MAX-PLANCK-INSTITUT
FÜR PHYSIK

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NNLO predictions for bottom quark pair production in MiNNLO$_{\mathrm{PS}}$

QCD@LHC2022
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Outlook

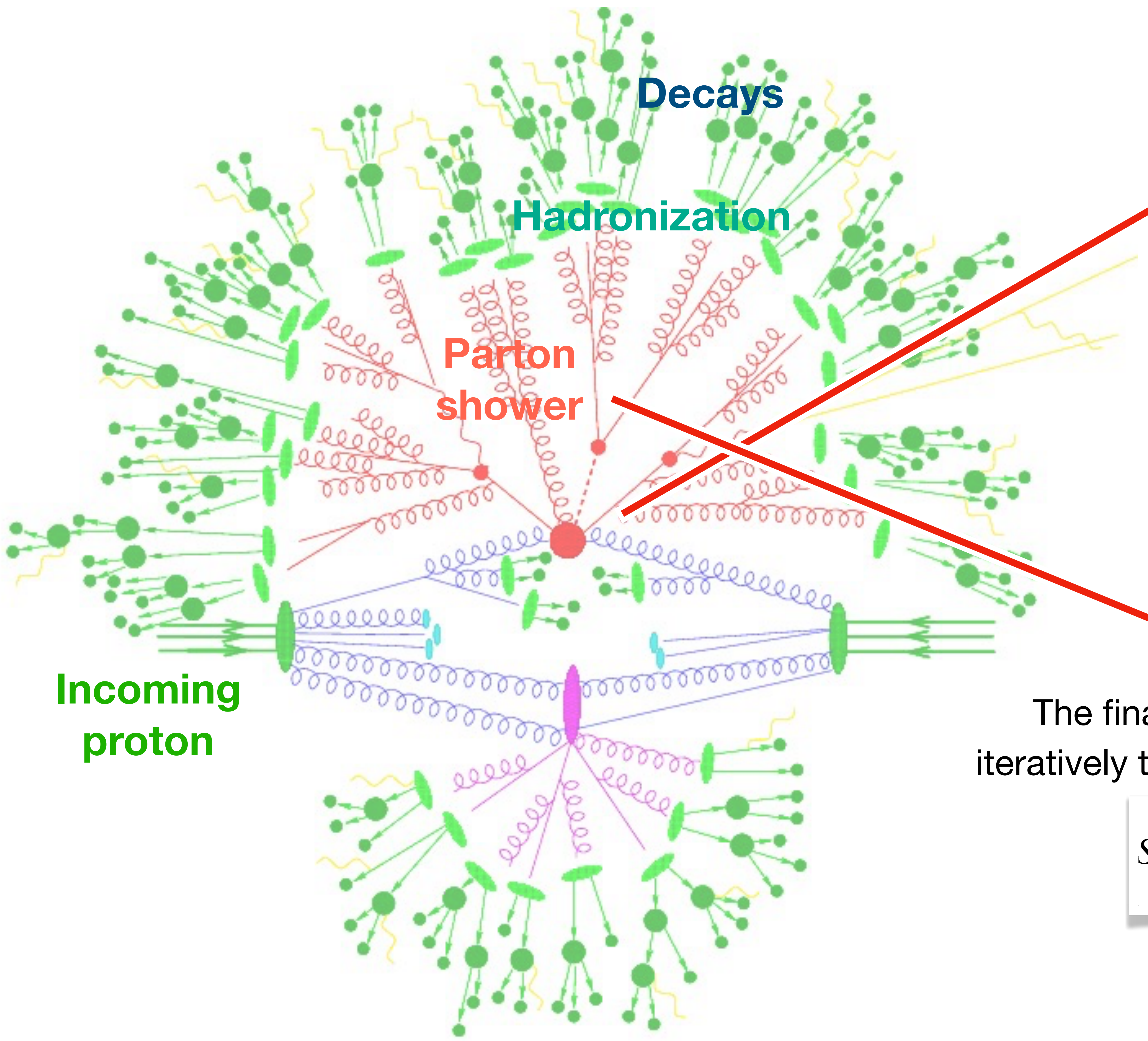
○ Event generation with MiNNLOps

- Basic aspects of the **POWHEG** framework (matching NLO+PS)
- Merging with no merging scale: the **MiNLO'** method
- The **MiNNLOps** method

○ Bottom pair production in MiNNLOps

- **Motivations**
- Settings and design of the code
- **Validation** against fixed order **NNLO** results from MATRIX
- Preliminary results for **B meson distributions** and comparison to LHC data
- Some final considerations

Events at LHC: theoretical perspective



Hard scattering

Described through the **factorization formula** for hadron collisions

$$\sigma = \sum_{a,b} \int dx_1 dx_2 \; f_{1,a}(x_1) f_{2,b}(x_2) \; \sigma_{ab}(x_1, x_2) \quad + \quad O(1/\Lambda)$$

Differential cross section for the elementary parton process (evaluated in perturbation theory)

The final state multiplicity for the hard scattering is then increased iteratively trough a **Parton shower algorithm** (down to a certain scale t_0)

$$S(t_I) \;=\; \Delta(t_I, t_0)\langle 1 | \,+\, \int_{t_0}^{t_I} dt \int_0^1 dz \; \Delta(t_I, t) \, F(t,z) \, S(z^2 t) \, S((1-z)^2 t)$$

$S(t_I) \cdot |\Psi\rangle$ ➡ Probability to generate some final state configuration from PS

Nason P. (2005)

Frixione S., Nason P., Oleari C. (2007)

The POWHEG method

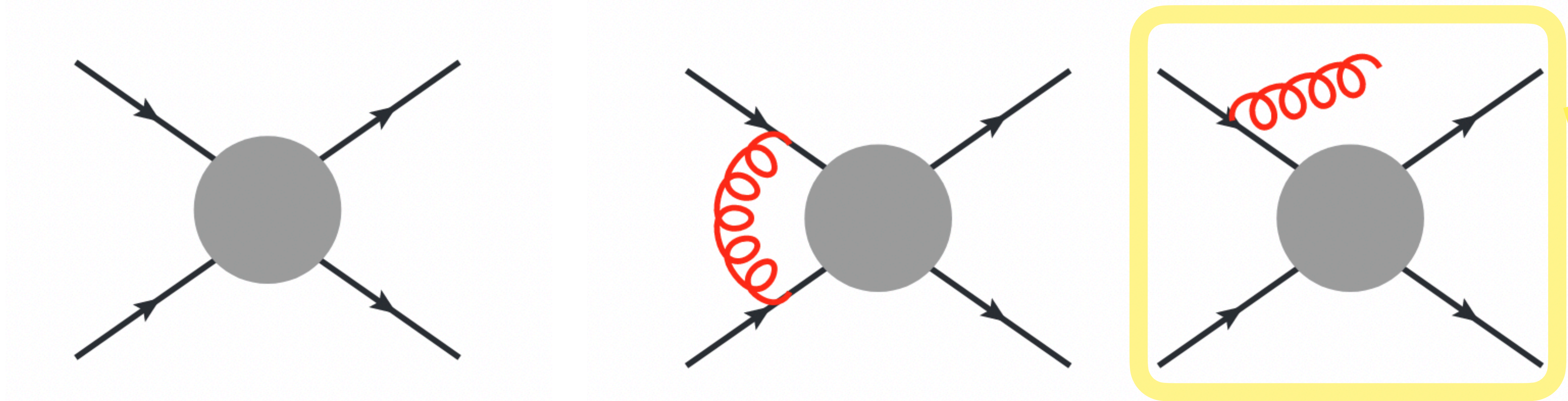
Matching NLO computations with PS is not trivial

Exact NLO computation

Amplitudes $A_2 = A_2^{(0)} + g_s A_2^{(1)} + O(g_s^2)$ $\quad A_3 = g_s A_3^{(0)} + O(g_s^2)$

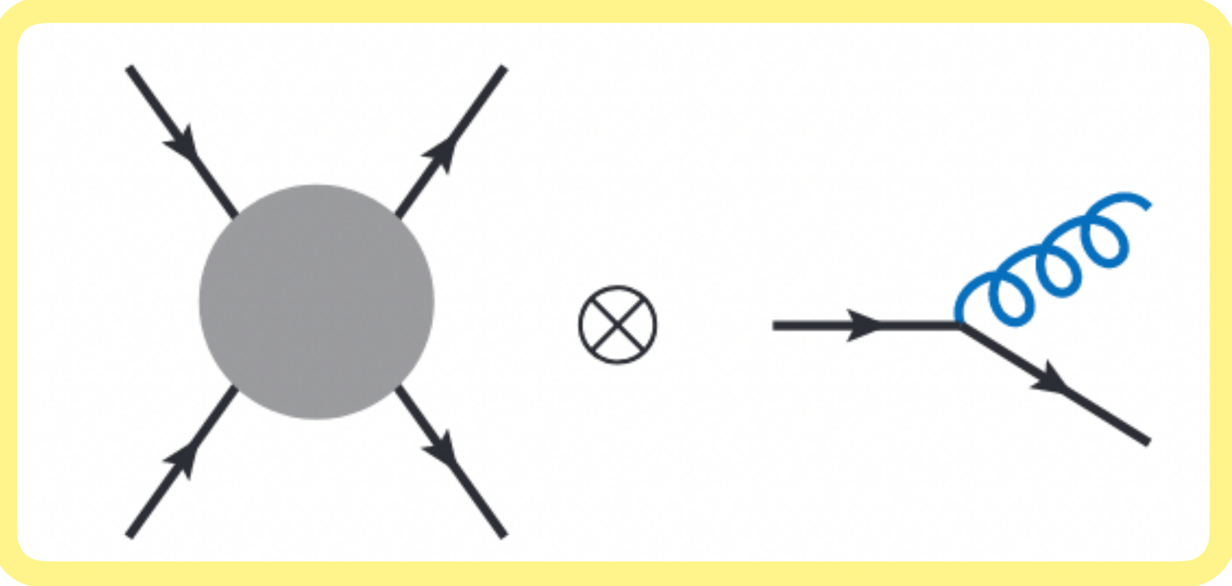
Matrix elements $B \equiv |A_2^{(0)}|^2$ $\quad V \equiv \alpha_s 2\,Re\left(A_2^{(0)} \cdot A_2^{(1)*}\right)$ $\quad R \equiv \alpha_s |A_3^{(0)}|^2$

Born matrix element — Virtual correction — Real correction



Parton shower at LO

$$d\sigma = d\Phi_n\, B(\Phi_n) \left[\Delta(t_I, t_0) + d\phi_{rad} F(z,t) \Delta(t_I, t) + \dots\right]$$



DOUBLE COUNTING!

The POWHEG approach: generating the **hardest radiation first and with NLO accuracy**, then attaching a pt-ordered Parton shower

POWHEG master formula

$$d\sigma = d\Phi_n\, \overline{B}(\Phi_n) \left[\Delta^{(NLO)}(t_I, t_0) + d\phi_{rad} \frac{R}{B} \Delta^{(NLO)}(t_I, t)\right]$$

$$\overline{B} \equiv B + V^{(fin)} + \int d\phi_{rad} R^{(sub)}$$

$$\Delta^{NLO} = e^{-\int d\phi_{rad} R/B}$$

The MiNLO' method

Unfortunately, there are still some issues with observables which are exclusive in QCD radiation, like $\frac{d\sigma}{dp_{T,jet}}$

Color singlet production

$$pp \;\rightarrow\; F$$

$$\overline{B}(\Phi_F) \;=\; B(\Phi_F) \,+\, \alpha_s \left\{ V(\Phi_F) + \int d\phi_{rad} R(\Phi_{FJ}) \right\} \,+\, O(\alpha_s^2)$$

Jet transverse momentum distribution is well defined (IRC-safe), but just at LO accuracy

Color singlet production with one jet

$$pp \;\rightarrow\; F + J$$

$$\overline{B}(\Phi_{FJ}) \;=\; \alpha_s B(\Phi_{FJ}) \,+\, \alpha_s^2 \left\{ V(\Phi_{FJ}) + \int d\phi_{rad} R(\Phi_{FJJ}) \right\} \,+\, O(\alpha_s^3)$$

Formally NLO accurate, but the matrix elements used in this formula show **large logarithmic enhancements** $L = \log(p_T/\mu)$, namely for low jet transverse momentum

How to "**merge**" the two generators to get the NLO accuracy for $\frac{d\sigma}{dp_{T,jet}}$ over the full p_T spectrum?

Implementing **resummation** ingredients in $\overline{B}(\Phi_{FJ})$

- **Sudakov** form factor $e^{-\tilde{S}(p_T)}$ to suppress $\overline{B}$ at low p_T
- New **scale choice** for the strong couplings ($\mu = p_T$)

BUT

Retaining NLO accuracy for both F and $F+J$ inclusive events

$$\bar{B}(\Phi_{\rm FJ}) = e^{-\tilde{S}(p_{\rm T})} \left[B(\Phi_{\rm FJ}) \left(1 + \frac{\alpha_s(p_{\rm T})}{2\pi} [\tilde{S}(p_{\rm T})]^{(1)} \right) + V(\Phi_{\rm FJ}) \right] + \int {\rm d}\Phi_{\rm rad} R(\Phi_{\rm FJ}, \Phi_{\rm rad}) e^{-\tilde{S}(p_{\rm T})} \,.$$

Hamilton K., Nason P., Oleari C., Zanderighi G. (2012)

The MiNNLOPS method

From MiNLO', we can even go further and adapt $\overline{B}(\Phi_{FJ})$ so to achieve **NNLO accuracy when computing F inclusive distributions!**

Monni P., Nason P., Re E., Wiesemann M., Zanderighi G. (2019)

1) Resummation formula in p_T

$$\frac{d\sigma}{d\Phi_F dp_T} = \frac{d}{dp_T}\left\{ e^{-\tilde{S}(p_T)} L \right\} + R_f(p_T)$$

Singular terms — Regular terms

Luminosity L, which includes:

1. PDFs convoluted to Coefficient functions
2. Hard function H (including B, V, and VV for $pp \to b\bar{b}$)

2) Expanding to get $O(\alpha_s^2)$ after integration over dp_T

$$\frac{\mathrm{d}\sigma}{\mathrm{d}\Phi_\mathrm{F}\mathrm{d}p_\mathrm{T}} = \exp[-\tilde{S}(p_\mathrm{T})]\left\{\frac{\alpha_s(p_\mathrm{T})}{2\pi}\left[\frac{\mathrm{d}\sigma_\mathrm{FJ}}{\mathrm{d}\Phi_\mathrm{F}\mathrm{d}p_\mathrm{T}}\right]^{(1)}\left(1+\frac{\alpha_s(p_\mathrm{T})}{2\pi}[\tilde{S}(p_\mathrm{T})]^{(1)}\right) + \left(\frac{\alpha_s(p_\mathrm{T})}{2\pi}\right)^2\left[\frac{\mathrm{d}\sigma_\mathrm{FJ}}{\mathrm{d}\Phi_\mathrm{F}\mathrm{d}p_\mathrm{T}}\right]^{(2)} + \left(\frac{\alpha_s(p_\mathrm{T})}{2\pi}\right)^3[D(p_\mathrm{T})]^{(3)} + \text{regular terms}\right\}$$

3) Embedding in POWHEG $\bar{B}$ function

$$\tilde{B}(\Phi_\mathrm{FJ},\Phi_\mathrm{rad}) = \exp[-\tilde{S}(\Phi_\mathrm{F},p_\mathrm{T})]\Big[B(\Phi_\mathrm{FJ})\left(1+\frac{\alpha_s(p_\mathrm{T})}{2\pi}[\tilde{S}(\Phi_\mathrm{F},p_\mathrm{T})]^{(1)}\right) + V(\Phi_\mathrm{FJ}) + R(\Phi_\mathrm{FJ},\Phi_\mathrm{rad}) + D^{(\geq 3)}(\Phi_\mathrm{F},p_\mathrm{T})F^\mathrm{corr}(\Phi_\mathrm{FJ})\Big],$$

MiNNLOps

$pp \to F$	$A_F = A_F^{(tree)} + g_s A_F^{(1-loop)} + g_s^2 A_F^{(2-loops)} + \ldots$
$pp \to F+J$	$A_{FJ} = g_s A_{FJ}^{(tree)} + g_s^2 A_{FJ}^{(1-loop)} + \ldots$
$pp \to F+JJ$	$A_{FJJ} = g_s^2 A_{FJJ}^{(tree)} + \ldots$

POWHEG

Accuracy for inclusive distributions →

	MiNLO'	MiNNLOPS
F	NLO	**NNLO**
F+J	NLO	**NLO**

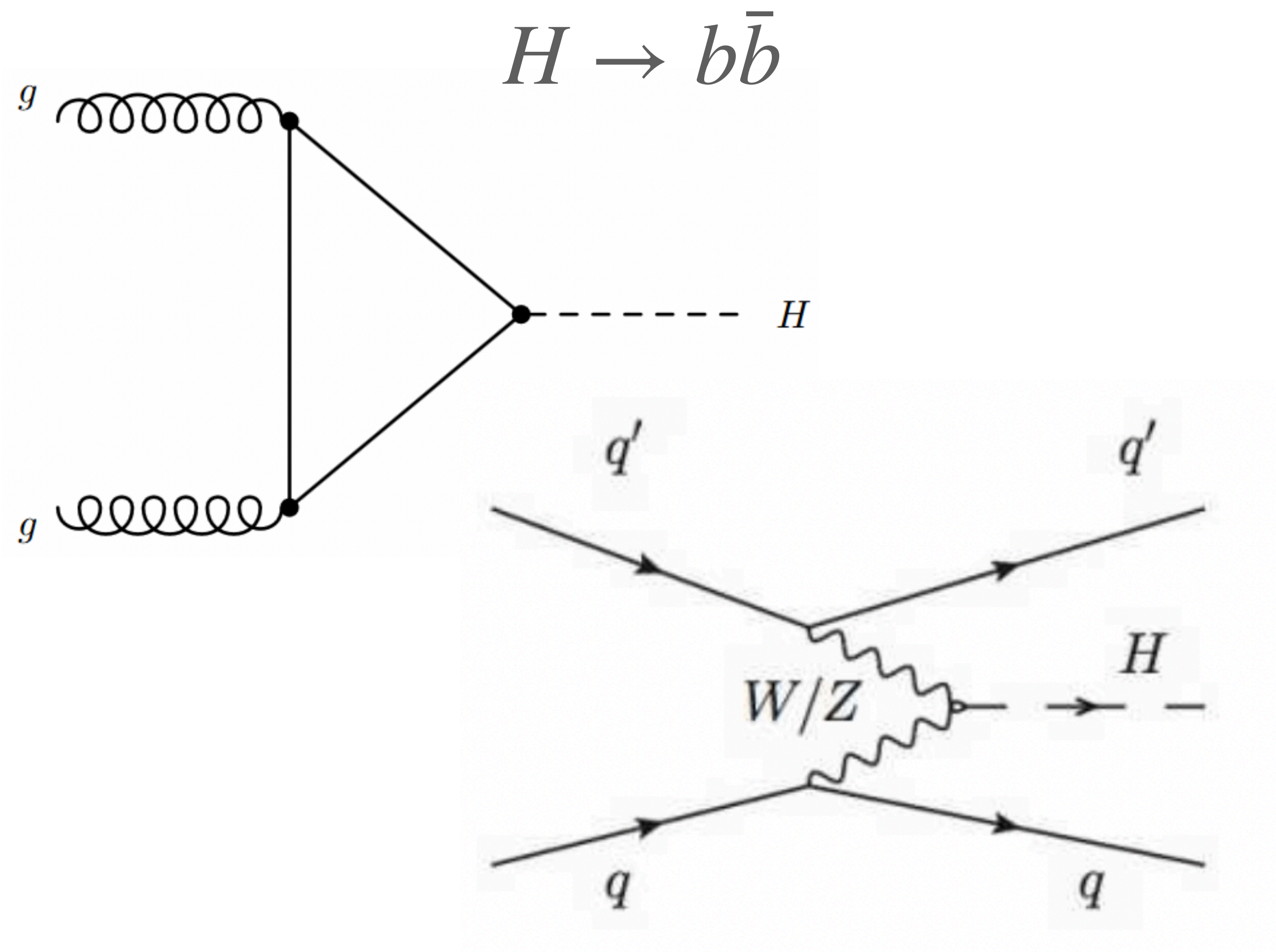
Bottom pair production

Motivations and status

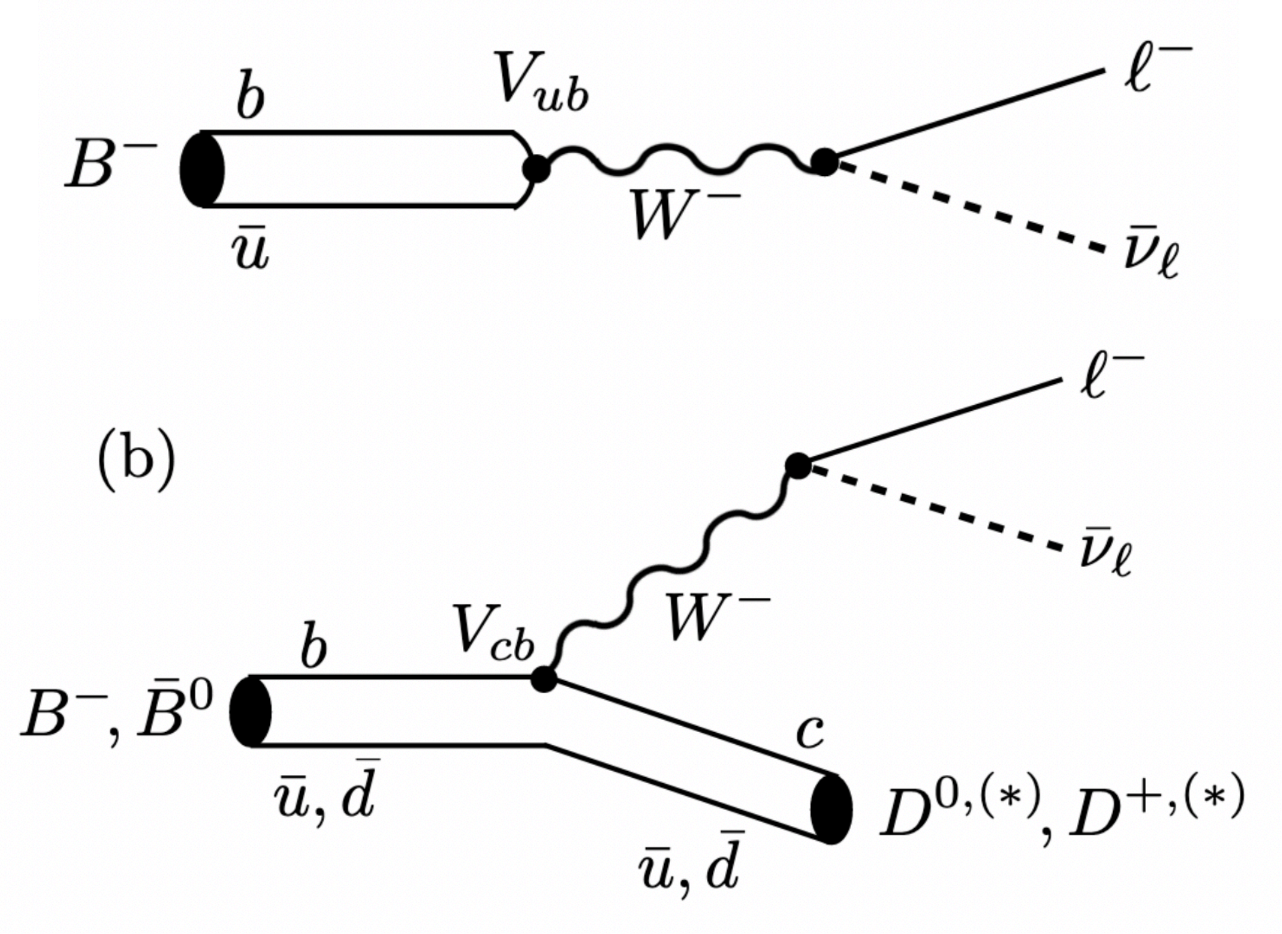
Bottom pair production

Why is this process interesting?

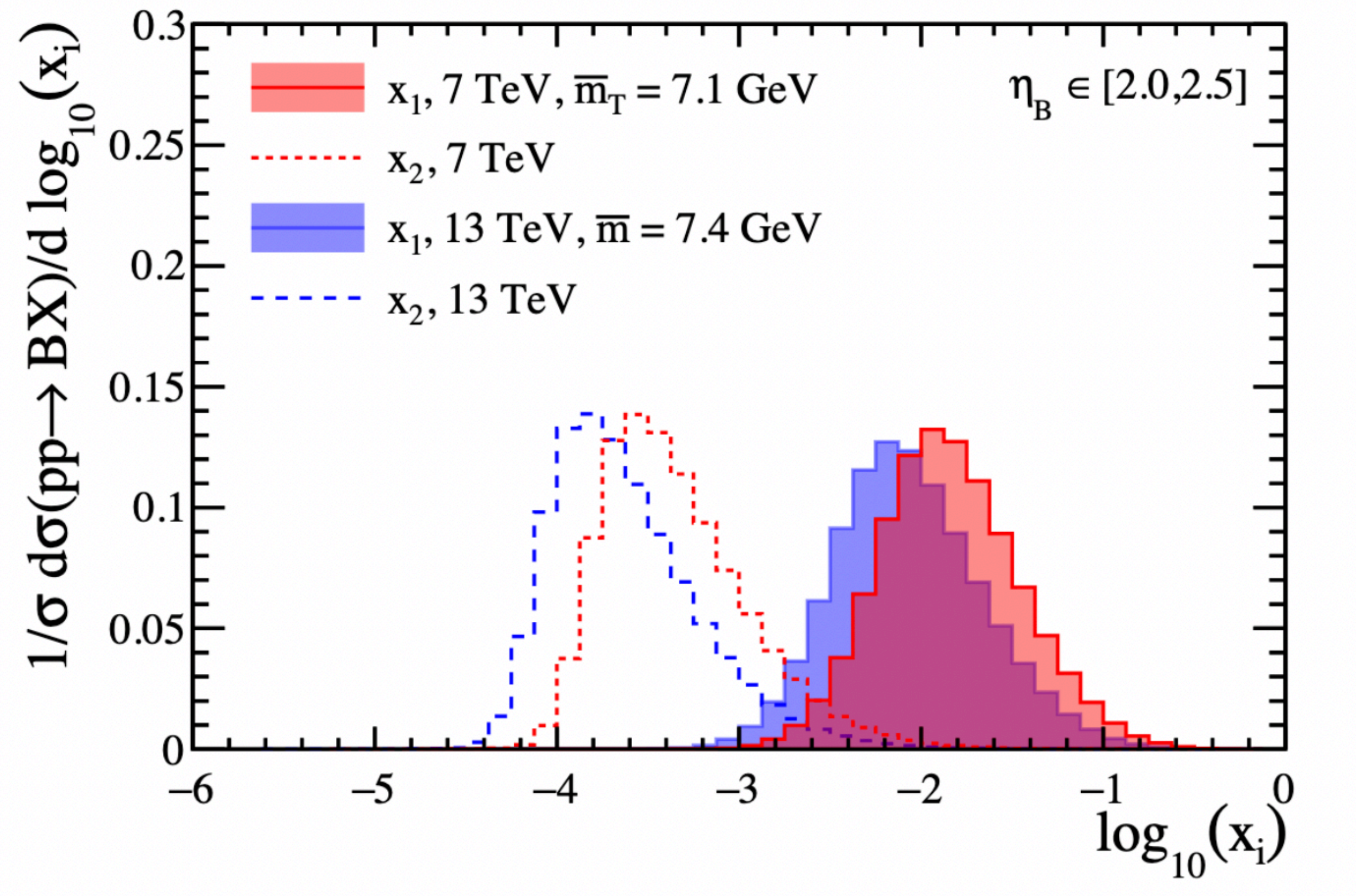
Relevant **background** for several SM and BSM processes (especially in the Higgs sector)



Important in predicting the **prompt atmospheric neutrino flux** (background in neutrino telescopes)



Useful to constrain **gluon PDF** at low x values



Event generators for $pp \to b\bar{b}$ are needed

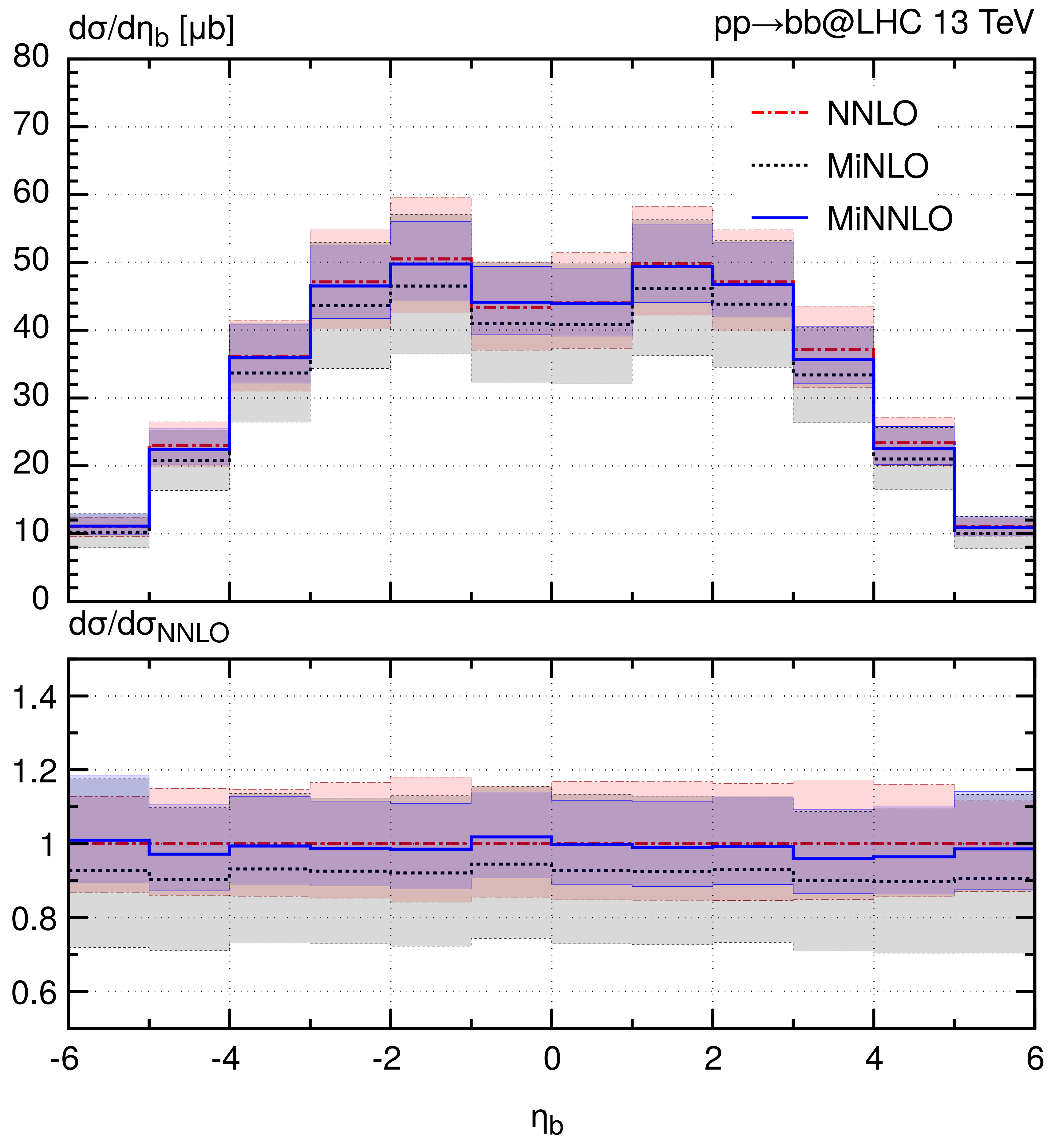
MiNNLOps in the POWHEG framework can provide a NNLO+LL event generator at the level of $b\bar{b}$ inclusive distributions. At the moment:

- Code validated against NNLO predictions from MATRIX
- Code interfaced to PYTHIA8 to get B hadron distributions (comparison to experimental data)

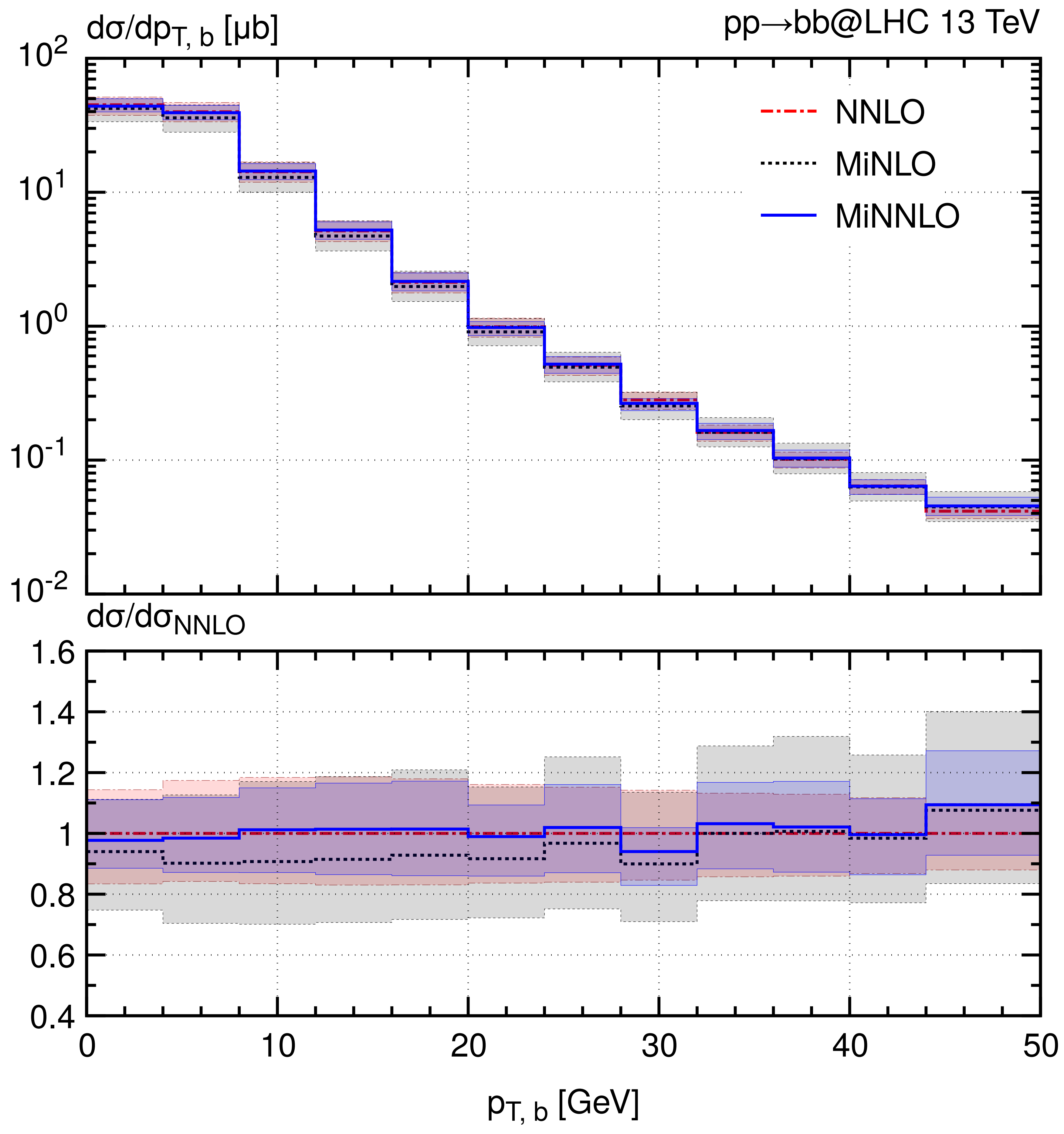
Comparison to fixed order results

Validation against NNLO results from MATRIX ($b\bar{b}$ inclusive)

Bottom quark pseudorapidity



Bottom transverse momentum



Total cross section

NLO	$348.5(3)^{+27\%}_{-24\%}\ \mu b$
MiNLO'	$399.7(5)^{+22\%}_{-21\%}\ \mu b$
NNLO	$435(2)^{+16\%}_{-15\%}\ \mu b$
MiNNLO$_{PS}$	$428.7(5)^{+13\%}_{-11\%}\ \mu b$

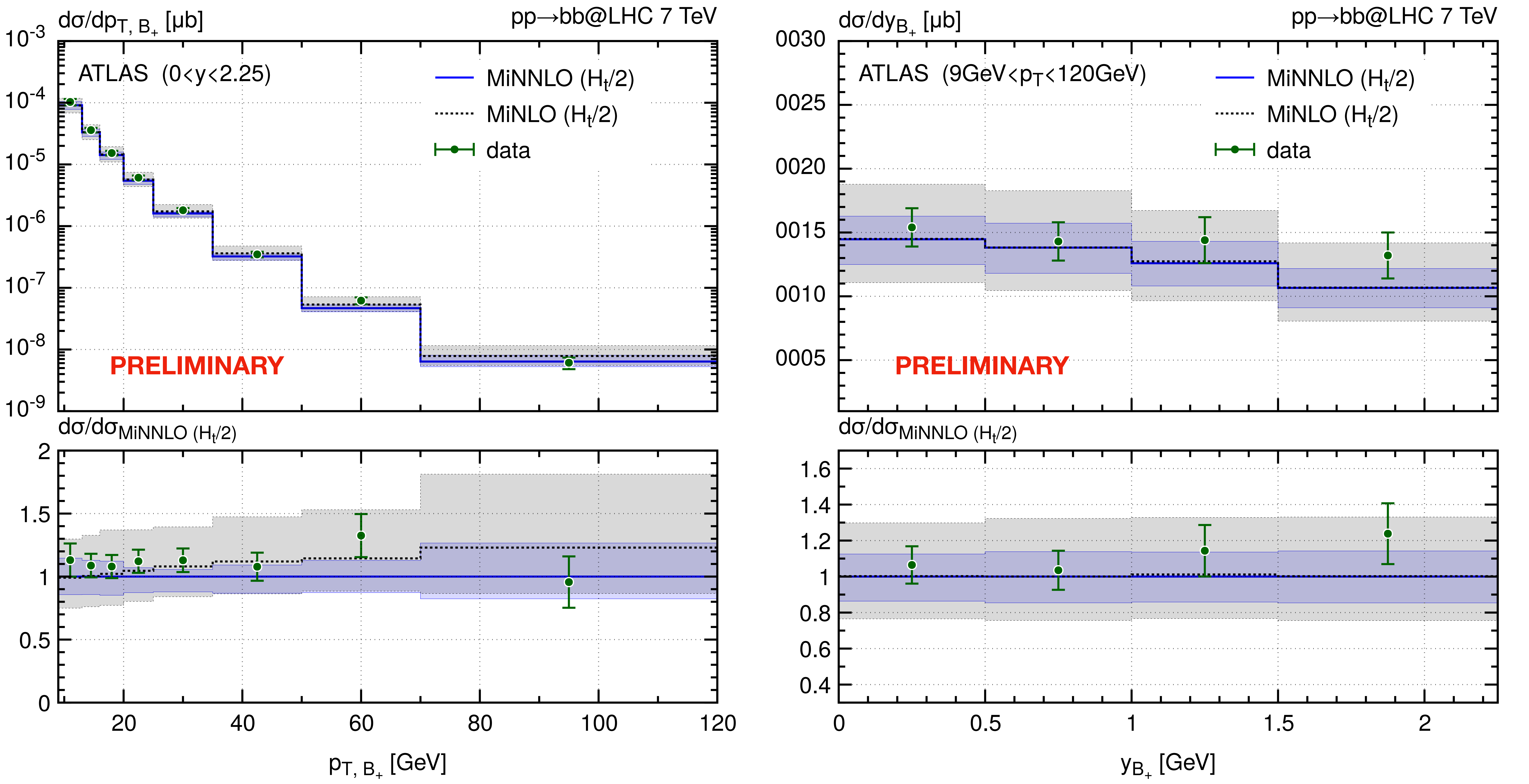
Comparison with fixed order NNLO predictions from MATRIX. 7-point scale variation has been performed choosing the central scale:

$$\mu_R = \mu_F = m_{b\bar{b}}$$

Comparison to experimental data

Comparison with experimental hadron distributions. Standard settings for PYTHIA8 (Parton shower and hadronization). 7-point scale variation has been performed around $\mu_R = \mu_F = H_t/2$ (with $H_t \equiv \sqrt{m_b^2 + p_{T,b}^2} + \sqrt{m_{\bar{b}}^2 + p_{T,\bar{b}}^2}$)

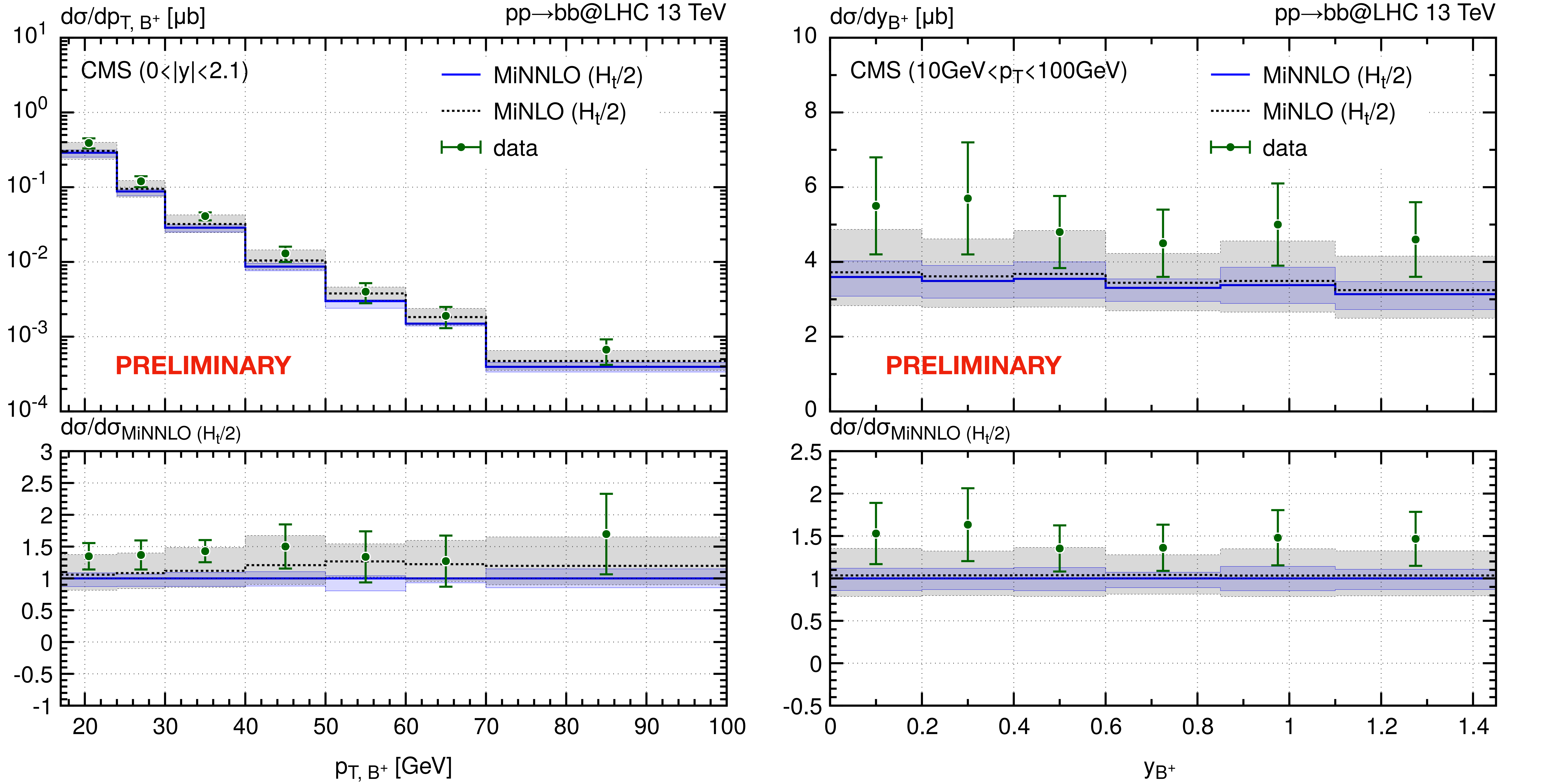
ATLAS



Comparison to experimental data: ATLAS

Comparison with experimental hadron distributions. Standard settings for PYTHIA8 (Parton shower and hadronization). 7-point scale variation has been performed around $\mu_R = \mu_F = H_t/2$ (with $H_t \equiv \sqrt{m_b^2 + p_{T,b}^2} + \sqrt{m_{\bar{b}}^2 + p_{T,\bar{b}}^2}$)

CMS, 2017



Comparison to experimental data

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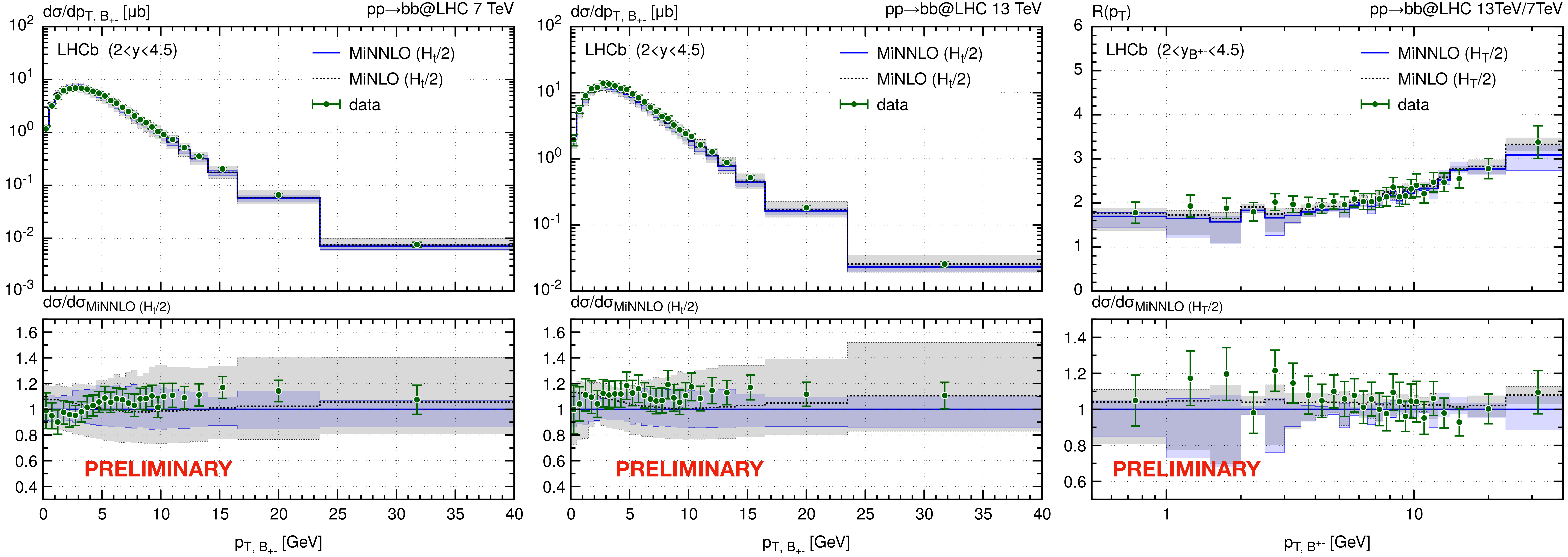
LHCb, 2018



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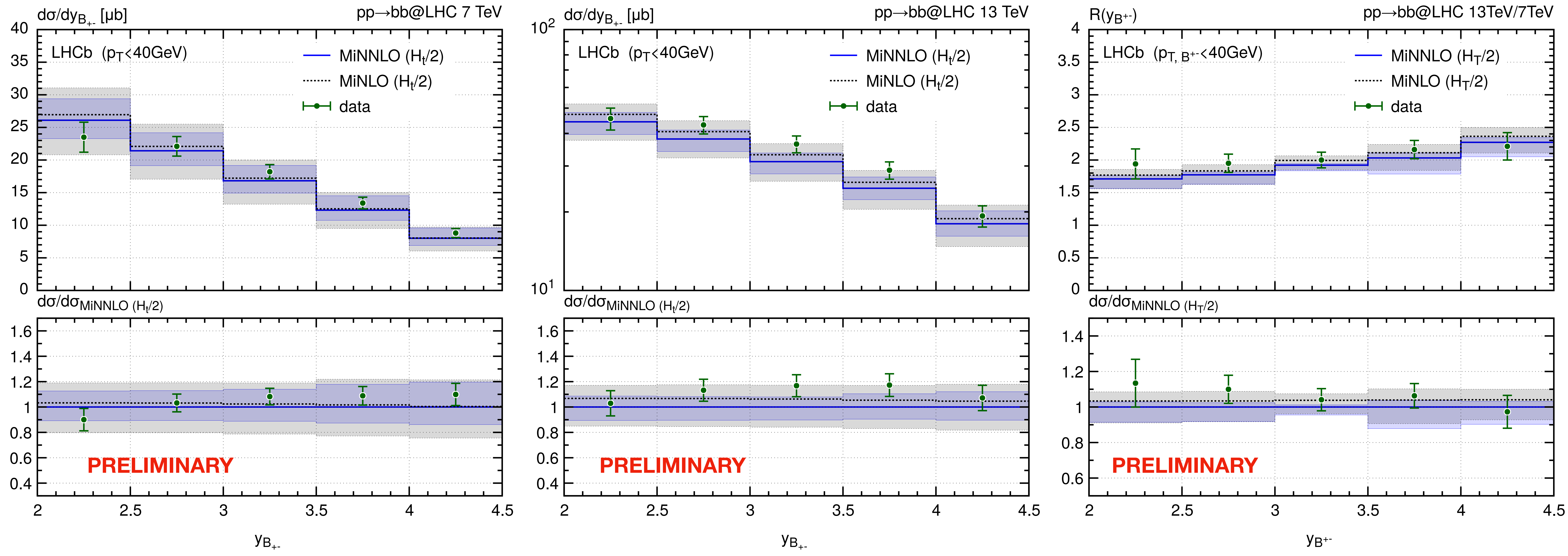
LHCb, 2018



Summary

Status of the bottom pair production with MiNNLOps

- MiNNLOps for $b\bar{b}$ production has been **implemented** in POWHEG
- Checks against inclusive **fixed order NNLO** distributions
- Comparison with experimental **B meson distributions** from ATLAS, CMS, LHCb
- Including the uncertainties over **pole mass** m_b **and PDFs (13TeV/7TeV analysis)**
- Comparison to experimental data for b **jet distributions**

Thank you!

Backup

The MiNNLOps method

From MiNLO', we can even go further and adapt $\overline{B}(\Phi_{FJ})$ so to achieve NNLO accuracy when computing F inclusive distributions!

$$\frac{d\sigma}{d\Phi_F dp_T} = \frac{d}{dp_T}\left\{ e^{-\tilde{S}(p_T)} L \right\} + R_f(p_T)$$

$$\frac{d\sigma}{d\Phi_F dp_T} = \underbrace{e^{-\tilde{S}(p_T)} D(p_T)}_{\text{div}} + \underbrace{R_f(p_T)}_{\text{fin}}$$

We expand the right-hand side so to get $O(\alpha_s^2)$ after integration over dp_T

- $R_f(p_T)$: it's regular, we can just expand it up to $O(\alpha_s^2)$

$$R_f(p_T) = \frac{d\sigma}{d\Phi_F dp_T} - e^{-\tilde{S}} D \longrightarrow \alpha_s \left(\frac{d\sigma}{d\Phi_F dp_T} \right)^{(1)} + \alpha_s^2 \left(\frac{d\sigma}{d\Phi_F dp_T} \right)^{(2)} + e^{-\tilde{S}} \left[\alpha_s D^{(1)} + \alpha_s^2 D^{(2)} \right]$$

- $e^{-\tilde{S}} D$ more subtle: not straightforward achieving $O(\alpha_s^2)$ after integration over p_T (divergent terms), but we can notice that:

$$\int_\Lambda^Q dp_T\, \alpha_s^M(p_T) \frac{1}{p_T} \ln^n \frac{Q}{p_T}\, e^{-\tilde{S}(p_T)} \approx O\left(\alpha_s^{m-\frac{n+1}{2}}(Q)\right)$$

Doing the right "power counting", we end up with:

$$\frac{\mathrm{d}\sigma}{\mathrm{d}\Phi_{\mathrm{F}}\mathrm{d}p_{\mathrm{T}}} = \exp[-\tilde{S}(p_{\mathrm{T}})] \left\{ \frac{\alpha_s(p_{\mathrm{T}})}{2\pi} \left[\frac{\mathrm{d}\sigma_{\mathrm{FJ}}}{\mathrm{d}\Phi_{\mathrm{F}}\mathrm{d}p_{\mathrm{T}}} \right]^{(1)} \left(1 + \frac{\alpha_s(p_{\mathrm{T}})}{2\pi} [\tilde{S}(p_{\mathrm{T}})]^{(1)} \right) + \left(\frac{\alpha_s(p_{\mathrm{T}})}{2\pi} \right)^2 \left[\frac{\mathrm{d}\sigma_{\mathrm{FJ}}}{\mathrm{d}\Phi_{\mathrm{F}}\mathrm{d}p_{\mathrm{T}}} \right]^{(2)} + \left(\frac{\alpha_s(p_{\mathrm{T}})}{2\pi} \right)^3 [D(p_{\mathrm{T}})]^{(3)} + \text{regular terms} \right\}$$

$$\tilde{B}(\Phi_{\mathrm{FJ}}, \Phi_{\mathrm{rad}}) = \exp[-\tilde{S}(\Phi_{\mathrm{F}}, p_{\mathrm{T}})] \Big[B(\Phi_{\mathrm{FJ}}) \left(1 + \frac{\alpha_s(p_{\mathrm{T}})}{2\pi} [\tilde{S}(\Phi_{\mathrm{F}}, p_{\mathrm{T}})]^{(1)} \right) + V(\Phi_{\mathrm{FJ}}) + R(\Phi_{\mathrm{FJ}}, \Phi_{\mathrm{rad}}) + D^{(\geq 3)}(\Phi_{\mathrm{F}}, p_{\mathrm{T}}) F^{\mathrm{corr}}(\Phi_{\mathrm{FJ}}) \Big],$$

MiNNLOps master formula

	MiNLO'	MiNNLOps
F	NLO	**NNLO**
F+J	NLO	**NLO**

Monni P., Nason P., Re E., Wiesemann M., Zanderighi G. (2019)

Bottom pair production with MiNNLO$_{PS}$

Settings

- We consider **7 and 13 TeV** LHC collisions
- **Four-flavour scheme**
- **Pole mass** of bottom quarks set to $m_b = 4.92\,GeV$
- PDF choice: NNPDF31_nnlo_as_0118_nf_4
- For the factorization and renormalization scales entering the MiNNLOps formula, we tested m_{bb}, $m_{bb}/2$, $H_t/2$ and $H_t/4$
- OpenLoops2 for tree level and **1-loop contributions**, and evaluated the genuinely **2-loops contributions** using analytical grids

$pp \to Q\bar{Q}$	$A_{Q\bar{Q}} = A^{(tree)}_{Q\bar{Q}} + g_s A^{(1-loop)}_{Q\bar{Q}} + g_s^2 A^{(2-loops)}_{Q\bar{Q}} + \dots$
$pp \to Q\bar{Q} + J$	$A_{Q\bar{Q}J} = g_s A^{(tree)}_{Q\bar{Q}J} + g_s^2 A^{(1-loop)}_{Q\bar{Q}J} + \dots$
$pp \to Q\bar{Q} + JJ$	$A_{Q\bar{Q}JJ} = g_s^2 A^{(tree)}_{Q\bar{Q}JJ} + \dots$

Bottom pair production with MiNNLO$_{PS}$

Extension of the MiNNLOps method to $pp \to Q\bar{Q} + J$

Mazzitelli J., Monni P., Nason P., Re E., Wiesemann M., Zanderighi G. (2020)

$pp \to F + J$

1) Resummation formula in p_T

$$\frac{d\sigma}{d\Phi_F dp_T} = \frac{d}{dp_T}\left\{ e^{-\tilde{S}(p_T)} L \right\} + R_f(p_T)$$

2) Inclusion of relevant terms at required accuracy

$$\frac{\mathrm{d}\sigma}{\mathrm{d}\Phi_\mathrm{F}\mathrm{d}p_\mathrm{T}} = \exp[-\tilde{S}(p_\mathrm{T})]\left\{ \frac{\alpha_s(p_\mathrm{T})}{2\pi}\left[\frac{\mathrm{d}\sigma_\mathrm{FJ}}{\mathrm{d}\Phi_\mathrm{F}\mathrm{d}p_\mathrm{T}}\right]^{(1)}\left(1+\frac{\alpha_s(p_\mathrm{T})}{2\pi}[\tilde{S}(p_\mathrm{T})]^{(1)}\right) + \left(\frac{\alpha_s(p_\mathrm{T})}{2\pi}\right)^2\left[\frac{\mathrm{d}\sigma_\mathrm{FJ}}{\mathrm{d}\Phi_\mathrm{F}\mathrm{d}p_\mathrm{T}}\right]^{(2)} + \left(\frac{\alpha_s(p_\mathrm{T})}{2\pi}\right)^3[D(p_\mathrm{T})]^{(3)} + \text{regular terms}\right\}$$

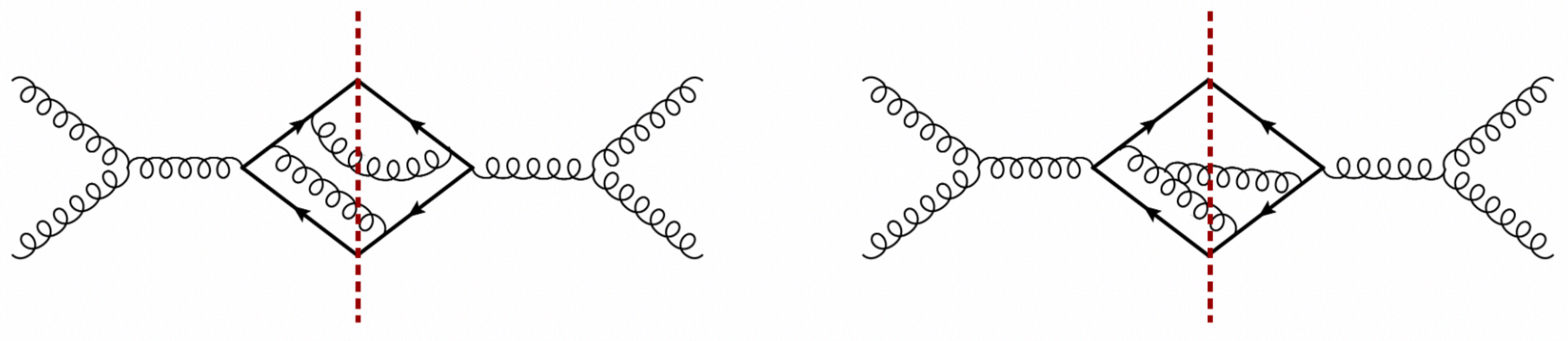
3) Embedding in POWHEG $\bar{B}(\Phi_{FJ})$

$$\tilde{B}(\Phi_\mathrm{FJ},\Phi_\mathrm{rad}) = \exp[-\tilde{S}(\Phi_\mathrm{F},p_\mathrm{T})]\left[B(\Phi_\mathrm{FJ})\left(1+\frac{\alpha_s(p_\mathrm{T})}{2\pi}[\tilde{S}(\Phi_\mathrm{F},p_\mathrm{T})]^{(1)}\right) + V(\Phi_\mathrm{FJ}) + R(\Phi_\mathrm{FJ},\Phi_\mathrm{rad}) + D^{(\geq 3)}(\Phi_\mathrm{F},p_\mathrm{T})F^\mathrm{corr}(\Phi_\mathrm{FJ})\right],$$

$pp \to Q\bar{Q} + J$

1) Resummation formula in b-space taking into account

- Both $q\bar{q}$ and gg channels in the initial state
- Color structures in the amplitudes
- More sophisticated infrared IR structure (soft final state gluons and initial-final state exchange of soft gluons)



2) Derivation of (approximated) resummation formula in p_T

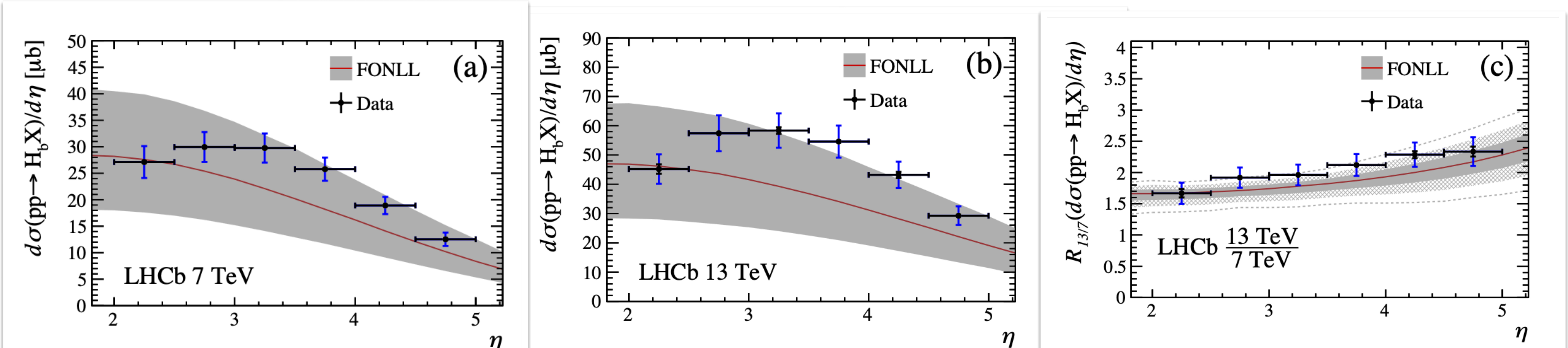
$$\frac{\mathrm{d}\sigma}{\mathrm{d}p_\mathrm{T}\,\mathrm{d}\Phi_{Q\bar{Q}}} = \frac{\mathrm{d}}{\mathrm{d}p_\mathrm{T}}\left\{ \sum_c \frac{e^{-\tilde{S}_{c\bar{c}}(p_\mathrm{T})}}{2m_{Q\bar{Q}}^2}\langle M_{c\bar{c}}^{(0)}|(\mathbf{V}_\mathrm{NLL})^\dagger\,\mathbf{V}_\mathrm{NLL}|M_{c\bar{c}}^{(0)}\rangle \times \sum_{i,j}\left[\mathrm{Tr}(\tilde{\mathbf{H}}_{c\bar{c}}\mathbf{D})\,(\tilde{C}_{ci}\otimes f_i)\,(\tilde{C}_{\bar{c}j}\otimes f_j)\right]_\phi \right\} + R_\mathrm{finite}(p_\mathrm{T}) + \mathcal{O}(\alpha_s^5)$$

3) Inclusion of relevant terms at required accuracy

4) Embedding in POWHEG $\bar{B}(\Phi_{Q\bar{Q}J})$

Already done for $pp \to t\bar{t}$

Experimental results for B meson production



LHCb collaboration, 2018

Average B meson **pseudorapidity** distributions $d\sigma(pp \to H_b X)/d\eta$, compared to FONLL predictions

Here it has been defined:

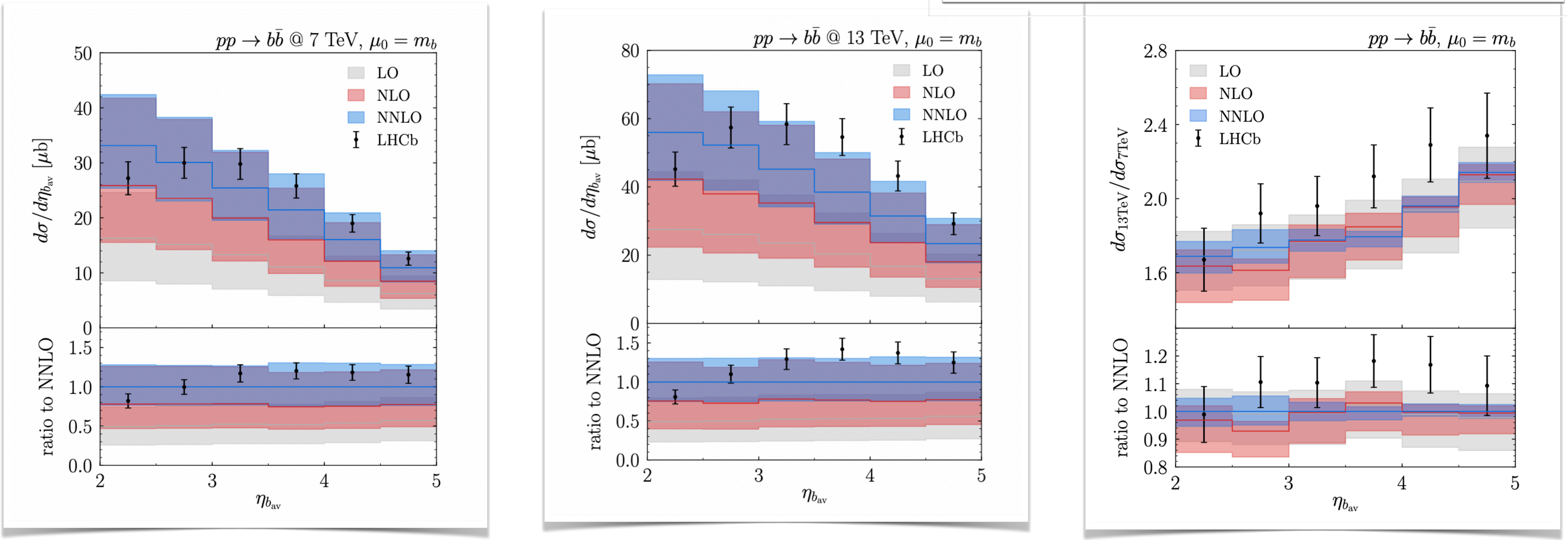
$$\sigma(pp \to H_b X) = \frac{1}{2}\left[\sigma(B^0) + \sigma(\overline{B}^0)\right] + \frac{1}{2}\left[\sigma(B^+) + \sigma(B^-)\right] + \frac{1}{2}\left[\sigma(B_s^0) + \sigma(\overline{B}_s^0)\right] + \frac{1+\delta}{2}\left[\sigma(\Lambda_b^0) + \sigma(\overline{\Lambda}_b^0)\right]$$

We notice that:

- Data and FONLL predictions are **compatible** within their respective uncertainties
- There are some **shape differences** between data and experimental predictions
- Such shape disagreements compensate in the ratio of distributions

Fixed order NNLO bottom pair production

S. Catani, S. Devoto, M. Grazzini, S. Kallweit, J. Mazzitelli; JHEP 03 (2021) 029



Average bottom and anti-bottom pseudorapidity distributions at 7TeV and 13TeV (LO, NLO, NNLO) with $\mu_F = \mu_R = m_b$

Comparison to experimental data

Comparison with experimental hadron distributions. Standard settings for PYTHIA8 (Parton shower and hadronization). 7-point scale variation has been performed around $\mu_R = \mu_F = H_t/2$ (with $H_t \equiv \sqrt{m_b^2 + p_{T,b}^2} + \sqrt{m_{\bar{b}}^2 + p_{T,\bar{b}}^2}$)

LHCb