cumulants are THERMO-meters

e.g.:
$$\frac{\chi^{(4)}_{B,\mu}}{\chi^{(2)}_{B,\mu}} = \frac{\chi^{(4)}_{B,0} + \frac{1}{2}\chi^{(6)}_{B,0}(\mu_B/T)^2 + \dots}{\chi^{(2)}_{B,0} + \frac{1}{2}\chi^{(4)}_{B,0}(\mu_B/T)^2 + \dots}$$

$$= \frac{\chi^{(4)}_{B,0}(T)}{\chi^{(2)}_{B,0}(T)} \left[\frac{1 + \frac{1}{2}\frac{\chi^{(6)}_{B,0}}{\chi^{(4)}_{B,0}}(\mu_B/T)^2 + \dots}{1 + \frac{1}{2}\frac{\chi^{(4)}_{B,0}}{\chi^{(2)}_{B,0}}(\mu_B/T)^2 + \dots} \right]$$

electric charge fluctuations

similar/better

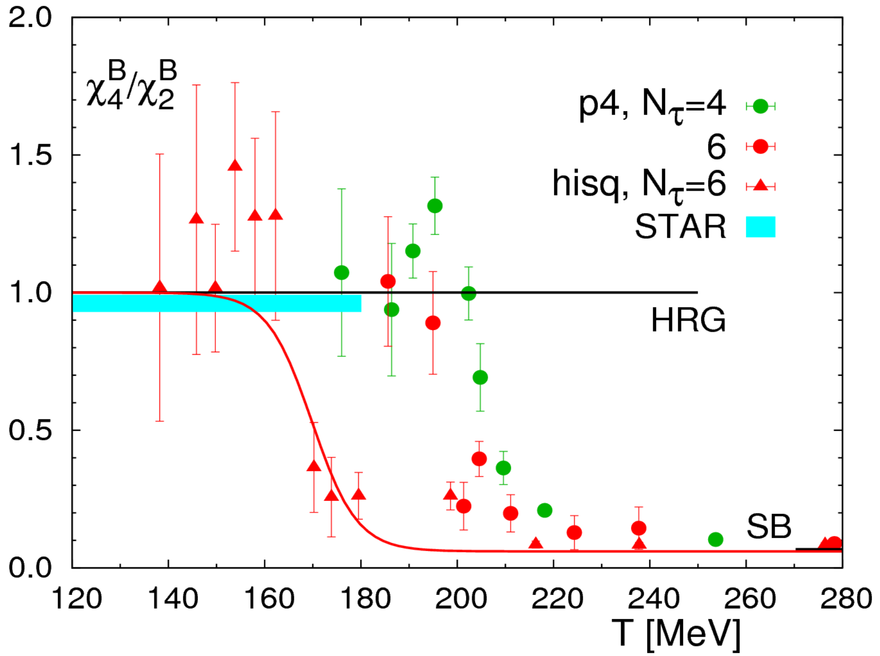
$$\chi^{(4)}_{Q,\mu}/\chi^{(2)}_{Q,\mu}$$

→ at 200 GeV: $\frac{\chi^{(4)}_P}{\chi^{(2)}_P} = 0.96(3)$ (STAR)

NLO contributions require control over $\chi^{(6)}_{B,0}$ (statistically demanding!!)

THERMO-meter

LGT: Even-even ratios of cumulants are THERMO-meters



BNL-Bielefeld preliminary

at 200 GeV:

$$\frac{\chi_P^{(4)}}{\chi_P^{(2)}} = 0.96(3) \qquad \text{(STAR)}$$

$$\Rightarrow \quad T_f \sim (150 - 170)\ \mathrm{MeV}$$

LO LGT;
NLO may contribute
20% to $\chi_B^{(4)}/\chi_B^{(2)}$

we assume here: $\chi_P^{(2)}/\chi_P^{(1)} = \chi_B^{(2)}/\chi_B^{(1)}$

need to improve statistics
and control NLO contributions

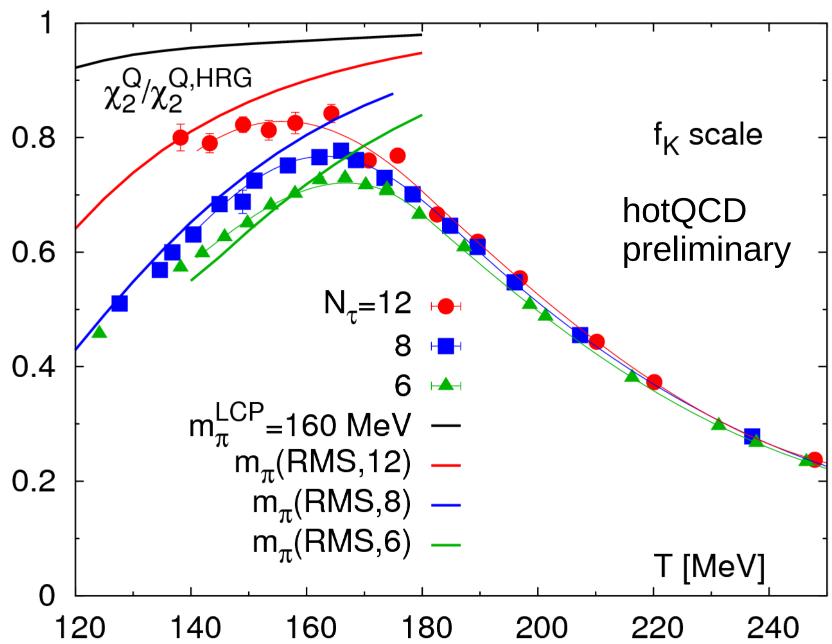
Electric charge fluctuations: a better THERMO-meter?

Cumulants of net electric charge fluctuations $\chi^{(4)}_{Q,\mu}/\chi^{(2)}_{Q,\mu}$

HRG: $\chi^{(4)}_{Q,\mu}/\chi^{(2)}_{Q,\mu} > 1$ due to double charged hadrons and quantum statistics (pions)

ideal gas: $2/\pi^2$ → larger variation with T in the transition region

QCD:



hotQCD preliminary: $\chi^{(2)}_{Q,0}/\chi^{(2,HRG)}_{Q,0}$

large cut-off effects for $T \lesssim 170$ MeV due to taste-violation in the staggered discretization scheme (large modifications of the pion spectrum)

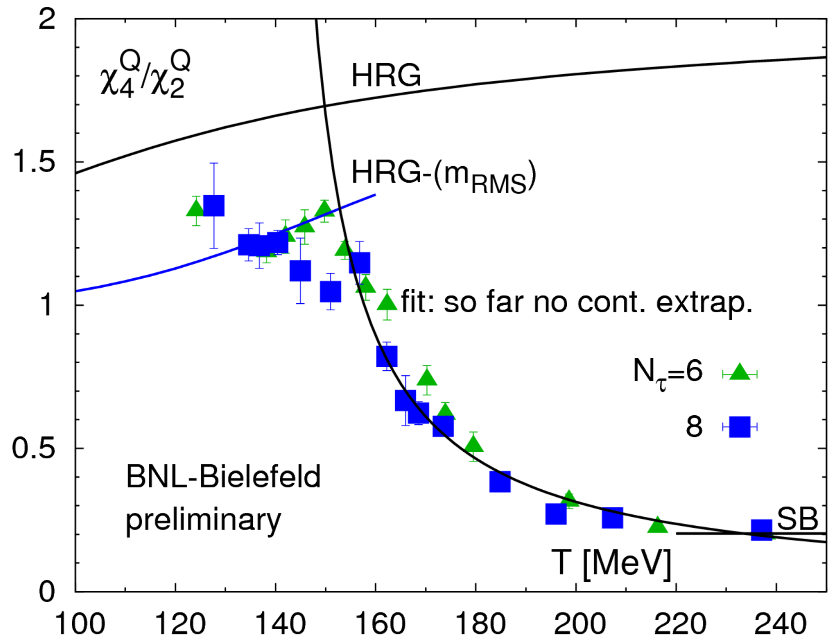
a 'cleaner' observable, but systematic errors are more difficult to control in lattice calculations

A better THERMO-meter

Cumulants of net electric charge fluctuations

HRG: $\chi^{(4)}_{Q,\mu}/\chi^{(2)}_{Q,\mu} > 1$ due to double charged hadrons and quantum statistics (pions)

QCD:



cut-off effects arise from taste symmetry violations; rapidly reduce for $T \gtrsim 160$ MeV

estimates for continuum values

T [MeV]	$\chi^{(4)}_{Q,0}/\chi^{(2)}_{Q,0}$
155	~ 1.1
160	~ 0.9
165	~ 0.7

this will unambiguously determine T_f

Relating net baryon number and net proton susceptibilities

- experiment: only net-proton rather than net-baryon numbers are accessible;

- as far as critical behavior is concerned, proton number fluctuations capture the same singular behavior

  (Stephanov et al, 1998)

- proton and baryon number fluctuations may differ quantitatively

  (Asakawa, Kitazawa, 2011)

- given experimental data on $\chi_Q^{(2)}, \chi_P^{(2)}$ we can obtain $\chi_B^{(2)}$ using QCD results on $\chi_Q^{(2)}, \chi_B^{(2)}$ at freeze-out

Relating net baryon number and net proton susceptibilities

- given experimental data on $\chi_Q^{(2)}, \chi_P^{(2)}$ we can obtain $\chi_B^{(2)}$ using QCD results on $\chi_Q^{(2)}, \chi_B^{(2)}$ at freeze-out:

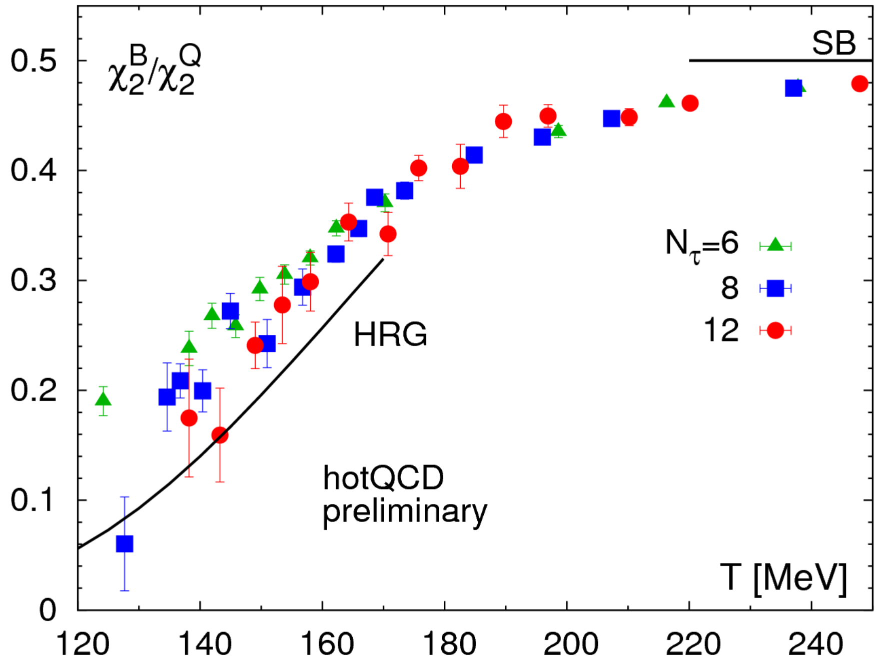
at freeze-out in thermal equilibrium:

$$\left[\frac{\chi_B^{(2)}}{\chi_Q^{(2)}}\right]_{AA} = \left[\frac{\chi_B^{(2)}}{\chi_Q^{(2)}}\right]_{QCD}$$

may vary with time

experiment: $$\left[\frac{\chi_P^{(2)}}{\chi_Q^{(2)}}\right]_{AA} = R_{PQ}$$

conserved

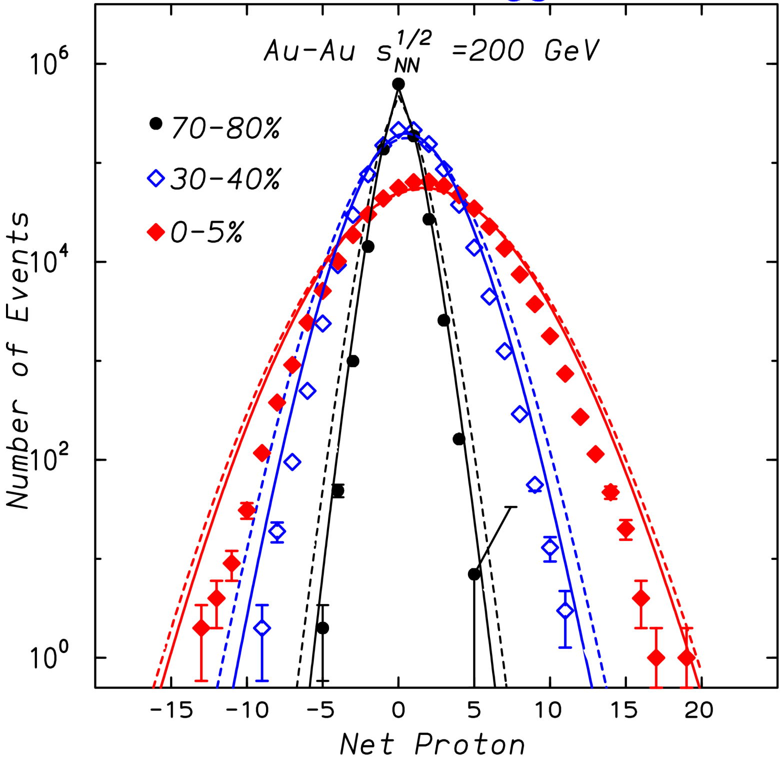


$$\Rightarrow \left[\frac{\chi_B^{(2)}}{\chi_P^{(2)}}\right]_{AA} = R_{PQ}^{-1}\left[\frac{\chi_B^{(2)}}{\chi_Q^{(2)}}\right]_{QCD} \simeq (0.25 - 0.35) R_{PQ}^{-1}$$

The freeze-out volume

the probability distribution of net charges, P(N), may be expressed in terms of a cumulant expansion;

$$P_P(N) = \frac{1}{2\pi}\int_{-\infty}^{\infty} \mathrm{d}k\ \cos(kx)\exp\left[VT^3\left(-\frac{k^2}{2}\chi_2^P + \frac{k^4}{4!}\chi_4^P - \frac{k^6}{6!}\chi_6^P \dots\right)\right]$$



leading order corresponds to Gaussian approximation:

$$P_P(N) \sim \exp[-N^2/(2V_f T_f^3 \chi_P^{(2)})]$$

higher order cumulants modify tail of P(N) ➡ 'bulk' structure determines the freeze out volume, once T_f, $\chi_B^{(2)}$ are given

P. Braun-Munziger et al., arXiv:1107.4267

get it from $\chi_P^{(2)}/\chi_B^{(2)}$

in order to extract V_f from experiment we need $\chi_P^{(2)}$ or analyze directly $P(Q)$

Conclusion

- higher moments of charge fluctuations are increasingly sensitive to critical behavior, even at $\mu_B = 0$; differences between HRG and QCD will inevitably show up

- experimental results on moments of B or Q charge fluctuations up to 4^{th} order can be used to determine freeze out paramters from QCD;
  results are similar to HRG calculations, BUT they are based on QCD!!

- comparing this analysis with calculations of the QCD transition temperature allows to quantify the relation between freeze out and transition temperatures

Establishing this at $\mu_B \simeq 0$ provides an anchor point for the analysis of the entire phase diagram