Refractive Index of Light in the QGP with the HTL perturbation Theory

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arXiv:1109.4083

Motivations

- QGP is believed as a new state of matter of the strong interaction
- Transport coefficients for the QGP, e.g. viscosity (stress-stress), conductivity (current-current)…
- The Refractive Index (RI) of the QGP is a theoretical motivated observable for the current-current correlation function
- Negative RI has been predicted by AdS/CFT in strongly coupled system with finite chemical potential
- Question: What does the RI behaves in QGP at temperature several times higher than the critical temperature where the perturbative approach is thought to be trustable ?

Refractive Index of Light

Refractive Index in a medium

The Refractive Index (RI) measures the speed of light in the medium w.r.t. the vacuum.

n = speed of light in vacuum / speed of light in medium

The medium effect: the interacting of photon with the medium gives rise to a correction to the vacuum action

$$S_{eff} = -\frac{1}{2}\int \frac{d^4K}{(2\pi)^4}\left[\tilde{E}^{\mu}\tilde{E}_{\mu} - \tilde{B}^{\mu}\tilde{B}_{\mu}\right] - \frac{1}{2}\int \frac{d^4K}{(2\pi)^4} A^{\mu}\Pi_{\mu\nu}A^{\nu} + \ldots$$

$$\Pi_{\mu\nu}(K) = -i\int d^4x e^{-iK\cdot x}\theta(t)\langle[J_{\mu}(x), J_{\nu}(0)]\rangle$$

Framework: the electric permittivity ε and magnetic permeability μ can be read out from the S_{eff} , and gives the RI by $n^2 = \varepsilon\mu$

Depine-Lakhtakia Index

The ordinary refractive index $n^2 = \varepsilon\mu$ is quadratic defined

The phase velocity: $v_p = \frac{\omega}{k}\hat{k} = \frac{1}{\mathrm{Re}(n)}\hat{k}$

The energy flow: $S = v_g U\hat{k} = \frac{d\omega}{dk}U\hat{k}$

The sign of the n represents the direction of phase velocity w.r.t. k, but the group velocity is not affected by the sign, so the direction of phase- and group- velocity could be opposite, when

$$v_p < 0, v_g > 0$$

which equivalent to the negative value of the Depine-Lakhtakia index (DL index)

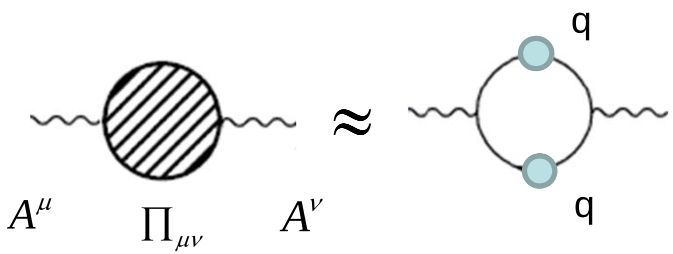
$$n_{DL} = |\varepsilon|\mathrm{Re}(\mu) + |\mu|\mathrm{Re}(\varepsilon) < 0$$

The DL index encodes the extra information of the relative directions of phase- and group- velocity.

The Hard-Thermal-Loop Perturbation theory

The HTL self-energy

The QGP is thought to be weakly coupled in the temperature region far from the critical temperature, the Hard-Thermal-Loop approximation is trustable.



$$\approx \ldots + \ldots = \quad \Pi_{\mu\nu}(K) = g^2 \int \frac{d^4P}{(2\pi)^4} Tr\left[\gamma_\mu S(K-P)\gamma_\nu S(P)\right]$$

The quark loops are hard when $T \gg k, \omega$ the integral can be obtained

$$\Pi_{\mu\nu} = \Pi_T P_{\mu\nu} + \Pi_L Q_{\mu\nu}$$

The transverse and longitudinal projectors:

$$P_{\mu\nu} = g_{\mu\nu} - u_\mu u_\nu + \frac{1}{k^2}\left(K_\mu - \omega u_\mu\right)\left(K_\nu - \omega u_\nu\right)$$

$$Q_{\mu\nu} = \frac{-1}{K^2 k^2}\left(\omega K_\mu - K^2 u_\mu\right)\left(\omega K_\nu - K^2 u_\nu\right)$$

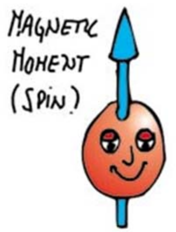
$$\Pi_T = \frac{1}{2} m_D^2 \left[\frac{\omega^2}{k^2} + \left(1 - \frac{\omega^2}{k^2}\right)\frac{\omega}{2k}\log\frac{\omega+k}{\omega-k}\right]$$

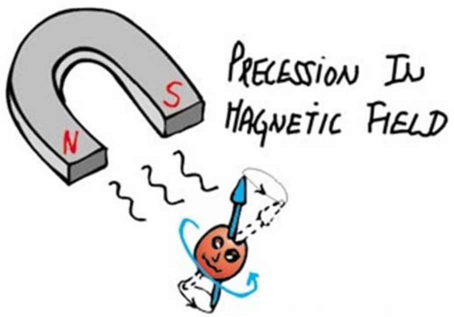
$$\Pi_L = m_D^2 \left(1 - \frac{\omega^2}{k^2}\right)\left[1 - \frac{\omega}{2k}\log\frac{\omega+k}{\omega-k}\right]$$

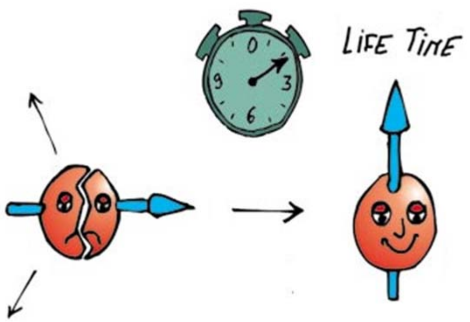
Debye mass: $m_D^2 \equiv e^2 N_c \sum_f \left(\frac{1}{3}T^2 + \frac{\mu_f^2}{\pi^2}\right) Q_f^2$

Two Types of Plasmas

The magnetic response of a plasma to the external magnetic fields is crucial for considering a electromagnetic wave propagating in it.







(1) Magnetizable Plasma:

The magnetic moment density is meaningful, so does the magnetic permeability.

$$S_{eff} = -\frac{1}{2}\int \frac{d^4K}{(2\pi)^4}\left[\varepsilon \cdot \tilde{E} \cdot \tilde{E} - \frac{1}{\mu}\tilde{B} \cdot \tilde{B}\right]$$

(2) Non-magnetizable Plasma:

The magnetic moment density loses its physical meaning. The magnetic permeability is set to be unity.

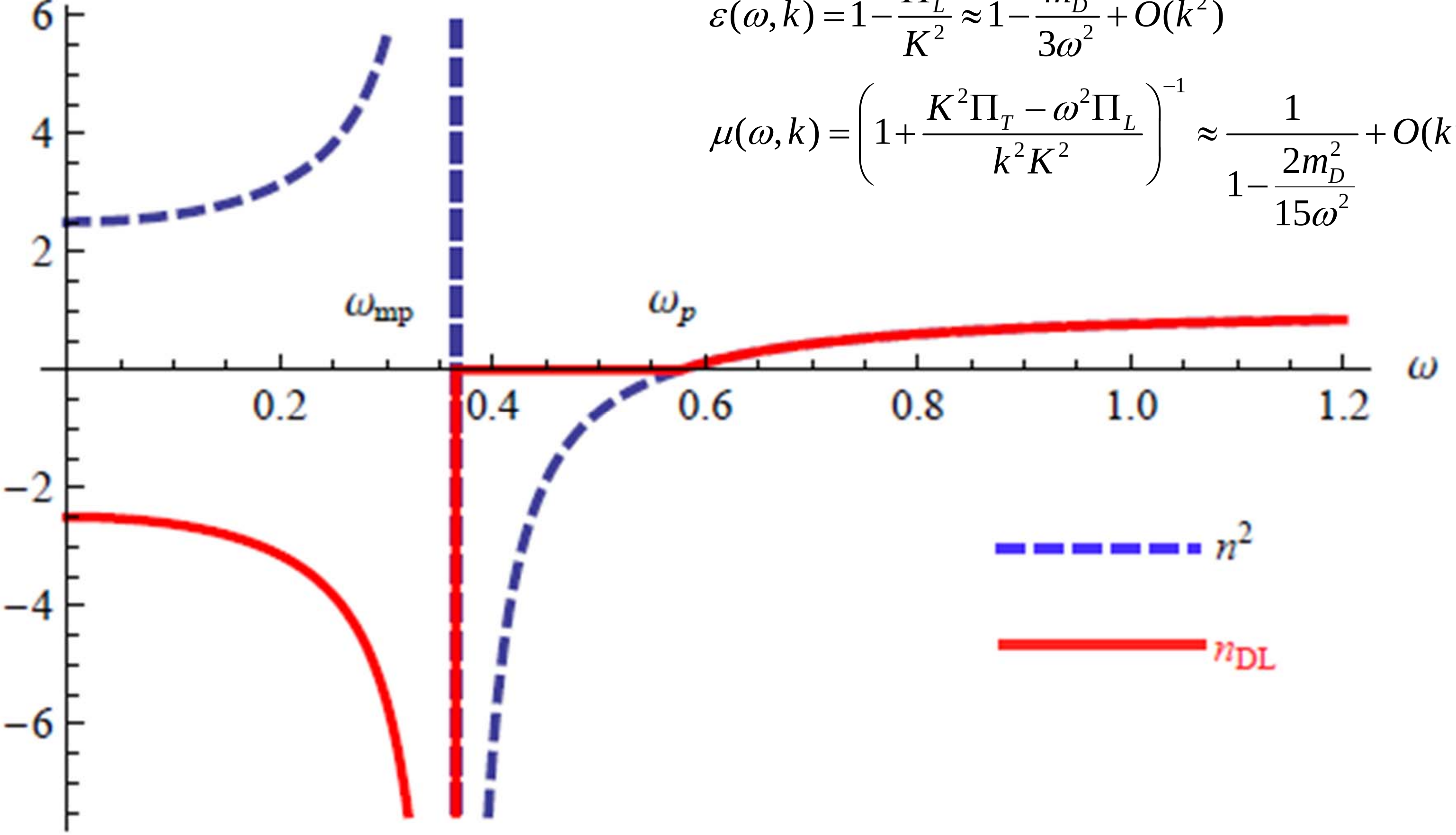
$$S_{eff} = -\frac{1}{2}\int \frac{d^4K}{(2\pi)^4}\left[\tilde{\varepsilon} \cdot \tilde{E} \cdot \tilde{E} - \tilde{B} \cdot \tilde{B}\right] \qquad \tilde{\varepsilon} = \varepsilon + \frac{k^2}{\omega^2}\left[1 - \frac{1}{\mu}\right]$$

Magnetizable Plasma

Small spatial dispersion

$$\varepsilon(\omega,k)=1-\frac{\Pi_L}{K^2}\approx 1-\frac{m_D^2}{3\omega^2}+O(k^2)$$

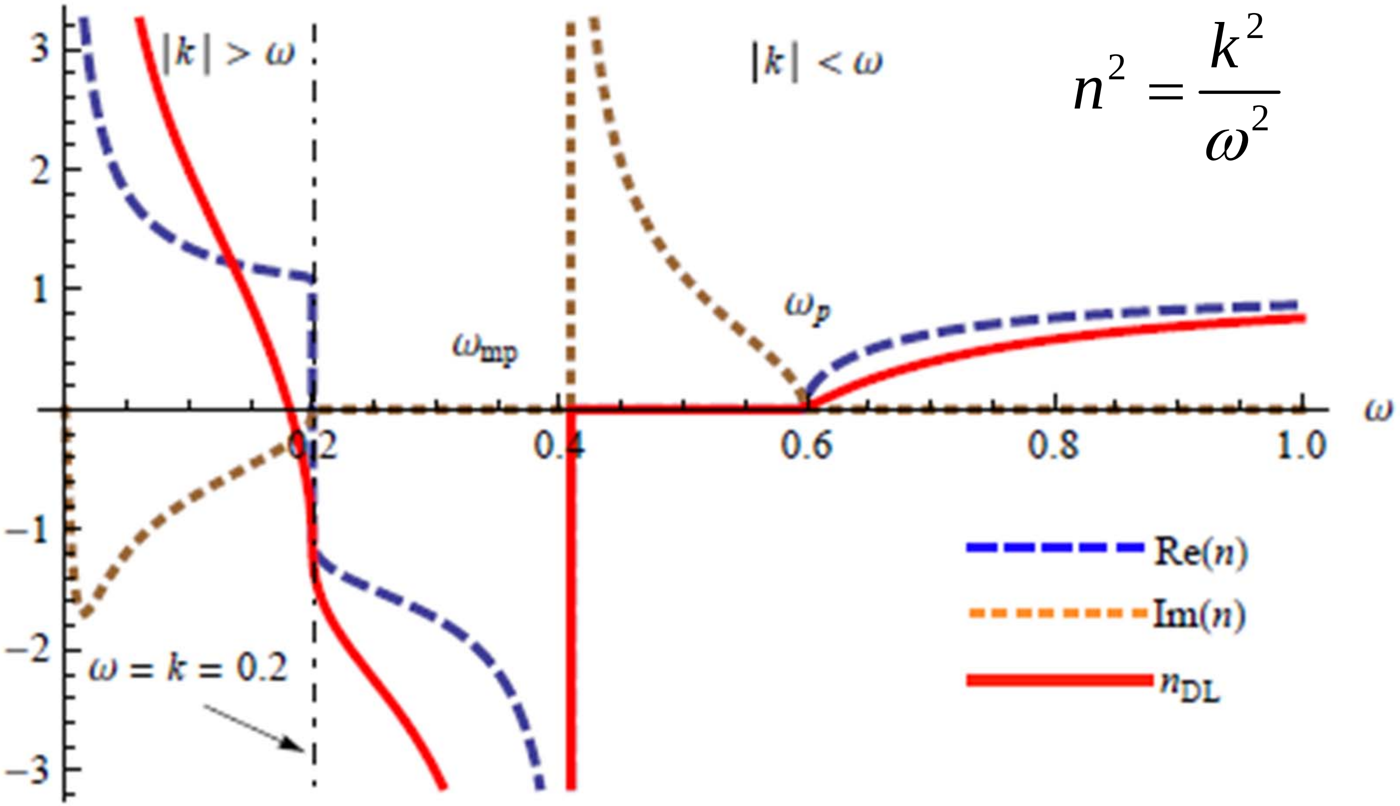
$$\mu(\omega,k)=\left(1+\frac{K^2\Pi_T-\omega^2\Pi_L}{k^2K^2}\right)^{-1}\approx\frac{1}{1-\frac{2m_D^2}{15\omega^2}}+O(k^2)$$



Frequency structure of RI

dispersion relation:

$$n^2 = \frac{k^2}{\omega^2}$$

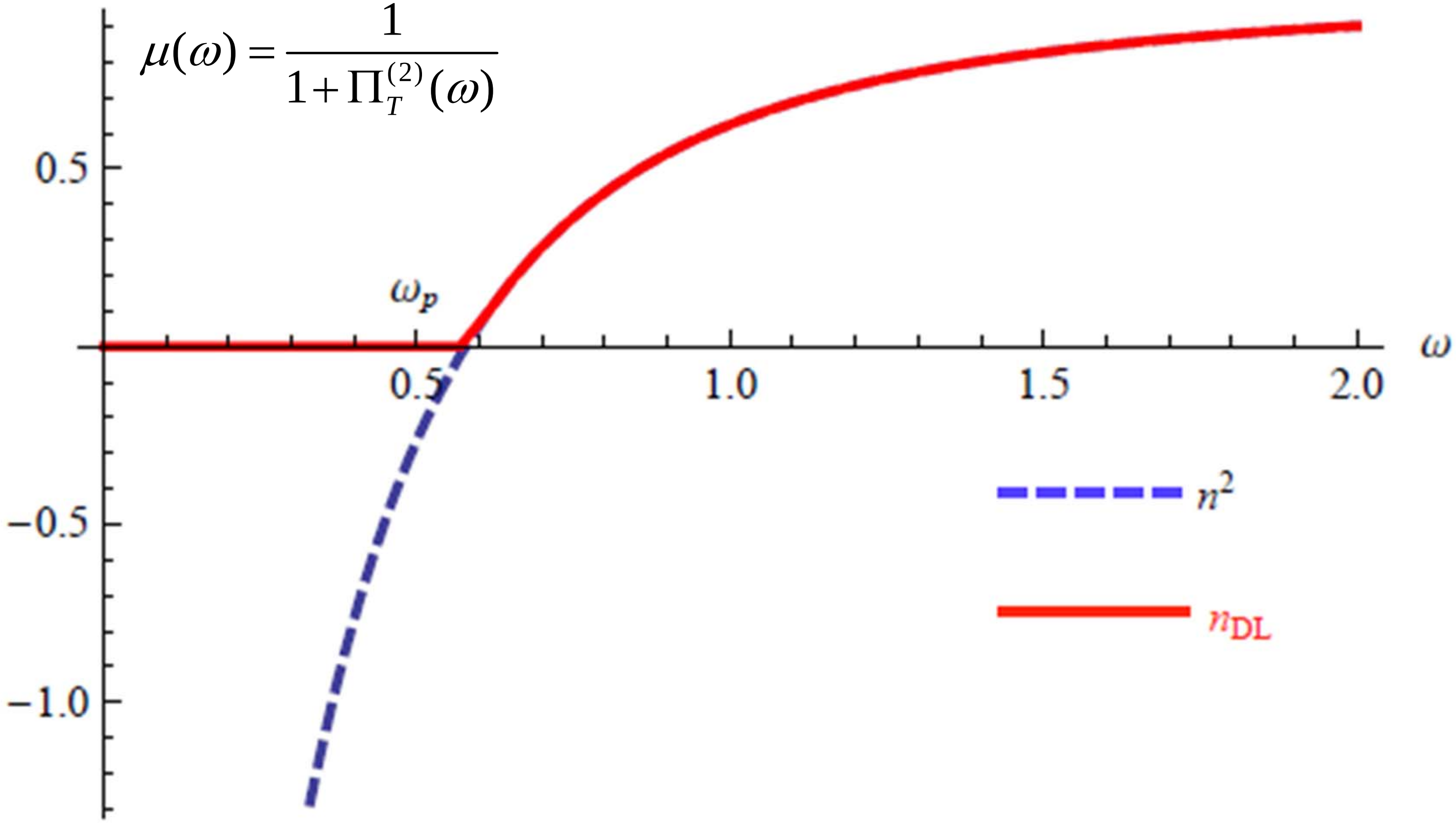


Non-magnetizable Plasma

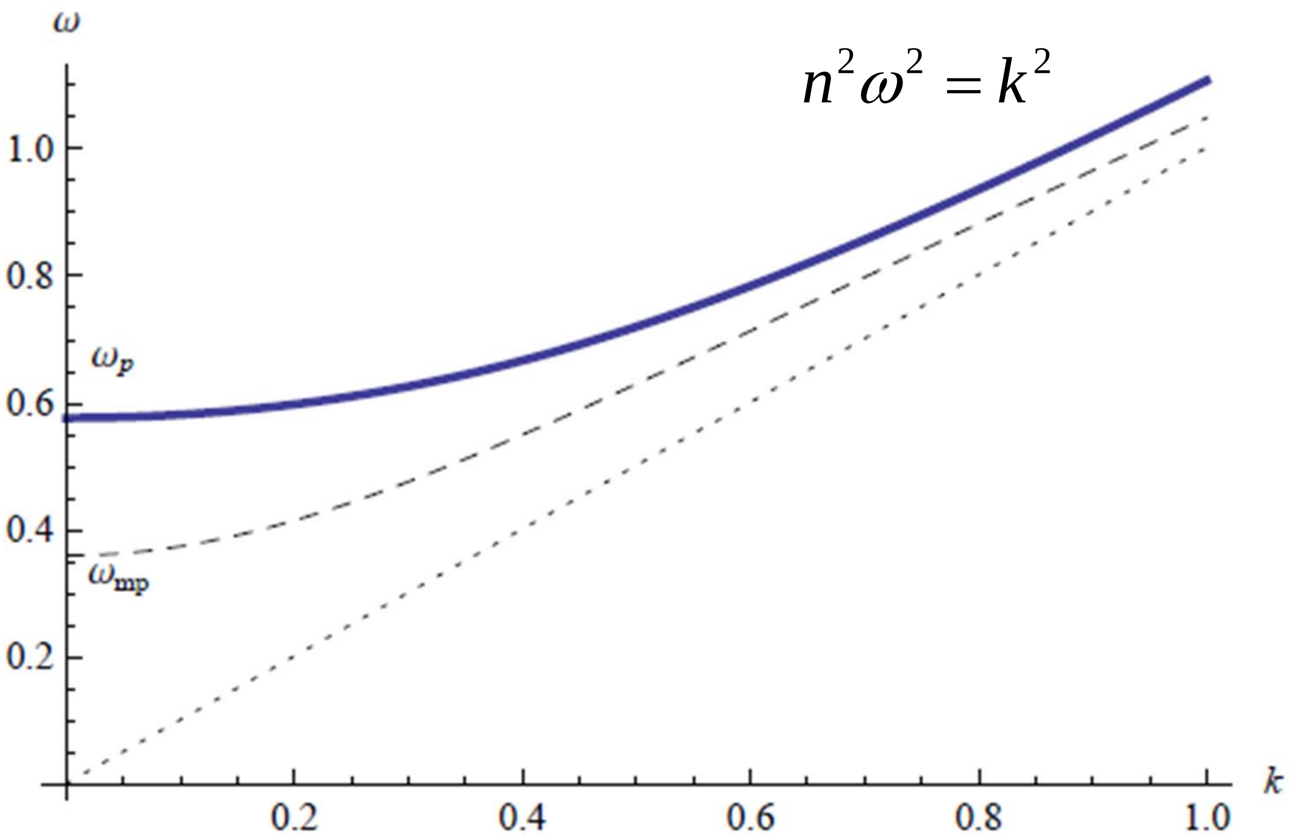
Small spatial dispersion

$$\varepsilon(\omega) = 1 - \frac{\Pi_T^{(0)}(\omega)}{\omega^2} \qquad \Pi_T(\omega,k) = \Pi_T^{(0)}(\omega) + k^2 \Pi_T^{(2)}(\omega) + O(k^4)$$

$$\mu(\omega) = \frac{1}{1 + \Pi_T^{(2)}(\omega)}$$

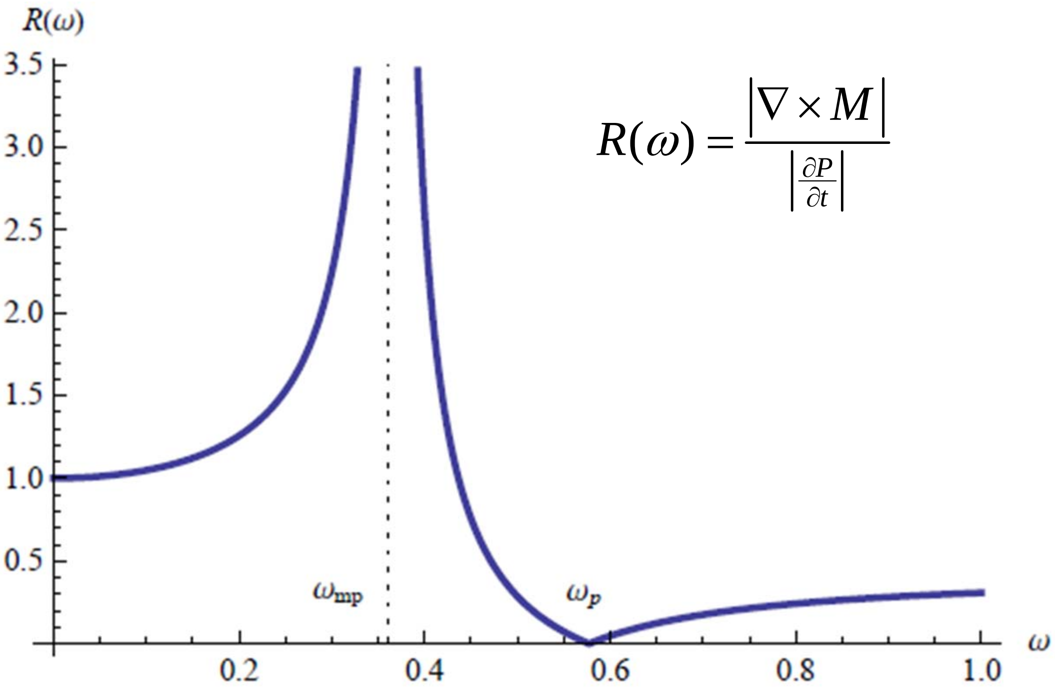


Dispersion Relation



A criterion: The Induced Current

The electromagnetic induced current : $J = \nabla \times M + \frac{\partial P}{\partial t}$



$$R(\omega) = \frac{\left|\nabla \times M\right|}{\left|\frac{\partial P}{\partial t}\right|}$$

When $R(\omega) \gg 1$ the induced current is dominated by the magnetization, but the dielectric polarization, the magnetic moment is physically meaningful.

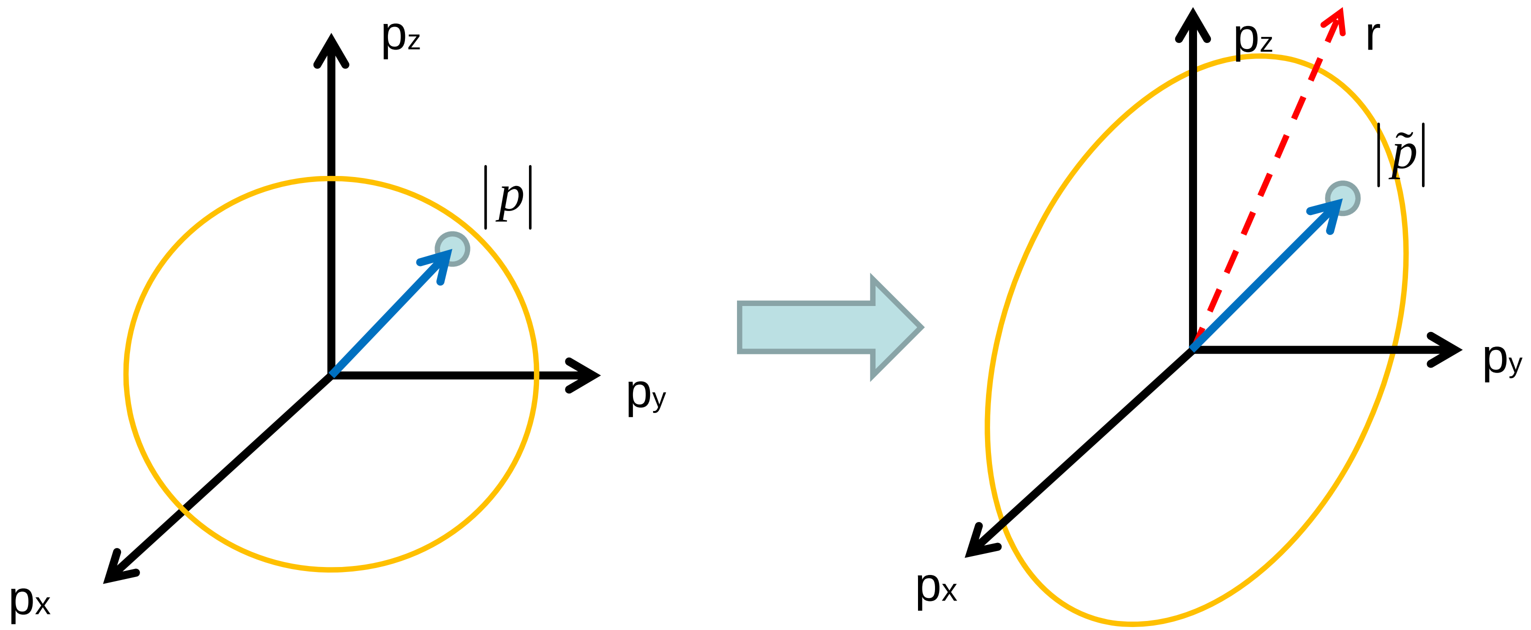
Anisotropic Quark Matter

Anisotropic self-energy (I)

The self-energy of photon in terms of distribution function can be expressed as:

$$\Pi_{\mu\nu}(K) = -e^2 N_c \sum_a Q_a^2 \int \frac{d^3p}{(2\pi)^3} V_\mu \frac{\partial f_a(p)}{\partial p_i}\left(g_{\nu i} - \frac{K_i V_\nu}{K\cdot V + i\eta}\right)$$

Do the replacement: $p \rightarrow \tilde{p} = \sqrt{p^2 + \xi(p\cdot r)^2}$



Anisotropic self-energy (II)

Small ξ expansion,

$$\Pi_{ij} = -m_D^2 \int \frac{d\Omega}{4\pi} \frac{v_l + \xi (v\cdot r) r_l}{\left[1+\xi (v\cdot r)^2\right]^2} v_i \left(\delta_{jl} + \frac{k_l v_j}{K\cdot V + i\eta} \right)$$

$$= \Pi_{ij}^{(HTL)} + \xi m_D^2 \int \frac{d\Omega}{4\pi} \left[(v\cdot r) r_l - 2 (v\cdot r)^2 v_l \right] v_i \left(\delta_{jl} + \frac{k_l v_j}{K\cdot V + i\eta} \right) + O(\xi^2)$$

4 tonsorial basis,

$$\left\{ P_{\mu\nu}, Q_{\mu\nu}, C_{\mu\nu}, G_{\mu\nu} \right\}$$

$$C_{\mu\nu} = \frac{\tilde{r}_\mu \tilde{r}_\nu}{\tilde{r}^2}, G_{\mu\nu} = \left(K_\mu \tilde{r}_\nu + K_\nu \tilde{r}_\mu \right) - \frac{K^2}{\omega} \left(\tilde{r}_\mu u_\nu + \tilde{r}_\nu u_\mu \right) \qquad \tilde{r}_\mu = P_{\mu\nu} r^\nu$$

Projections,

$$\Pi_{\mu\nu} = \alpha P_{\mu\nu} + \beta Q_{\mu\nu} + \gamma C_{\mu\nu} + \delta G_{\mu\nu}$$

Anisotropic self-energy (III)

The structure functions:

$$\alpha = \Pi_T(z) + \xi \left\{ \frac{z^2}{12} (3 + 5\cos 2\theta)\, m_D^2 - \frac{1}{6} (1 + \cos 2\theta)\, m_D^2 + \frac{1}{4}\Pi_T(z) \left[(1 + 3\cos 2\theta) - z^2 (3 + 5\cos 2\theta) \right] \right\},$$

$$\beta = \Pi_L(z) + \xi \left\{ \frac{1}{6}(z^2 - 1)(1 + 3\cos 2\theta) m_D^2 + \Pi_L(z) \times \left(\cos 2\theta - \frac{z^2}{2}(1 + 3\cos 2\theta) \right) \right\},$$

$$\gamma = \frac{\xi}{3}[3\Pi_T(z) - m_D^2](z^2 - 1)\sin^2\theta,$$

$$z = \omega / k \qquad \cos\theta = k \cdot r$$

An analytic solvable result

The RI in an anisotropic case is in general a 2-rank tensor,

Magnetizable plasma:

$$\varepsilon_{ij} = \left(1 - \frac{\beta}{K^2}\right)\delta_{ij} - \frac{\gamma}{\omega^2}\frac{r_{Ti}r_{Tj}}{r_T^2}$$

The left- and right- rotating photon

$$\varepsilon_L = 1 - \frac{\beta}{K^2}$$

$$\varepsilon_R = 1 - \frac{\beta}{K^2} - \frac{\gamma}{\omega^2}$$

$$\frac{1}{\mu} = 1 + \frac{K^2\alpha - \omega^2\beta}{k^2K^2}$$

Non-magnetizable plasma:

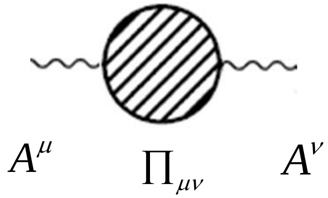
$$\tilde{\varepsilon}_{ij} = \left(1 - \frac{\alpha}{\omega^2}\right)\delta_{ij} - \frac{\gamma}{\omega^2}\frac{r_{Ti}r_{Tj}}{r_T^2}$$

The left- and right- rotating photon

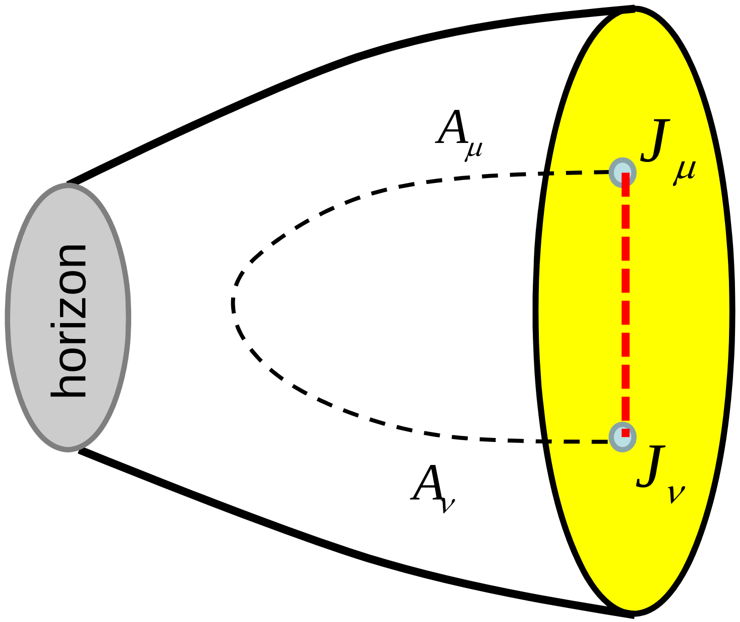
$$\tilde{\varepsilon}_L = 1 - \frac{\alpha}{\omega^2}$$

$$\tilde{\varepsilon}_R = 1 - \frac{\alpha}{\omega^2} - \frac{\gamma}{\omega^2}$$

Holographic Approach



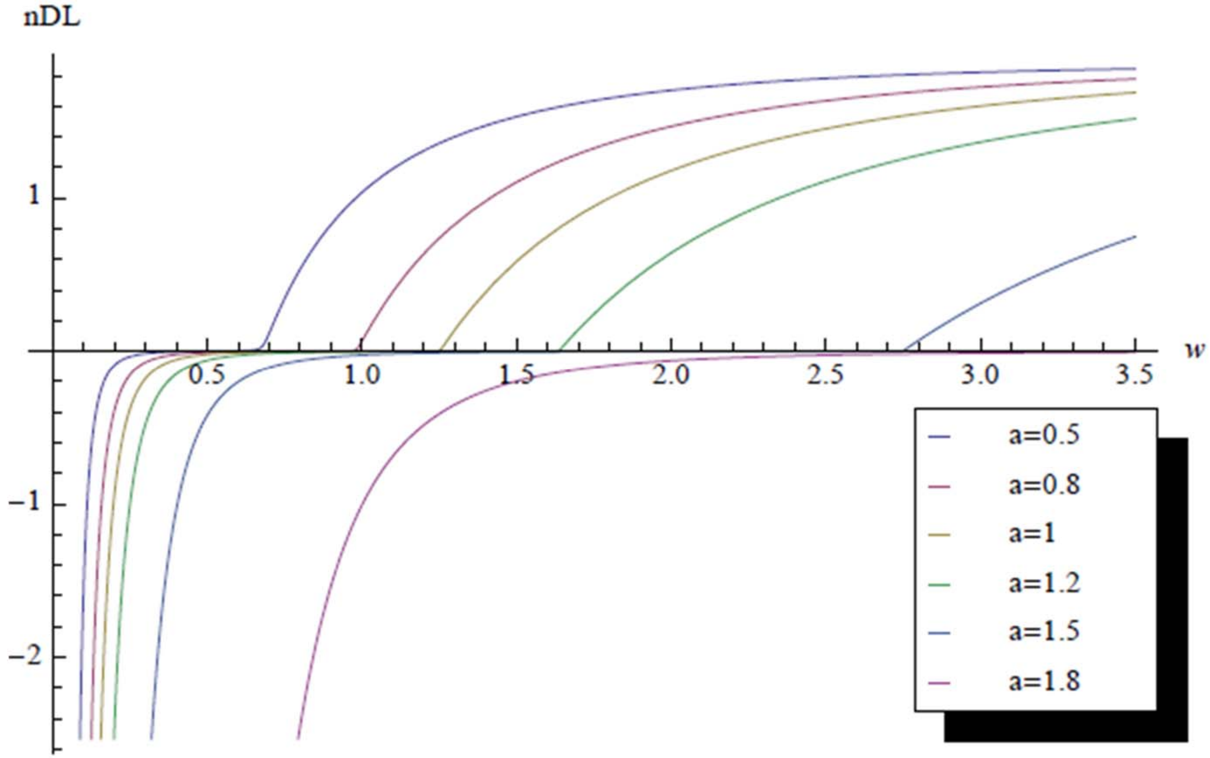
$$= \Pi_{\mu\nu}(K) = -i\int d^4x e^{-iK\cdot x}\theta(t)\langle[J_\mu(x), J_\nu(0)]\rangle$$



$$\Pi_{xx} = \frac{i\omega R}{e^2}\left(\frac{3a}{2b^2\left[2i\omega(1+a) - bk^2\right]} - \frac{(2-a)^2}{8(1+a)^2 b}\right)$$

$$T = \frac{2-a}{4\pi b}, \qquad \Sigma = \frac{1}{2b}\sqrt{\frac{3}{2}a}$$

Holographic results



arXiv:1006.5714

Conclusions

- We study the refractive index of electromagnetic wave in a QGP within the HTL perturbation theory.
- We find two types of magnetic response of the plasmas depending on the ratio $R(\omega)$ of the contributions from the magnetic and electric part to the induced current.
- The magnetic permeability loses its physical meaning in a non-magnetizable plasma, but it is meaningful in a magnetizable one, which has a pole in the RI.
- There are complicated structure of RI below plasma frequency $\omega_p \sim \sqrt{\frac{1}{3}} m_D$ e.g. gap, screening, dissipation and dissipation-less…
- We do not find propagating modes with NRI in HTL perturbation theory which is found in holographic approach

Thank you for your attention!