How Wide is the Transition to the Deconfinement

—— A matrix model for the deconfining phase transition

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In collaboration with: A. Dumitru, Y. Hidaka, C. P. Korthals-Altes and R. D. Pisarski

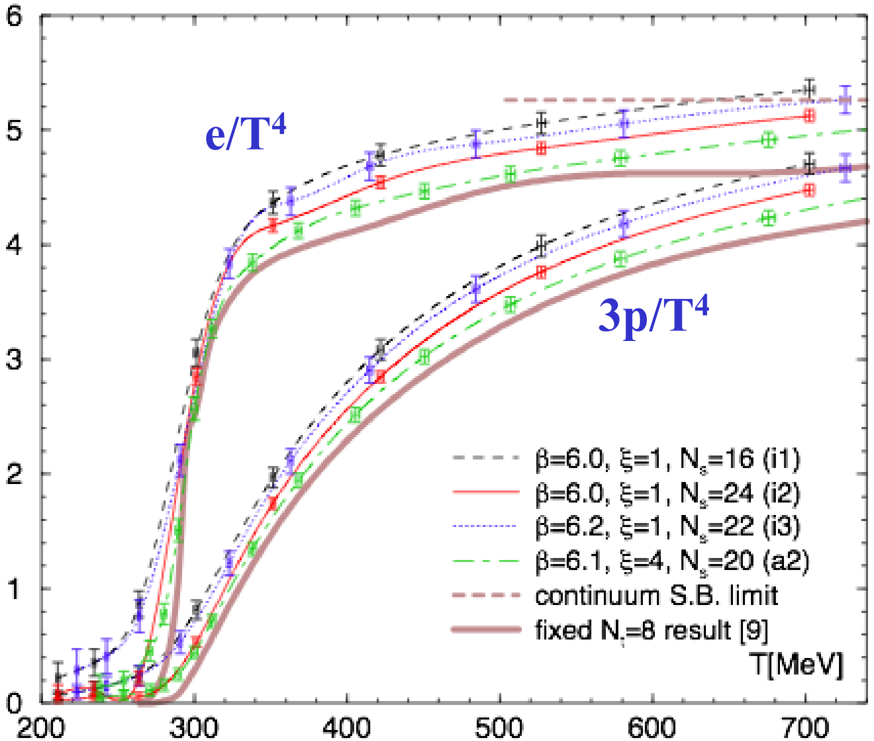
- What can we learn from Lattice QCD
- How to construct the matrix model
- Comparison with Lattice

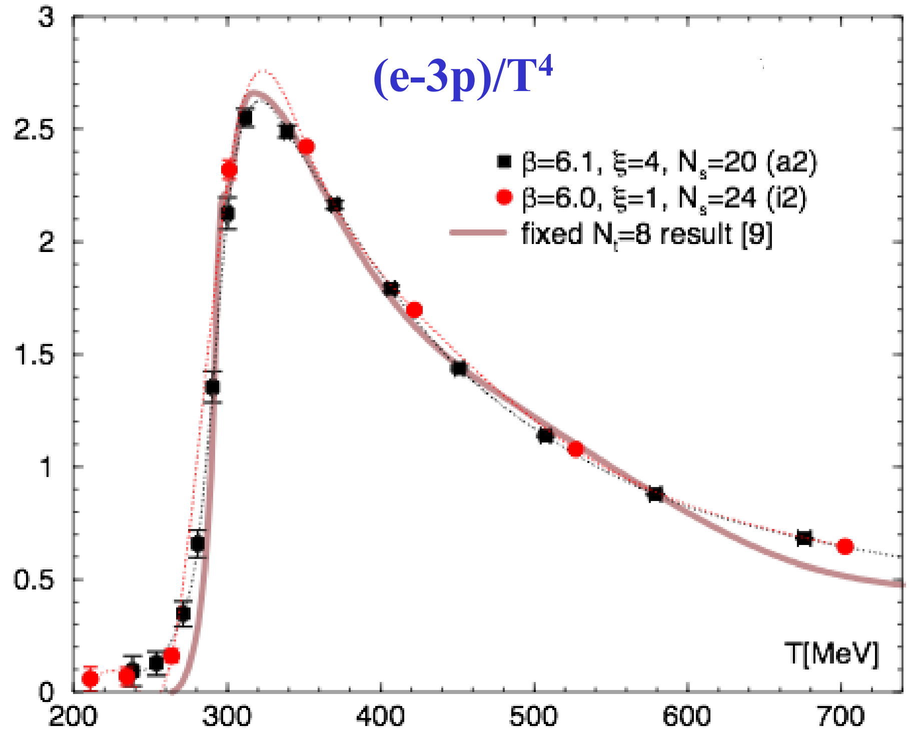
- Meisinger, Miller, Ogilvie(MMO), hep-ph/0108009
- Dumitru, YG, Hidaka, Korthals-Altes, Pisarski, arXiv:1011.3820

CPOD, Wuhan, 10 Nov. 2011

SU(3) gauge theory with*out* quarks

WHOT: **(Umeda, Ejiri, Aoki, Hatsuda, Kanaya, Maezawa & Ohno, arXiv:0809.2842)**





(Weakly) first order transition at $T_c \sim 290$ MeV

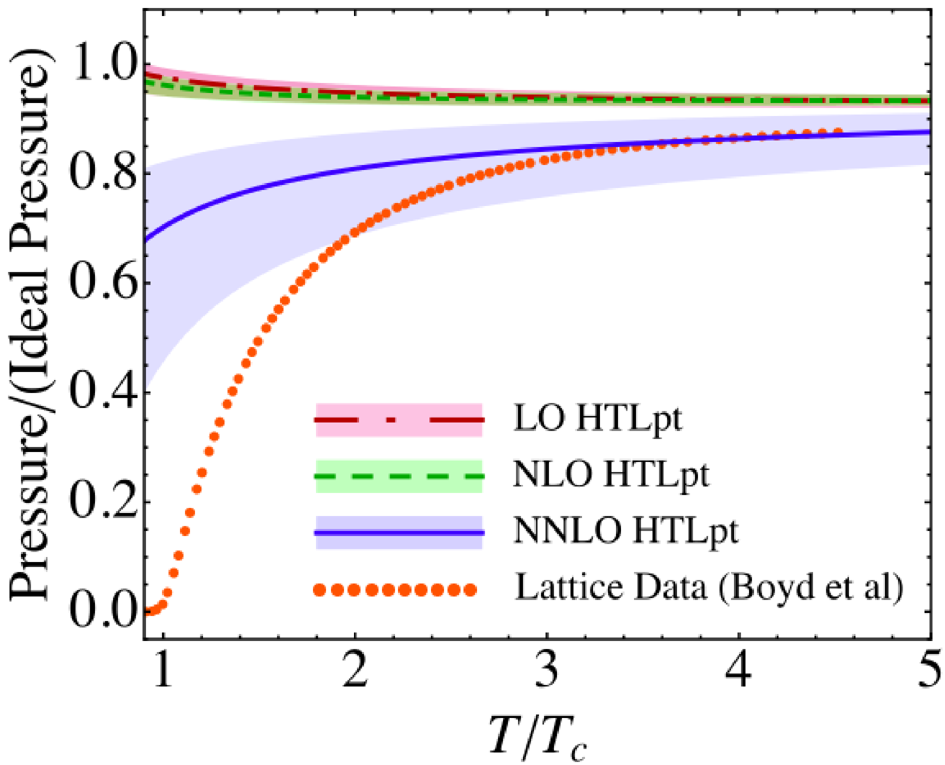
Conformal anomaly, $(e-3p)/T^4$: large peak above T_c

pertubation theory *vs* Lattice

- Resummed perturbation theory at 3-loop order works down to ~ 3 T_c.
  (Andersen, Su & Strickland, arXiv:1005.1603)

- Intermediate coupling: $\alpha_s(T_c) \sim 0.3$.
  (Braaten & Nieto, hep-h/9501375 Laine & Schröder, hep-ph/0503061 & 0603048)

  From two loop calculation, matching original to effective theory: *Not* so big...



What happens below ~ 3 T_c?

Ansatz: **constant background filed, diagonal matrix**

$$A_0^{ij} = \frac{2\pi T}{g}\, q_i\, \delta^{ij} \qquad \mathbf{L}_{ij} = \mathrm{e}^{2\pi i\, q_j}\, \delta_{ij} \qquad i, j = 1\ldots N$$

For SU(N), $\sum_{j=1\ldots N} q_j = 0$, modulo 1. Hence N-1 independent q_j's.

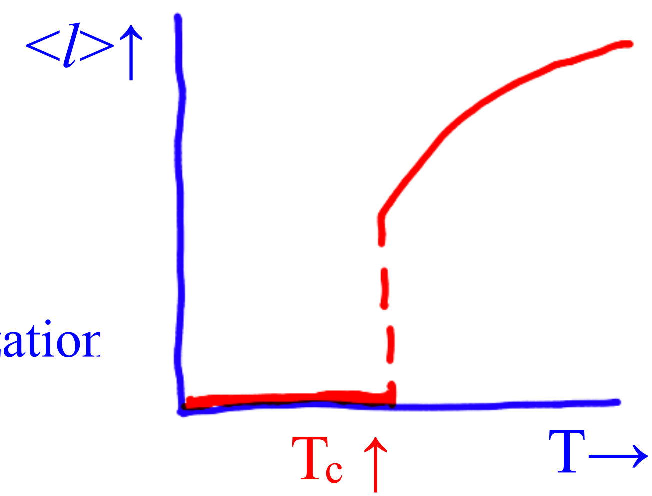
Polyakov loop : $$\ell = \frac{1}{N}\, \mathrm{tr}\, \mathbf{L}$$

measures ionization of color:

Confinement: $\langle l \rangle = 0$

Complete QGP: $\langle l \rangle = 1$

"semi"-QGP: $0 < \langle l \rangle < 1$ *partial* ionizatior



perturbative *plus* non-pertubative potential

At one loop, the potential in a constant A_0 background field is given by:

$$V_{pert}(q) = \frac{2\pi^2}{3} T^4 \left(-\frac{4}{15}(N^2-1) + \sum_{i,j} q_{ij}^2 (1-q_{ij})^2 \right) \; , \; q_{ij} = |q_i - q_j|_1$$

- **Polyakov loop thus predicts a gas of gluons would always be in the deconfined phase. $\langle q \rangle = 0$**

- **Higher orders in perturbation theory does not modify this result.**

Add *non*-perturbative terms, by *hand*, to generate $\langle q \rangle \neq 0$

Symmetries of the q's

Periodic: $q \to q + 1$ Z(N) transformation.

$$V_{non}(q) \sim q(1-q)$$

O*rdering* of Polyakov loop's eigenvalues irrelevant.

1-parameter matrix model, N = 2

(Dumitru, YG, Hidaka, Korthals-Altes, Pisarski, arXiv:1011.3820)

$$V_{pert}(q) = \frac{4\pi^2}{3} T^4 \left(-\frac{1}{20} + q^2 (1-q)^2 \right)$$

Add - *by hand* - a non-pert. potential $V_{non} \sim T^2\, T_c^2$. *Also* add a term like V_{pert}:

$$V_{non}(q) = \frac{4\pi^2}{3} T^2\, T_c^2 \left(-\frac{c_1}{5} q(1-q) - c_2\, q^2 (1-q)^2 + \frac{c_3}{15} \right)$$

Now just like any other mean field theory. $\langle q \rangle$ given by minimum of V_{eff}:

$$V_{eff}(q) = V_{pert}(q) + V_{non}(q) \qquad \left. \frac{d}{dq} V_{eff}(q) \right|_{q=\langle q \rangle} = 0$$

$\langle q \rangle$ depends nontrivially on temperature.

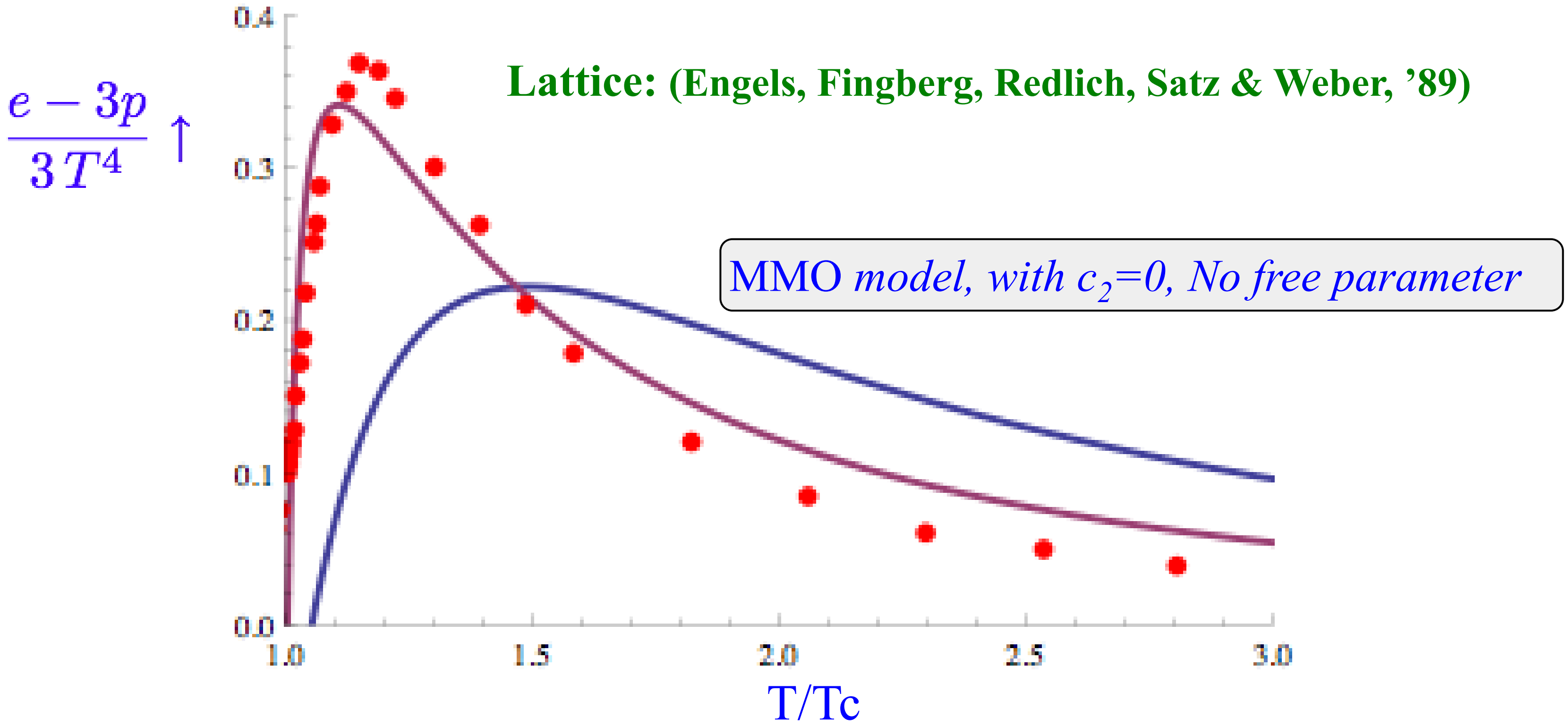
Pressure value of potential at minimum: $$p(T) = -V_{eff}(\langle q \rangle)$$

Three parameters in the model

Two conditions: transition occurs at T_c & $p(T_c) = 0$

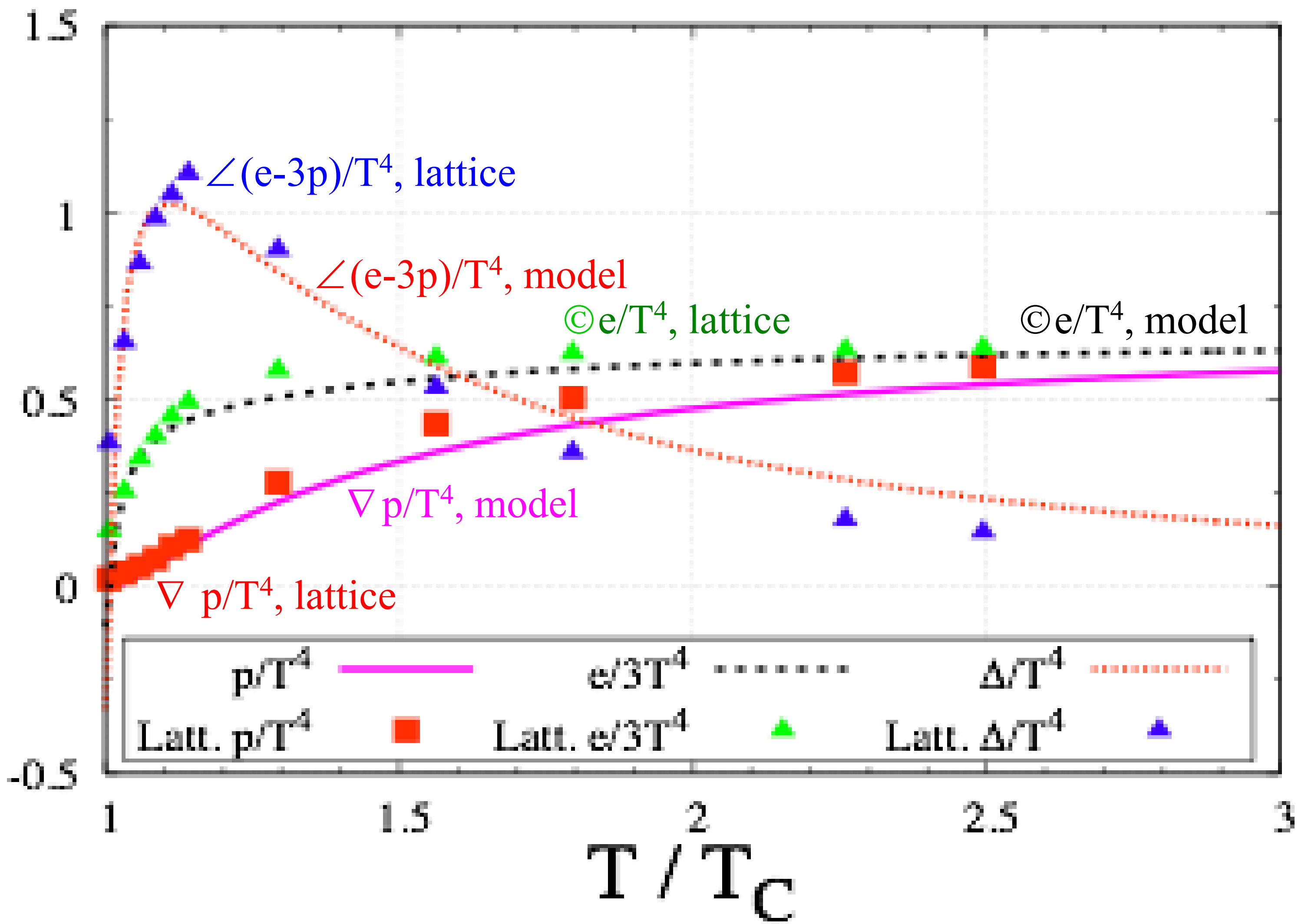
Only one free parameter ⟶ choose c_2 to fit $(e-3p)/T^4$: optimal choice

$$c_1 = 0.23\,,\; c_2 = .91\,,\; c_3 = 1.11$$



Lattice vs 1-parameter model, N = 2

$c_1 = 0.23\,,\; c_2 = .91\,,\; c_3 = 1.11$



Polyakov loop: 1-parameter matrix model ≠ lattice

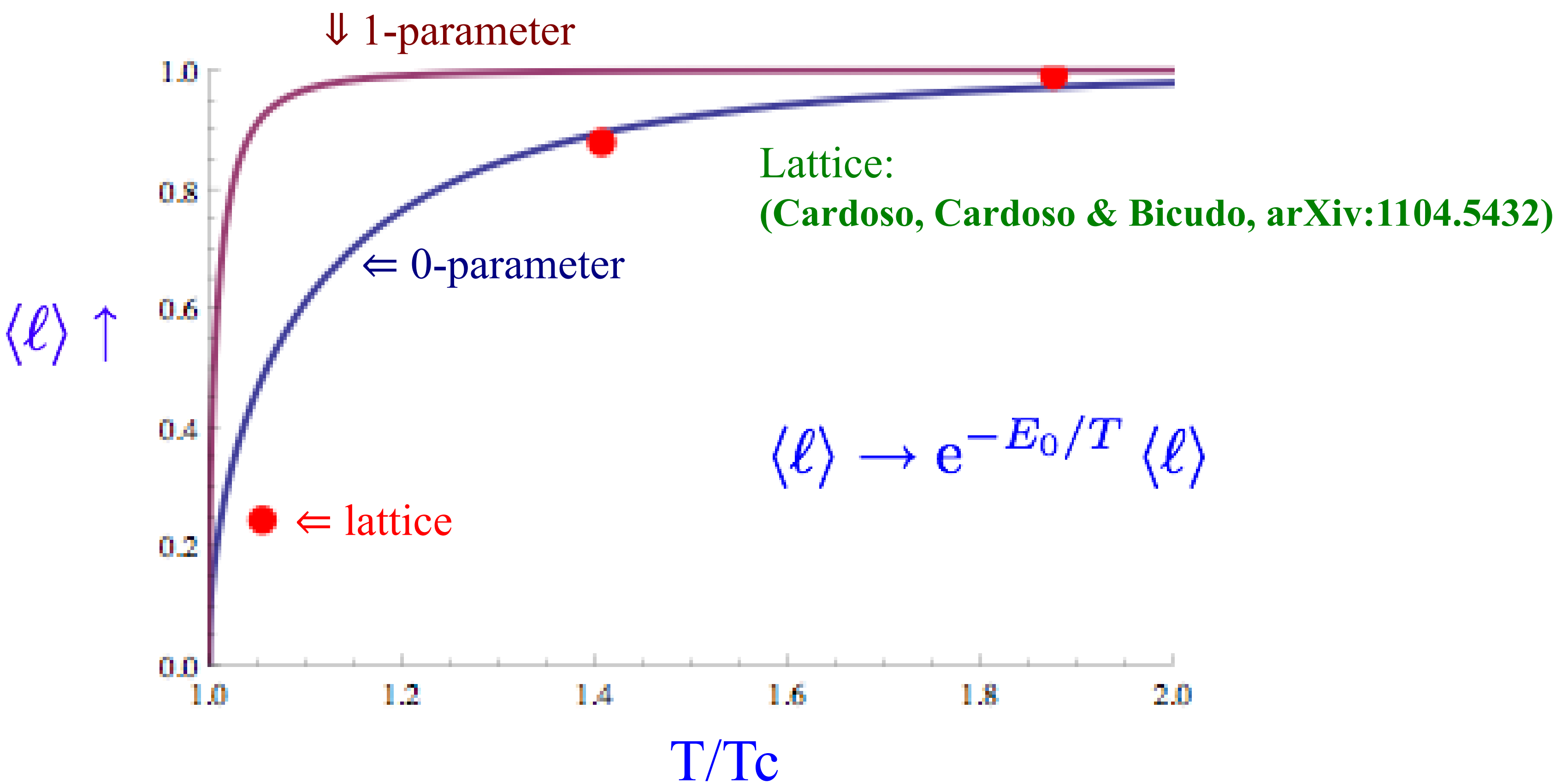
Lattice: *renormalized* Polyakov loop.

0-parameter model: close to lattice.

1-parameter model: *sharp* disagreement. $\langle l \rangle$ rises to ~ 1 *much* faster?

Sharp rise also found using Functional Renormalization Group (FRG):

(Braun, Gies & Pawlowski, arXiv:0708.2413; Marhauser & Pawlowski, arXiv:0812.1144)

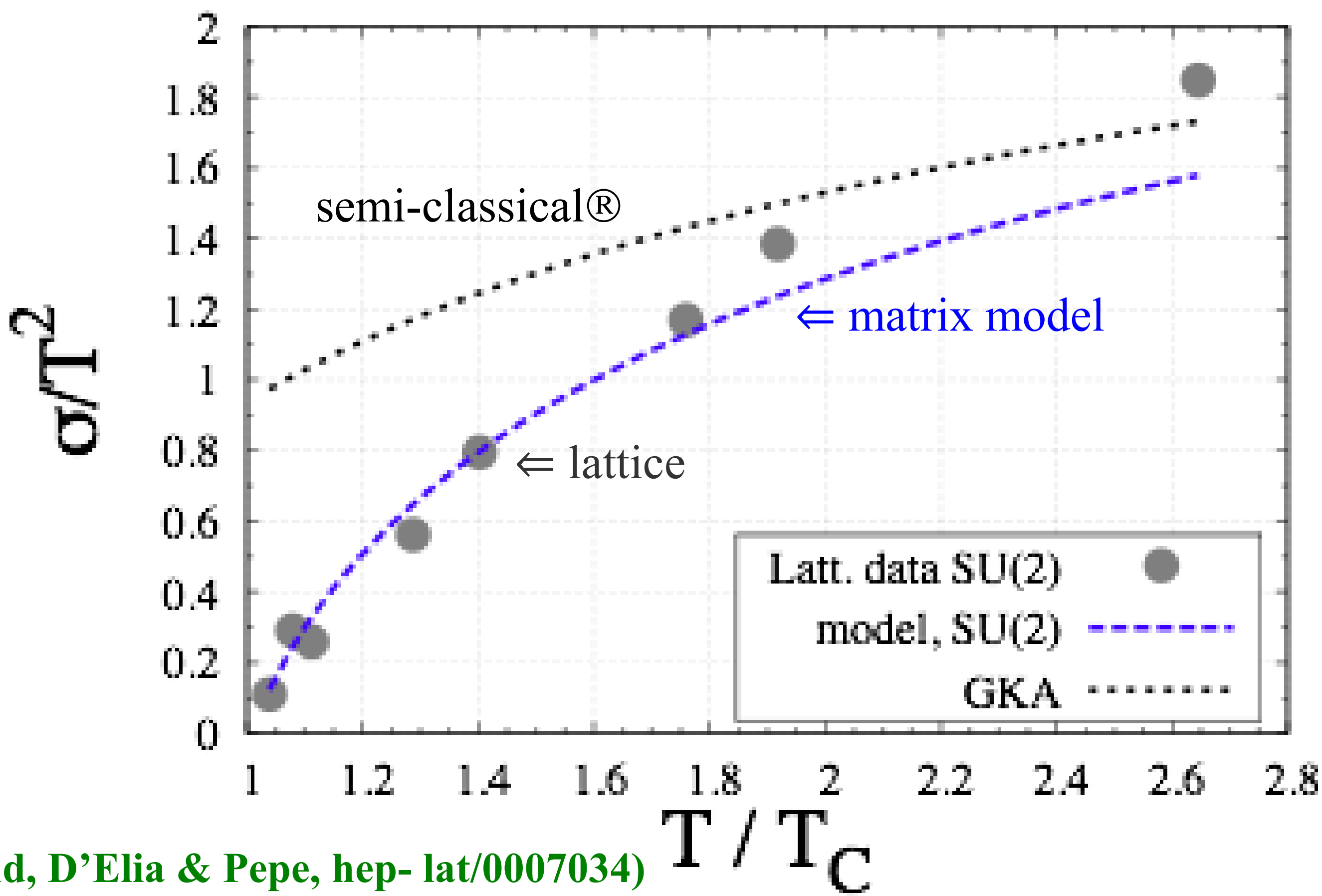


Interface tension, N = 2

σ vanishes as $T \to T_c$, $\sigma \sim (t-1)^{2\nu}$.
Ising $2\nu \sim 1.26$; Lattice: ~ 1.32.
Matrix model: ~ 1.5: c_2 important.

$$\sigma(T) = \frac{4\pi^2 T^2}{3\sqrt{6g^2}} \frac{(t^2-1)^{3/2}}{t\,(t^2-c_2)} \, , \; t = \frac{T}{T_c}$$

Semi-class.: GKA '04. *Include* corr.'s $\sim g^2$ in matrix $\sigma(T)$ (ok when $T > 1.2\, T_c$)



Lattice:
(de Forcrand, D'Elia & Pepe, hep- lat/0007034)

Adjoint Higgs phase, N = 2

$A_0^{cl} \sim q\,\sigma_3$, so $\langle q \rangle \neq 0$ generates an (adjoint) Higgs phase:

(Pisarski, hep-ph/0608242; Unsal & Yaffe, arXiv:0803.0344; Simic & Unsal, arXiv:1010.5515)

In background field, $A = A_0^{cl} + A^{qu}$: $D_0^{cl} A^{qu} = \partial_0 A^{qu} + i\,g\,[A_0^{cl}, A^{qu}]$

Fluctuations $\sim \sigma_3$ not Higgsed, $\sim \sigma_{1,2}$ Higgsed, get mass $\sim 2\pi T \langle q \rangle$

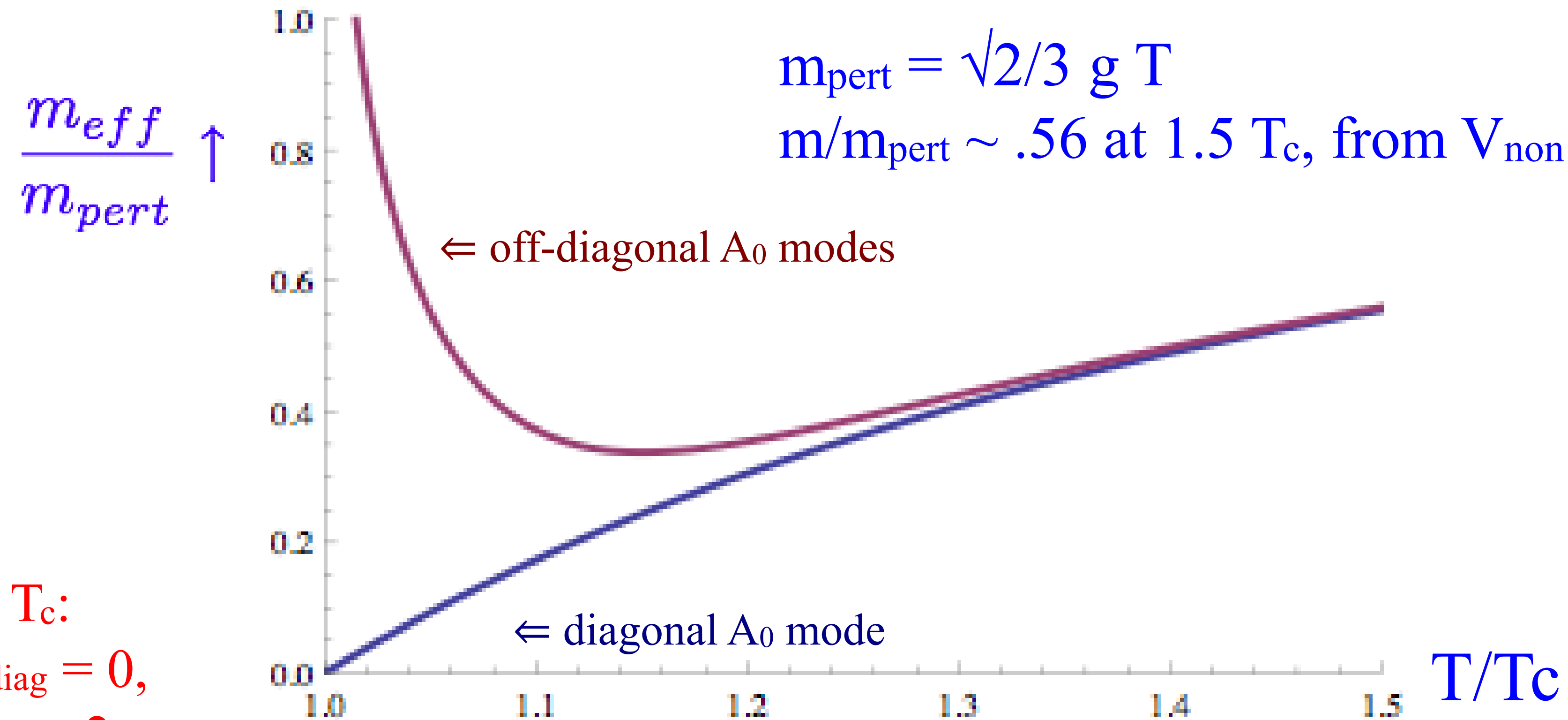
Hence when $\langle q \rangle \neq 0$, for $T < 1.2\, T_c$, *splitting* of masses:

$$\langle (A_0^{qu})_{ab}(\vec{x})(A_0^{qu})_{ba}(0)\rangle \sim \int \frac{d^3p}{(2\pi)^3} e^{i\vec{p}\cdot\vec{x}} \sum_{n=-\infty}^{+\infty} \Delta_{00}$$

$$\Delta_{00} = \frac{e^{-ip_0\tau}}{(\vec{p})^2 + p_0^2 + m_D^2(q)}; \quad p_0 = 2\pi T(n + q_a - q_b).$$

$$p_0 = 2\pi T n \rightarrow 2\pi T(n + q_a - q_b),$$

Adjoint Higgs phase, N = 2



At T_c:
$m_{diag} = 0$,
$m_{off} \sim 2\, m_{pert}$.

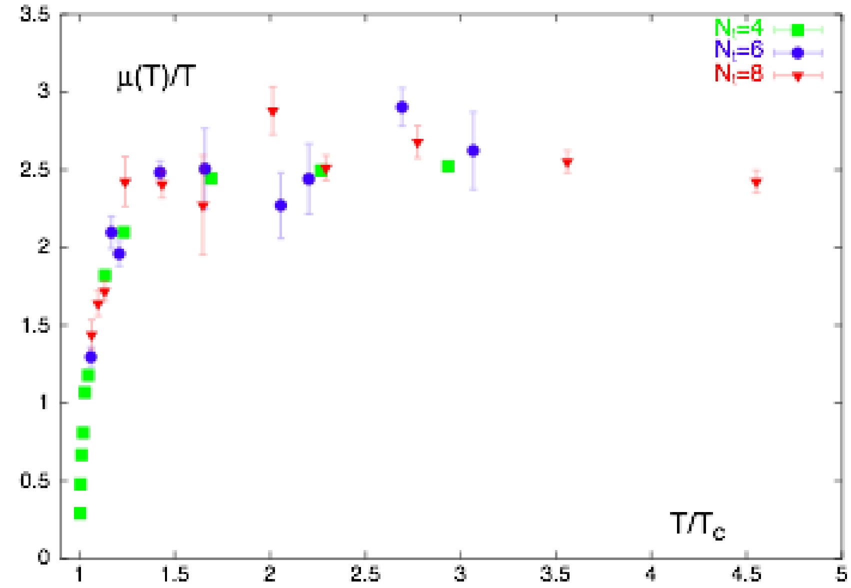
Lattice: A_0 mass as $T \to T_c$ - *up or down*?

Gauge invariant: 2 pt function of loops:

$$\langle \mathrm{tr}\, \mathbf{L}^\dagger(x)\, \mathrm{tr}\, \mathbf{L}(0)\rangle \sim e^{-\mu x}/x^d$$

μ/T goes *down* as $T \to T_c$

(Kaczmarek, Karsch, Laermann & Lutgemeier, hep-lat/9908010)

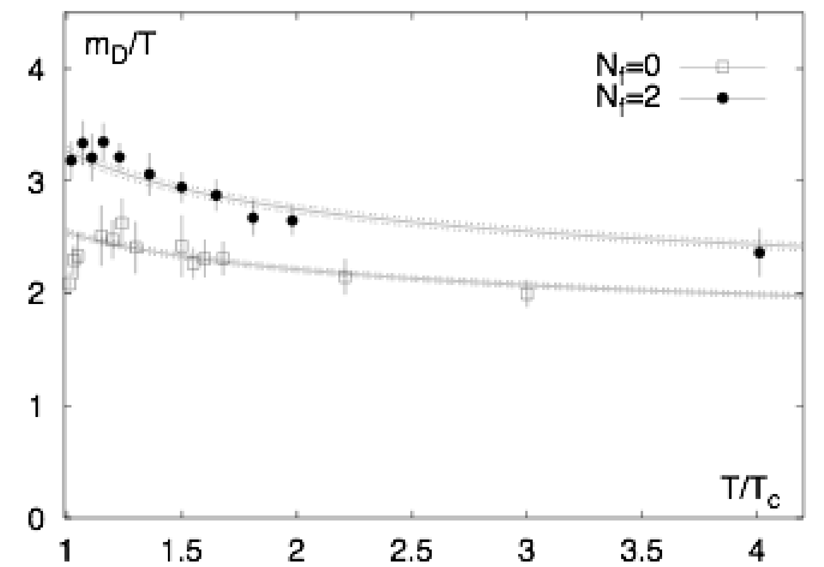


Gauge dependent: singlet potential

$$\langle \mathrm{tr}\, \left(\mathbf{L}^\dagger(x)\, \mathbf{L}(0)\right)\rangle \sim e^{-m_D x}/x$$

m_D/T goes *up* as $T \to T_c$

(Cucchieri, Karsch & Petreczky, hep-lat/0103009; Kaczmarek & Zantow, hep-lat/0503017)



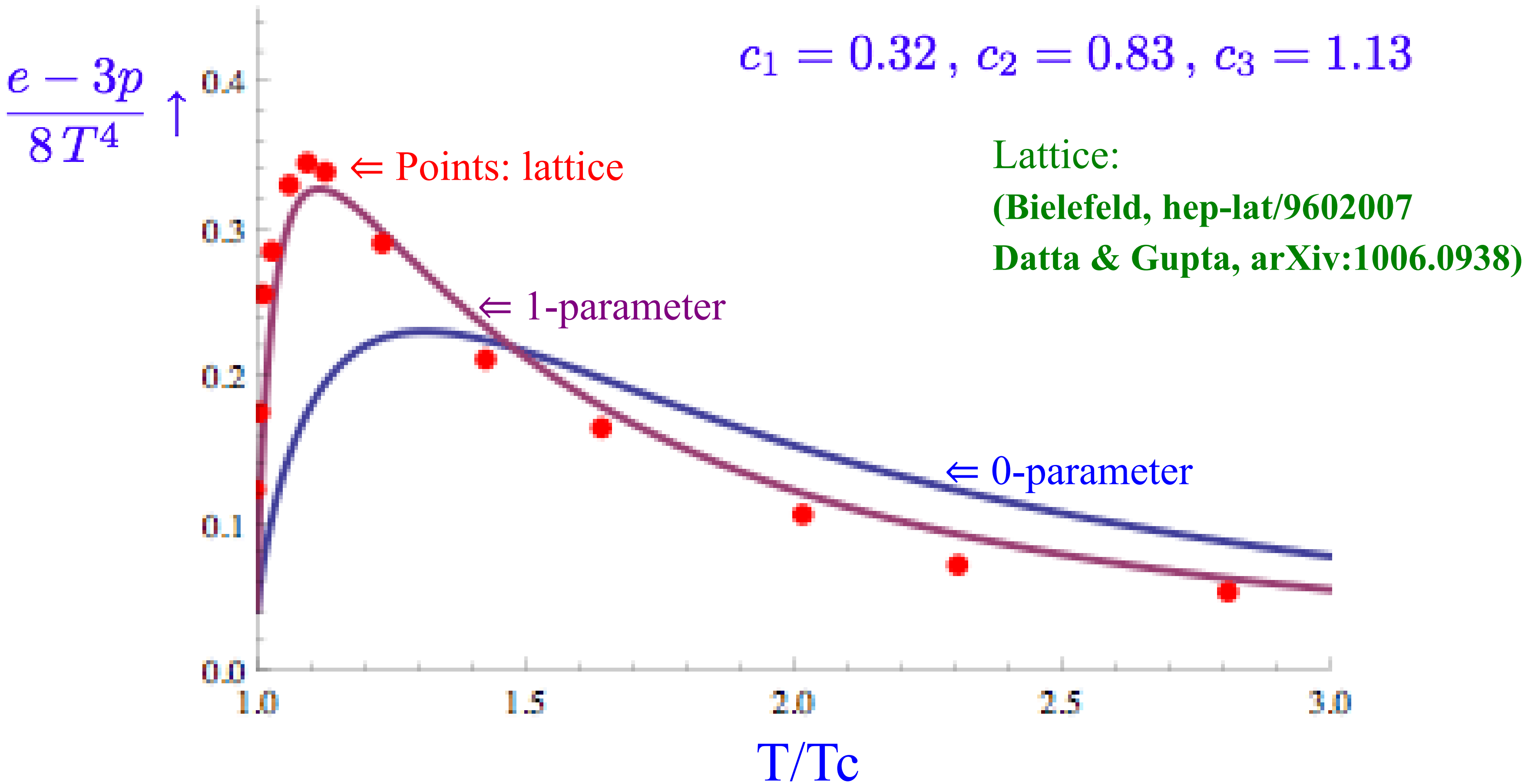
Which way do masses go as $T \to T_c$?

Both are constant above ~ 1.5 T_c.

Lattice vs 0- and 1- parameter matrix models, N = 3

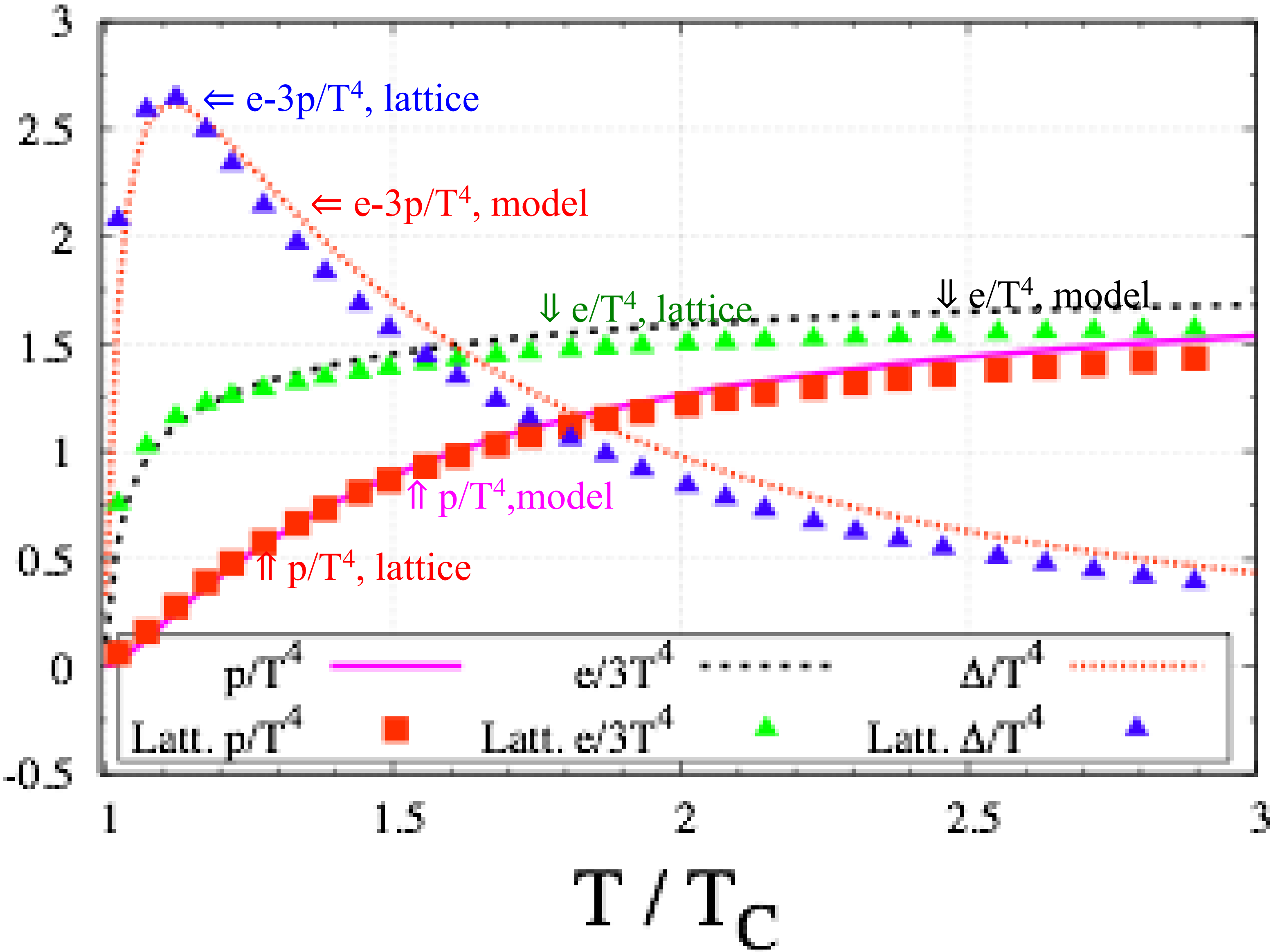
Results for N=3 similar to N=2.
0-parameter model way off.
Good fit $(e-3p)/T^4$ for 1-parameter model,

Again, $c_2 \sim 1$, so at T_c, terms $\sim q^2(1-q)^2$ *almost* cancel.



Lattice vs 1- parameter model, N = 3

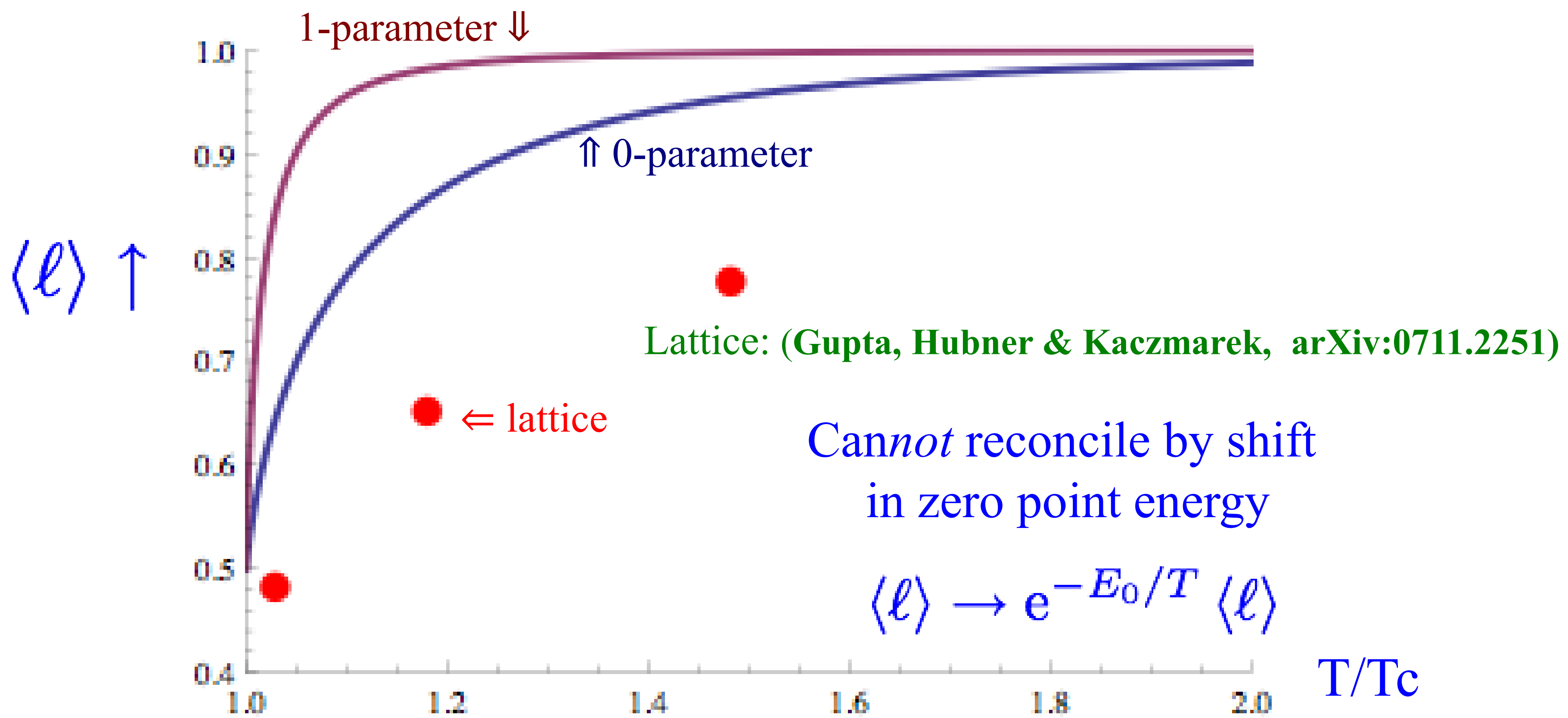
$c_1 = 0.32\,,\; c_2 = 0.83\,,\; c_3 = 1.13$



Polyakov loop: matrix models ≠ lattice

Renormalized Polyakov loop from lattice does *not* agree with *either* matrix model
$\langle l \rangle - 1 \sim \langle q \rangle^2$, by 1.2 T_c, $\langle q \rangle \sim .05$, negligible.

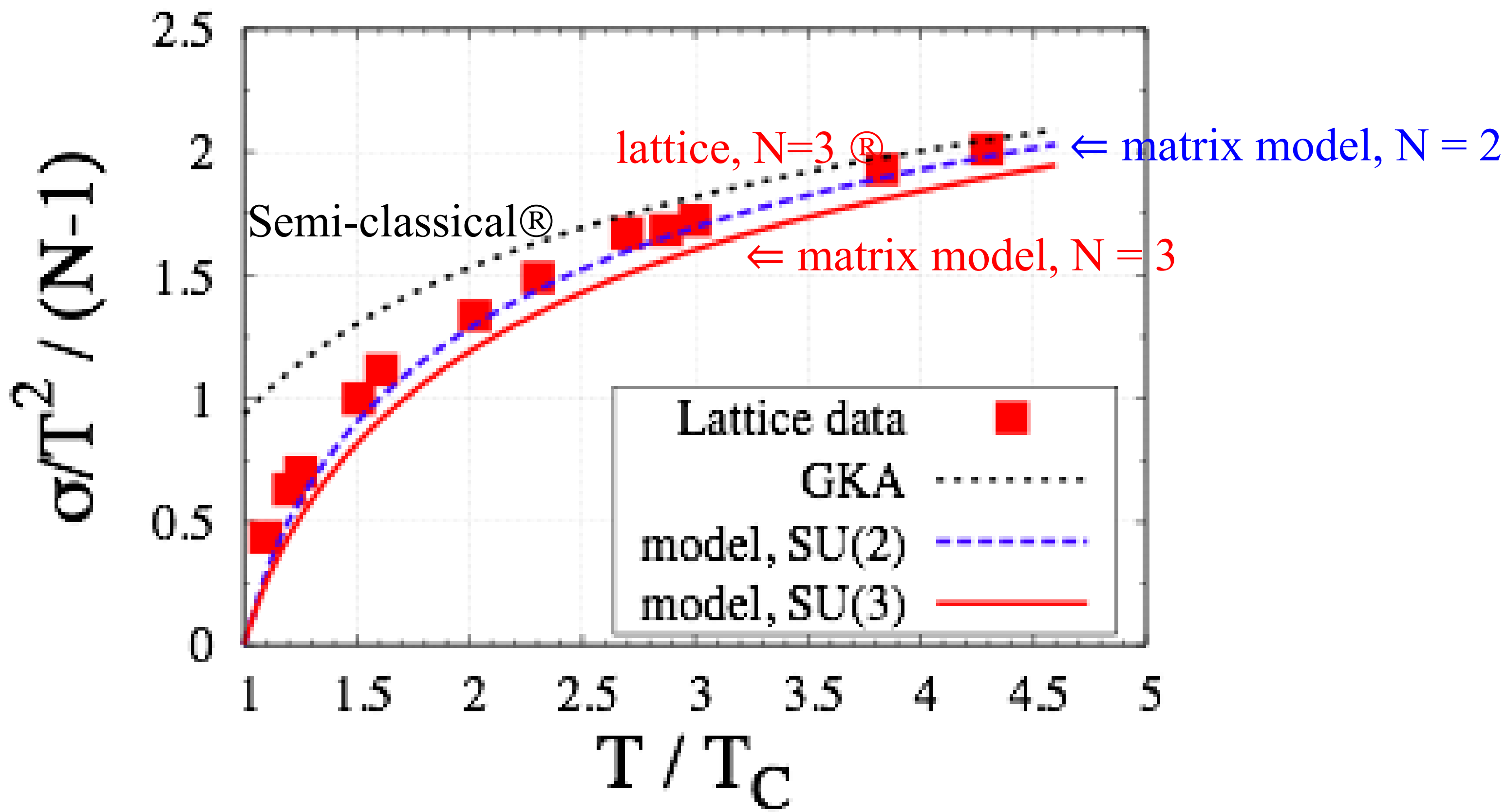
Again, for T > 1.2 T_c, the T^2 term in pressure due *entirely* to the *constant* term, c_3!
Rapid rise of $\langle l \rangle$ as with FRG. **(Braun, Gies & Pawlowski, arXiv: 0708.2413)**



Interface tension, N = 3

Order-order interface tension, σ, from matrix model close to lattice.

$\sigma(T_c)/T_c^2$ nonzero but *small*, ~ .02. Results for N =2 and N = 3 similar - ?



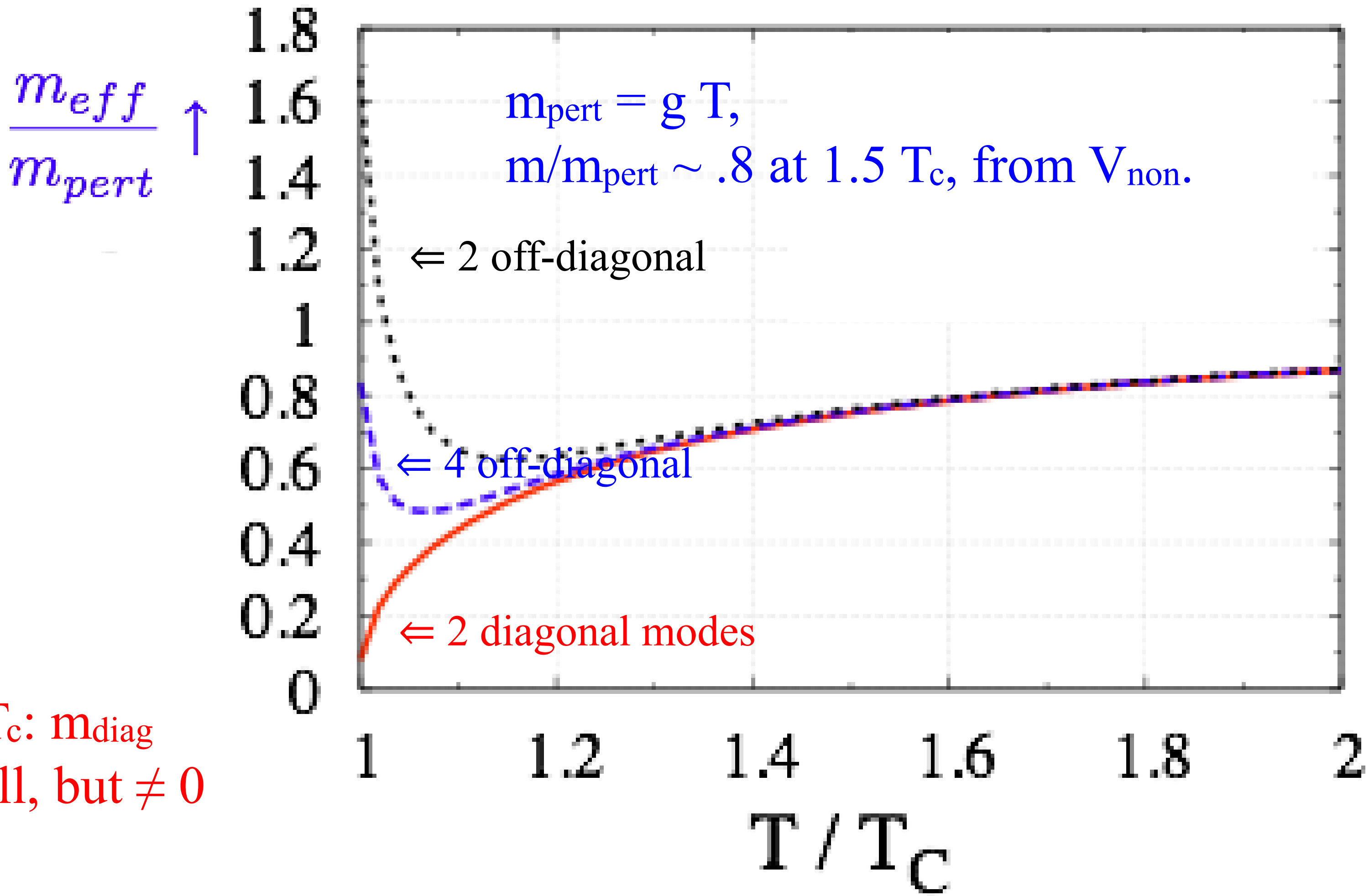
Lattice:

(de Forcrand, D'Elia & Pepe, hep-lat/0007034, de Forcrand & Noth, hep-lat/0506005)

Adjoint Higgs phase, N = 3

Splitting of masses only for $T < 1.2\ T_c$:

Measureable from singlet potential, $\langle \mathrm{tr}\, L^\dagger(x)\, L(0) \rangle$, over *all* x.



At T_c: m_{diag} small, but $\neq 0$

Matrix model for N $\geq$ 3

Latent heat, $e(T_c)/T_c^4$: 1-parameter model too small:
1-para.: 0.33. BPK: 1.40 $\pm$ .1; DG: 1.67 $\pm$ .1.

Latttice latent heat: **(Beinlich, Peikert & Karsch (BPK), hep-lat/9608141; Datta & Gupta (DG), arXiv:1006.0938)**

To get the latent heat right, two parameter model.

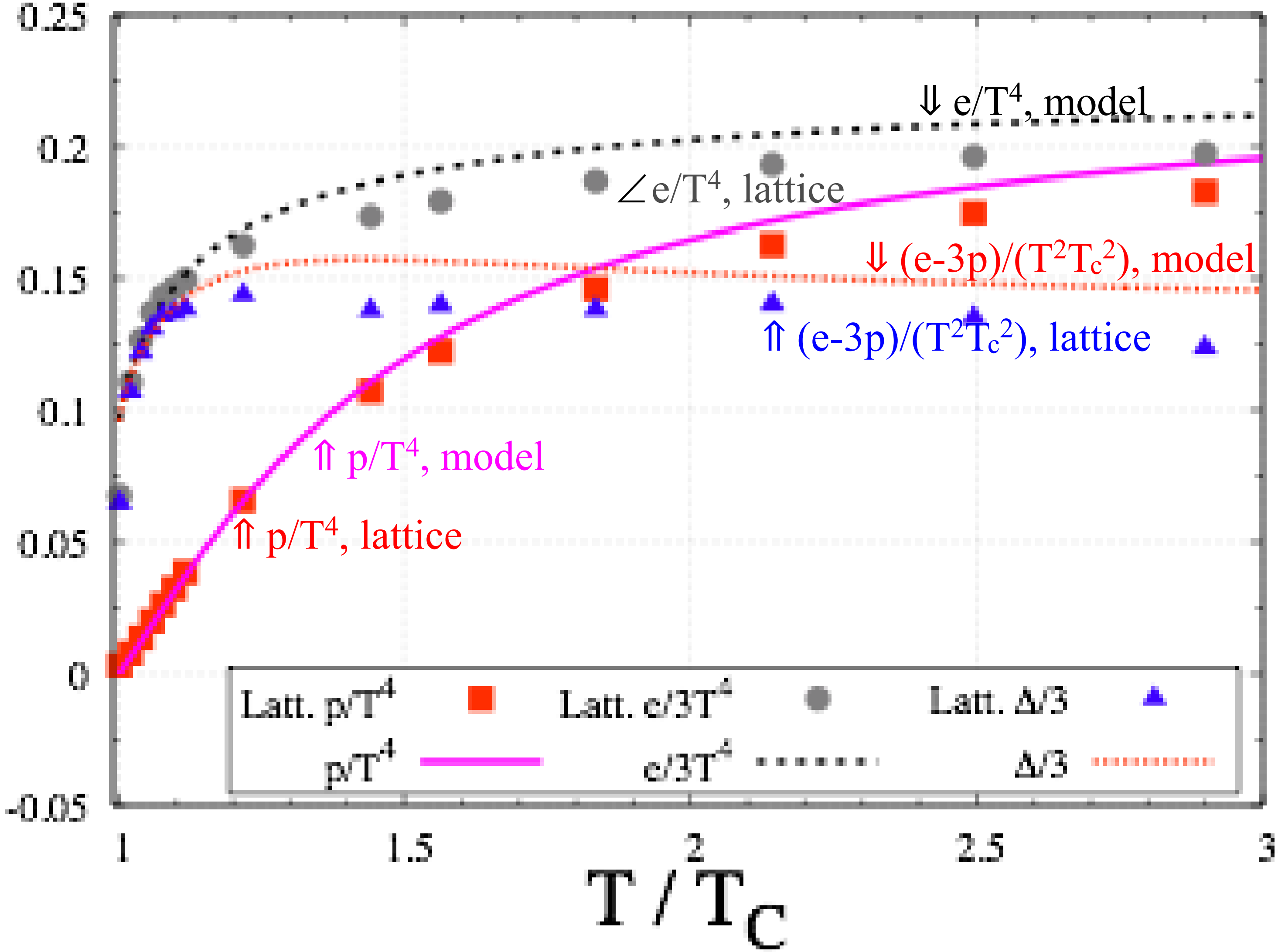
$$c_3(T) = c_3(\infty) + \frac{c_3(1) - c_3(\infty)}{t^2} \, , \, t = \frac{T}{T_c}$$

2-parameter model, $c_3(T)$. Like MIT bag constant.
WHOT: $c_3(1) \sim 1$. Fit $c_3(1)$ to DG latent heat.

Thermodynamics, order-disorder interface tensions improve.

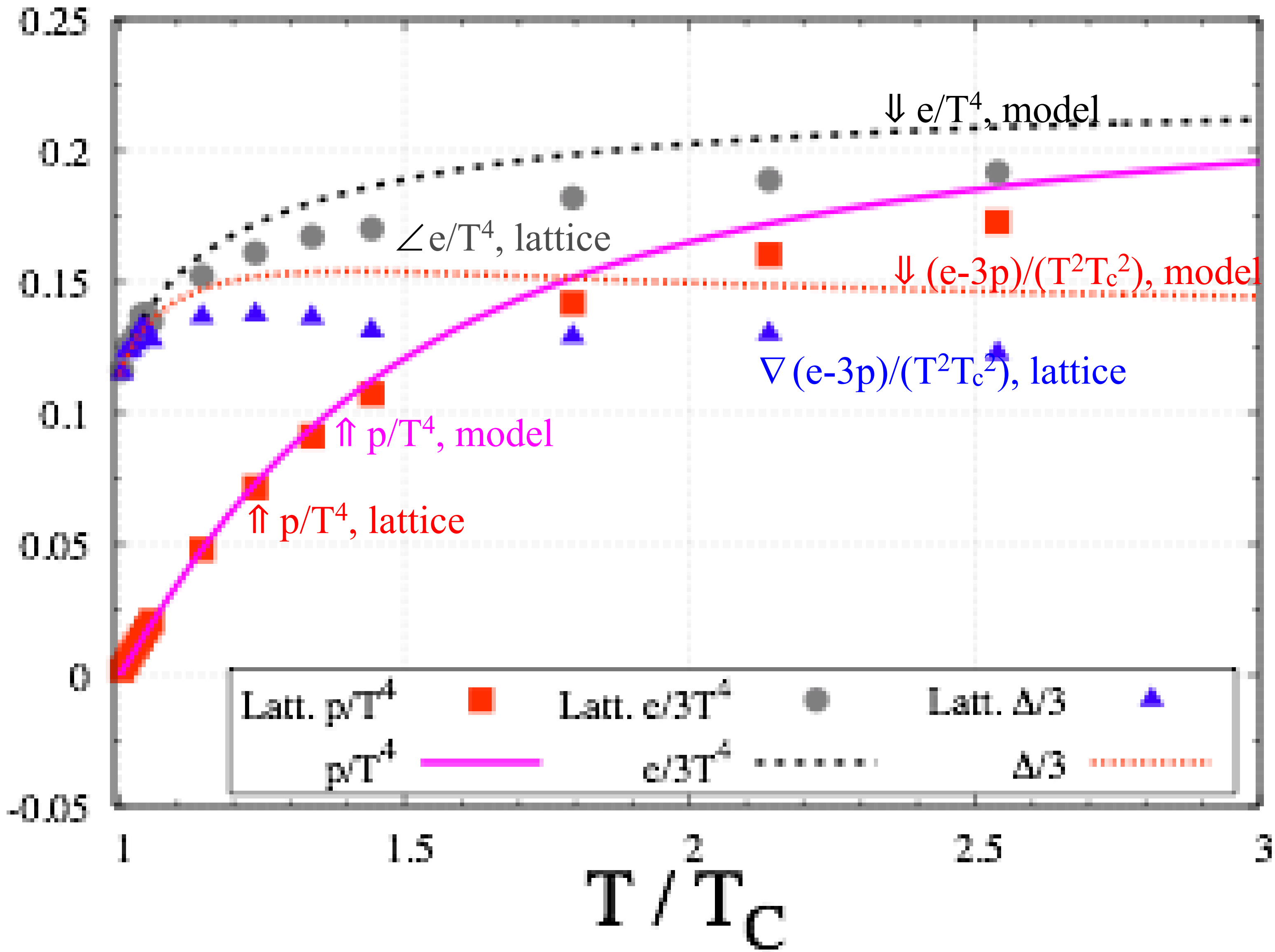
Thermodynamics of 2-parameter model, N = 4

$c_3(1) = 1.38\,,\ c_3(\infty) = .95\,,\ c_1 = 1.025\,,\ c_2 = 0.39$



Thermodynamics of 2-parameter model, N = 6

$c_3(1) = 1.42\,,\ c_3(\infty) = .95\,,\ c_1 = 1.21\,,\ c_2 = 0.23$



Conclusions

We construct an effective model as a function of the expectation values of the Polyakov loop.

We fixed the parameters in this model by fitting the lattice data of conformal anomaly, then use the model to compute other quantities.

Transition region: from model $< 1.2\ T_c$!
from lattice data $< 4\ T_c$...

Above $1.2\ T_c$, pressure dominated by *constant* term $\sim T^2$. (due to small expectation value of q.)

Need to include quarks!

Can then compute temperature dependence of:

shear viscosity, energy loss of light quarks, damping of quarkonia...

Thank You for Your Attention

Backup

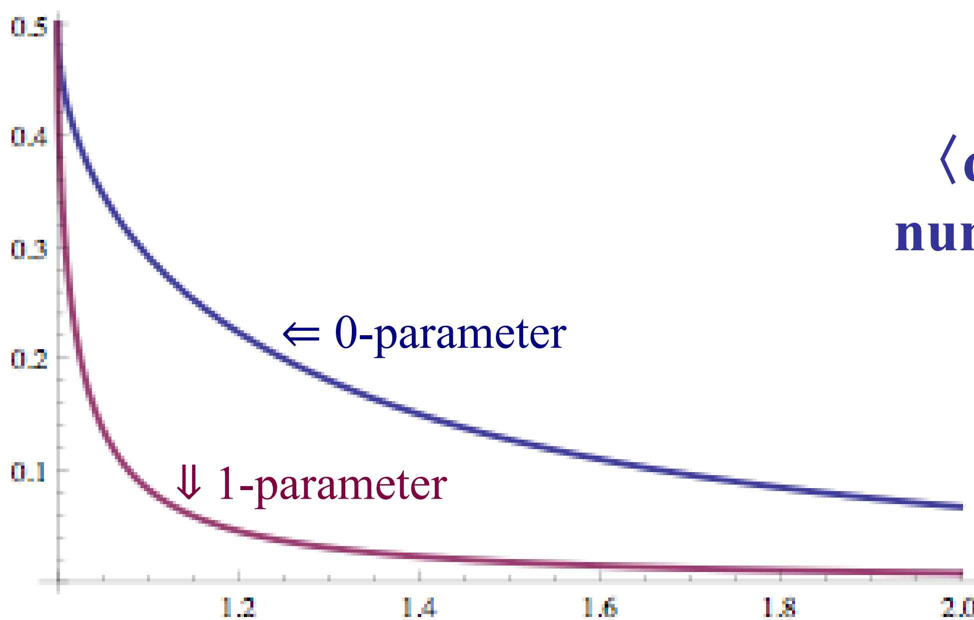
Width of transition region, 0- vs 1-parameter

1-parameter model:

⟨q⟩ ! 0 *much* quicker above T_c ⟶ sharper (e-3p)/T^4

Physically: sharp (e-3p)/T^4 implies region where ⟨q⟩ is significant is *narrow*.

$\langle q \rangle \uparrow$



T/Tc

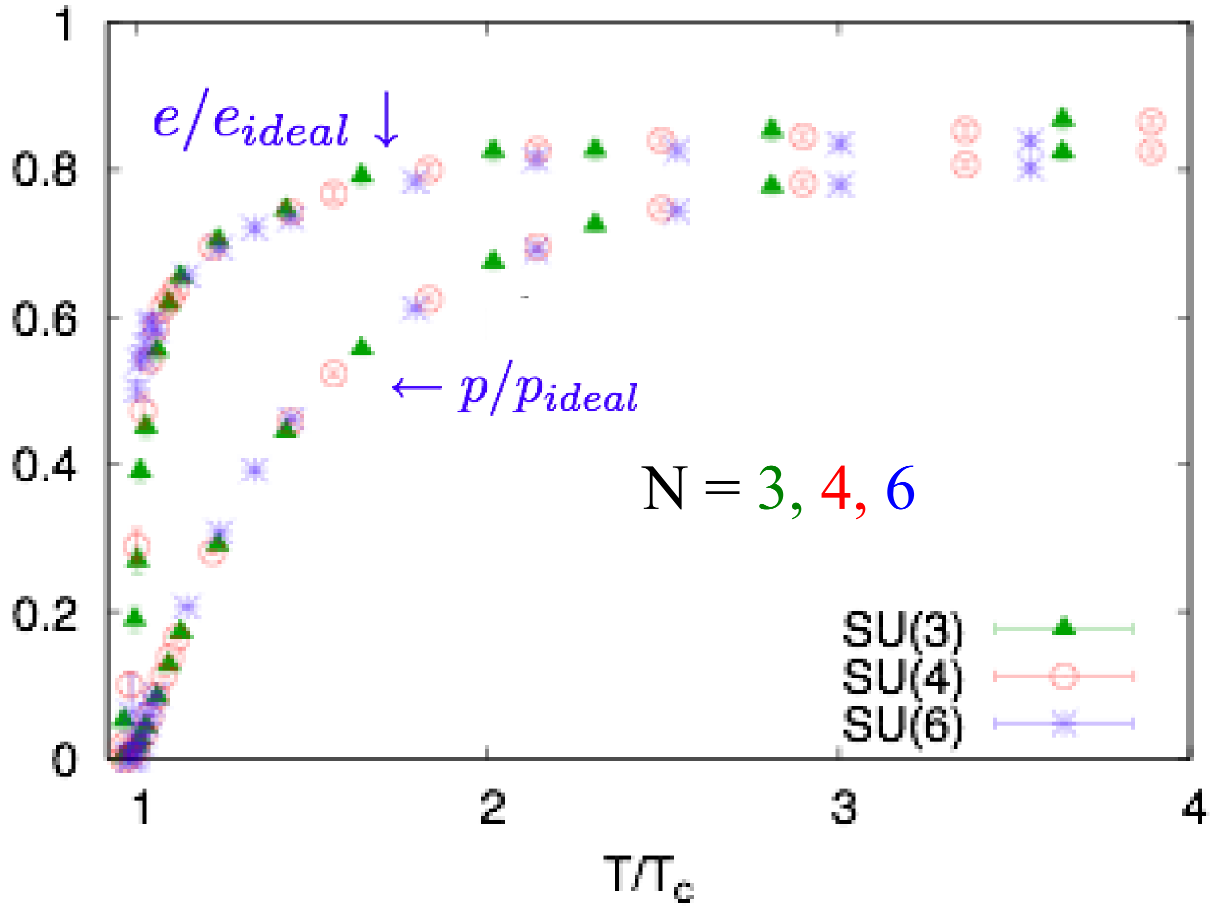
⟨q⟩ ≠ 0 at all T, but numerically, *negligible* above ~ 1.2 T_c

Above ~1.2 T_c, the T^2 term in the pressure is due *entirely* to the *constant* term, c_3! This agrees with the lattica data (WHOT).

$$p(T) \approx \#\,(T^4 - c\,T^2 T_c^2)\,,\; T/T_c : 1.2 \to 2.0$$

Lattice data for N ≥ 3

Scaled by ideal gas values, e and p for N = 3, 4 and 6 look *very* similar

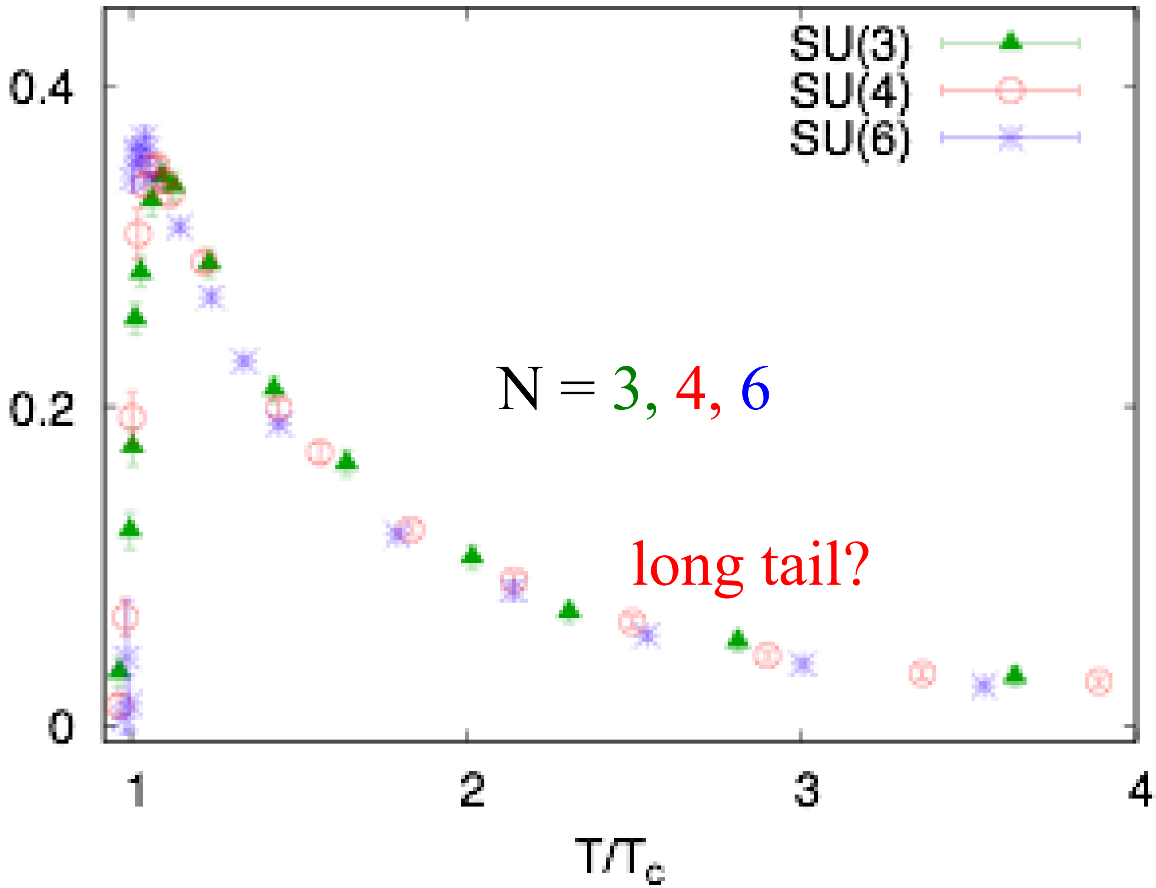


N=3: (**Boyd et al, hep-lat/9602007**); N = 4 & 6: (**Datta & Gupta, arXiv:1006.0938**)

Conformal anomaly ≈ N independent

For SU(N), "peak" in (e-3p)/T^4 just above T_c. *Approximately* uniform in N.

$$\frac{1}{N^2-1}\,\frac{e-3p}{T^4}\uparrow$$

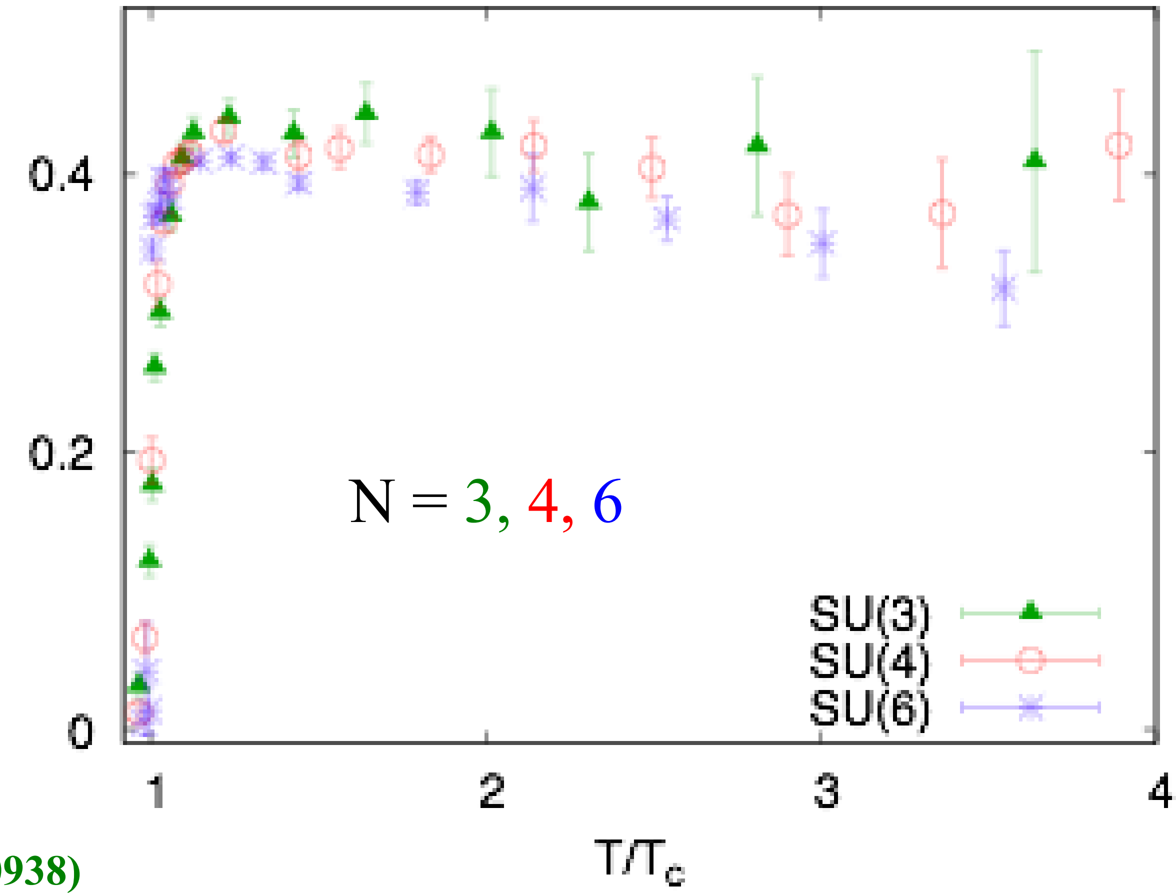


(Datta & Gupta, arXiv:1006.0938)

Tail in the conformal anomaly

To study the tail in $(e-3p)/T^4$, multiply by T^2 (divided by $(N^2-1)\,T_c^2$):
$(e-3p)/T^2$ *approximately* constant

$$\frac{1}{N^2-1}\,\frac{e-3p}{T^2\,T_c^2}\uparrow$$



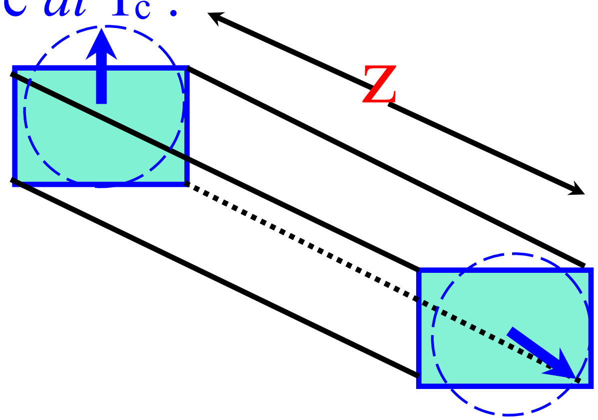
(Datta & Gupta, arXiv:1006.0938)

Interface tensions: order-order & order-disorder

Interface tension: box long in z.
Each end: distinct but *degenerate* vacua.
Interface forms, action ~ interface tension:

$T > T_c$: order-order interface = '*t Hooft loop*:
measures response to *magnetic* charge
(Korthals-Altes, Kovner & Stephanov, hep-ph/9909516)

Also: *if* trans. 1st order, order-*dis*order interface *at* T_c .



$$Z \sim \mathrm{e}^{-\sigma_{int} V_{tr}}$$