# Back-to-back dijets production in DIS at small-$x$: Sudakov suppression and gluon saturation at NLO

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**SUBATECH**

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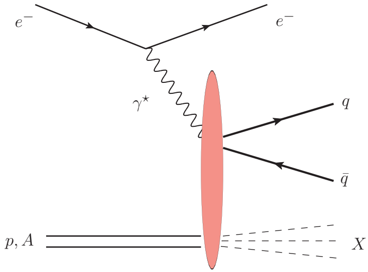
JHEP 2021 (11), 1-108, arXiv:2208.13872 and work in progress

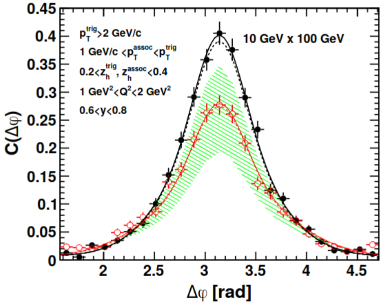
# Inclusive dijet production in DIS at small-$x$

⇒ probe of the saturated regime of QCD

⇒ access to the Weizsäcker-Williams gluon TMD in the back-to-back limit.

⇒ application to LHC phenomenology in the photo-production limit (ultra-peripheral Pb-Pb collisions)




Zheng, Aschenauer, Lee, Xiao, 1403.2413

# Dijets in DIS at NLO and small-$x$: many recent progresses!

- Dihadrons production. Bergabo, Jalilian-Marian, 2207.03606, Iancu, Mulian, 2211.04837
- Photo-production limit. Taels, Altinoluk, Beuf, Marquet, 2204.11650
- Related processes: exclusive dijet, Boussarie, Grabovsky, Szymanowski, Wallon. 1606.00419 - 1905.07371,
  Single inclusive hadron production Bergabo, Jalilian-Marian, 2210.03208 ,
  Diffractive dihadron Fucilla, Grabovsky, Li, Szymanowski, Wallon, 2211.05774
- Results from different approaches: cross-check of a challenging computation!

**In this talk: NLO impact factor for inclusive dijet production in DIS**

- Reliable QCD prediction requires to account for NLO corrections.
- Systematic determination of the theoretical uncertainties.
- Analytic expressions in back-to-back kinematics that simplify the numerical calculation.

# LO cross-section at small-$x$

- Differential cross-section at leading order:

$$\left.\frac{\mathrm{d}\sigma^{\gamma_\lambda^*+A\to q\bar{q}+X}}{\mathrm{d}^2\boldsymbol{k}_\perp \mathrm{d}^2\boldsymbol{p}_\perp \mathrm{d}\eta_q \mathrm{d}\eta_{\bar{q}}}\right|_{\mathrm{LO}} = \frac{\alpha_{\mathrm{em}} e_f^2 N_c}{(2\pi)^6}\int \mathrm{d}^8\boldsymbol{X}_\perp e^{-i\boldsymbol{k}_\perp \boldsymbol{r}_{xx'}} e^{-i\boldsymbol{p}_\perp \boldsymbol{r}_{yy'}} \Xi_{\mathrm{LO}}(\boldsymbol{x}_\perp, \boldsymbol{y}_\perp; \boldsymbol{y}'_\perp, \boldsymbol{x}'_\perp)\mathcal{R}^\lambda_{\mathrm{LO}}(\boldsymbol{r}_{xy}, \boldsymbol{r}'_{xy})$$

- **Factorization** between perturbative factor describing the $\gamma^\star \to q\bar{q}$ splitting...

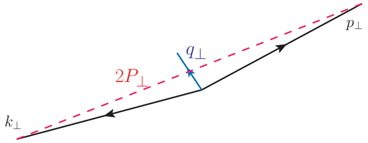
$$\mathcal{R}^{\mathrm{L}}_{\mathrm{LO}}(\boldsymbol{r}_{xy}, \boldsymbol{r}'_{xy}) = 8 z_q^3 z_{\bar{q}}^3 Q^2 K_0(\bar{Q} r_{xy}) K_0(\bar{Q} r_{xy'})$$

- ... and a color structure describing the interaction of $q\bar{q}$ with the dense target

$$\Xi_{\mathrm{LO}}(\boldsymbol{x}_\perp, \boldsymbol{y}_\perp; \boldsymbol{x}'_\perp, \boldsymbol{y}'_\perp) = \left\langle \underbrace{Q(\boldsymbol{x}_\perp, \boldsymbol{y}_\perp; \boldsymbol{y}'_\perp, \boldsymbol{x}'_\perp)}_{quadrupole} - D(\boldsymbol{x}_\perp, \boldsymbol{y}_\perp) - \underbrace{D(\boldsymbol{y}'_\perp, \boldsymbol{x}'_\perp)}_{dipole} + 1 \right\rangle_Y$$

$$\text{Dipole:} \qquad D(\boldsymbol{x}_\perp, \boldsymbol{y}_\perp) = \frac{1}{N_c}\langle \mathrm{Tr}(V(\boldsymbol{x}_\perp) V^\dagger(\boldsymbol{y}_\perp))$$

# The back-to-back limit




- Def: $|\boldsymbol{P}_\perp| = |z_{\bar{q}}\boldsymbol{k}_\perp - z_q \boldsymbol{p}_\perp| \gg |\boldsymbol{q}_\perp| = |\boldsymbol{k}_\perp + \boldsymbol{p}_\perp|$

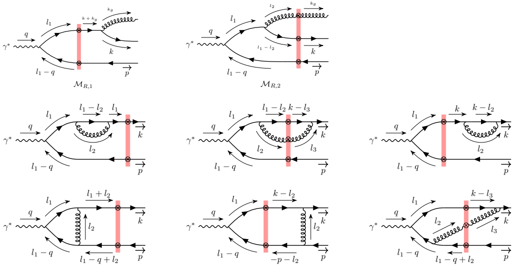
- LO: TMD factorization Dominguez, Marquet, Xiao, Yuan, 1101.0715

$$\left.\frac{\mathrm{d}\sigma^{\gamma^\star \to q\bar{q}+X}}{\mathrm{d}^2\boldsymbol{P}_\perp \mathrm{d}^2\boldsymbol{q}_\perp}\right|_{\mathrm{LO}} \propto \mathcal{H}(\boldsymbol{P}_\perp) \int \mathrm{d}^2\boldsymbol{b}_\perp \mathrm{d}^2\boldsymbol{b}'_\perp e^{-i\boldsymbol{q}_\perp(\boldsymbol{b}_\perp - \boldsymbol{b}'_\perp)} \underbrace{G_{\mathrm{WW}}(\boldsymbol{b}_\perp, \boldsymbol{b}'_\perp)} + \mathcal{O}\left(\frac{q_\perp}{P_\perp}\right) + \mathcal{O}\left(\frac{Q_s}{P_\perp}\right)$$

$$\searrow \left\langle \frac{1}{N_c} \mathrm{Tr}\left[\partial_i V^\dagger(\boldsymbol{b}_\perp) V(\boldsymbol{b}'_\perp) \partial_j V^\dagger(\boldsymbol{b}'_\perp) V(\boldsymbol{b}_\perp)\right]\right\rangle_Y$$

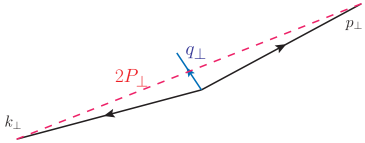
# Inclusive dijet at NLO and small-$x$ in the CGC

- We have done the full computation for general kinematics in JHEP 2021 (11), 1-108
- The diagrams are




- The cross-section is UV finite, IR divergences cancel provided one uses jets, light cone singularity absorbed into the JIMWLK evolution of the LO color structure.

# Back-to-back limit at NLO: emergence of large Sudakov logarithms




- NLO: large Sudakov logarithms vs small-x logarithm.

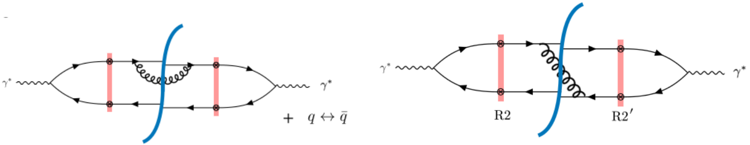
$$\left.\frac{\mathrm{d}\sigma^{\gamma^\star\to q\bar{q}+X}}{\mathrm{d}^2\boldsymbol{P}_\perp \mathrm{d}^2\boldsymbol{q}_\perp}\right|_{\mathrm{NLO}} \propto \mathcal{H}(\boldsymbol{P}_\perp)\int \mathrm{d}^2\boldsymbol{b}_\perp \mathrm{d}^2\boldsymbol{b}'_\perp e^{-i\boldsymbol{q}_\perp(\boldsymbol{b}_\perp-\boldsymbol{b}'_\perp)}$$
$$\times\left[1-\frac{\alpha_s N_c}{4\pi}\ln^2\left(\frac{\boldsymbol{P}_\perp^2(\boldsymbol{b}_\perp-\boldsymbol{b}'_\perp)^2}{c_0^2}\right)+\ldots+\alpha_s\ln\left(\frac{1}{x_{\mathrm{Bj}}}\right)\mathcal{K}_{\mathrm{LL}}\otimes\right]G_{\mathrm{WW}}(\boldsymbol{b}_\perp-\boldsymbol{b}'_\perp)$$

See Mueller, Xiao, Yuan, 1308.2993 based on Higgs production in pA.

- $\boldsymbol{b}_\perp-\boldsymbol{b}'_\perp$ conjugate to $\boldsymbol{q}_\perp \Rightarrow$ double log of the form $\ln^2(P_\perp/q_\perp)$

# Sudakov logarithms in our computation

- Real diagrams with soft divergences.




- However: the integration over the soft gluon gives the Sudakov with a positive sign!

$$\mathrm{d}\sigma_{\mathrm{NLO}}^{\gamma^\star \to q\bar{q}+X} \sim \mathcal{H}(\boldsymbol{P}_\perp) \int \mathrm{d}^2\boldsymbol{b}_\perp \mathrm{d}^2\boldsymbol{b}'_\perp e^{-i\boldsymbol{q}_\perp(\boldsymbol{b}_\perp - \boldsymbol{b}'_\perp)}$$

$$\times \left[1 + \frac{\alpha_s N_c}{4\pi} \ln^2\left(\frac{\boldsymbol{P}_\perp^2(\boldsymbol{b}_\perp - \boldsymbol{b}'_\perp)^2}{c_0^2}\right) + \ldots + \alpha_s \ln\left(\frac{k_f^-}{\Lambda^-}\right) \mathcal{K}_{\mathrm{LL}} \otimes\right] G_{\mathrm{WW}}(\boldsymbol{b}_\perp - \boldsymbol{b}'_\perp)$$

- Problem: overlapping phase space between soft gluons and slow gluons included in $\mathcal{K}_{\mathrm{LL}}$.

# Solution: collinearly improved small-$x$ evolution of the WW

- Kinematic improvement: impose both $k_g^-$ and $k_g^+$ ordering (lifetime ordering).
  $\Longrightarrow$ Resum large transverse double logarithms to all orders.
  $\Longrightarrow$ Solve the instability of NLO B-JIMWLK evolution.
  Beuf, 1401.0313, Taels, Altinoluk, Beuf, Marquet, 2204.11650
- In practice, add an additional constraint in the LL evolution kernel

$$k_g^+ \geq k_f^+ \Longrightarrow k_g^- \leq \frac{\boldsymbol{k}_{g\perp}^2}{Q_f^2} k_f^-$$

with $Q_f^2 \sim Q^2 \sim \boldsymbol{P}_\perp^2$.

- With this modification $\mathcal{K}_{\mathrm{LL}} \to \mathcal{K}_{\mathrm{LL,coll}}$ , one recovers the expected double logarithm.

$$\mathrm{d}\sigma_{\mathrm{NLO}}^{\gamma^\star \to q\bar{q}+X} \sim \mathcal{H}(\boldsymbol{P}_\perp) \int \mathrm{d}^2\boldsymbol{b}_\perp \mathrm{d}^2\boldsymbol{b}'_\perp e^{-i\boldsymbol{q}_\perp(\boldsymbol{b}_\perp - \boldsymbol{b}'_\perp)}$$
$$\times \left[1 - \frac{\alpha_s N_c}{4\pi} \ln^2\left(\frac{\boldsymbol{P}_\perp^2(\boldsymbol{b}_\perp - \boldsymbol{b}'_\perp)^2}{c_0^2}\right) + \ldots + \alpha_s \mathcal{K}_{\mathrm{LL,coll}} \otimes\right] G_{\mathrm{WW}}(\boldsymbol{b}_\perp - \boldsymbol{b}'_\perp)$$

# Sudakov resummation at single log accuracy

- Exponentiation of the Sudakov logarithms $G_{\rm WW}(\boldsymbol{r}_{bb'}) \to G_{\rm WW}(\boldsymbol{r}_{bb'})\mathcal{S}(\boldsymbol{P}_\perp^2, \boldsymbol{r}_{bb'}^2)$

$$\mathcal{S}(\boldsymbol{P}_\perp^2, \boldsymbol{r}_{bb'}^2) = \exp\left(-\int_{c_0^2/\boldsymbol{r}_{bb'}^2}^{\boldsymbol{P}_\perp^2} \frac{\mathrm{d}\mu^2}{\mu^2}\frac{\alpha_s(\mu^2)N_c}{\pi}\left[\underbrace{\frac{1}{2}\ln\left(\frac{\boldsymbol{P}_\perp^2}{\mu^2}\right)}_{\text{double log}} + \underbrace{\frac{C_F}{N_c}s_0 - s_f}_{\text{single log}}\right]\right)$$

- Double and **single** Sudakov logarithms with **exact** $N_c$ dependence:
- Dijet geometry single log $s_0$

$$s_0 = \ln\left(\frac{2(1+\cosh(\Delta Y_{12}))}{R^2}\right) + \mathcal{O}(R^2)$$

See also Hatta, Xiao, Yuan, Zhou, 2106.05307

- Single log from the interplay between small-$x$ and Sudakov resummation:

$$s_f = \ln\left(\frac{\boldsymbol{P}_\perp^2 x_{\rm Bj}}{z_1 z_2 Q^2 c_0^2 x_f}\right) \quad \Rightarrow x_f \text{ factorization scale dependence!}$$

# Finite terms in $\alpha_s$ in the back-to-back limit

## Azimuthal anisotropies from soft gluon radiations

- We can also access pure $\alpha_s$ (and non power suppressed) corrections.
- Some of them are coming from soft gluon radiations. Hatta, Xiao, Yuan, Zhou, 2010.10774
- Azimuthally averaged x-section sensitive to the linearly polarized gluon TMD at NLO!

$$\langle \mathrm{d}\sigma \rangle = \ldots + \mathcal{H}(\boldsymbol{P}_\perp) \times \int \frac{\mathrm{d}^2 \boldsymbol{r}_{bb'}}{(2\pi)^4} e^{-i\boldsymbol{q}_\perp \cdot \boldsymbol{r}_{bb'}} \hat{h}(\boldsymbol{r}_{bb'}) \mathcal{S}(\boldsymbol{P}_\perp^2, \boldsymbol{r}_{bb'}^2) \times \frac{\alpha_s}{\pi} \left\{ \frac{N_c}{2} + C_F \ln(R^2) - \frac{1}{2N_c} \ln(z_1 z_2) \right\}$$

- The $\cos(2\phi)$ anisotropy is also sensitive to the unpolarized gluon TMD.

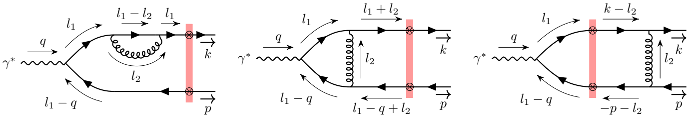
$$\langle \cos(2\phi) \mathrm{d}\sigma \rangle = \ldots + \mathcal{H}(\boldsymbol{P}_\perp) \times \int \frac{\mathrm{d}^2 \boldsymbol{r}_{bb'}}{(2\pi)^4} e^{-i\boldsymbol{q}_\perp \cdot \boldsymbol{r}_{bb'}} \cos(2\theta) \hat{G}(\boldsymbol{r}_{bb'}) \mathcal{S}(\boldsymbol{P}_\perp^2, \boldsymbol{r}_{bb'}^2) \times \frac{\alpha_s}{\pi} \left\{ N_c + 2C_F \ln(R^2) - \frac{1}{N_c} \ln(z_1 z_2) \right\}$$

# NLO hard factors

- Some other pure $\alpha_s$ involve new "hard factor" and do not break TMD factorization:

$$\left.\frac{\mathrm{d}\sigma^{\gamma^\star\to q\bar{q}+X}}{\mathrm{d}^2\boldsymbol{P}_\perp\mathrm{d}^2\boldsymbol{q}_\perp}\right|_{\mathrm{NLO}} \propto \alpha_s\mathcal{H}^{ij}_{\mathrm{NLO}}(\boldsymbol{P}_\perp)\int\mathrm{d}^2\boldsymbol{b}_\perp\mathrm{d}^2\boldsymbol{b}'_\perp e^{-i\boldsymbol{q}_\perp(\boldsymbol{b}_\perp-\boldsymbol{b}'_\perp)}G^{ij}_{\mathrm{WW}}(\boldsymbol{b}_\perp,\boldsymbol{b}'_\perp)$$

- They come from virtual graphs in which the gluon does not cross the SW:




- Do they appear in the TMD litterature for this process?

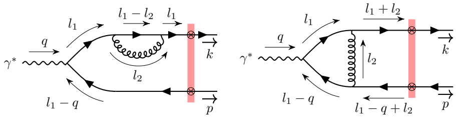
# NLO hard factors

- Remarkably, they can be computed fully analytically!
- They have the form

$$\mathcal{H}_{\mathrm{NLO}}(\boldsymbol{P}_\perp) = \mathcal{H}_{\mathrm{LO}}(\boldsymbol{P}_\perp) \times f\left(\frac{Q}{P_\perp}, z_1, z_2\right)$$

with $f$ linear combination of logarithms and dilogarithms.

- Example:




$$\mathcal{H}^{\lambda=\mathrm{L},ij}_{\mathrm{SE}_1+\mathrm{V}_1}(\boldsymbol{P}_\perp) = \mathcal{H}^{ij}_{\mathrm{LO}}(\boldsymbol{P}_\perp) \times \left[4 - 4\ln\left(\frac{u}{c_0}\right) - 4\ln\left(1 + \frac{1}{u^2}\right)\right]$$

with $u = \sqrt{z_1 z_2}\, Q/P_\perp$.

# TMD factorization at NLO and small-$x$

- Can we put the full result in a TMD factorized form?
- The small-$x$ evolution of the WW is not closed. Dominguez, Mueller, Munier, Xiao, 1108.1752
- At NLO, non-trivial color correlators, e.g.

$$\frac{N_c}{2}\langle 1 - D_{y'x'} + Q_{zy,y'x'}D_{xz} - D_{xz}D_{zy}\rangle_Y$$

which does not reduce to the WW gluon TMD, unless at least $Q_s^2 \ll \boldsymbol{k}_{g\perp}^2 \sim$ dilute limit.

- Maybe there is an other argument to neglect these complicated finite terms beyond the dilute limit...

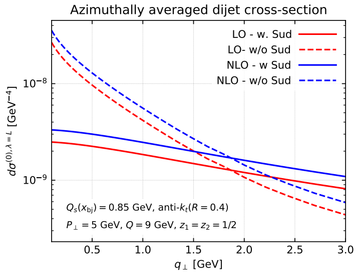
# Preliminary numerical results (with many caveats)

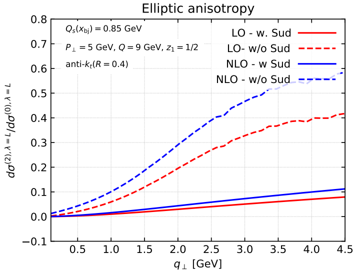
✓ Include:

- Sudakov with double and single log **but** at fixed coupling.
- **All** finite terms that do not manifestly break TMD factorization.

X Do not include:

- Proper small-$x$ evolution. The WW is parametrized by $Q_s(x_{\rm Bj})$ in the Gaussian approx.
- Factorization breaking terms.



# Summary and outlook

- NLO results for inclusive dijet production at small-$x$ in the CGC EFT.
- Back-to-back limit: Sudakov double and single log at exact $N_c$, and impact factor.
- Necessity to use a collinearly improved small-$x$ evolution to find the correct Sudakov double log.
- Towards a numerical evaluation of the impact factor with saturation corrections: **very challenging...**
  But recent progresses with the complete evaluation of all non factorization breaking terms!

**THANK YOU!**