Multiple parton scattering: the Pythia model and sum rules

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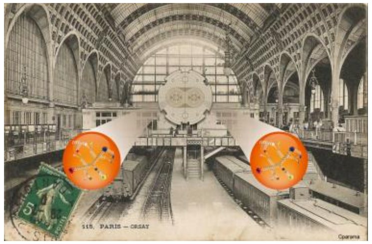
Based on arXiv:2208.08197 with Oleh Fedkevych



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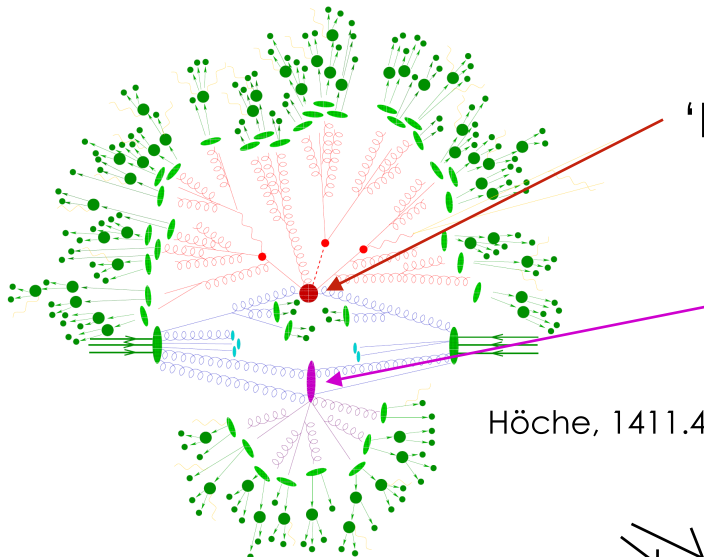
MULTIPLE INTERACTIONS: INTRODUCTION

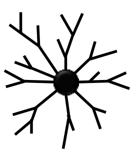
Protons are ‘bags’ of quarks and gluons – multiple interactions likely!

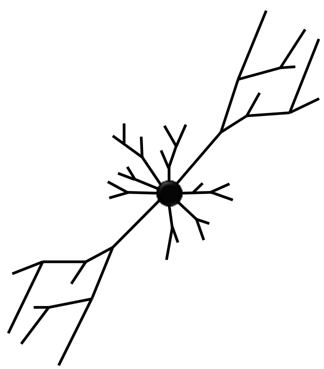


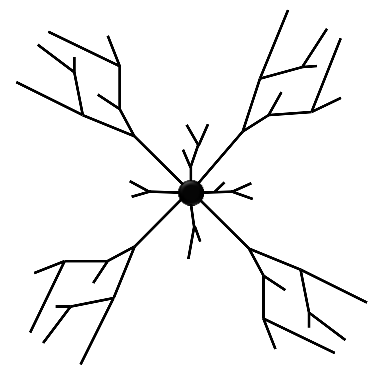
‘Primary process’

Additional scatters

Höche, 1411.4085







‘Minimum bias’

‘Underlying event’

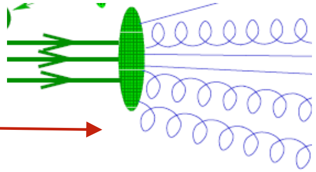
Double(/triple…) hard parton scattering. Rarer…but DPS measured for several processes, first indications of TPS! CMS, arXiv:2111.05370

<u>Ubiquitous!</u>

For a summary see Gaunt, Bartalini (eds.), *Adv.Ser.Direct.High Energy Phys.* 29 (2018)

MULTI-PARTON PDFS

Any Monte Carlo event generator needs a model of multi-parton PDFs (mPDFs)

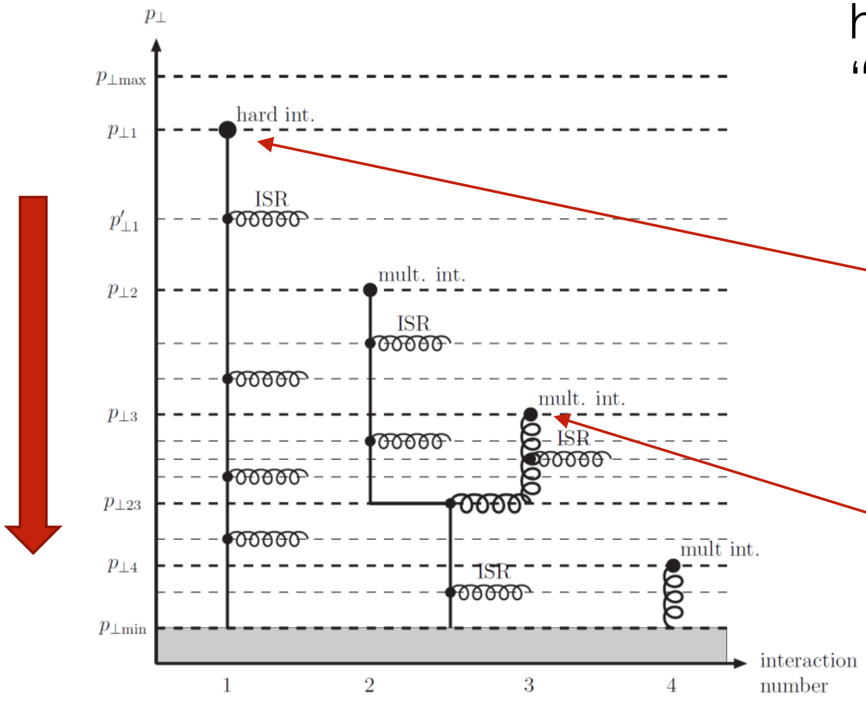


Here: study Pythia mPDFs. How good are they from a theory perspective? In particular, how well do they satisfy number and momentum sum rules? Compare Pythia double PDFs (dPDFs) to a set of purpose-built dPDFs (GS09).

PYTHIA MULTIPLE INTERACTIONS MODEL

How does the Pythia multiple interactions model work?

Backwards evolution from hardest interaction, "interleaving" both ISR and MPI



Start with hardest interaction, using usual PDFs $f_i^r(x, Q)$

PDFs are modified in "subsequent" interactions, $f_i^{m \leftarrow j_1, x_1 \dots j_{n-1}, x_{n-1}}(x, Q)$

Sjostrand, Skands, hep-ph/0402078, hep-ph/0408302

PYTHIA MULTIPLE INTERACTIONS MODEL

Sjostrand, Skands, hep-ph/0402078, hep-ph/0408302

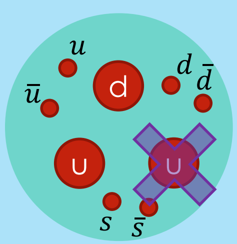
Modifications:

(I) MOMENTUM "SQUEEZING": $f_i^{m \leftarrow j_1, x_1 \dots j_{n-1}, x_{n-1}}(x, Q) = \frac{1}{X} f_i^r\left(\frac{x}{X}, Q\right)$ $\left[X = 1 - \sum_{i=1}^{n-1} x_i\right]$

Ensures $\sum_i \int dx\, x\ f_i^{m \leftarrow j_1, x_1 \dots j_{n-1}, x_{n-1}}(x, Q) = X$.

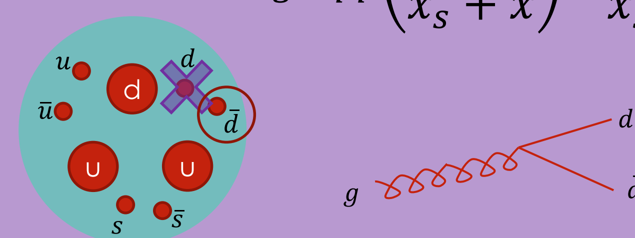
(II) VALENCE NUMBER SUBTRACTION:

$$f_{i_v}^{m \leftarrow j_1, x_1 \dots j_{n-1}, x_{n-1}}(x, Q) = \frac{N_{i_v n}}{N_{i_v 0}} \frac{1}{X} f_{i_v}^r\left(\frac{x}{X}, Q\right)$$



(III) COMPANION QUARK ADDITION:

$$q_c(x, x_s) = C(x_s) P_{g \to q\bar{q}}\left(\frac{x_s}{x_s + x}\right) \frac{g(x_s + x)}{x_s + x}$$



(IV) SEA QUARK AND GLUON RESCALING: Steps (II) and (III) break (I). To fix, rescale all sea quark and gluon distributions by a factor "a".

PYTHIA MULTIPLE INTERACTIONS MODEL

Model for multi-parton PDFs in Pythia:

$$D_{j_1 \dots j_n}(x_1 \dots x_n, Q_1 \dots Q_n) = f_{j_1}^{r}(x_1, Q_1) f_{j_2}^{m \leftarrow j_1, x_1}(x_2, Q_2) \dots f_{j_n}^{m \leftarrow j_1, x_1 \dots j_{n-1}, x_{n-1}}(x_n, Q_n)$$

Do we have any constraints on these objects from the theory side?

THE SUM RULES

JG, Stirling, 0910.4347
Blok, Dokshitzer, Frankfurt, Strikman, 1306.3763
Diehl, Plößl, Schäfer, 1811.00289

For DPS case ($n = 2$), we have for $Q_1 = Q_2 = Q$ the **sum rules**:

Momentum rule: $$\sum_{j_2} \int dx_2\, x_2\;\; D_{j_1 j_2}(x_1, x_2, Q) = (1 - x_1)\, f_{j_1}(x_1, Q)$$

Available momentum after "taking out" j_1

Number rule: $$\int dx_2\;\; D_{j_1 j_{2v}}(x_1, x_2, Q) = \left(N_{j_{2v}} - \delta_{j_1 j_2} + \delta_{j_1 \bar{j}_2}\right) f_{j_1}(x_1, Q)$$

Number of j_2 quarks - number of $\bar{j}_2$ quarks after "taking out" j_1

For TPS case ($n = 3$), we showed that the following sum rules hold:

JG, Fedkevych, 2208.08197

Momentum rule: $$\sum_{j_3} \int dx_3\, x_3\;\; T_{j_1 j_2 j_3}(x_1, x_2, x_3, Q) = (1 - x_1 - x_2)\, D_{j_1 j_2}(x_1, x_2, Q)$$

Number rule: $$\int dx_3\;\; T_{j_1 j_2 j_{3v}}(x_1, x_2, x_3, Q) = \left(N_{j_{3v}} - \delta_{j_1 j_3} - \delta_{j_2 j_3} + \delta_{j_1 \bar{j}_3} + \delta_{j_2 \bar{j}_3}\right) D_{j_1 j_2}(x_1, x_2, Q)$$

How well do the Pythia double and triple PDFs satisfy these?

PYTHIA MPDFS AND THE SUM RULES

dPDFs: $D_{j_1 j_2}(x_1, x_2, Q) = f_{j_1}^{r}(x_1, Q_1) f_{j_2}^{m \leftarrow j_1, x_1}(x_2, Q_2)$

Sum rules satisfied exactly when integrating over second parton (can show analytically or numerically)…

…but the same is not true when integrating over the first parton! Crucially, dPDFs not symmetric under $1 \leftrightarrow 2$.

Simplest proposal to restore symmetry:

$$D_{j_1 j_2}^{sym}(x_1, x_2, Q) = \frac{D_{j_1 j_2}(x_1, x_2, Q) + D_{j_2 j_1}(x_2, x_1, Q)}{2}$$

$$T_{j_1 j_2 j_3}^{sym}(x_1, x_2, x_3, Q) = \frac{1}{3!} \sum_{\{1,2,3\}} T_{j_1 j_2 j_3}(x_1, x_2, x_3, Q)$$

PYTHIA MPDFS AND THE SUM RULES

dPDF case: satisfies sum rules reasonably well (~10-25% level). Bigger deviations in places!

x_1	Momentum sum rule ($j_1 = u$). Should = 1.	$u\bar{u}$ number sum rule. Should = -1.	$\bar{u}u$ number sum rule. Should = 3.
10^{-6}	0.979	-1.227	2.961
10^{-3}	0.980	-0.847	3.351
10^{-1}	1.014	-0.925	3.491
0.2	1.047	-0.928	3.580
0.4	1.133	-0.884	3.858
0.8	1.679	-0.740	7.048

Connected to companion quark mechanism when both quarks have large x

Triple PDF (tPDF) case: roughly similar picture.

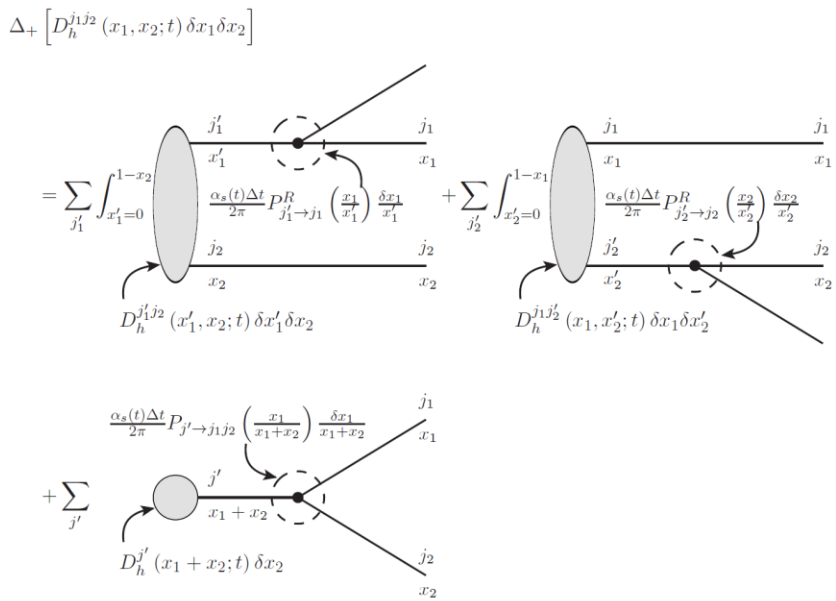
$x_2 = 10^{-4}$

x_1	Momentum sum rule ($j_1 = j_2 = u$). Should = 1.	uuu number sum rule. Should = 0.	$u\bar{u}u$ number sum rule. Should = 2.
10^{-6}	0.965	0.108	2.542
10^{-3}	0.967	-0.276	2.154
10^{-1}	0.998	-0.232	2.188
0.2	1.029	-0.242	2.189
0.4	1.117	-0.317	2.161
0.8	1.719	-0.589	2.079

COMPARISON OF PYTHIA WITH GS09

Compare Pythia dPDFs to another set of dPDFs constructed to (approximately) satisfy the sum rules: GS09 dPDFs. JG, Stirling, 0910.4347

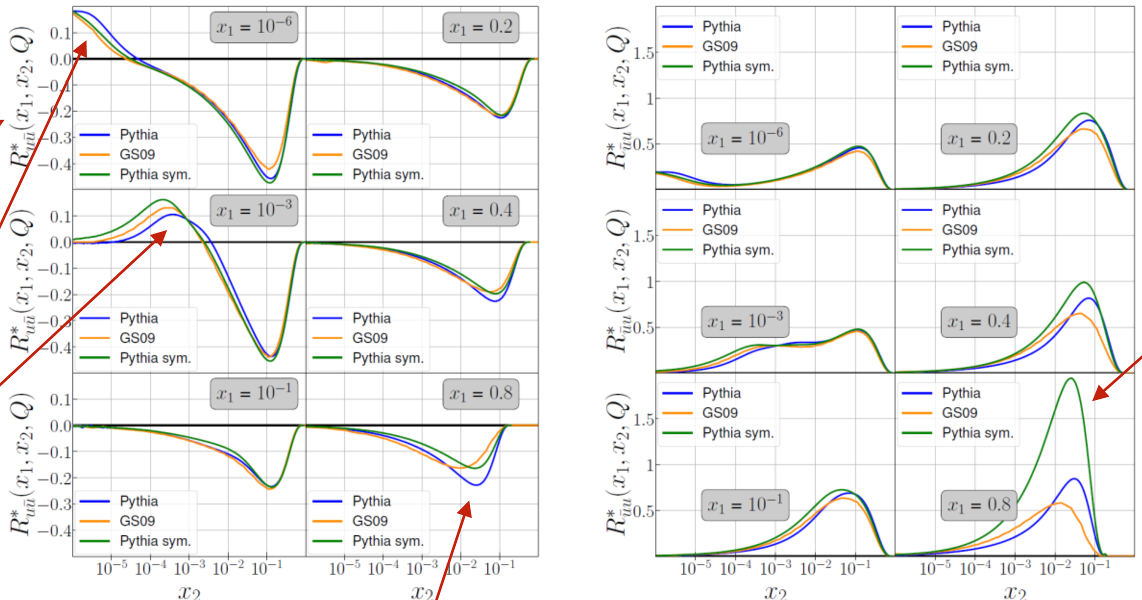
Inputs ($Q = 1$ GeV) designed to approximately satisfy sum rules, evolved to higher scales using "homogeneous double DGLAP" (preserves sum rules)



COMPARISON OF PYTHIA WITH GS09

"Response functions" (~ integrands of sum rules)

$$\frac{x_2 D_{u\bar{u}_v}(x_1, x_2, Q)}{f_u(x_1)}$$



Companion $\bar{u}$ accompanying u with $x_2 \approx x_1$

Removal of a valence u

Large violation of sum rule by Pythia sym. here

<u>Response functions very similar!</u>

Comparison of PYTHIA with GS09

Compare dPDFs themselves. Let's do this in the context of a toy pheno study.

DY process (to probe quarks & antiquarks). Only cut is $|y_{\text{leptons}}| < 5$.
Set $\mu_R = \mu_F = 91$ GeV ($\sim M_Z$).

Use following formula to compute DPS cross sections:

$$\sigma_{DPS} = \frac{1}{1+\delta_{AB}} \frac{1}{\sigma_{eff}} \sum_{j_i} \int dx_i D_{j_1 j_2}(x_1, x_2, Q) D_{j_3 j_4}(x_3, x_4, Q) \hat{\sigma}_{j_1 j_3 \to A} \hat{\sigma}_{j_2 j_4 \to B}$$

NOTE! This is actually not the proper DPS formula – includes only longitudinal correlations, neglects transverse ones.

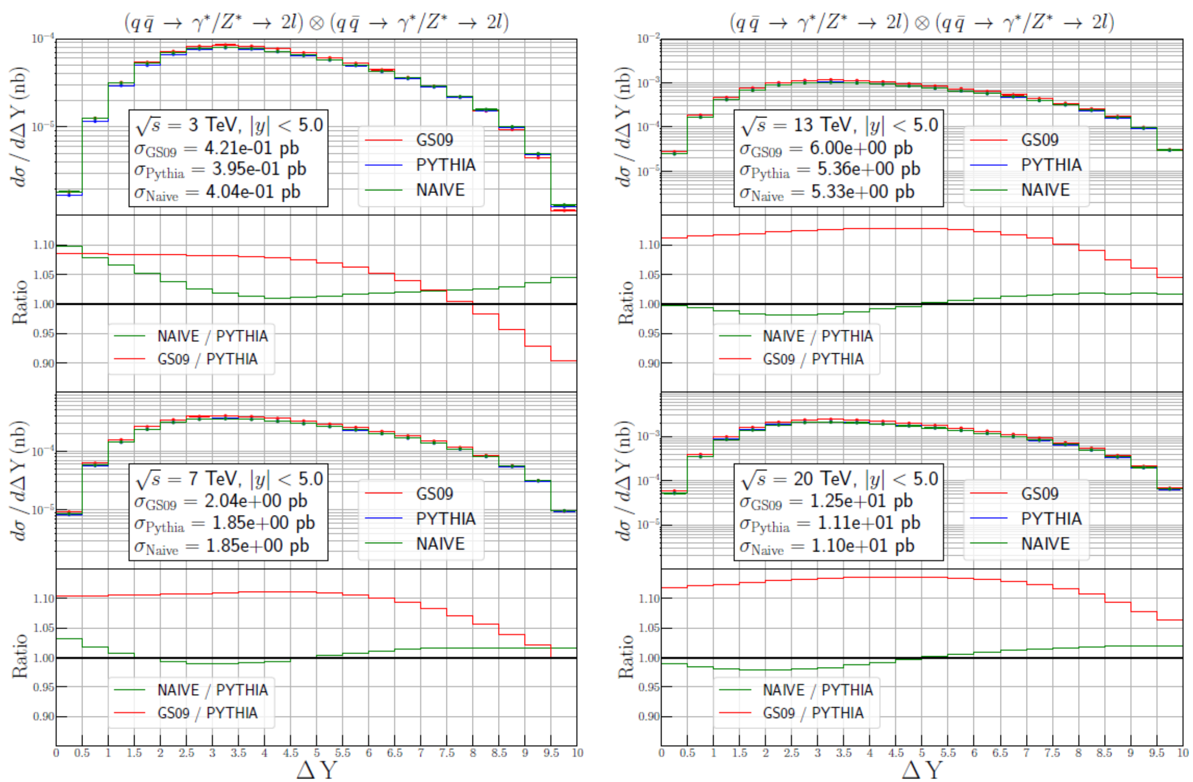
Blok, Dokshitzer, Frankfurt, Strikman, 1106.5533, 1306.3763, JG, Stirling, 1103.1888, JG, 1207.0480, JG, Diehl, Schönwald, 1702.06486, Ryskin, Snigirev, 1103.3495, 1203.2330, Manohar, Waalewijn, 1202.5034,…

Compare to also predictions with naïve dPDFs:

$$D^{\text{naive}}_{j_1 j_2}(x_1, x_2, Q) = f^r_{j_1}(x_1, Q) f^r_{j_2}(x_2, Q)$$

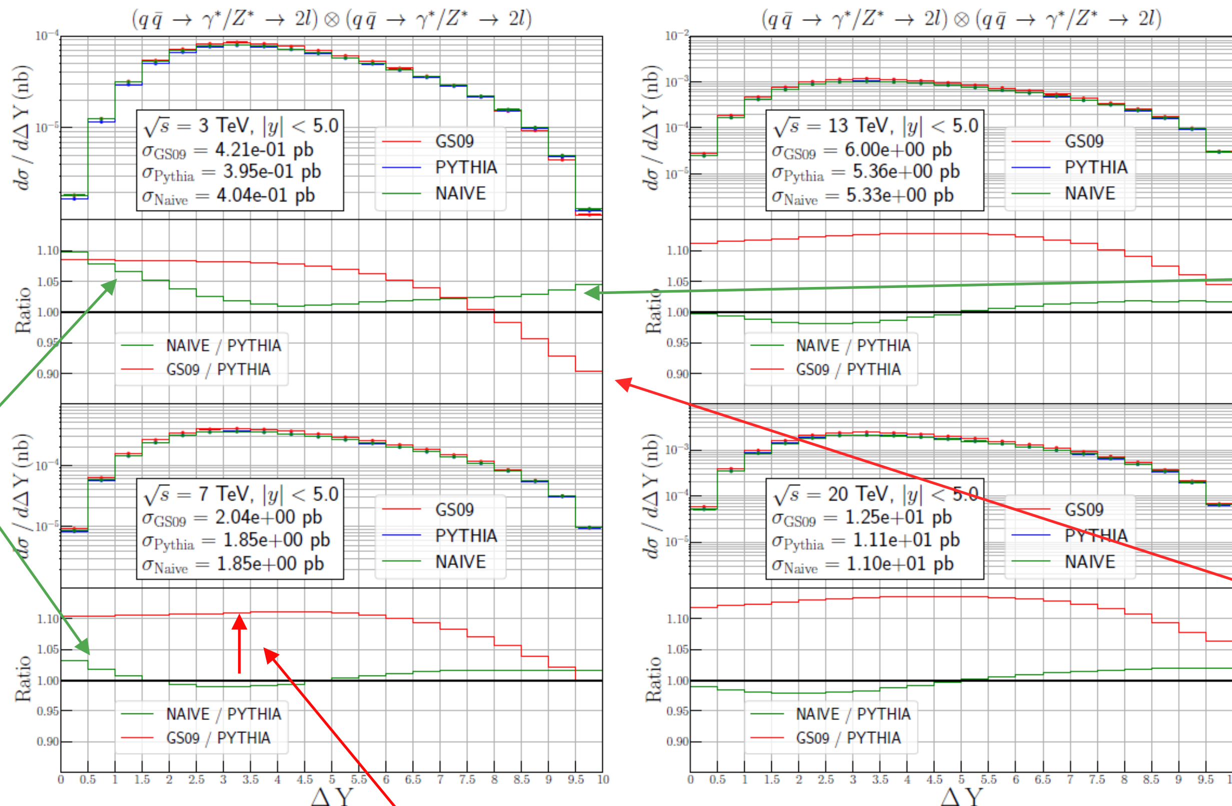
COMPARISON OF PYTHIA WITH GS09

Plot $\Delta Y = \max|y_i - y_j|$ for different $\sqrt{s}$:



<u>Interesting differences in shape, at the ~10% level!</u>

COMPARISON OF PYTHIA WITH GS09

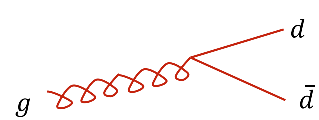


Driven by momentum squeezing

Driven by momentum squeezing + valence number subtraction

Driven by evolution effects 'transporting' sum rule subtractions to lower x values

Driven by '1→2 feed' in inhomogeneous double DGLAP



SUMMARY

- Multiple scattering is an important feature of pp collisions. Monte Carlo Event generators (e.g. Pythia) need a model of multiple scattering, and multi-parton PDFs.

- Number and momentum sum rules for double PDFs, and now triple PDFs, are known.

- Asymmetric Pythia dPDFs satisfy sum rules exactly in "final" argument. Symmetrising in a simple way leads to dPDFs/tPDFs that satisfy sum rules to 10-30% level for most x values.

- Comparing Pythia dPDFs to GS09 dPDFs:

  - Response functions (~ sum rule integrands) are quite similar!
  - dPDFs themselves/cross section predictions show some differences.