Local Unitarity

Computing cross-sections beyond NLO accuracy with Local Unitarity

Zeno Capatti

In collaboration with:

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Andrea Pelloni
Ben Ruijl

A talk for QCD@LHC 2022
Orsay, November 28th, 2022

Photo: posing in front of Monte Carlo's casino, 2021

"It was at that time that I suggested an obvious name for the statistical method—a suggestion not unrelated to the fact that Stan had an uncle who would borrow money from relatives because he "just had to go to Monte Carlo." The name seems to have endured."

Nicholas Metropolis, "The beginning of the Monte Carlo method",
Los Alamos Science Special Issue 1987

Local Unitarity: framing the problem

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A cross-section admits a perturbative expansion when $\alpha < 1$

$$\sigma = \sum_{L=1}^{\infty} \alpha^L \sigma^{(L)}$$

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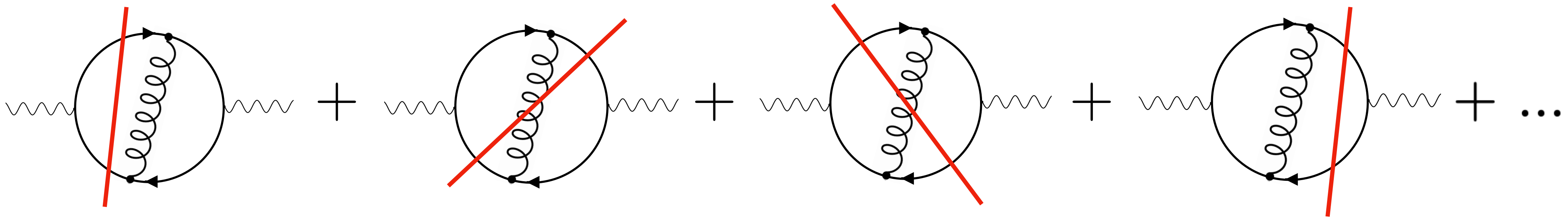

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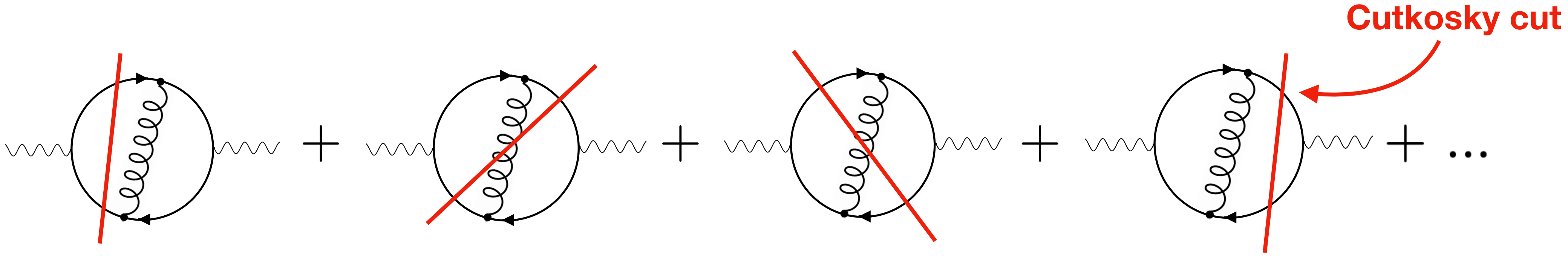

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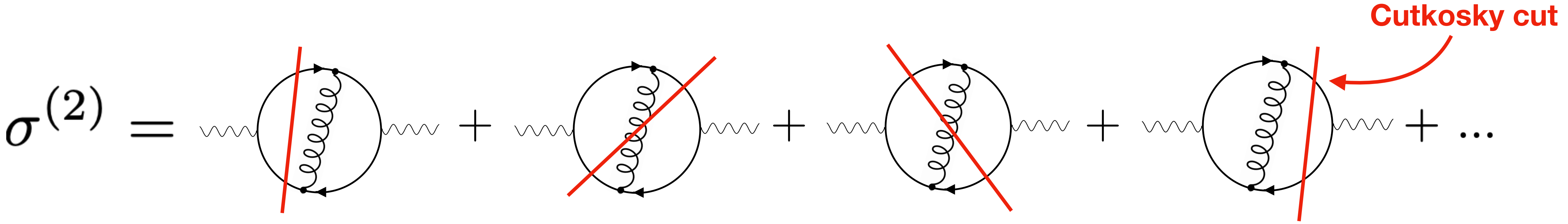
$\sigma^{(2)} =$ … $+$ … $+$ … $+$ … $+ \ldots$

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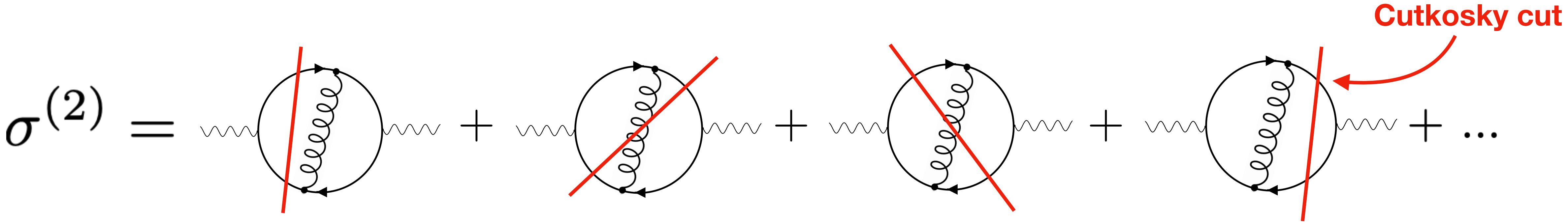

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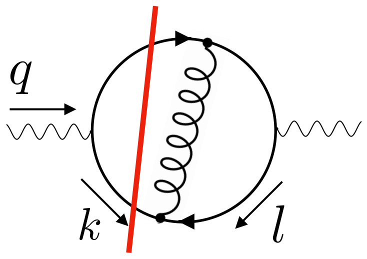

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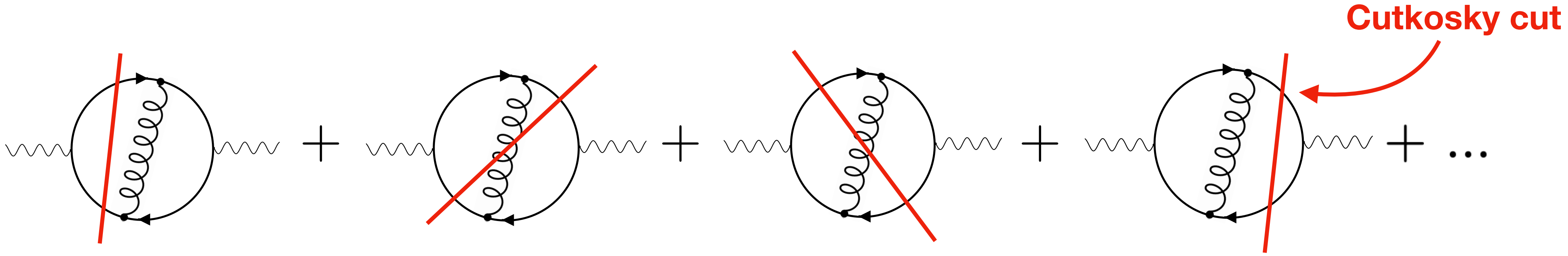

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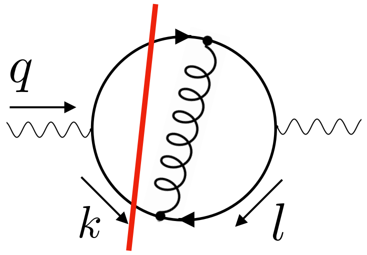

$$= \int \mathrm{d}^4 k \, \delta^+(k^2) \, \delta^+((q-k)^2)$$

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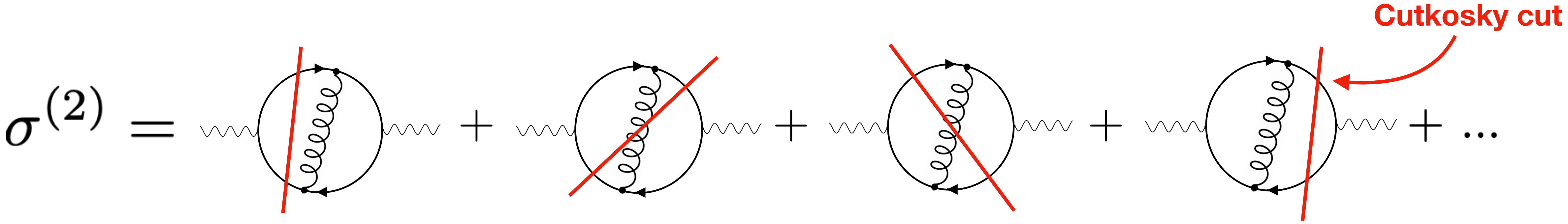

Cutkosky cut

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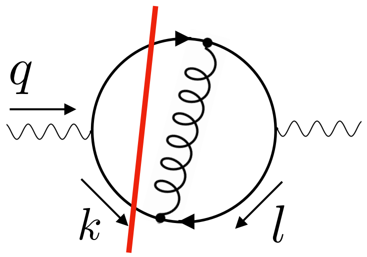

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Phase space integral

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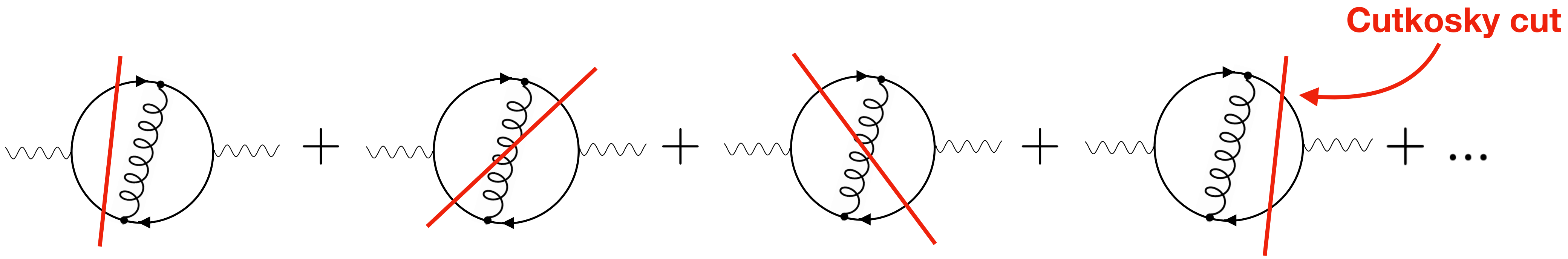

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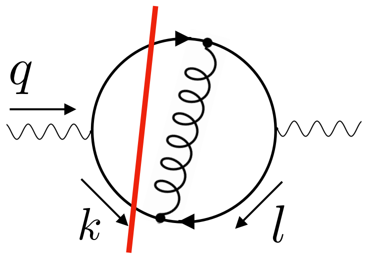

$$= \underbrace{\int \mathrm{d}^4k\, \delta^+(k^2)\, \delta^+((q-k)^2)}_{\text{Phase space integral}} \int \mathrm{d}^4l \frac{N(k,l,q)}{l^2(l+q)^2(k+l)^2}$$

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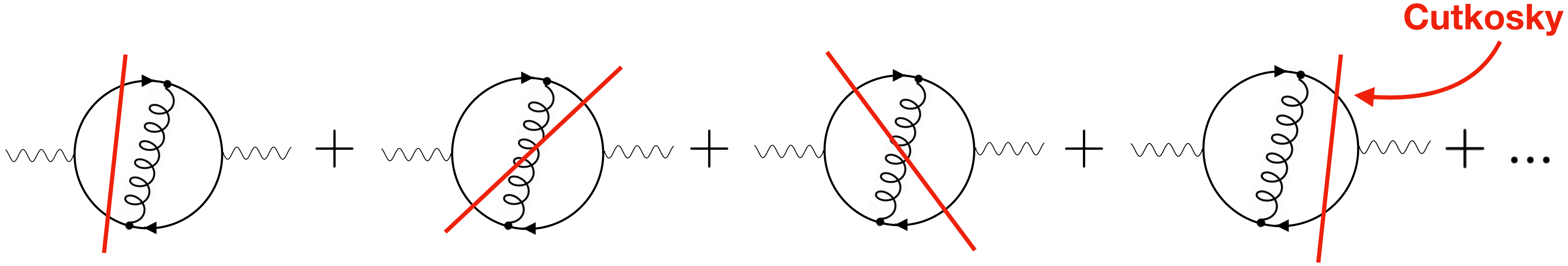

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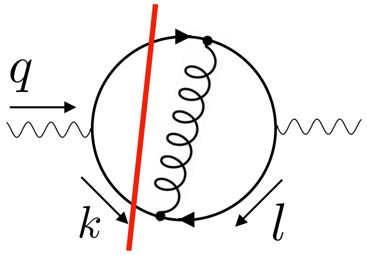

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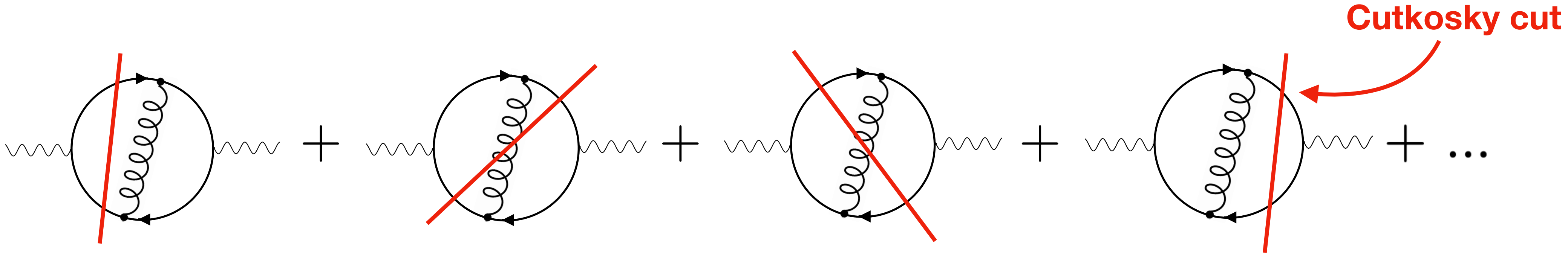

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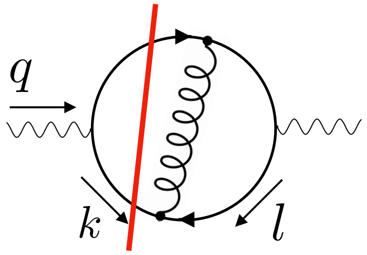

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Problem: both types of integrals have **infrared** (collinear, soft) **divergences** and thresholds

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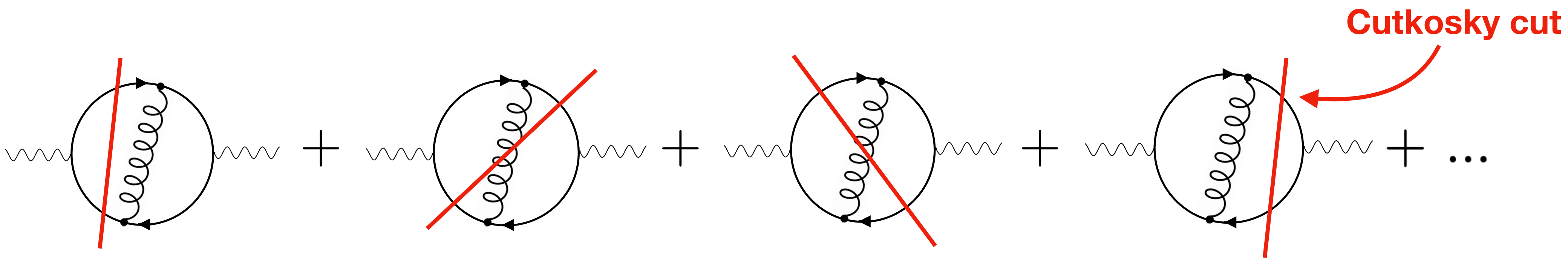

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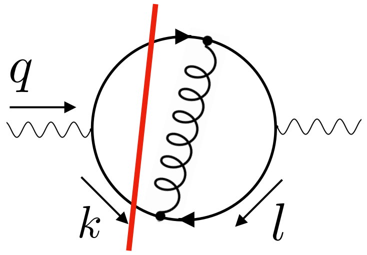

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Many good methods around to deal with this that work **either** for loop or for phase-space integrals

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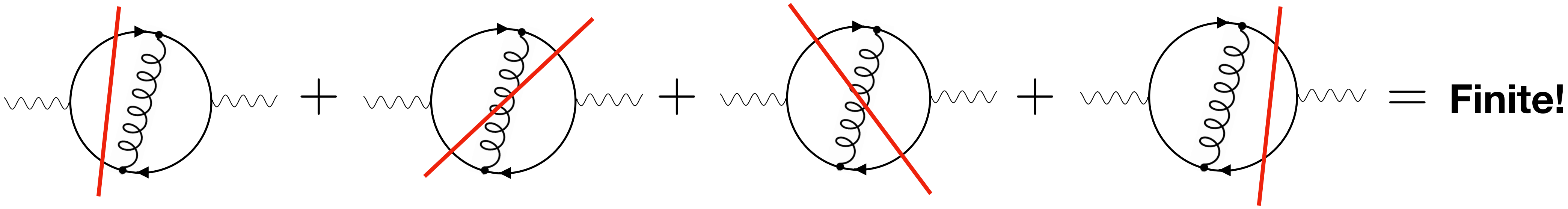

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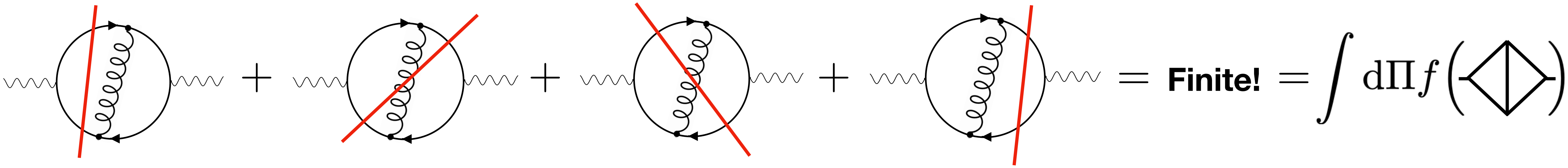

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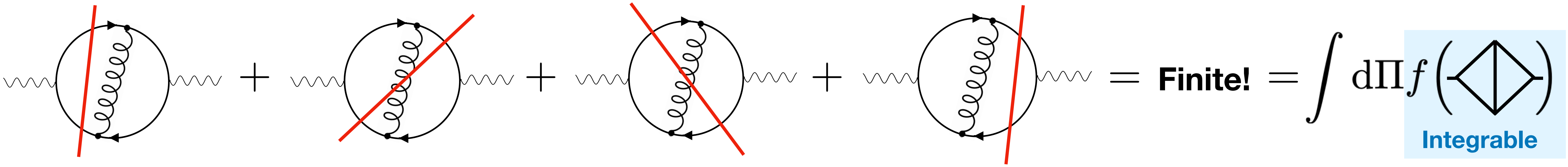

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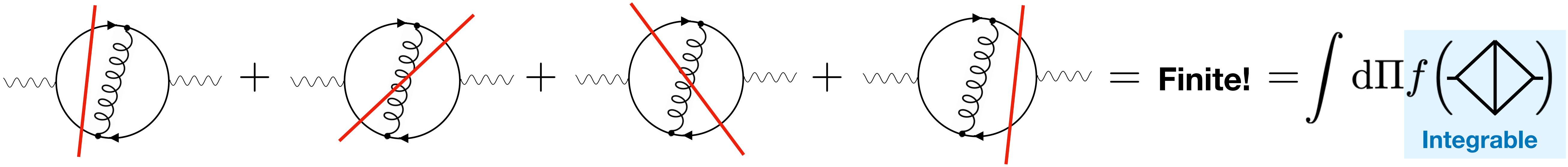

Rough idea:

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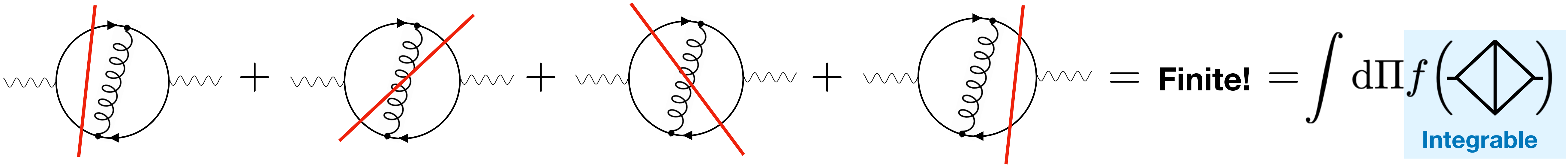

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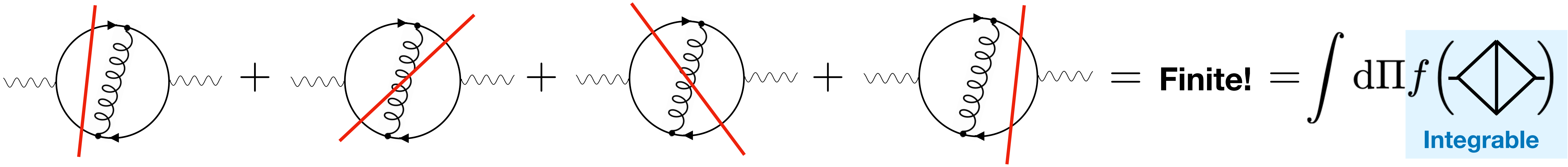

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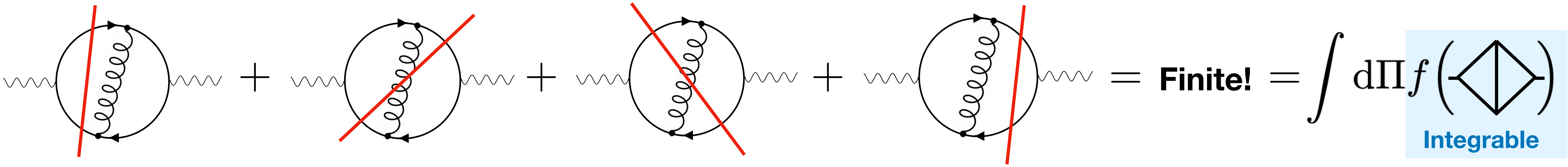

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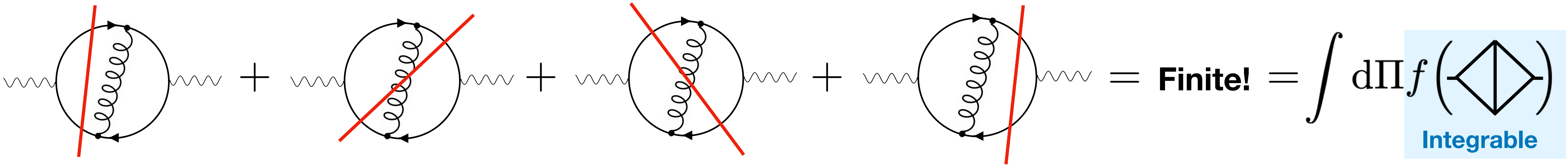

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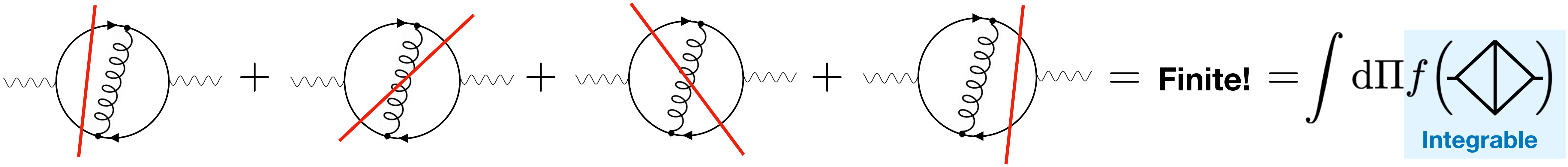

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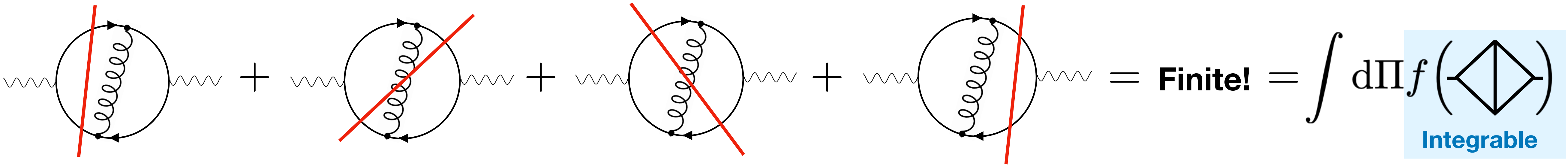

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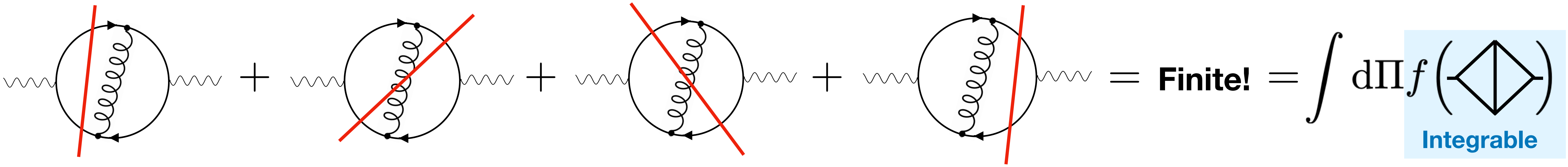

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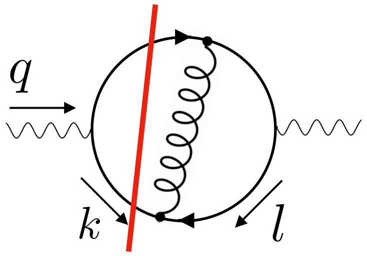

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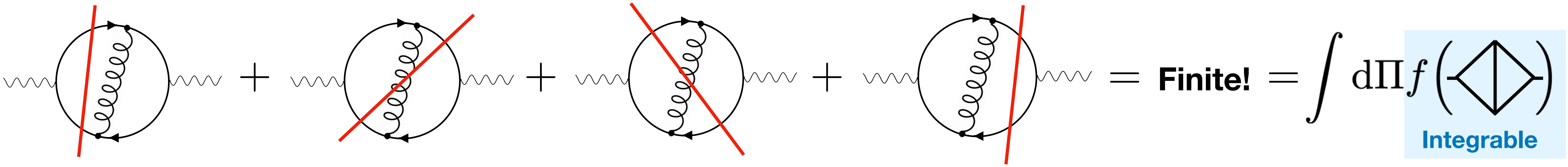

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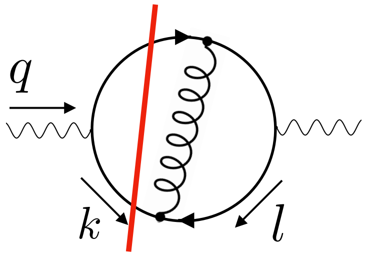

$$= \int \mathrm{d}^4k\mathrm{d}^4l\, \delta^{(+)}(k^2)\, \delta^{(+)}((k-q)^2)\ \frac{N}{l^2(l+q)^2(k+l)^2}$$

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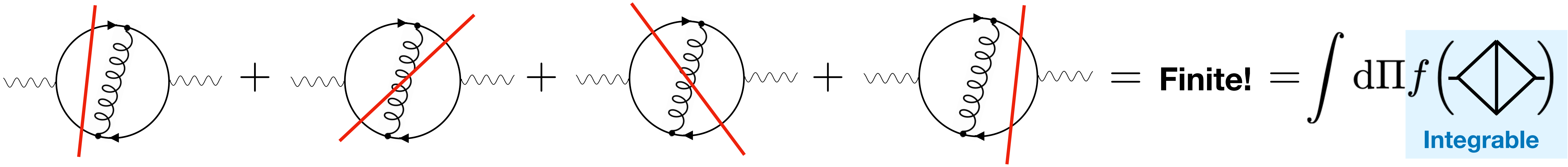

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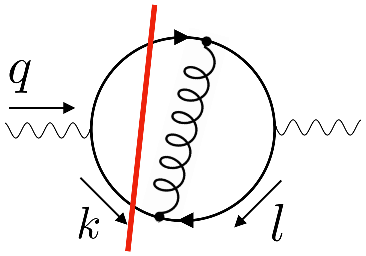

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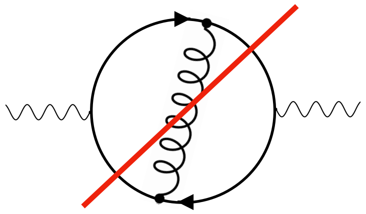

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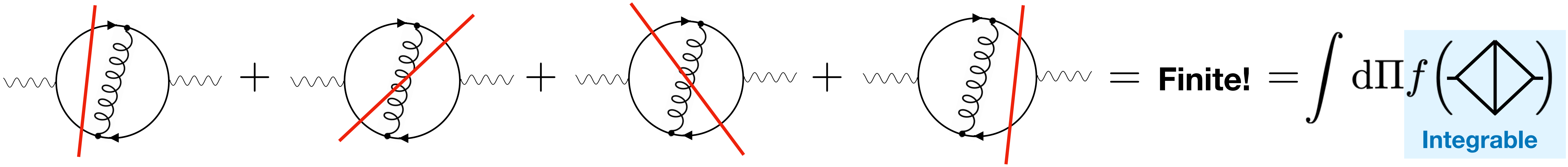

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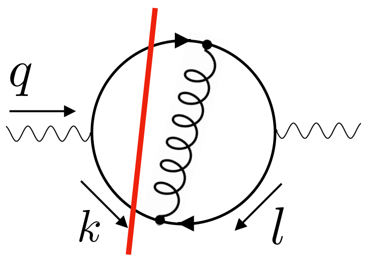

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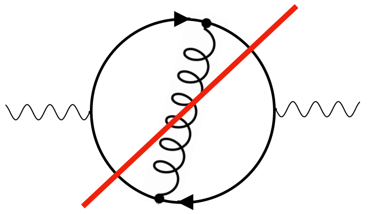

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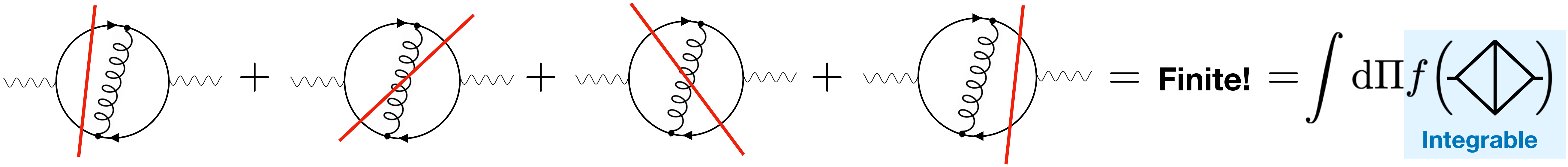

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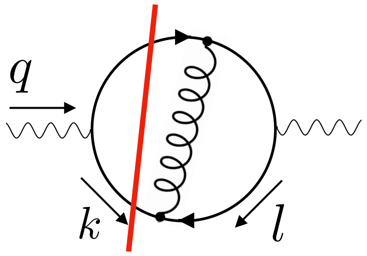

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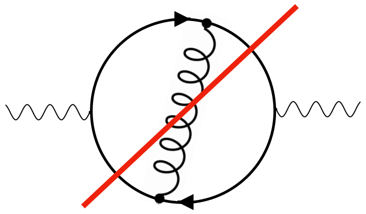

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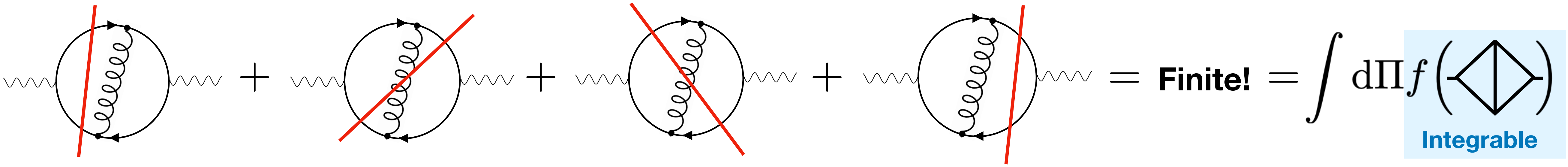

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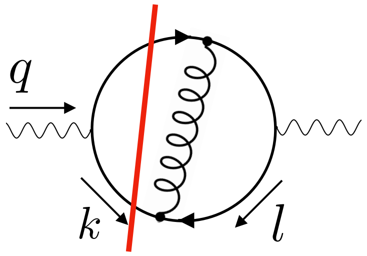

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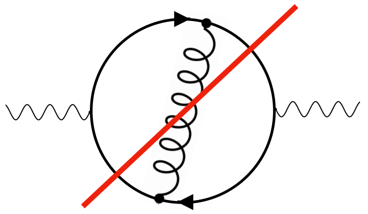

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Problem 1: Different number of deltas

Problem 2: Too few energy variables to solve the deltas

Problem 1: Different number of deltas

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Observation:

Problem 1: Different number of deltas

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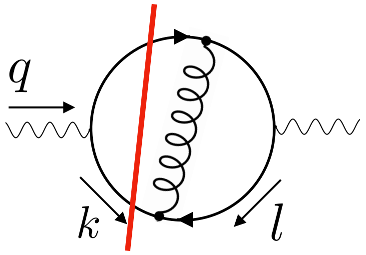

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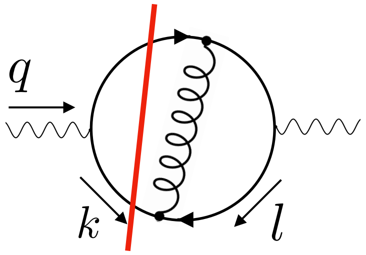

$$= \int \mathrm{d}^4k\, \delta^+(k^2)\, \delta^+((q-k)^2) \int \mathrm{d}^4l \frac{N(k,l,q)}{l^2(l+q)^2(k+l)^2}$$

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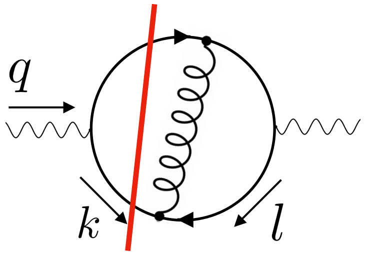

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unconstrained integration over l^0

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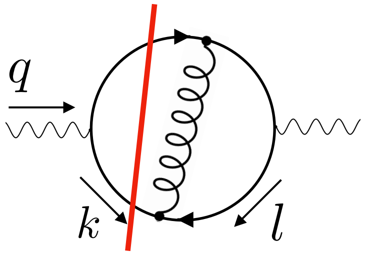

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unconstrained integration over l^0

Solution 1: Perform integration over l^0

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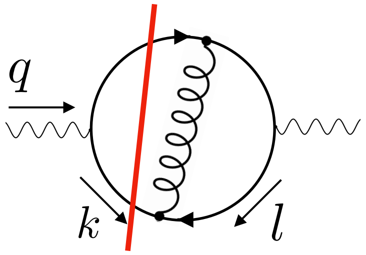

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(LTD, cLTD, TOPT…)

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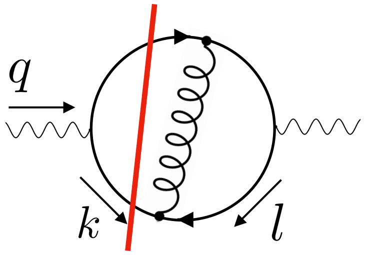

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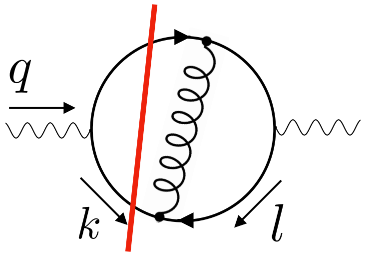

$$=$$

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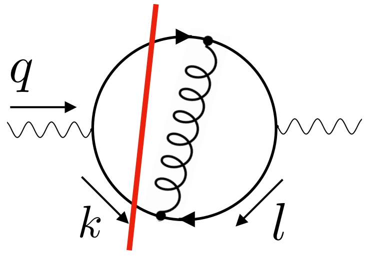

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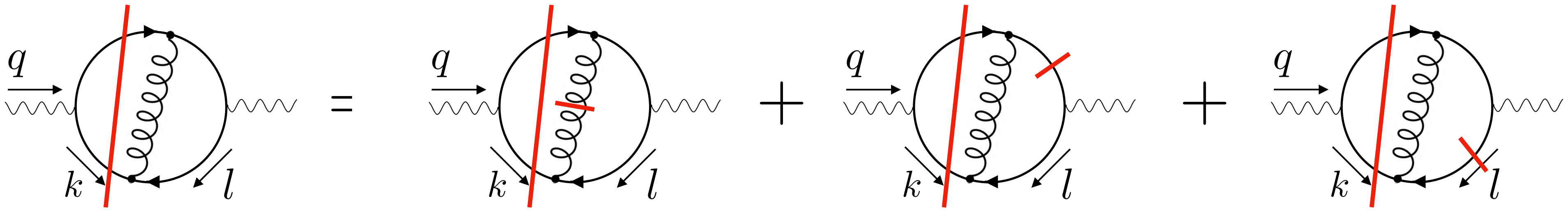

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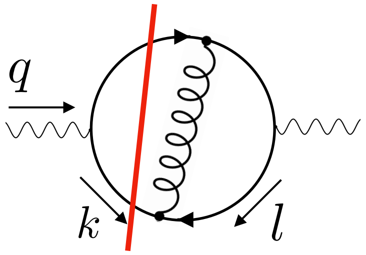

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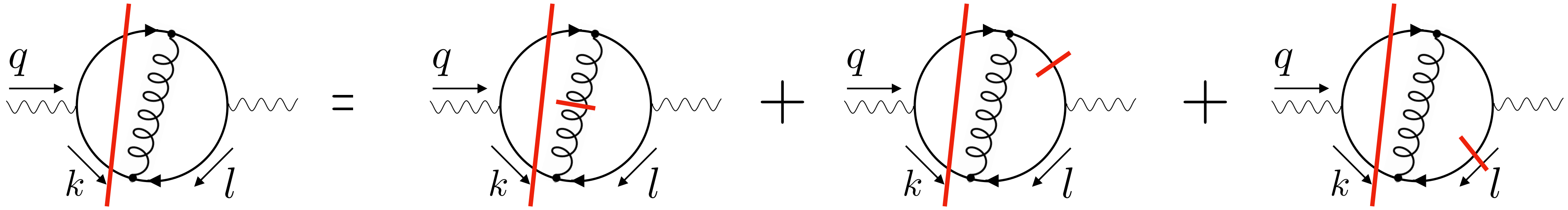

Catani, Gleisberg, Krauss, Rodrigo, Winter
arXiv: 0804.3170 (2008)

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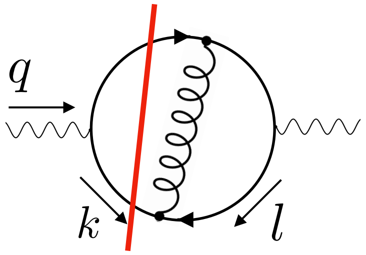

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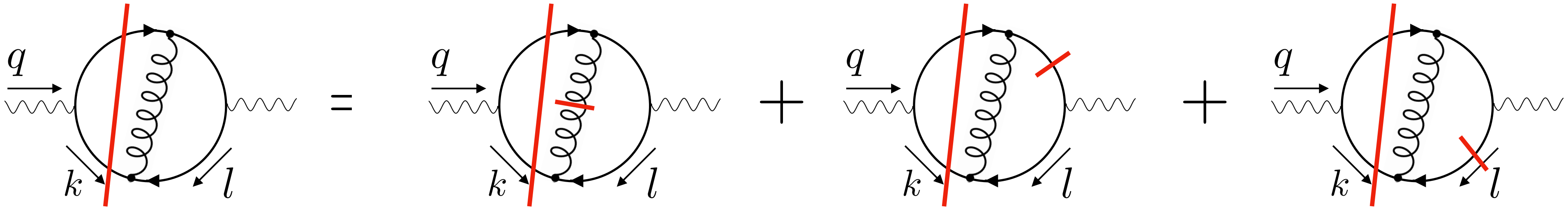

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Observation: Now real and virtual contributions have the same amount of deltas

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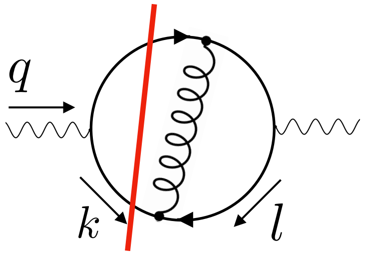

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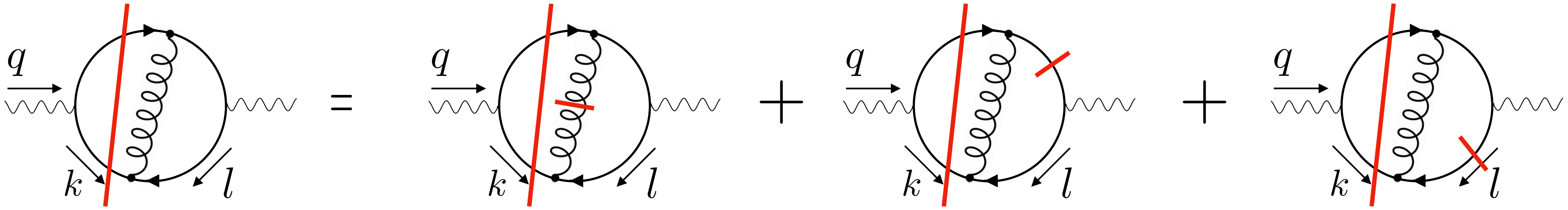
(LTD, cLTD, TOPT…)



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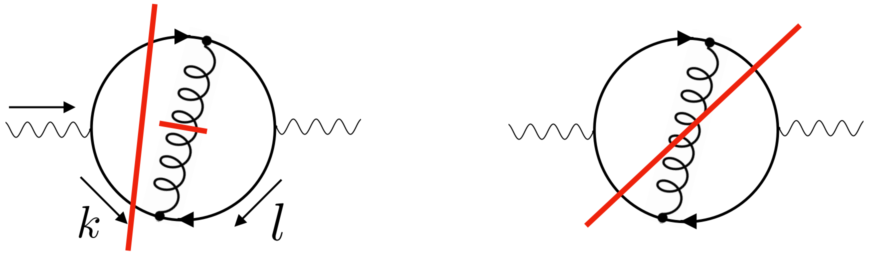

both have three cut lines!

Problem 2: Delta enforcing on-shell energy conservation

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After killing all
energy integrations…

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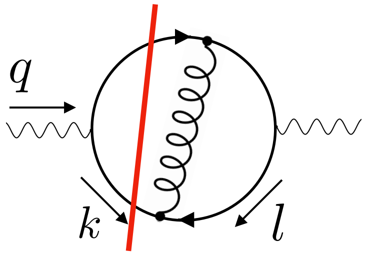

$$= \int \mathrm{d}^3\vec{k}\,\mathrm{d}^3\vec{l}\,\delta(2|\vec{k}| - Q_0)\, f_{\mathrm{v},1}$$

Problem 2: Delta enforcing on-shell energy conservation

After killing all
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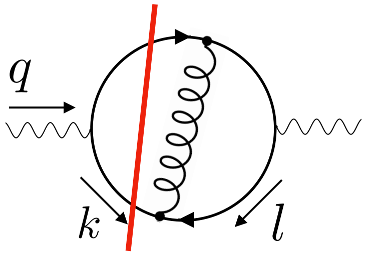

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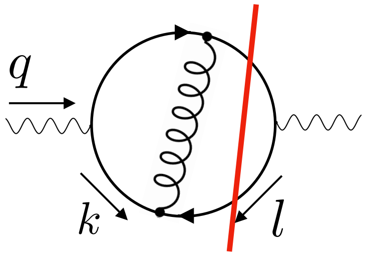

$$= \int \mathrm{d}^3\vec{k}\,\mathrm{d}^3\vec{l}\,\delta(2|\vec{l}| - Q_0)\, f_{\mathrm{v},2}$$

Problem 2: Delta enforcing on-shell energy conservation

After killing all
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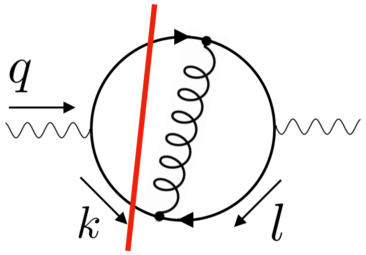
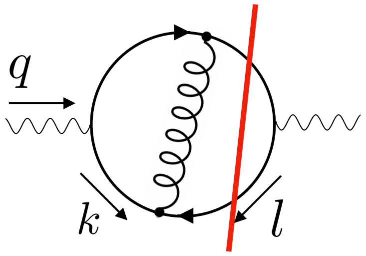


$$= \int \mathrm{d}^3\vec{k}\, \mathrm{d}^3\vec{l}\, \delta(2|\vec{k}| - Q_0)\, f_{\mathrm{v},1}$$

Solving in the norm
misaligns measure



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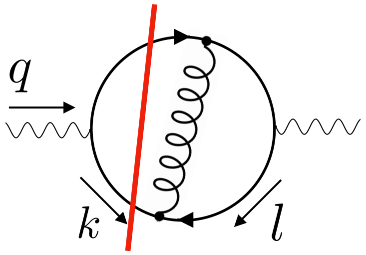

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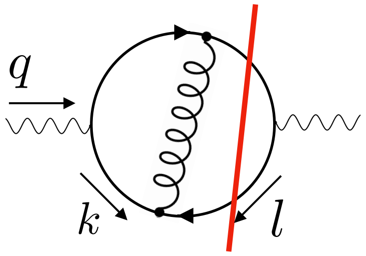

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Solution 2: Introduce fictitious variable to solve deltas

Problem 2: Delta enforcing on-shell energy conservation

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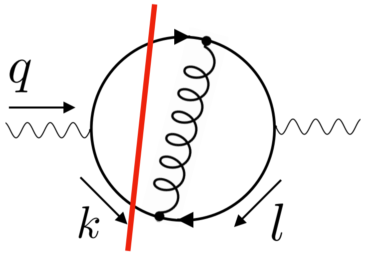

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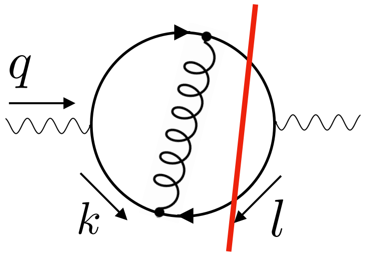

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Solution 2: Introduce fictitious variable to solve deltas

$$\int \mathrm{d}^3\vec{k}\,\delta(|\vec{k}| - Q_0)f(\vec{k})$$

Problem 2: Delta enforcing on-shell energy conservation

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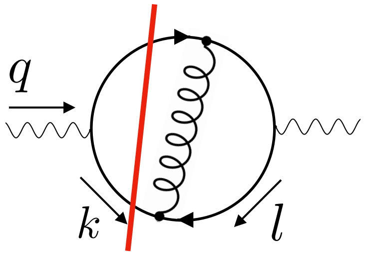

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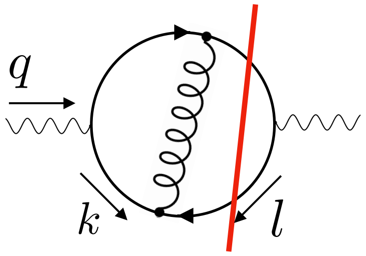

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Problem 2: Delta enforcing on-shell energy conservation

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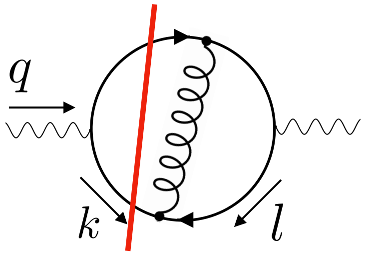

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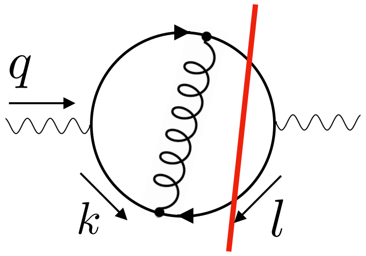

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$$\vec{k} \to t\vec{k}$$

Problem 2: Delta enforcing on-shell energy conservation

After killing all
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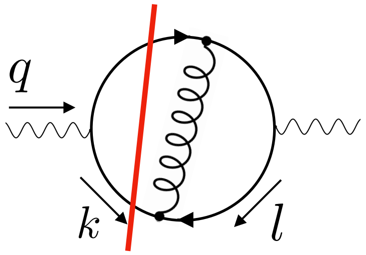

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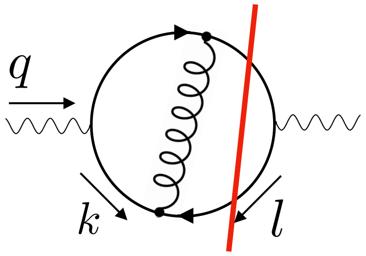

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Problem 2: Delta enforcing on-shell energy conservation

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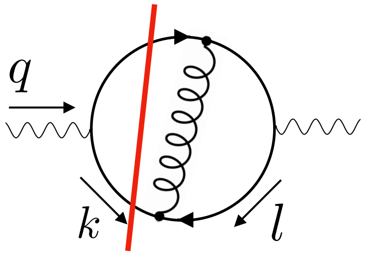

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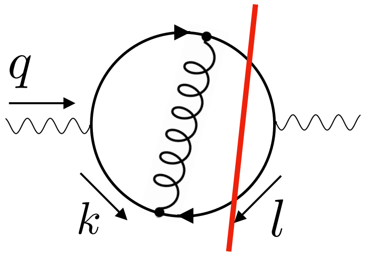

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Soper,
arXiv: 9804454 (1998)

Soper,
arXiv: 0102031 (2001 @ RADCOR)

ZC, Hirschi, Pelloni, Ruijl
arXiv: 2010.01068 (2020)

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After killing all
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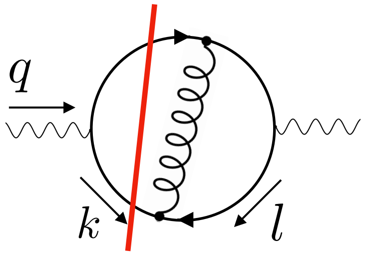

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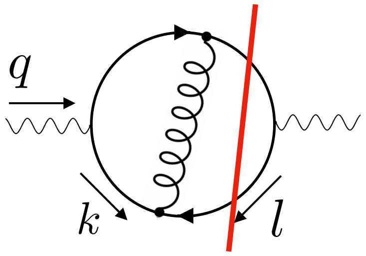

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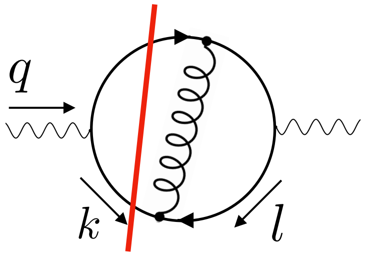
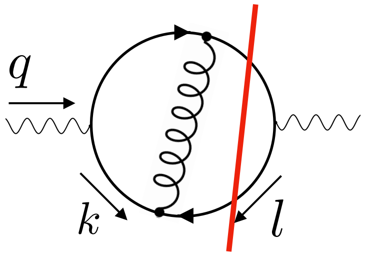
In the end:



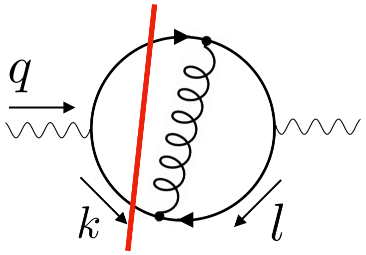
$+$



$$= \int \mathrm{d}^3\vec{k}\, \mathrm{d}^3\vec{l} \left[g_{\mathrm{v},1}(\vec{k},\vec{l}) + g_{\mathrm{v},2}(\vec{k},\vec{l}) \right]$$

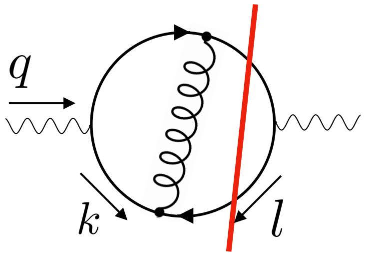
Problem 2: Delta enforcing on-shell energy conservation

After killing all energy integrations…



$$= \int \mathrm{d}^3\vec{k}\,\mathrm{d}^3\vec{l}\,\delta(2|\vec{k}| - Q_0)\, f_{\mathrm{v},1}$$

Solving in the norm misaligns measure



$$= \int \mathrm{d}^3\vec{k}\,\mathrm{d}^3\vec{l}\,\delta(2|\vec{l}| - Q_0)\, f_{\mathrm{v},2}$$

Solution 2: Introduce fictitious variable to solve deltas

$$\int \mathrm{d}^3\vec{k}\,\delta(|\vec{k}| - Q_0) f(\vec{k}) = \int \mathrm{d}^3\vec{k} \int \mathrm{d}t\, h(t)\,\delta(|\vec{k}| - Q_0) f(\vec{k})$$

$$1 = \int \mathrm{d}t\, h(t)$$

$$= \int \mathrm{d}^3\vec{k}\,\mathrm{d}t\, t^3\, h(t)\,\delta(t|\vec{k}| - Q_0) f(t\vec{k}) = \int \mathrm{d}^3\vec{k}\, h\left(\frac{Q_0}{|\vec{k}|}\right) \frac{Q_0^3}{|\vec{k}|^4} f\left(\frac{Q_0}{|\vec{k}|}\vec{k}\right)$$

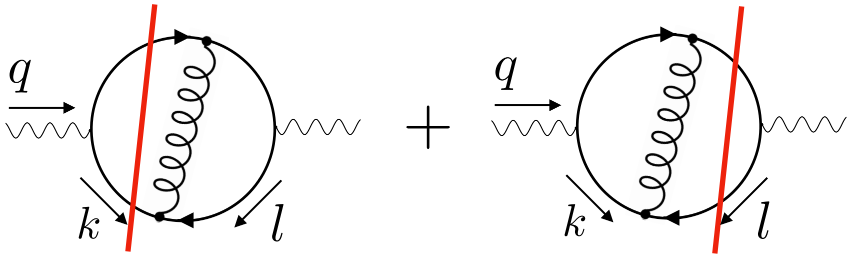
$$\vec{k} \to t\vec{k}$$

Soper,
arXiv: 9804454 (1998)

Soper,
arXiv: 0102031 (2001 @ RADCOR)

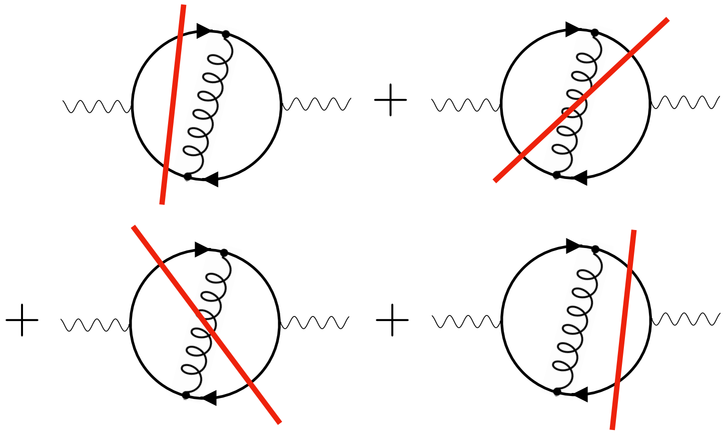
ZC, Hirschi, Pelloni, Ruijl
arXiv: 2010.01068 (2020)

In the end:

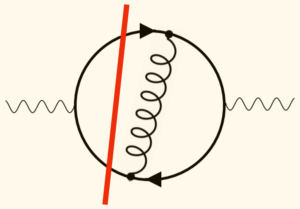


$$= \int \mathrm{d}^3\vec{k}\,\mathrm{d}^3\vec{l}\left[g_{\mathrm{v},1}(\vec{k},\vec{l}) + g_{\mathrm{v},2}(\vec{k},\vec{l})\right]$$

Observation: solved deltas, phase-space has same dimensionality (redundant dimension)



$$= \int \mathrm{d}^3\vec{k}\,\mathrm{d}^3\vec{l}\,\Big[g_{\mathrm{v},1}(\vec{k},\vec{l}) + g_{\mathrm{v},2}(\vec{k},\vec{l}) + g_{\mathrm{r},1}(\vec{k},\vec{l}) + g_{\mathrm{r},2}(\vec{k},\vec{l})\Big]$$



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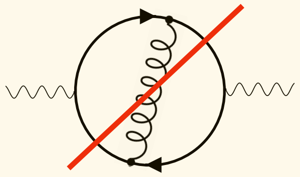

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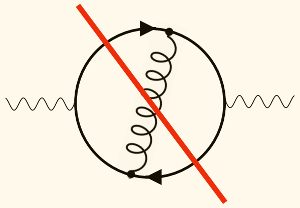

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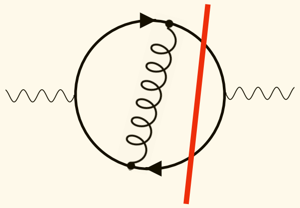

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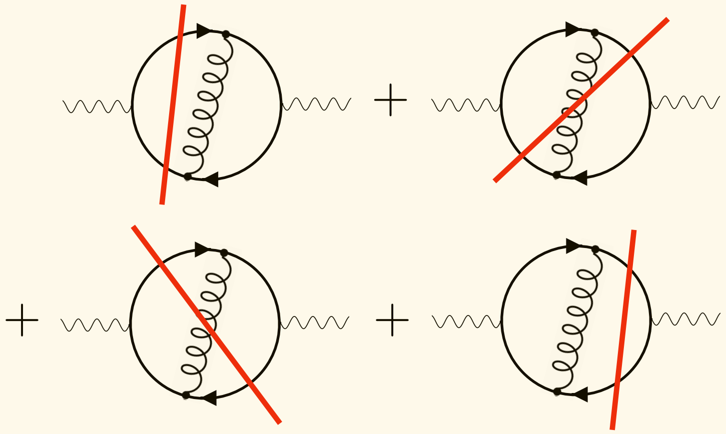
Measure is aligned now!



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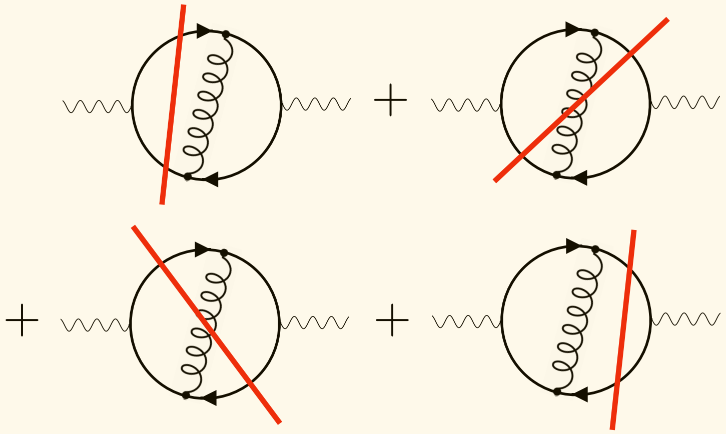

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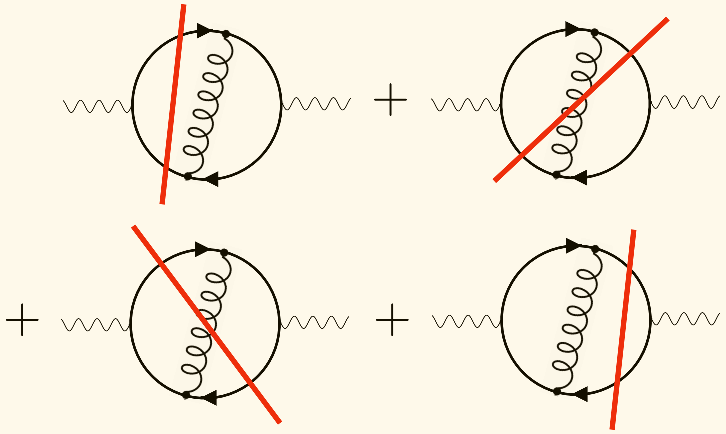

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- 3D representations hold at N loops
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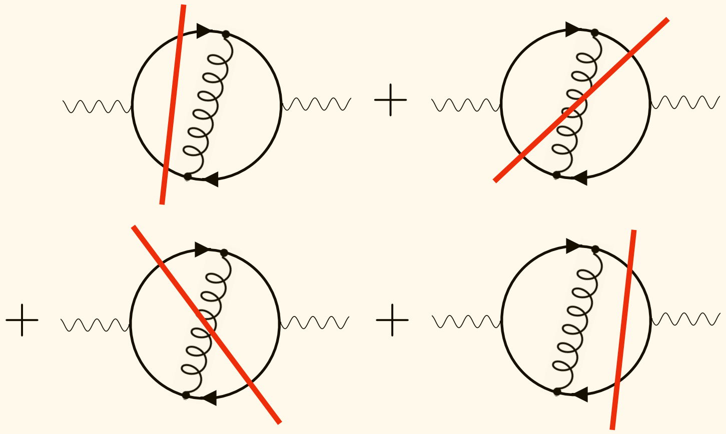

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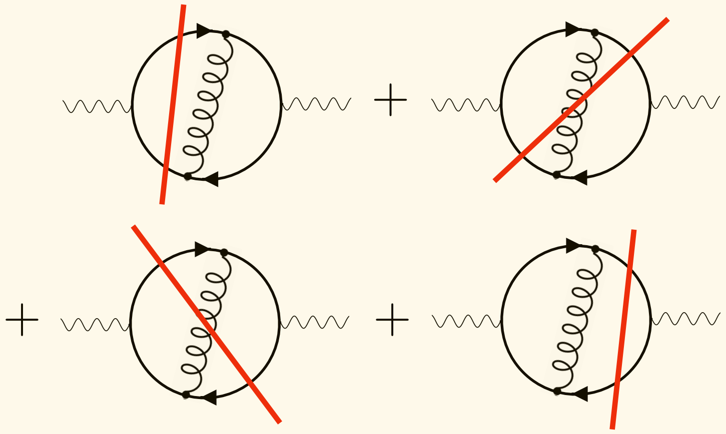

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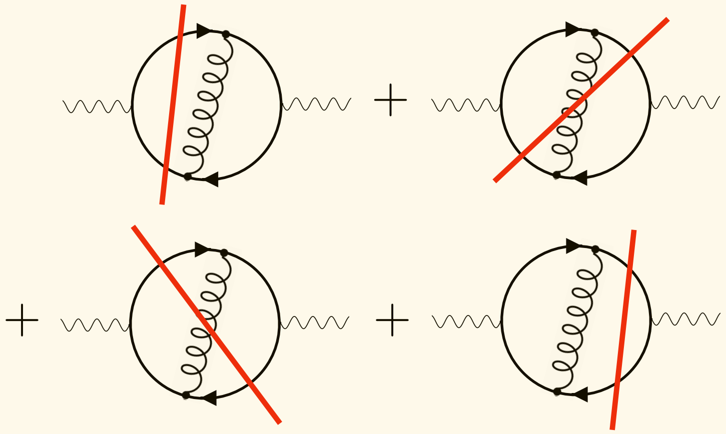

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Proof $\Rightarrow$ ZC, Hirschi, Pelloni, Ruijl
arXiv: 2010.01068 (2020)

Local IR cancellations, what next?

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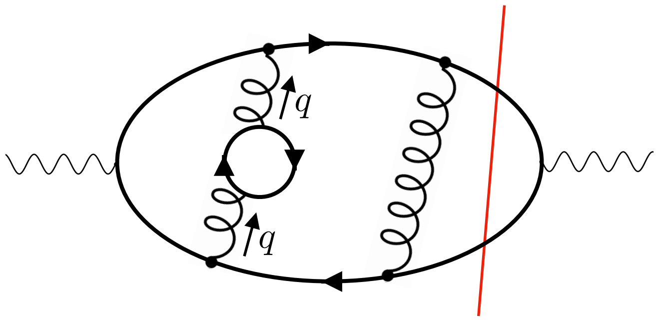

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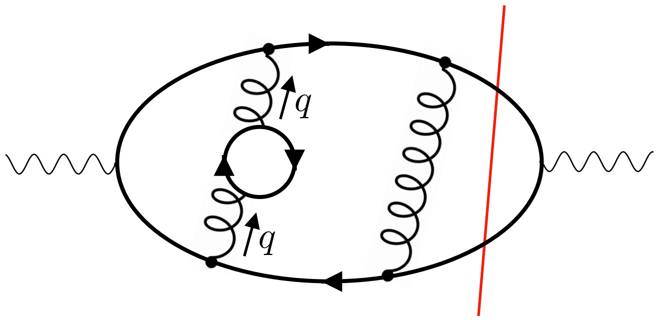

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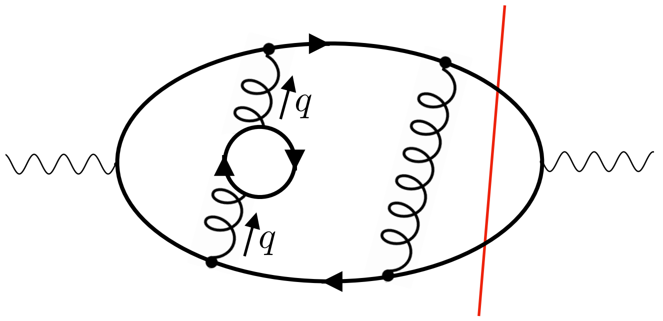

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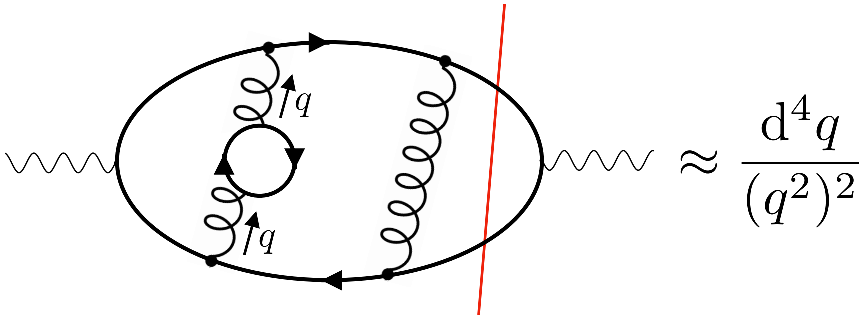

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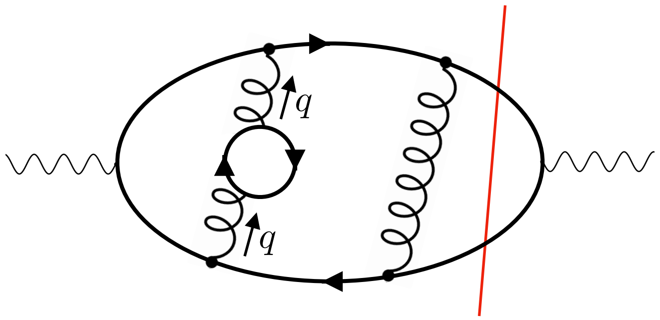

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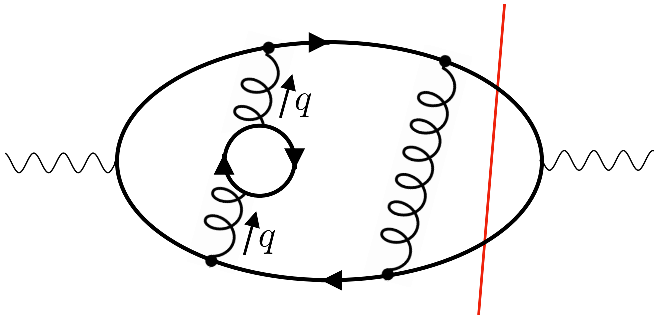

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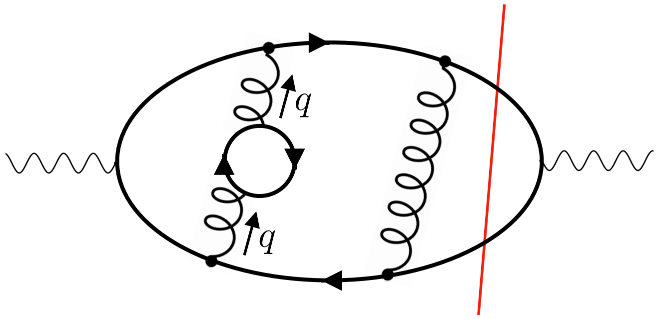

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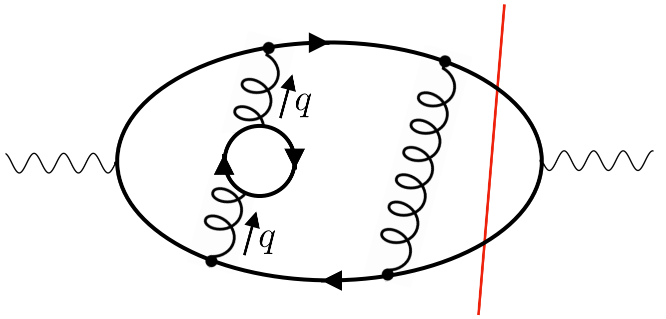

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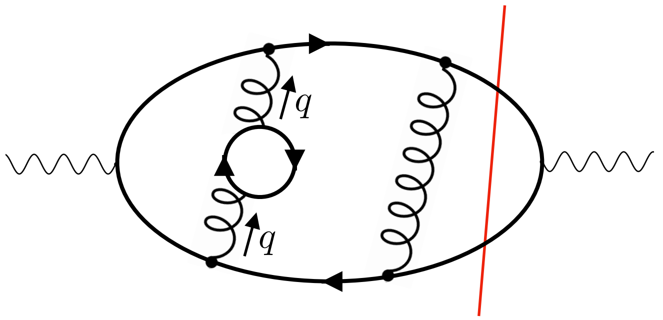

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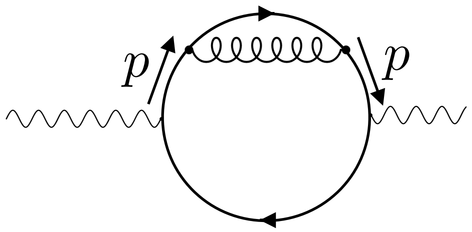

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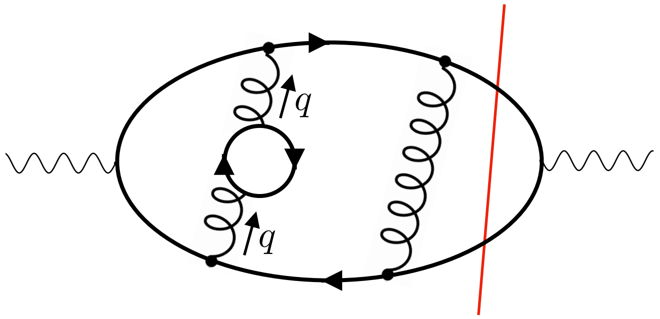

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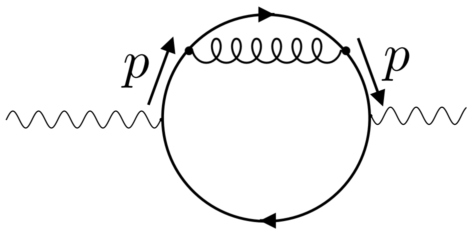

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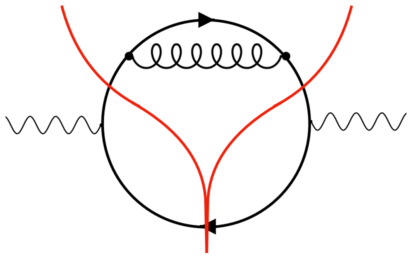

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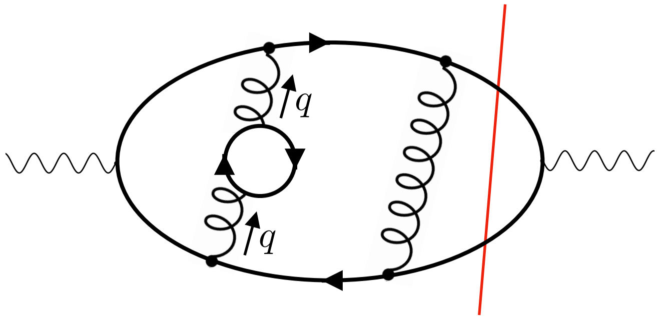

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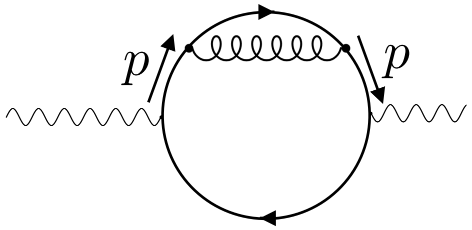

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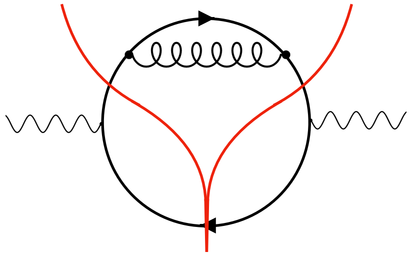

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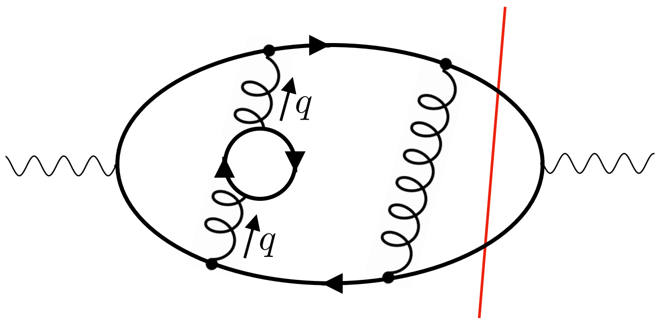

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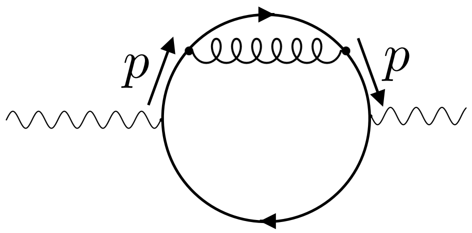

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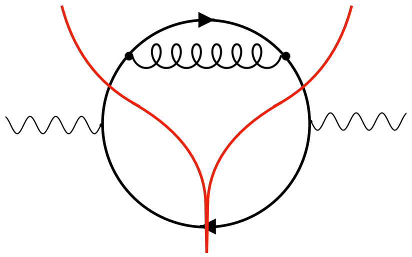

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→ Retain local IR cancellations: more complicated cancellation patterns

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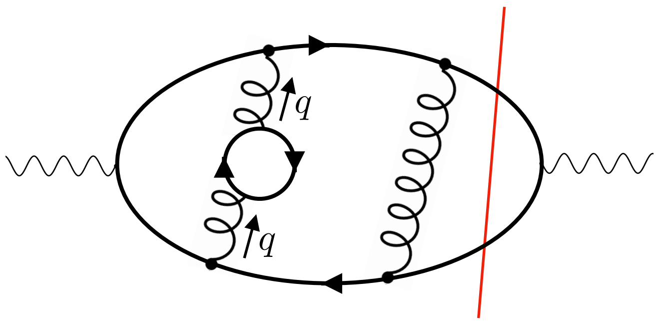

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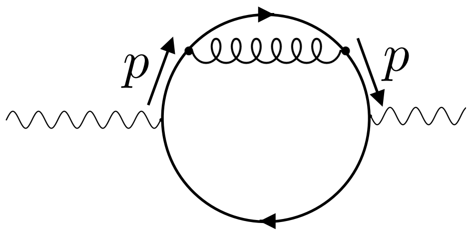

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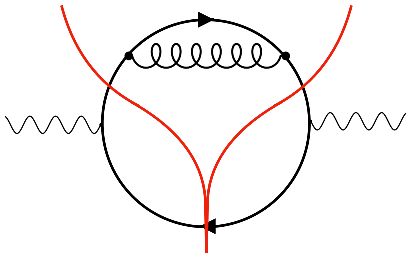

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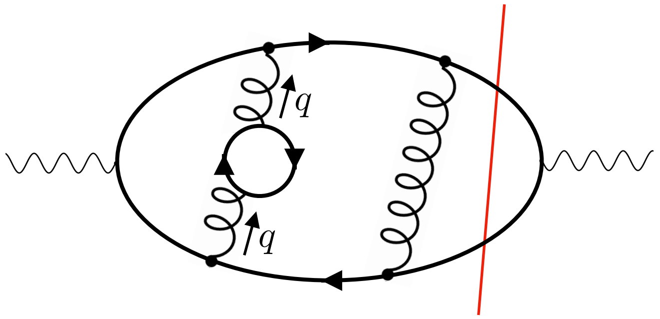

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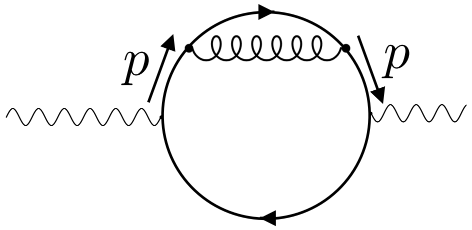

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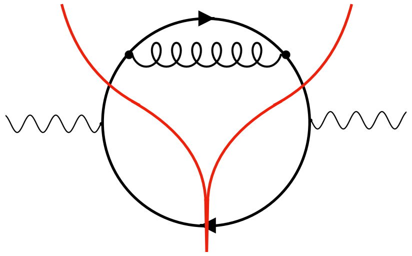

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→ Untangle mangling of scheme choice (OS vs MSbar) and IR structure

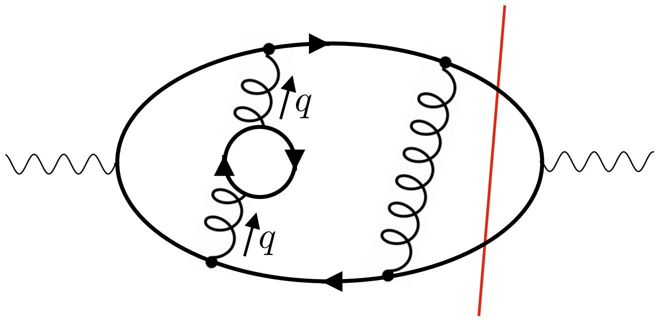
→ A new perspective on the LSZ formalism

Local IR cancellations, what next?

- **Automated UV renormalisation:**

Improved BPHZ

- → Local subtraction of UV divergences
- → Local subtraction of spurious soft divergences
- → Retain local IR cancellations



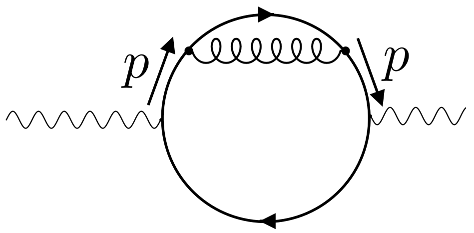
$$\approx \frac{\mathrm{d}^4 q}{(q^2)^2}$$

Localised Renormalisation

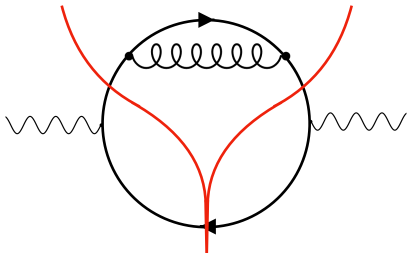
- → Minimal analytic complexity: only requires computation of vacuum bubbles
- → Hybrid MSbar+OS schemes

Method is only limited by computation of vacuum diagrams (known up to 4 loops)

- **Raised propagators:**



$$= \frac{1}{(p^2)^2} \quad \Rightarrow$$



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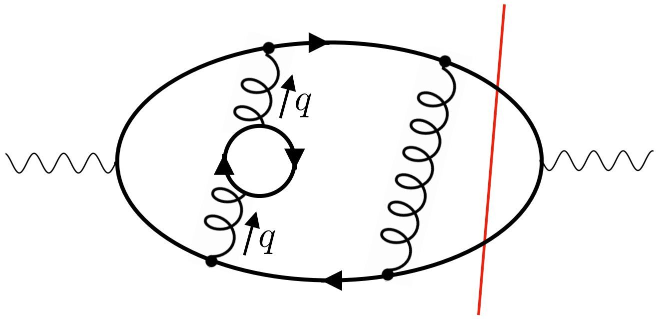

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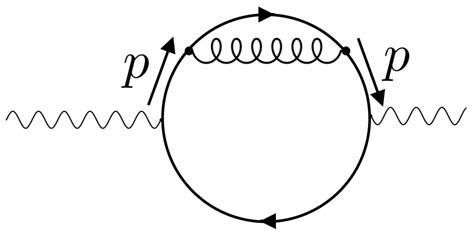

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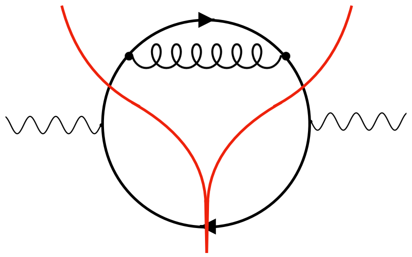

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Everything solved generically in arXiv: 2203.11038

Tests and results

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NNLO $e^+e^- \to jj$

and $e^+e^- \to t\bar{t}$

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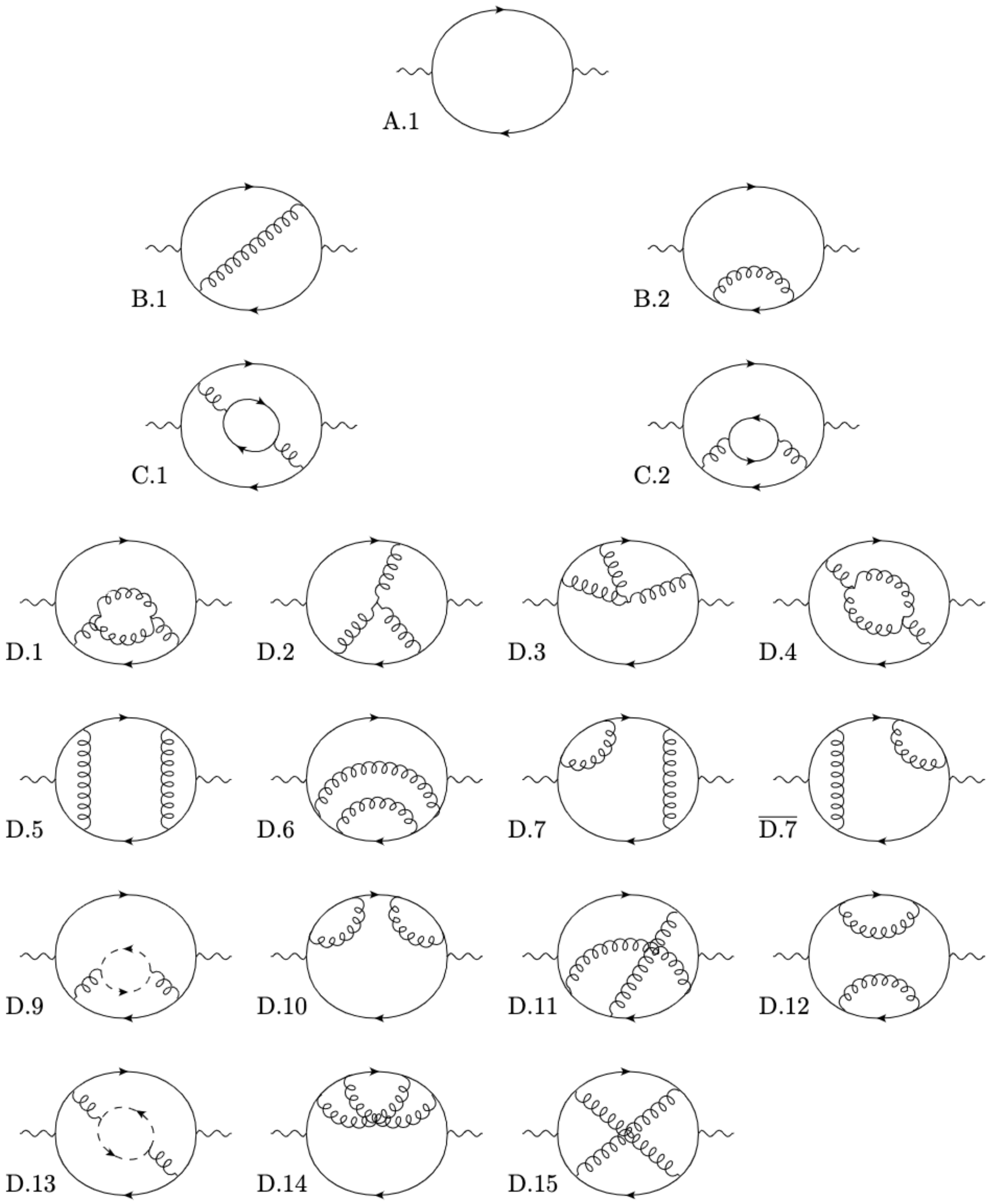

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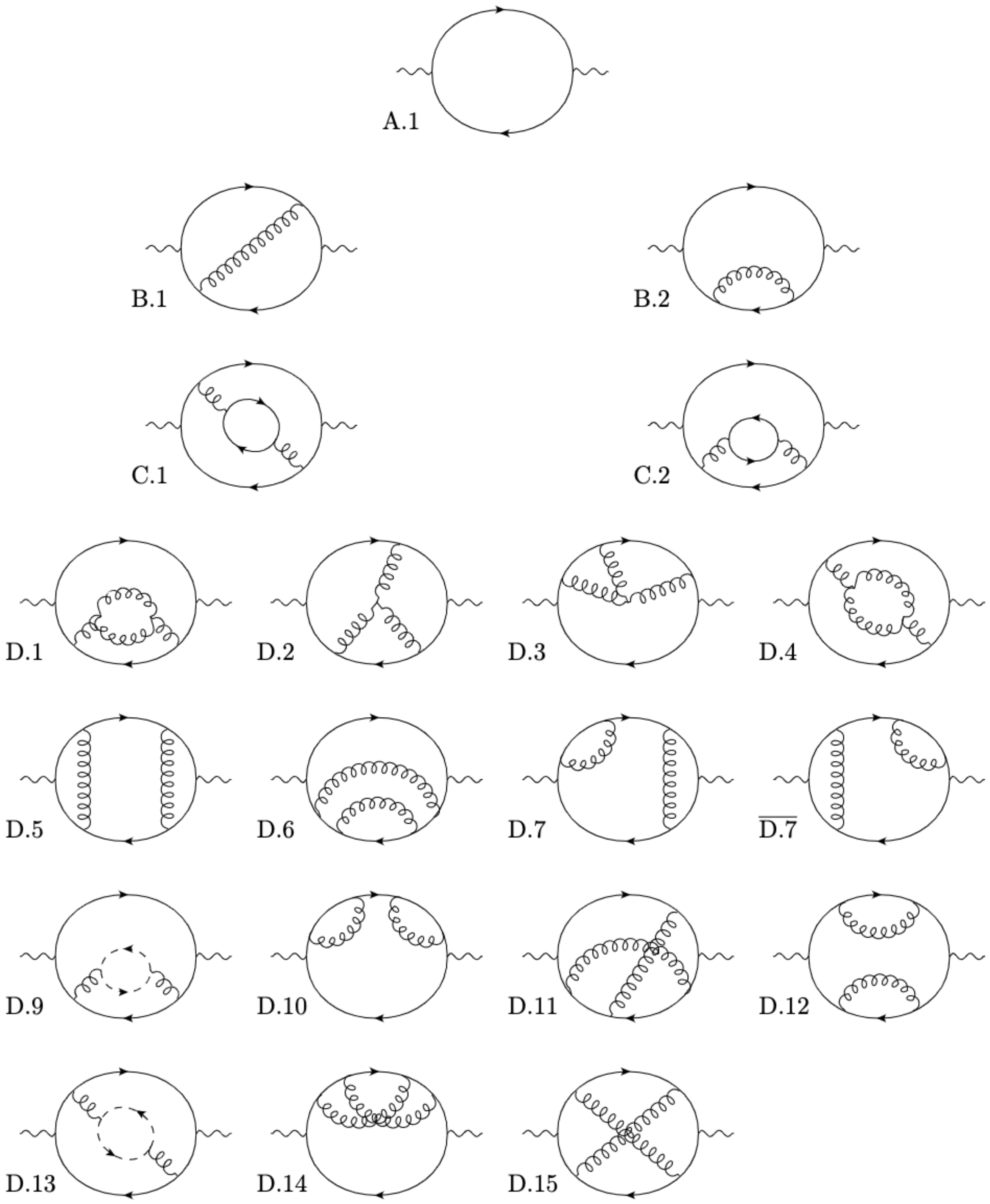

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LO		$\mathcal{O}(\alpha_s^0)$			
A.1	1	$1.387586 \cdot 10^{+00}$	0.0011	$1.509262 \cdot 10^{+01}$	0.000064
Total		$1.387586 \cdot 10^{+00}$	0.0011	$1.509262 \cdot 10^{+01}$	0.000064
NLO		$\mathcal{O}(\alpha_s)$			
B.1	1	$2.52705 \cdot 10^{-01}$	0.034	$-6.3725 \cdot 10^{-01}$	0.071
B.2	2	$1.80050 \cdot 10^{-01}$	0.049	$1.22702 \cdot 10^{+00}$	0.039
Total		$4.3276 \cdot 10^{-01}$	0.028	$5.8977 \cdot 10^{-01}$	0.11
Benchmark		$4.32831 \cdot 10^{-01}$	-0.018	$5.9047 \cdot 10^{-01}$	-0.12
NNLO		$\mathcal{O}(\alpha_s^2)$ ($n_f = 1$ contribution)			
C.1	1	$-1.0022 \cdot 10^{-03}$	0.17	$2.6658 \cdot 10^{-02}$	0.059
C.2	2	$-4.6982 \cdot 10^{-03}$	0.081	$-8.388 \cdot 10^{-03}$	0.30
Total		$-5.7004 \cdot 10^{-03}$	0.073	$1.8270 \cdot 10^{-02}$	0.16
Benchmark		$-5.6982 \cdot 10^{-03}$	0.038	$1.8296 \cdot 10^{-02}$	-0.15
NNLO		$\mathcal{O}(\alpha_s^2)$ (all other contributions)			
D.1	2	$3.8886 \cdot 10^{-02}$	0.031	$6.3163 \cdot 10^{-02}$	0.11
D.2	2	$5.6351 \cdot 10^{-03}$	0.14	$-3.52337 \cdot 10^{-01}$	0.027
D.3	2	$1.76075 \cdot 10^{-02}$	0.055	$5.6646 \cdot 10^{-02}$	0.14
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D.5	1	$9.200 \cdot 10^{-04}$	0.79	$-7.9531 \cdot 10^{-02}$	0.054
D.6	2	$5.1058 \cdot 10^{-03}$	0.15	$1.1244 \cdot 10^{-02}$	0.51
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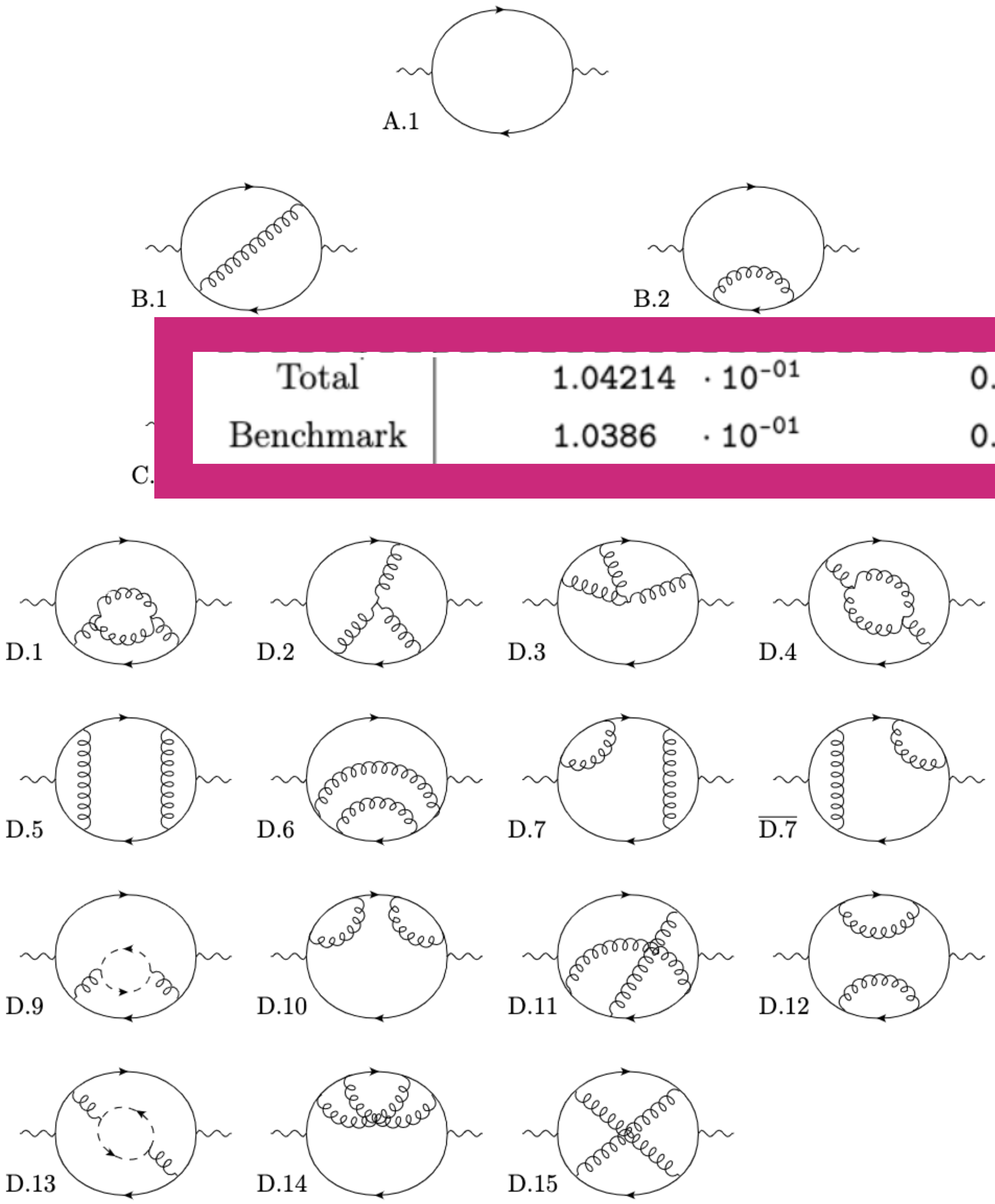

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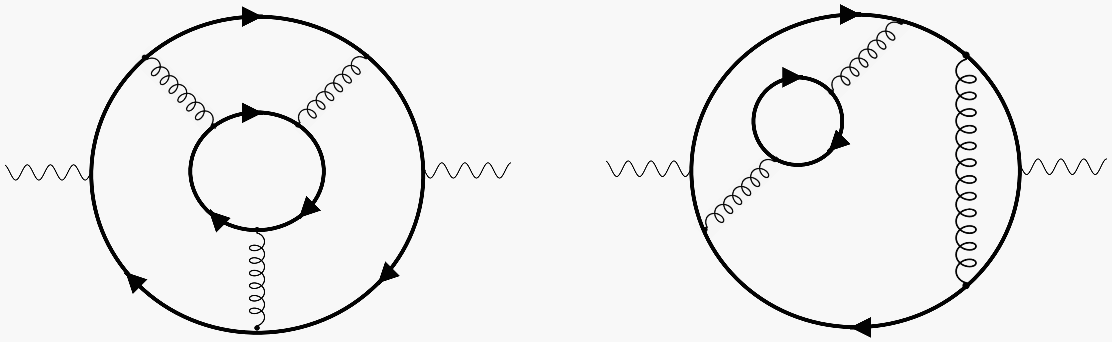

(+ many more)

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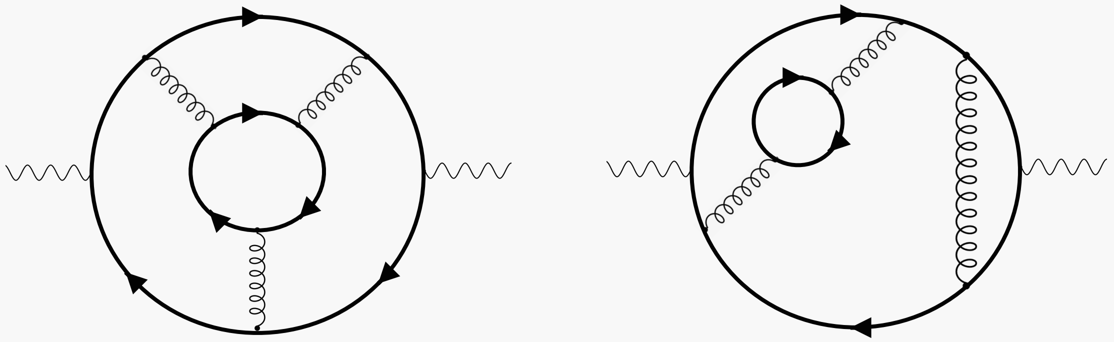

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N_f^2 part @N^{3}LO $e^+e^- \to jj$

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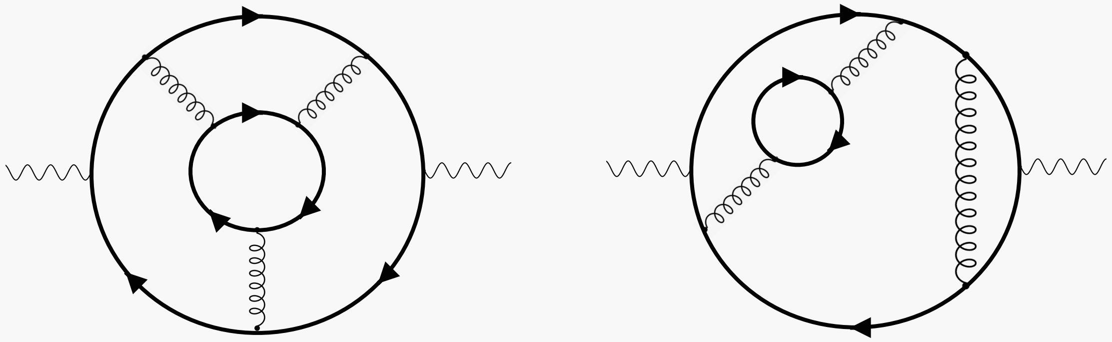

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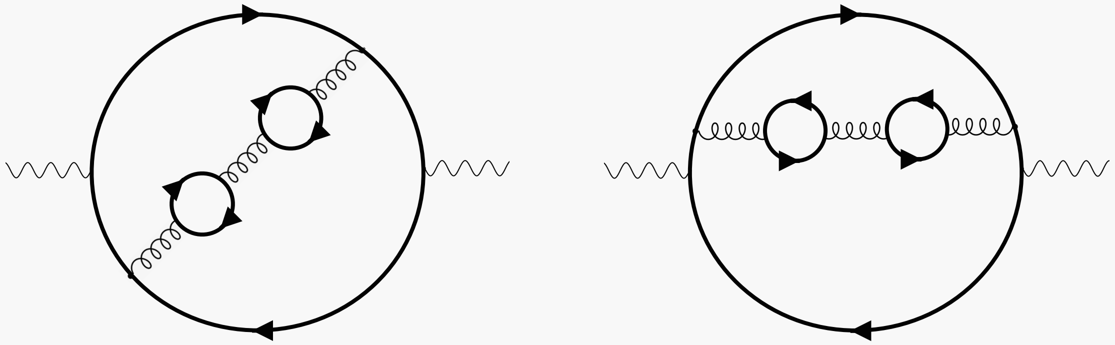

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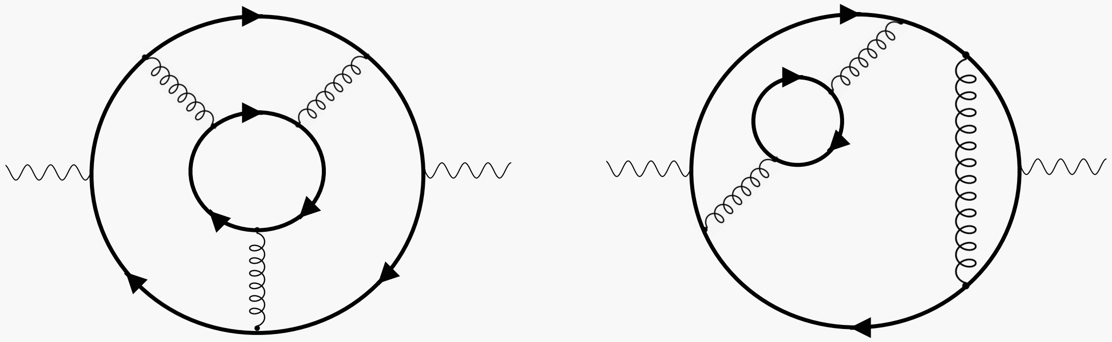

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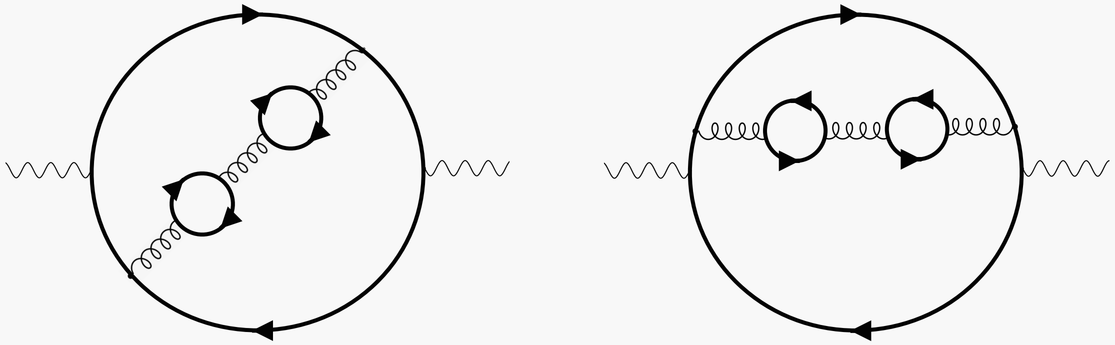

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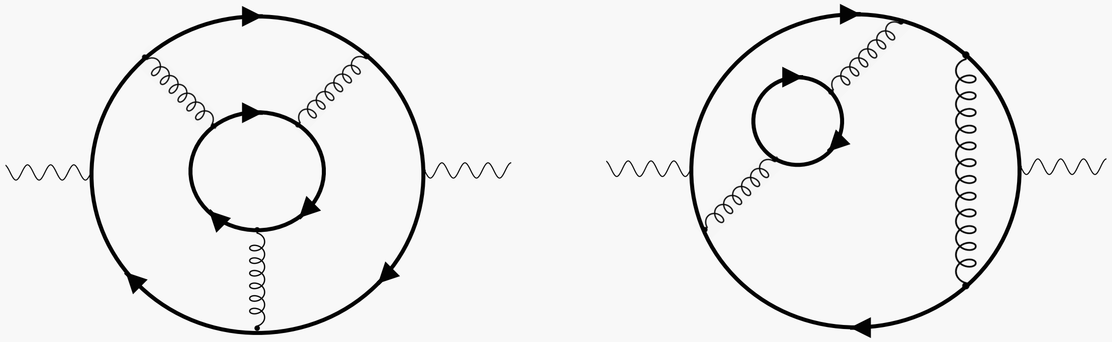

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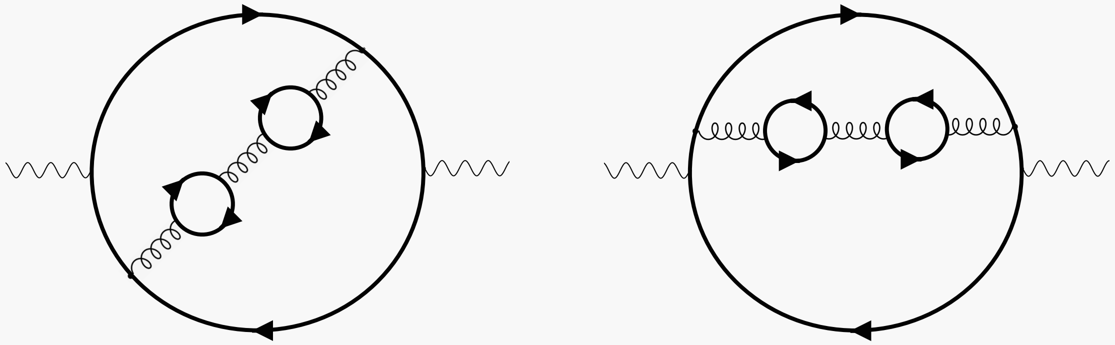

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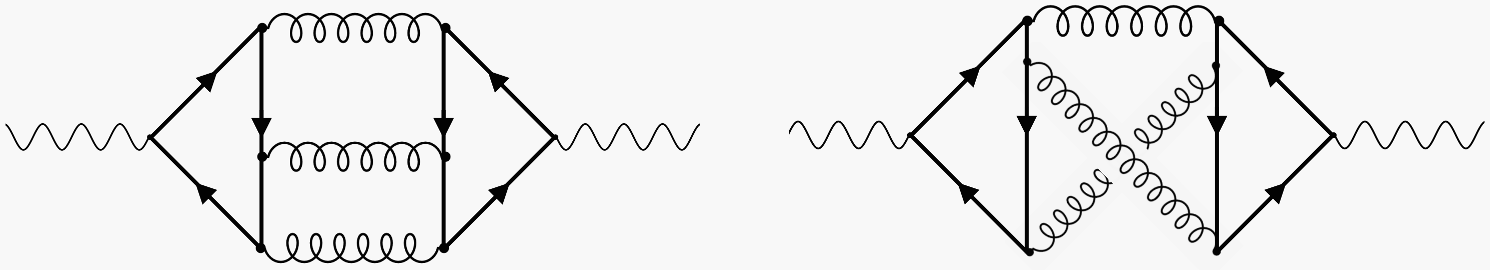

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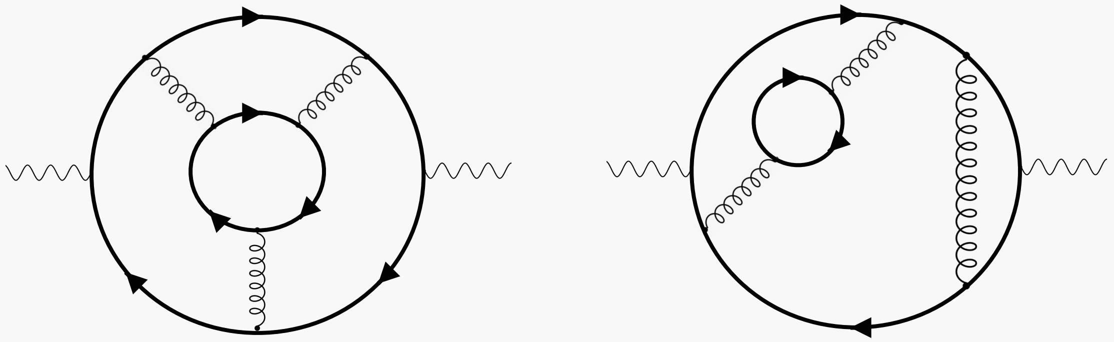

(+ many more)

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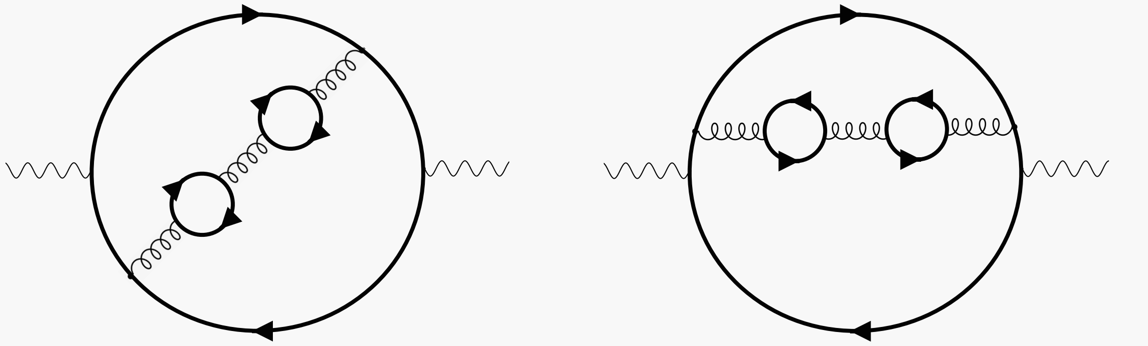

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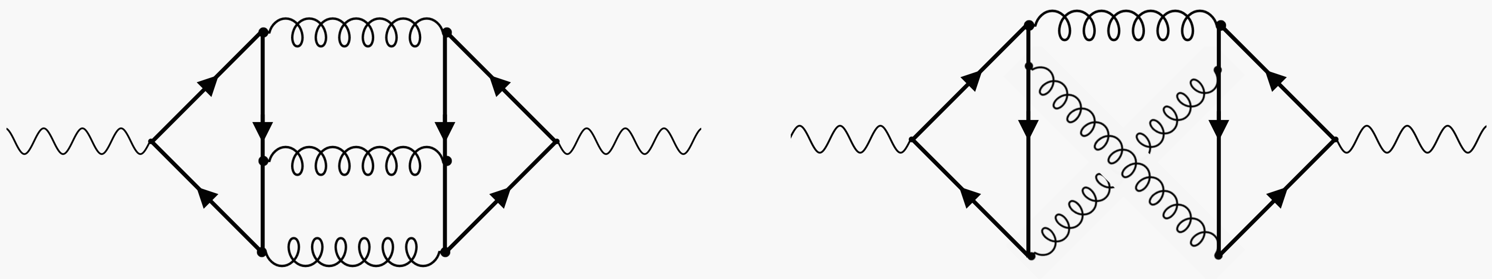

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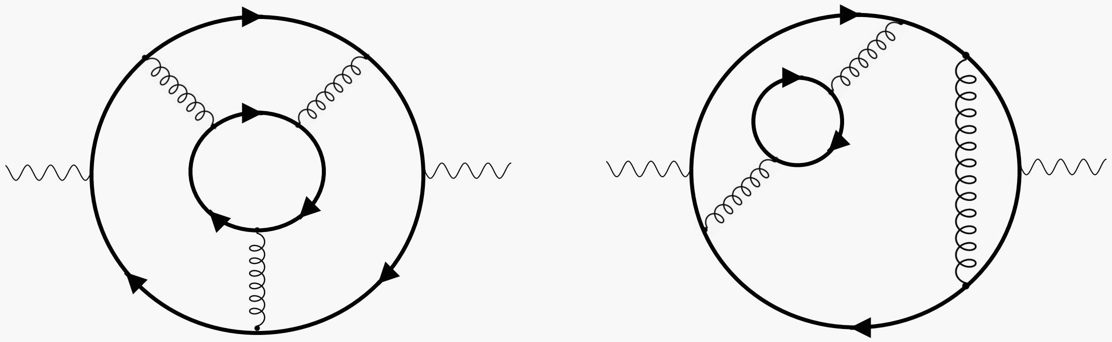

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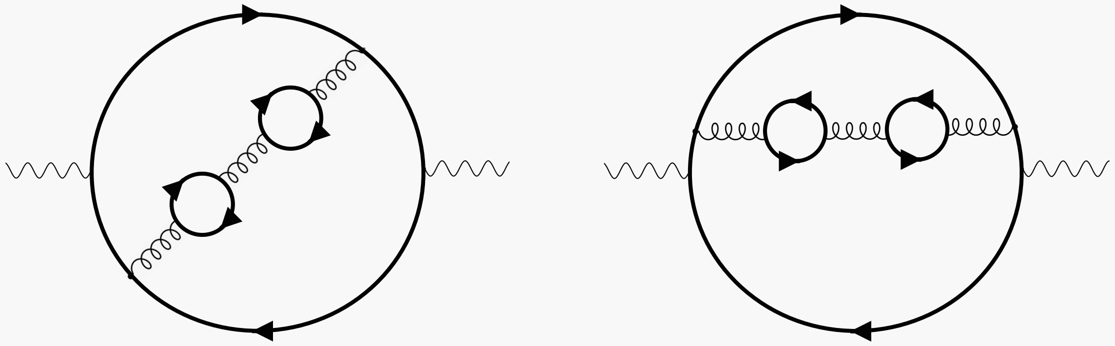

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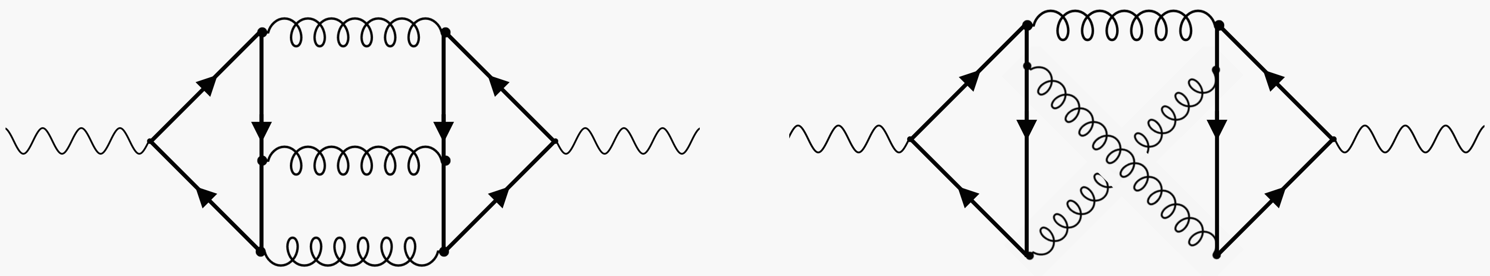

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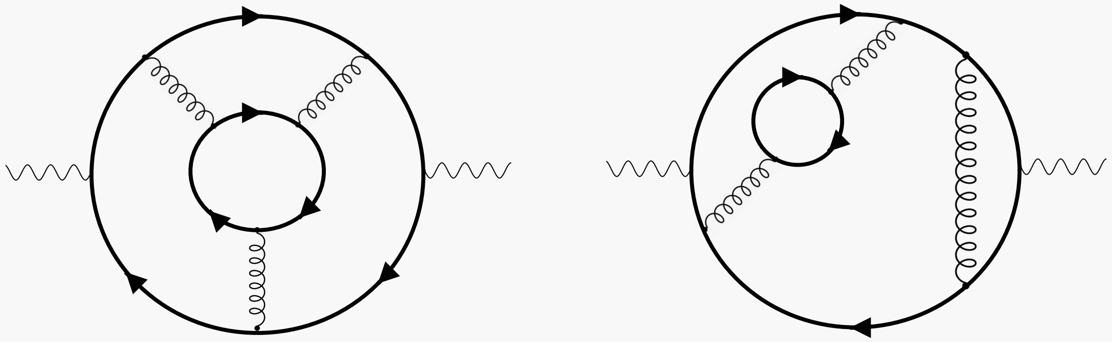

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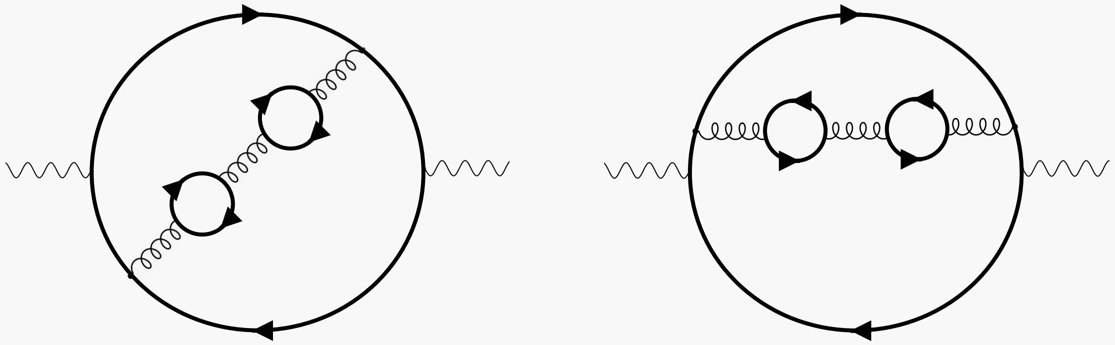

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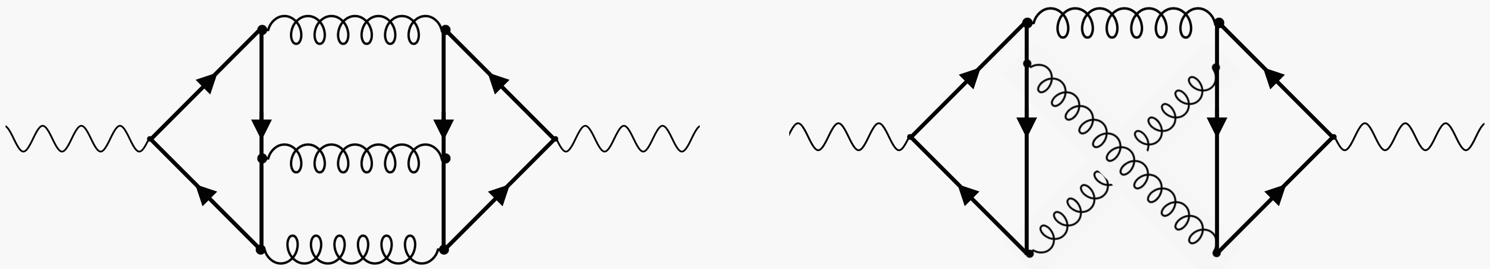

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Benchmarks:

Herzog, Ruijl, Ueda, Vermaseren, Vogt :
arXiv:1707.01044

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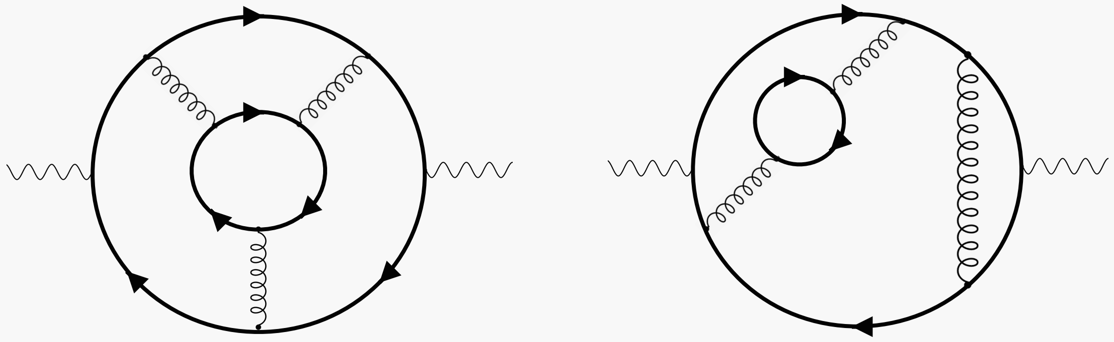

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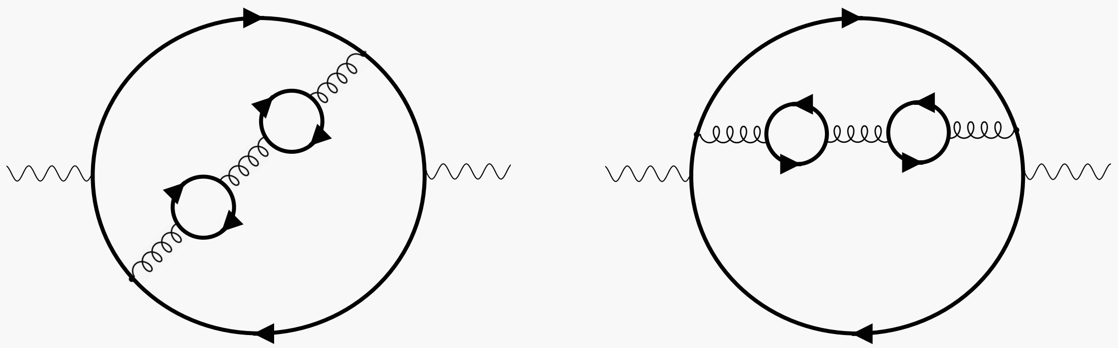

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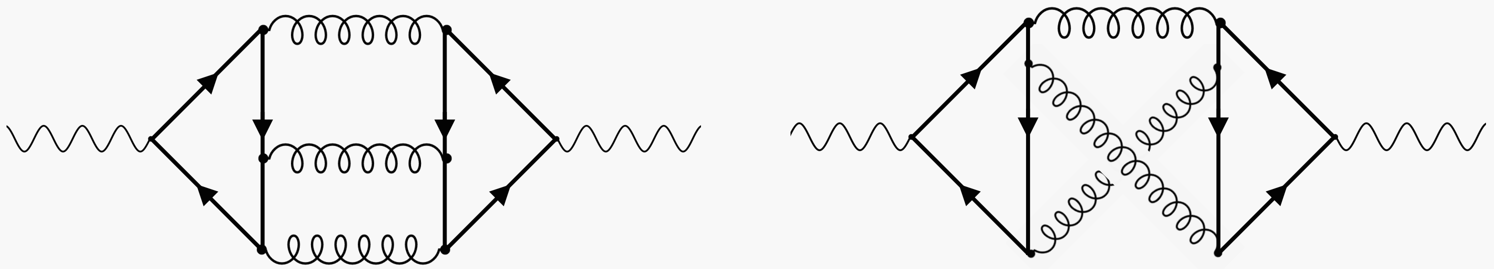
singlet part @N^{3}LO $e^+e^- \to jj$



$$K_{\alpha_s^3}^{\mathrm{singlet},\mathrm{LU}} = -25.6(1.5)$$

$$K_{\alpha_s^3}^{\mathrm{singlet},\mathrm{bm}} = -26.4435$$

Benchmarks:

Herzog, Ruijl, Ueda, Vermaseren, Vogt : arXiv:1707.01044

Testing

- ✓ N3LO IR cancellations
- ✓ 3-loop UV renormalisation
- ✓ 1,2,3-loop self-energies

Outlook

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Local Unitarity soon ready for automation of processes without initial-state singularities

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Outlook

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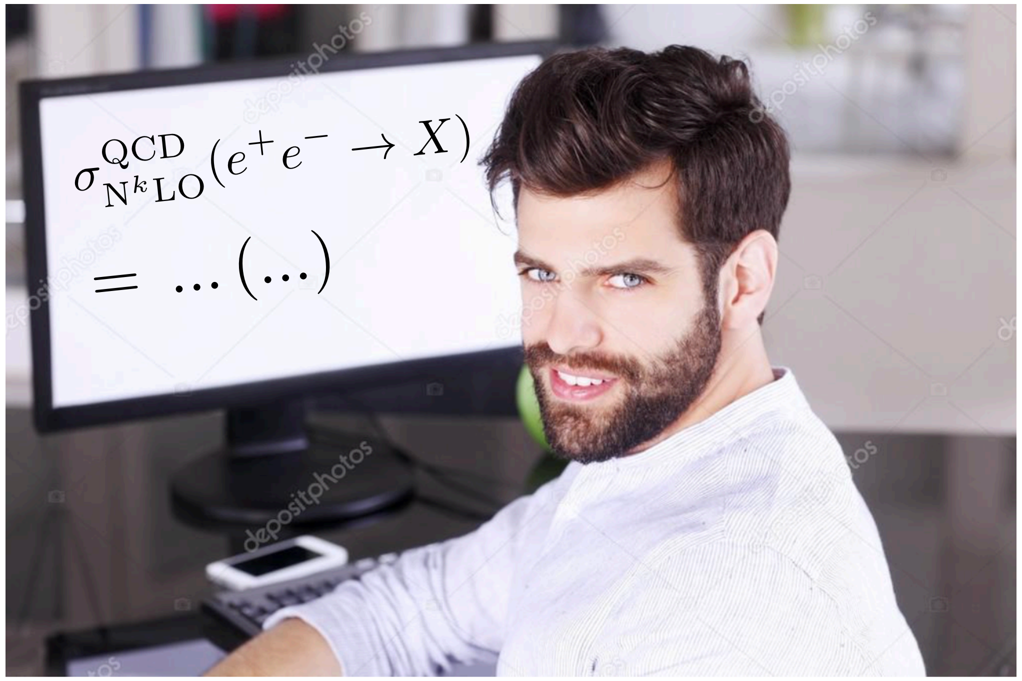

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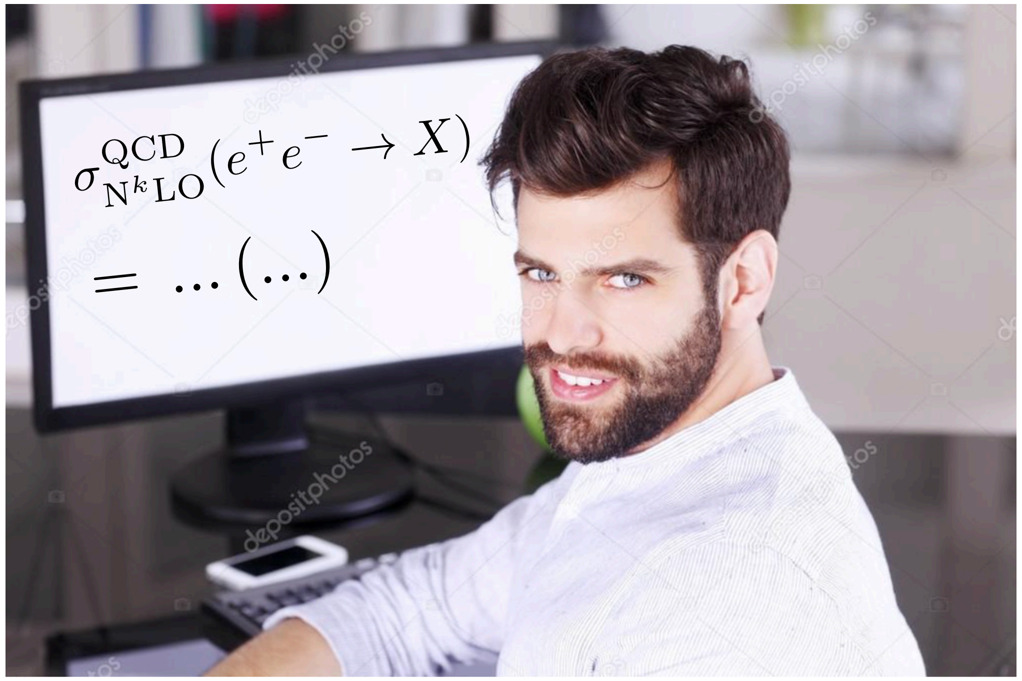

Future theory work:

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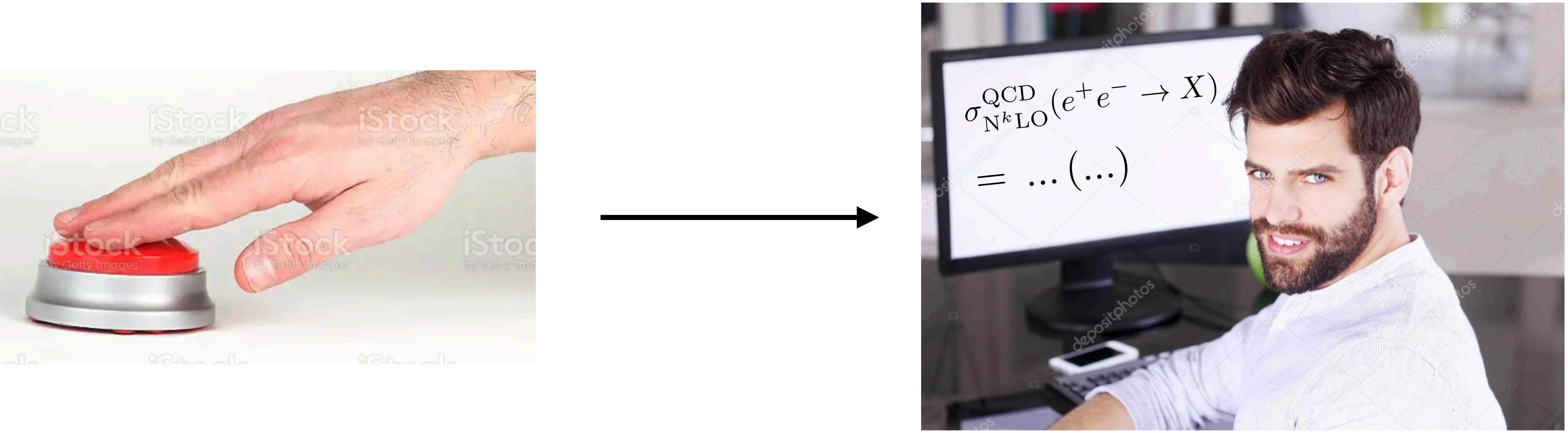

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Resummation: we would like an automatable solution, that only relies on the general factorisation properties of amplitudes

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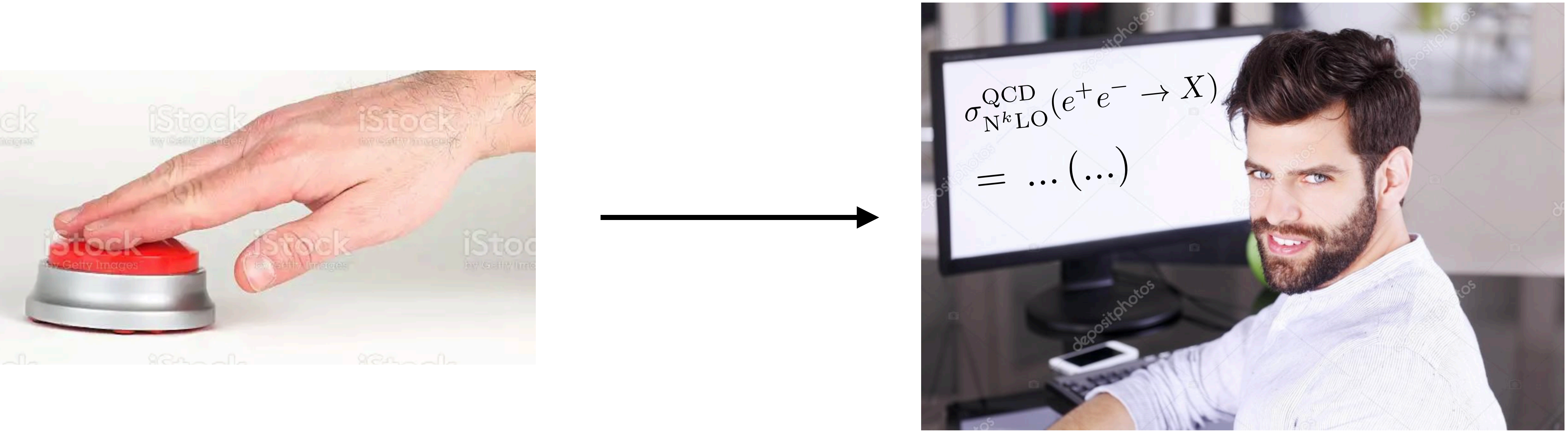

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Banfi, Salam, Zanderighi, arXiv:0304148

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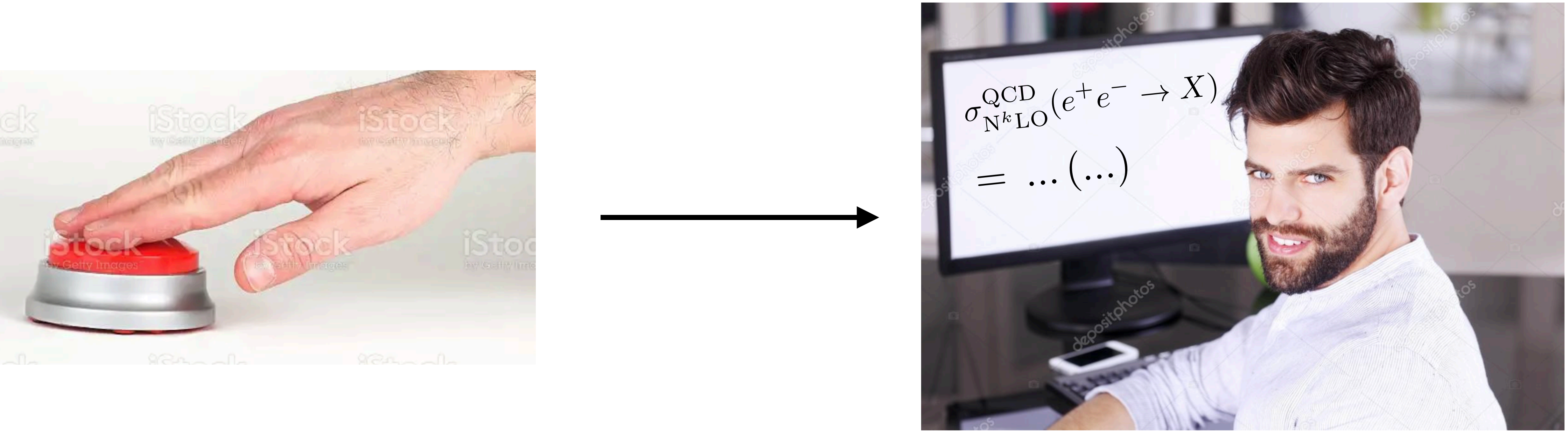

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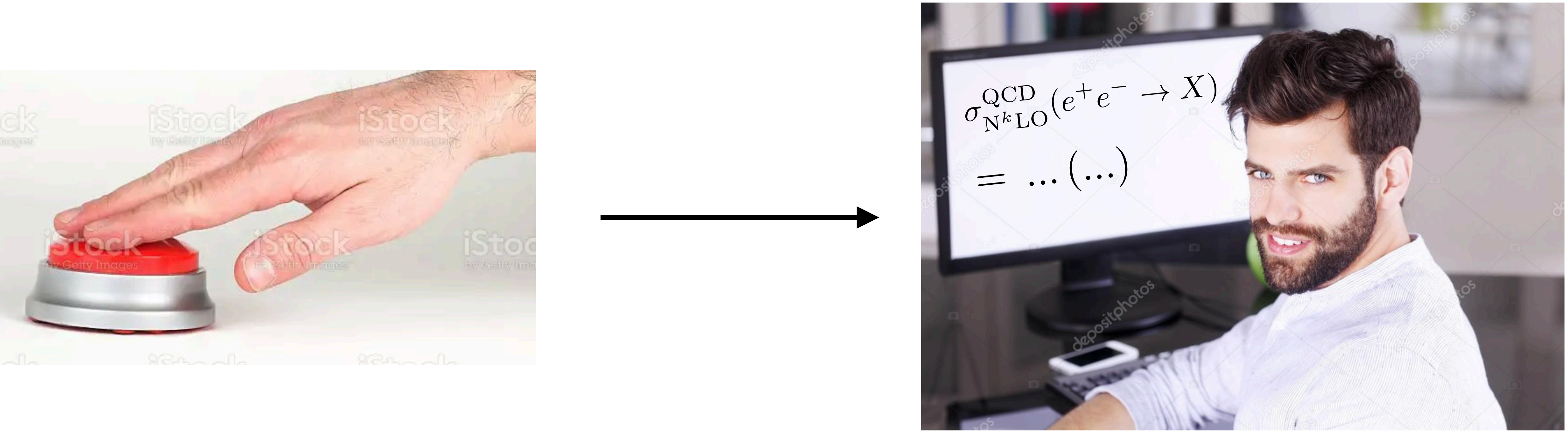

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- **KLN for initial states:** the KLN cancellation mechanism can be extended to include initial state singularities, but requires a significant departure from the Parton model

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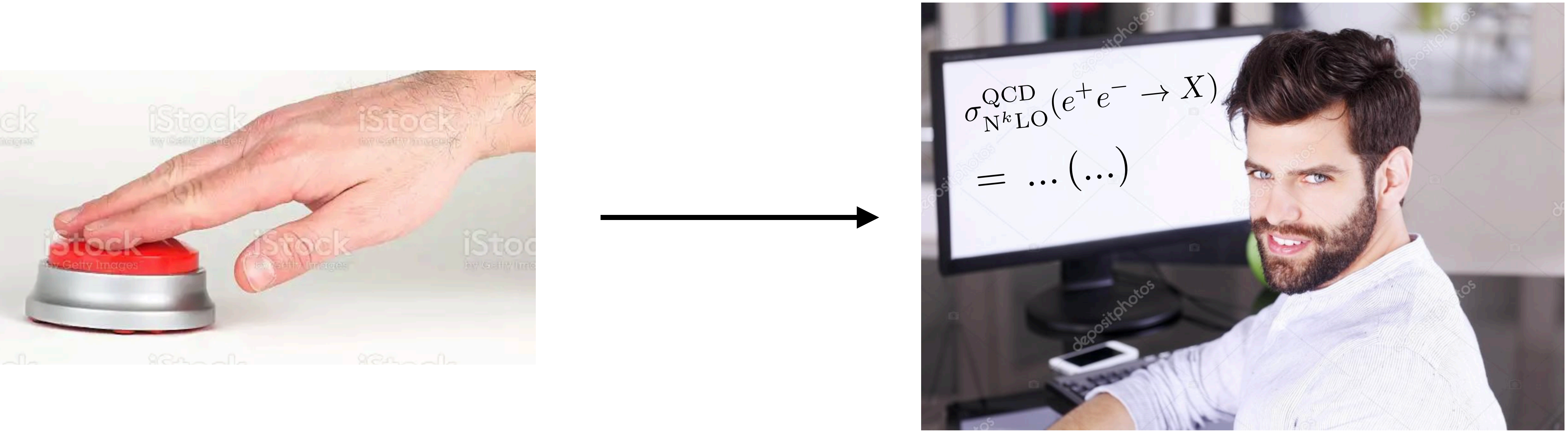

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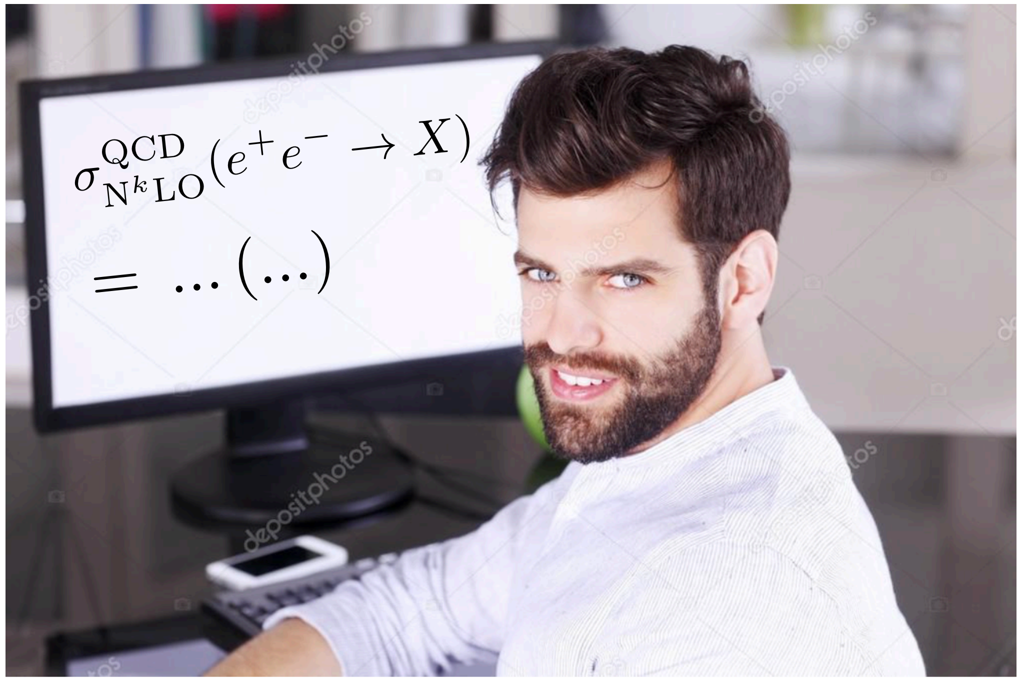

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- **KLN for initial states:** the KLN cancellation mechanism can be extended to include initial state singularities, but requires a significant departure from the Parton model

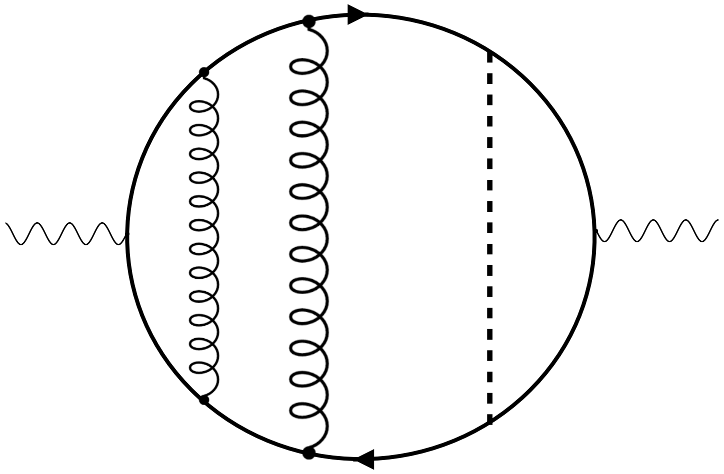
a lot of physics and cool mathematics!

- **Local PDF counterterms:** develop a competitive subtraction method or integrate existing ones

Thank you!

$$\text{N}^2\text{LO} \quad \gamma^\star \to t\bar{t}H$$

Top energy distribution for the supergraph



$$\sqrt{s} = 1\text{TeV}$$