A data-driven test of a quantum-statistics PDF parametrisation.

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QCD@LHC 2022   28 December 2022

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Consider Deep Inelastic Scattering (DIS)

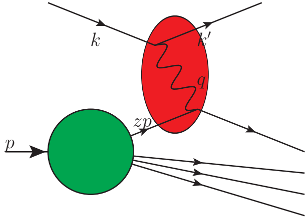
$$\sigma\left(x_{\mathrm{Bj}}, Q^2, \{P\}\right) = \sum_{i \in \{q, \overline{q}, g\}} \int_{x_{\mathrm{Bj}}}^{1} \frac{\mathrm{d}x}{x} C_i\left(\frac{x_{\mathrm{Bj}}}{x}, \alpha_s, Q^2\right) f_i\left(x, Q^2, \{P\}\right) \quad (1)$$

To fit $f_i(\ldots)$ at all scales:

1. Parametrise f at some (low) initial scale $Q_0 \rightarrow f_i\left(x, Q_0^2, \{P\}\right)$
2. Use DGLAP evolution from Q_0 to Q

$$f_i\left(x, \mathbf{Q_0^2}, \{P\}\right) \xrightarrow{\text{DGLAP}} f_i\left(x, \mathbf{Q^2}, \{P\}\right)$$

3. Take convolution with parton-level observable (1)
4. "Minimize" the difference $\sigma_{\mathrm{exp}}\left(x_{\mathrm{Bj}}, Q^2\right) - \sigma\left(x_{\mathrm{Bj}}, Q^2, \{P\}\right)$ by varying the free parameters $\{P\}$



PDF determination and parametrisation choice

No analytical way to pick the initial parametrisation nor Q_0

- HERAPDF $\rightarrow f_i\left(x, Q_0^2\right) = x^\alpha (1-x)^\beta P_n(z)$ using polynomials [1506.06042]
- NNPDF $\rightarrow f_i\left(x, Q_0^2\right) = x^\alpha (1-x)^\beta \mathrm{NN}(z)$ using neural networks [2109.02653]
- $\cdots$ many others [2207.04739],[1912.10053]

An alternative to a generic parametrisation is using physical arguments to model the structures of the proton...

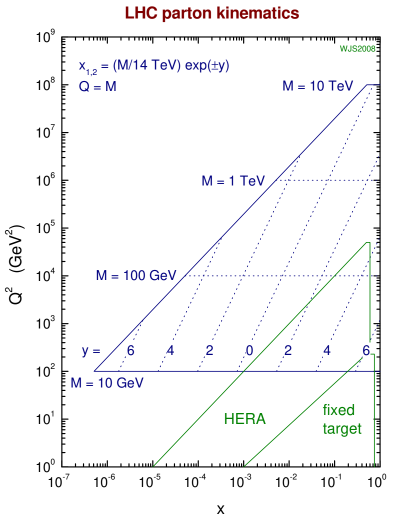
Proton modelling

Proton $\rightarrow$ gas mixture of massless partons **at equilibrium** [hep-ph/0109160]

- Quarks(gluons) distributions behave according to Fermi(Bose) statistics

$$f_i(x, Q_0^2) \supset \left[\exp\left(\frac{x - X_{0i}}{\bar{x}}\right) \pm 1\right]^{-1};$$

- "Chemical potentials" of quarks and antiquarks are related $X_{0q}^{\uparrow\downarrow} = -X_{0\bar{q}}^{\downarrow\uparrow}$
- Gluons behave like blackbody radiation $\rightarrow$ $X_{0g} = 0$
- Universal "temperature" parameter $\bar{x}$



Our QSPDF parametrisation (1)

We summarize the expressions from the model as [hep-ph/0109160]

$$h(x;b,\bar{x},X) = \frac{x^b}{\exp\left(\frac{x-X}{\bar{x}}\right)+1},$$

$$xf_{q^{\uparrow\downarrow}}(x,Q_0^2) = AX_q^{\uparrow\downarrow} h\left(x;b,\bar{x},X_q^{\uparrow\downarrow}\right) + \tilde{A}h\left(x;\tilde{b},\bar{x},0\right), \quad \text{(2a)}$$

$$xf_{\bar{q}^{\uparrow\downarrow}}(x,Q_0^2) = \bar{A}\frac{1}{X_q^{\downarrow\uparrow}} h\left(x;\bar{b},\bar{x},-X_q^{\downarrow\uparrow}\right) + \tilde{A}h\left(x;\tilde{b},\bar{x},0\right), \quad \text{(2b)}$$

$$\text{with} \quad q \in \{u,\, d\},$$

$$xf_g(x,Q_0^2) = \frac{A_g x^{b_g}}{\exp(x/\bar{x})-1}. \quad \text{(2c)}$$

An auxilliary term $\tilde{A}h\left(x;\tilde{b},\bar{x},0\right)$ is introduced to control the high-energy region.
Fitting only unpolarised DIS data $\rightarrow$ average over spin

$$f_q(x,Q_0^2) = f_{q^\uparrow}(x,Q_0^2) + f_{q^\downarrow}(x,Q_0^2)$$

Writing the unpolarised valence and sea contributions ($q \in \{u, d\}$)

$$\begin{aligned} xq_v(x, Q_0^2) &= q(x, Q_0^2) - \bar{q}(x, Q_0^2) \\ &= A\left[X_q^{\uparrow} h\left(x; b, \bar{x}, X_q^{\uparrow}\right) + X_q^{\downarrow} h\left(x; b, \bar{x}, X_q^{\downarrow}\right)\right] \\ &- \bar{A}\left[\frac{1}{X_q^{\downarrow}} h\left(x; \bar{b}, \bar{x}, -X_q^{\downarrow}\right) + \frac{1}{X_q^{\uparrow}} h\left(x; \bar{b}, \bar{x}, -X_q^{\uparrow}\right)\right] \end{aligned} \quad (3a)$$

$$\begin{aligned} x\bar{q}(x, Q_0^2) &= \bar{A}\left[\frac{1}{X_q^{\downarrow}} h\left(x; \bar{b}, \bar{x}, -X_q^{\downarrow}\right) + \frac{1}{X_q^{\uparrow}} h\left(x; \bar{b}, \bar{x}, -X_q^{\uparrow}\right)\right] \\ &+ 2\tilde{A} h\left(x; \tilde{b}, \bar{x}, 0\right), \end{aligned} \quad (3b)$$

$$xg(x, Q_0^2) = \frac{A_g x^{b_g}}{\exp(x/\bar{x}) - 1} \quad (3c)$$

$$s(x, Q_0^2) = \bar{s}(x; b, \bar{x}, X, Q_0^2) = \frac{f_s}{1 - f_s} \bar{d}(x; b, \bar{x}, X, Q_0^2) \quad \text{with} \quad f_s = 0.4\,. \quad (3d)$$

There are 13 parametres: $\left\{\bar{x}, A_g, A, \bar{A}, \tilde{A}, X_u^{\uparrow\downarrow}, X_d^{\uparrow\downarrow}, b, \bar{b}, b_g, \tilde{b}\right\}$

Constraints and other setups

Additional constriants to apply:

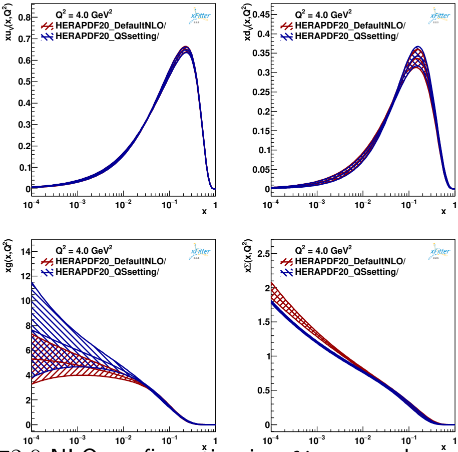
- Valence and momentum sum rules $\rightarrow$ fix normalisations $\{A_g, A, \bar{A}\}$
- $b_g = 1 + \tilde{b}$ (Regge theory)
- $\bar{b} = b$

This leaves 8 free parameters to fit: $\left\{\bar{x}, \tilde{A}, X_u^{\uparrow\downarrow}, X_d^{\uparrow\downarrow}, b, \tilde{b}\right\}$.

We use `xfitter` public framework with the HERA DIS dataset [1410.4412].

Some additional constraints are applied:

- Parametrisation scale $Q_0^2 = 4$ GeV
  $\rightarrow$ cut $Q^2 = 3.5$ GeV $< Q_0^2$
- Theory inputs:
  APFEL@NLO
  VFNS (FONLL-B)



Benchmark fit between default HERAPDF2.0 NLO configuration in `xfitter` and our settings

Fit setup and benchmark

We use `xfitter` public framework with the HERA DIS dataset [1410.4412].

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- Improved description of NCep 920

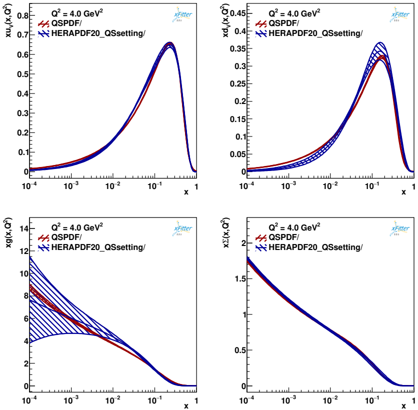
Dataset	HERAPDF20 Default-NLO	HERAPDF20 QSsetting
HERA1+2 CCem	54 / 42	54 / 42
HERA1+2 NCep 820	68 / 70	64 / 68
HERA1+2 NCep 460	217 / 204	216 / 200
→ HERA1+2 NCep 920	439 / 377	397 / 363 ←
HERA1+2 CCep	43 / 39	45 / 39
HERA1+2 NCem	222 / 159	221 / 159
HERA1+2 NCep 575	219 / 254	217 / 249
Correlated χ^2	86	67
Log penalty χ^2	+8.3	-4.68
Total χ^2 / dof	1357 / 1131	1275 / 1106
χ^2 p-value	0.00	0.00

Benchmark fit between default `HERAPDF2.0` NLO configuration in `xfitter` and our settings

Testing the QSPDF parametrisation: fits and χ^2

Now we fit QSPDF against the HERA DIS dataset.

- Reduced error bands
- Only experimental uncertainty is accounted for



In blue the benchmark fit with the HERAPDF2.0 and in red the fit of QSPDF

Testing the QSPDF parametrisation: fits and χ^2

Now we fit QSPDF against the HERA DIS dataset.

- Fit quality is good
- very minimal set of parametres

Dataset	QSPDF	HERAPDF20 QSsetting
HERA1+2 CCep	59 / 39	45 / 39
HERA1+2 CCem	69 / 42	54 / 42
HERA1+2 NCem	229 / 159	221 / 159
HERA1+2 NCep 820	71 / 68	64 / 68
HERA1+2 NCep 920	468 / 363	397 / 363
HERA1+2 NCep 460	231 / 200	216 / 200
HERA1+2 NCep 575	235 / 249	217 / 249
Correlated χ^2	104	67
Log penalty χ^2	-71.03	-4.68
Total χ^2 / dof	1397 / 1112	1275 / 1106
χ^2 p-value	0.00	0.00

	QSPDF	HERAPDF2.0
# param.	8	14
χ^2/D.O.F.	1.26	1.15

Testing the QSPDF parametrisation: parameter determination

We recover:

- $X_u^{\uparrow} > X_d^{\downarrow} \sim X_u^{\downarrow} > X_d^{\uparrow}$ like in [hep-ph/0109160]
- Qualitative agreement a determination of parameters
- No information on polarised PDF (unlike previous work)

Parameter	QSPDF	[hep-ph/0109160]
A	3.04	1.75
$\bar{A}$	0.12	1.91
A_g	33.52	14.28
$\tilde{A}$	0.133 ± 0.004	0.083
$X_d^{\uparrow}$	0.14 ± 0.02	0.23
$X_d^{\downarrow}$	0.284 ± 0.007	0.302
$X_u^{\uparrow}$	0.419 ± 0.007	0.461
$X_u^{\downarrow}$	0.21 ± 0.02	0.298
$b = \bar{b}$	0.52 ± 0.01	0.41
$\tilde{b} = b_g - 1$	-0.173 ± 0.003	-0.253
$\bar{x}$	0.092 ± 0.001	0.099

Testing the QSPDF parametrisation: $\bar{d} - \bar{u}$ distribution

We compare the $\bar{d} - \bar{u}$ distributions explicitly
$\rightarrow$ Interesting qualitative feature reproduced!

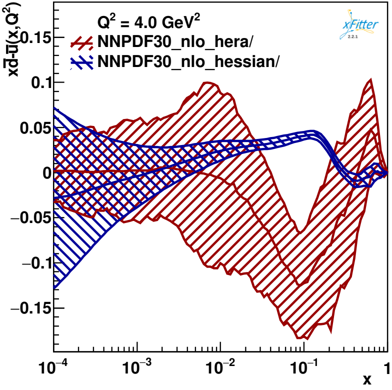


Figure: Comparison of NNNPDF3.0 fits with the HERA dataset only and the default dataset, (NLO theory)

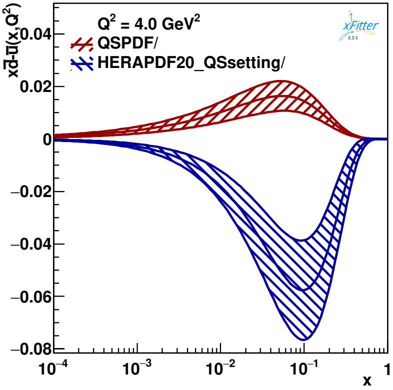


Figure: In blue the benchmark fit with the HERAPDF2.0 parametrisation and in red the fit of QSPDF

Conclusions and outlook

Fitting a statistical PDF model

- ✓ We performed a fit a custom PDF parametrisation (`QSPDF`) against the HERA DIS dataset.
- ↑ Acceptable agreement between data and model.
- ↑ Fit parameters match a previous attempts at fitting a similar parametrisation.
- ↓ This simplest iteration of the parametrisation isn't very competitive against more established models...
- ↑ ... but uses a smaller number of degrees of freedom.
- ↑ Qualitative shape of the $\bar{d} - \bar{u}$ distribution reproduced with HERA data only.

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Fitting a statistical PDF model

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Outlook

- Improve the current fit.
- Modify the parametrisation (relaxing $\bar{b}$, add transverse potentials) $\rightarrow$ building a "minimal" parameter set with state-of-the art performance.

Thank you for your attention

Backup slides

writing the unpolarised valence and sea contributions

$$xq_v^{\uparrow\downarrow}(x;b,\bar{x},X,Q_0^2) = x\left[f_{q^{\uparrow\downarrow}}(x;b,\bar{x},X,Q_0^2) - f_{\bar{q}^{\uparrow\downarrow}}(x;b,\bar{x},X,Q_0^2)\right], \quad (4)$$

$$xq_{\text{sea}}^{\uparrow\downarrow}(x;b,\bar{x},X,Q_0^2) = x\bar{q}^{\uparrow\downarrow}(x;b,\bar{x},X,Q_0^2) = xf_{\bar{q}^{\uparrow\downarrow}}(x;b,\bar{x},X,Q_0^2) \quad (5)$$

$$\begin{aligned} xq_v(x;b,\bar{x},X,Q_0^2) &= x\left[q_v^{\uparrow}(x;b,\bar{x},X,Q_0^2) + q_v^{\downarrow}(x;b,\bar{x},X,Q_0^2)\right] = \\ &= A\left[X_q^{\uparrow} f\left(x;b,X_q^{\uparrow}\right) + X_q^{\downarrow} f\left(x;b,X_q^{\downarrow}\right)\right] + \\ &- \bar{A}\left[\frac{1}{X_q^{\downarrow}} f\left(x;\bar{b},-X_q^{\downarrow}\right) + \frac{1}{X_q^{\uparrow}} f\left(x;\bar{b},-X_q^{\uparrow}\right)\right] \quad (6a) \\ x\bar{q}(x;b,\bar{x},X,Q_0^2) &= \bar{q}^{\uparrow}(x) + \bar{q}^{\downarrow}(x) = \\ &= \bar{A}\left[\frac{1}{X_q^{\downarrow}} f\left(x;\bar{b},-X_q^{\downarrow}\right) + \frac{1}{X_q^{\uparrow}} f\left(x;\bar{b},-X_q^{\uparrow}\right)\right] + 2\tilde{A}h(x), \quad (6b) \\ xf_g(x) &= \frac{A_g x^{b_g}}{\exp(x/\bar{x}) - 1}, \quad (6c) \end{aligned}$$

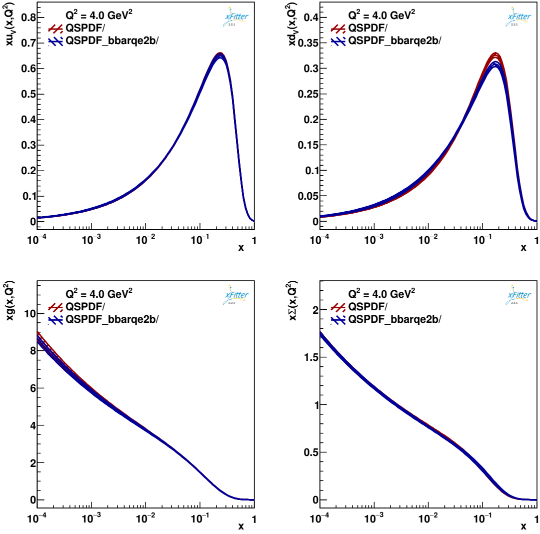

Figure: Effect of different constraint on $\bar{b}$: $\bar{b} = b$ and $= \bar{b} = 2b$

Gluon PDF error in default HERAPDF2.0 fit @NLO

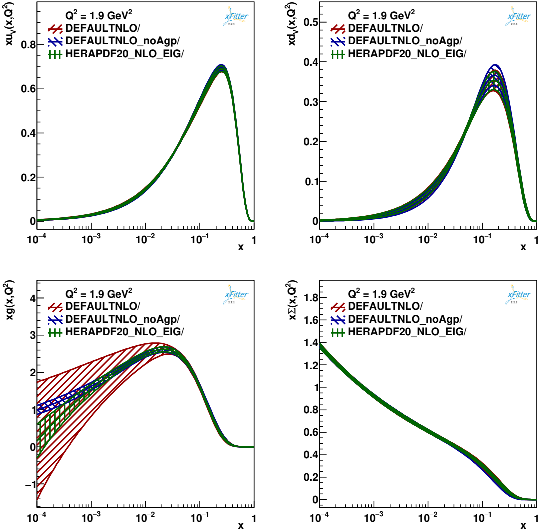

Figure: Large error in the gluon pdf induced by poor determination of $Agp = 0.23 \pm 0.29$.