Higher-order electroweak (EW) and mixed QCD-QED resummation effects for W and Z boson production at hadron colliders

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Introduction & Motivations

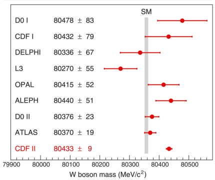
- Drell-Yan mechanism is a benchmark process at hadron colliders (SM and BSM studies, standard candle for detector calibration, m_W determination, ...) and is measured with high experimental precision

$\Rightarrow$ Need of **accurate theoretical predictions** $\Rightarrow$ **higher-order radiative corrections**

- Distribution in q_T, in the framework of RESUMMATION FORMALISM (G. Bozzi, S. Catani, D. de Florian, M. Grazzini [hep-ph/0508068]]), are known at the high-precision level of N3LL+N3LO in QCD (S. Camarda, L. Cieri, G. Ferrera [hep-ph/2103.04974 [hep-ph]]) $\Rightarrow$ EW corrections must be taken into account: $\alpha \sim \alpha_S^2$

- QED correction NLL+NLO was computed only for on-shell Z production (L. Cieri, G. Ferrera, G.F.R. Sborlini [hep-ph/1805.11948 [hep-ph]]) $\rightarrow$ We extend the formalism to reach $NLL_{QED} + NLO_{EW}$ accuracy for both charged and neutral current on-shell Drell-Yan processes

- The last extremely precise extraction of M_W at CDFII detector shows a substantial tension with SM prediction and past m_W measurements (ATLAS, DELPHI, OPAL, ...)

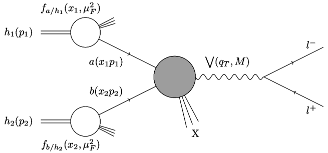


- CDF collaboration: High-precision measurement of the W boson mass with the CDF II detector 10.1126/science.abk1781
- theoretical studies should be directed to relevant kinematic distribution (e.g. $p_T(W/Z)$, ...)
- EW corrections could give $\mathcal{O}$(MeV) shift of m_W value extraction
- understand deeply the tension (underestimation of uncertainties, BSM effects, extension of the theory...)

Drell-Yan q_T distribution

$$\frac{d\sigma_V}{dq_T^2}(q_T, M, s) \overset{\text{factorization theorem}}{=}$$

$$= \sum_{a,b} \int_0^1 dx_1 \int_0^1 dx_2\, f_{a/h_1}(x_1, \mu_F^2) f_{b/h_2}(x_2, \mu_F^2) \times \frac{d\hat{\sigma}_{V_{ab}}}{dq_T^2}\left(q_T, M, \hat{s}, \alpha_S(\mu_R^2), \mu_R^2, \mu_F^2\right)$$



- In the region $q_T \gtrsim M_V$ the perturbative fixed-order expansion is reliable:

$$\frac{d\hat{\sigma}_{V_{ab}}}{dq_T^2} = \frac{d\hat{\sigma}^{(0)}_{V_{ab}}}{dq_T^2} + \frac{\alpha_S}{\pi}\frac{d\hat{\sigma}^{(1)}_{V_{ab}}}{dq_T^2} + \left(\frac{\alpha_S}{\pi}\right)^2 \frac{d\hat{\sigma}^{(2)}_{V_{ab}}}{dq_T^2} + \mathcal{O}\left[\left(\frac{\alpha_S}{\pi}\right)^3\right]$$

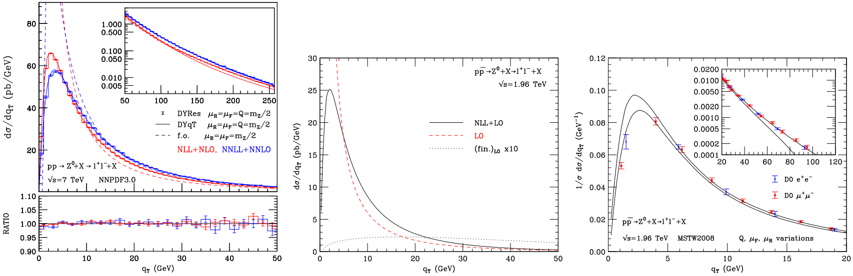
- In the region $q_T << m_V$ (bulk of the events) large logarithmic corrections of the type $\alpha_S^n \ln^m(M_V^2/q_T^2)$, due to soft and/or collinear parton radiations, spoils the convergence

$$\Downarrow$$

Resummation at all perturbative orders is mandatory:

$$\frac{d\hat{\sigma}_{V_{ab}}}{dq_T^2} = \frac{\hat{\sigma}^{(0)}_V}{q_T^2} \sum_{n=1}^{+\infty} \sum_{m=0}^{2n-1} A^V_{n,m} ln^m\left(\frac{M^2}{q_T^2}\right)\alpha_S^n(M^2), \quad \alpha_S^n \ln^m(M_V^2/q_T^2) >> 1$$

G. Bozzi, S. Catani, D. de Florian, M. Grazzini hep-ph/0508068



G. Bozzi, S. Catani, G. Ferrera, D. de Florian, M. Grazzini [1007.2351[hep-ph]], [1507.06937[hep-ph]]

- Partonic cross section is explicitly splitted as:

$$\frac{d\hat{\sigma}}{dq_T^2} = \frac{d\hat{\sigma}^{res}}{dq_T^2} + \frac{d\hat{\sigma}^{fin}}{dq_T^2}, \quad \text{with} \quad \lim_{q_T\to 0}\int_0^{Q_T} dq_T^2 \frac{d\hat{\sigma}^{fin.}}{dq_T^2} = 0$$

- Resummation is performed in impact parameter (**b**) space

$$\frac{d\hat{\sigma}^{res.}_{a_1 a_2\to V}}{dq_T^2}(q_T; M, \hat{s}; \alpha_S(\mu_R^2), \mu_F^2, \mu_R^2) = \frac{M^2}{\hat{s}}\int_0^\infty db\,\frac{b}{2} J_0(bq_T)\mathcal{W}^V_{a_1 a_2}(b; M, \hat{s}, \alpha_S(\mu_R^2), \mu_F^2, \mu_R^2).$$

- The resummed form factor $\mathcal{W}^{V}_{a_1 a_2}$ can be expressed in an exponential and factorized form in the Mellin space $\rightarrow z = M_V^2/\hat{s},\ f_N = \int_0^1 dz\, z^{N-1} f(z)$:

$$\mathcal{W}^{V}_{N,a_1 a_2} = \mathcal{H}_N(M_W, \alpha_S(\mu_R^2)) \times exp\, \mathcal{G}_N\left(\alpha_S(\mu_R^2), L; M^2/\mu_R^2, M^2/Q^2\right),$$

$$L = \ln\left(\frac{Q^2 b^2}{b_0^2} + 1\right),\ b_0 = 2exp(-\gamma_E), \gamma_E = 0.5772\ldots$$

Hard collinear coefficient function $\mathcal{H}^{V}_{N,a_1 a_2}$

- includes finite terms (e.g. virtual emission)
- process dependent

$$\mathcal{H}_{N,a_1 a_2}\left(M, \alpha_S, \frac{M^2}{\mu_R^2}, \frac{M^2}{\mu_F^2}, \frac{M^2}{Q^2}\right) = \hat{\sigma}^{V}_{0,a_1 a_2}(M)\left[1 + \sum_{n=1}^{\infty}\left(\frac{\alpha_S}{\pi}\right)^n \mathcal{H}_N^{(n)}\left(\frac{M^2}{\mu_R^2}, \frac{M^2}{\mu_F^2}, \frac{M^2}{Q^2}\right)\right]$$

Sudakov form factor $\mathcal{G}_N$

- contains the logarithmic enhanced contributions
- universal structure

$$\mathcal{G}^{V}_{N,a_1 a_2}\left(\alpha_S(\mu_R^2, L; \frac{M^2}{\mu_R^2}, \frac{M^2}{Q^2})\right) = L g_N^{(1)}(\alpha_S(\mu_R^2)L) + g_N^{(2)}\left(\alpha_S(\mu_R^2)L; \frac{M^2}{\mu_R^2}, \frac{M^2}{Q^2}\right) +$$
$$+ \frac{\alpha_S(\mu_R^2)}{\pi} g_N^{(3)}\left(\alpha_S(\mu_R^2); \frac{M^2}{\mu_R^2}, \frac{M^2}{Q^2}\right) + \sum_{n=4}^{\infty}\left(\frac{\alpha_S}{\pi}\right)^{n-2} g_N^{(n)}\left(\alpha_S(\mu_R^2)L; \frac{M^2}{\mu_R^2}, \frac{M^2}{Q^2}\right).$$

$$\Downarrow$$

Perturbative structure of the resummed component $\rightarrow$ LL accuracy ($\sim \alpha_S^n L^{n+1}$): $g_N^{(1)}$; NLL accuracy ($\sim \alpha_S^N L^n$): $g_N^{(2)}$, $\mathcal{H}_N^{(1)}$; NNLL accuracy: ($\sim \alpha_S^N L^{n-1}$): $g_N^{(3)}$, $\mathcal{H}_N^{(2)}$; N3LL accuracy ($\sim \alpha_S^N L^{n-2}$): $g_N^{(4)}$, $\mathcal{H}_N^{(3)}$

On-shell Z boson production

- QED corrections at NLL+NLO known
  - [L. Cieri, G. Ferrera, G. F. R. Sborlini 1805.11948[hep-ph]]
- We incorporated also weak corrections at one loop within the hard factor $\mathcal{H}$
  - We consider one-loop renormalized form factor
  - Modifications only in the hard factor $\mathcal{H}$ (massive loop corrections)

On-shell W boson production **(NEW)**

- Charged final state → a "naive abelianization" of QCD formulas for DY process is not suitable
- We use the formalism of $t\bar{t}$ production

1. Replacement: $t\bar{t} \to W$ (colour charged → electrically charged)
2. Abelianization of Casimir operators: $\mathbf{T_i} \to e_i$ (**S. Catani, S.Dittmaier, Zoltan Trocsany [9802439[hep-ph]], [0011222[hep-ph]]**)

On shell W boson production at $NLL_{QED} + NLO_{EW}$

- We start from the formalism of $t\bar{t}$ resummation programme (**S. Catani, M. Grazzini, A. Torre 1408.4564[hep-ph]**):

$$W_N^V(b,M) = \sum_{ca_1a_2} \sigma^{(0)}_{c\bar{c},V}(\alpha_S(M^2)) f_{a_1/h_1,N}(b_0^2/b^2) f_{a_2/h_2,N}(b_0^2/b^2) \times S_c(M,b) \times [(\mathbf{H}^V \boldsymbol{\Delta} C_1 C_2)]_{c\bar{c},a_1,a_2;N}(M^2, b_0^2/b^2)]$$

  - $[(\mathbf{H}^V \boldsymbol{\Delta} C_1 C_2)]$ hard factor; S_c Sudakov form factor
  - $\boldsymbol{\Delta}$ related to soft wide-angle radiation from final state ($\boldsymbol{\Delta} = 1$ for neutral final states)

- $\boldsymbol{\Delta}(\mathbf{b}, M, \boldsymbol{\Gamma}, y_{34}, \phi_3) = \mathbf{V}^\dagger(\mathbf{b}, M, \boldsymbol{\gamma}, y_{34}, \phi_3)\mathbf{D}(\alpha_S(b_0^2/b^2); y_{34}, \phi_3)\mathbf{V}(\mathbf{b}, M, \boldsymbol{\Gamma}, y_{34}, \phi_3)$
  - $\boldsymbol{V}$ evolution operator
  - $\mathbf{D}$ color operator: resums logarithms from final-state emission
  - explicit expressions: **1408.4564[hep-ph]**

- We can drop charge correlations involving intial and final state (abelian limit) $\rightarrow$ $\mathbf{V}$ is diagonal $\rightarrow$ commutation
- We are only interested on $\mathbf{D}$ at the lowest order

$$\mathbf{D}(\alpha_S; y_{34}, \phi_3) = 1 + \frac{\alpha_S}{\pi}\mathbf{D}^{(1)}(y_{34}, \phi_3) + \mathcal{O}(\alpha_S^2)$$

$$\mathbf{D}^{(1)}(y_{34}, \phi_3) = (\mathbf{T}_3^2 + \mathbf{T}_4^2) d_a(y_{34}, \phi_3) - (\mathbf{T}_3^2 + \mathbf{T}_4^2)^2 d_b(y_{34}, \phi_3) + \frac{1}{2v}\mathbf{T}_3 \cdot \mathbf{T}_4\, d_c(y_{34}, \phi_3)\,, \text{for } t\bar{t}$$

- Replacements and simplifications: $\mathbf{T}_3 \rightarrow e_W$, $\alpha_S \rightarrow \alpha$; $y_{34} = \phi_3 = 0 = \mathbf{T}_4$, $d_b(0,0) = \frac{1}{2}$,
- $\mathbf{D} \rightarrow D = 1 + \frac{\alpha}{\pi}\frac{-e_W}{2} + \mathcal{O}(\alpha^2)$ (diagonal)

Sudakov form factor

- The coefficient D can be absorbed in the colourless Sudakov form factor
- In mixed QCD-EW, we finally obtain the following generalization:

$$\tilde{S}_{cc'}(M,b) = \exp\left[-\int_{b_0^2}^{b^2} \frac{dq^2}{q^2}\left(A_{c,c'}(\alpha_S(q^2),\alpha(q^2)) log\left(\frac{M^2}{Q^2}\right)\right) + \right.$$

$$\left. B_{c,c'}(\alpha_S(q^2),\alpha(q^2)) + D_W(\alpha_S(q^2),\alpha(q^2))\right]$$

- A and B are obtainable from the QCD analogous of 0508068[hep-ph] by making an abelianization of Casimir operators, which leads to

$$2\, C_F \to (e_q^2 + e_{\bar{q}'}^2)$$

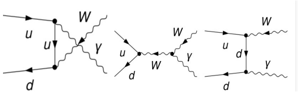
- D is instead an additional term, due to a charged final state, characteristic of final-state massive radiation
  - it generates (only) single-log terms (emission from massive leg: only-soft singularity!)

$$D^{(1)} = \frac{-eW^2}{2}$$

- Observation: the additional resummed contribution **implies** the replacement $B_1 \to B_1 + D_1$ in all the parts of the original formalism (Σ, $\mathcal{H}$, $\tilde{S}$)

Hard collinear coefficient function

- We started from $t\bar{t}$ subtraction operator of **1408.4564[hep-ph]**, transforming it properly
- The one-loop virtual renormalized form factor was included in $\mathcal{H}$



- This calculation reproduces the logarithmic structure of the fixed-order expansion of the Sudakov form factor
  - Cross-check of our formulas
  - Confirm the validity of the replacement $t\bar{t} \to W$ and abelianization procedures

Hadronic cross section : $\sigma = \sum_{ab} \tau \int_\tau^1 \frac{dz}{z} \mathcal{L}_{ab}\left(\frac{\tau}{z}\right) \frac{1}{z} \int dq_T^2 \frac{d\hat{\sigma}_{ab}(q_T, z)}{dq_T^2}$

Partonic inclusive cross section : $\hat{\sigma}_{ab}(z) = \int_{(q_T^{cut})^2}^{(q_T^{max})^2} dq_T^2 \frac{d\hat{\sigma}_{ab}(q_T, z)}{dq_T^2}$,

Inclusive hadronic cross section, introducing $f(a) = 2\sqrt{a}(\sqrt{1+a} - \sqrt{a})$, $a \equiv \frac{q_T^{(2)}}{Q^2}$:

$$\sigma_{u\bar{d}}^{>(1)} = \tau \int_0^{1-f(a)} \frac{dz}{z} \mathcal{L}_{u\bar{d}}\left(\frac{\tau}{z}\right) \frac{1}{z} \hat{\sigma}_{u\bar{d}}^{(1)}(z) = \tau \int_\tau^1 \frac{dz}{z} \mathcal{L}_{u\bar{d}}\left(\frac{\tau}{z}\right) \hat{\sigma}^{(0)} \hat{G}_{u\bar{d}}^{(1)}(z), \text{ with :}$$

$$\hat{G}_{u\bar{d}} = \sum_{m,r} \log^m(a) a^{\frac{r}{2}} \hat{G}_{u\bar{d}}^{(1,m,r)}(z), \text{ power series in the cutoff}$$

- Final expression obtained:

$$\hat{G}^{1}_{u\bar{d}} = \log(a)\left(\frac{3}{2}\delta(1-z)\frac{(e_D^2+e_U^2)}{2} - \frac{1}{2}\left(\left(P^{\mathrm{QED}}\right)_{\mathrm{dd}} + e_W^2\delta(1-z) - \left(P^{\mathrm{QED}}\right)_{\mathrm{uu}}\right)\right) +$$

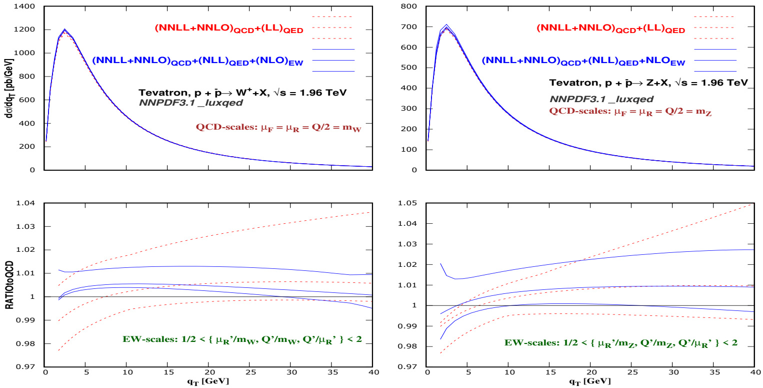
$$+\frac{1}{2}\log^2(a)\,\delta(1-z)\frac{(e_D^2+e_U^2)}{2} + \sqrt{a}\,\frac{1}{2}e_W^2\left(2\pi\delta'(1-z) - 3\pi\delta(1-z)\right)$$

$$+\textit{finite terms} + \textit{higher order terms}$$

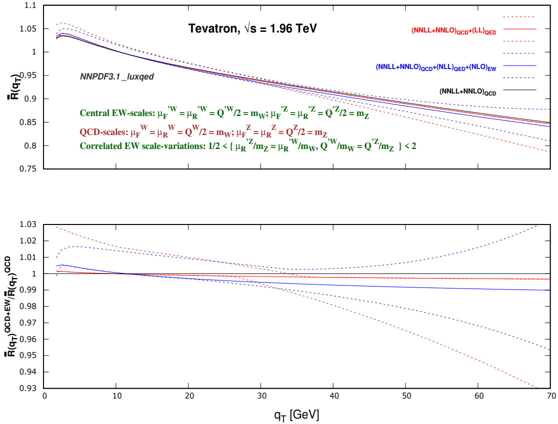
- $P^{QED}_{q\bar{q}}$ AP splitting functions in QED (D. de Florian, G. Rodrigo, G. F. R. Sborlini: 1611.04785[hep-ph]], 1512.00612[hep-ph], 1606.02887[hep-ph])
- We reproduce the known A and B perturbative coefficient of the QCD resummation formalism, modulo $C_F \to \frac{e_U^2+e_D^2}{2}$
- Additional logarithmic divergence from the charged final state $\sim D_1 \log(a)$, $D_1 = \frac{e_W^2}{2}$
- A linear power correction in the cutoff ($\sqrt{a}$) and proportional to the charged final state is present
  - Accordingly with L. Buonocore, M. Grazzini, F. Tramontano: 1911.10166 [hep-ph] (massive leg emission $\to$ linear power correction)

Numerical Results at hadron colliders

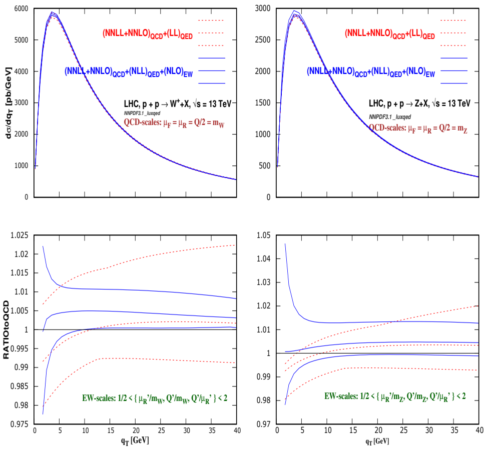
Code: DYQT G. Bozzi, S. Catani, G. Ferrera, D. de Florian, M. Grazzini, [1007.2351 [hep-ph]], [1007.2351 [hep-ph]], [0812.2862 [hep-ph]]



- EW corrections: effects at percent and per-thousand level (slightly greater in Z case)
- Scale variation band:
  - W: roughly $(2.5\% - 4\%)$ at LL and $(1\% - 1.5\%)$ at NLL+NLO
  - Z: roughly $(1\% - 5.5\%)$ at LL and (2%) at NLL+NLO
  - Inclusion of NLL+NLO reduces scale variation band (roughly $1.5\% - 3\%$)
  - W boson: small band-sizes (also at LL)
- Different q_T dependence at NLL+NLO (W, Z)
  - radiation from final state



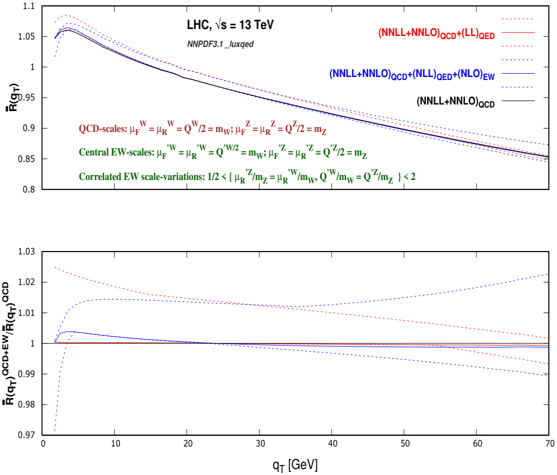
- $\bar{R}(q_T) = \dfrac{\frac{d\sigma^W}{dq_T^2}\frac{1}{\sigma^W}}{\frac{d\sigma^Z}{dq_T^2}\frac{1}{\sigma^Z}}$,
- LL: EW contributions simplify in the ratio
  - similar structure of leading logs terms
- NLL+NLO: EW effects up to $\mathcal{O}(1\%)$ (modification of sub-leading logs from massive charged legs)
- Scale variation band reduced at roughly 1% in low-intermediate q_T
  - Simplifications of common uncertainties in the ratio (especially at LL) (contribution from final-state radiation at NLL+NLO)



- Impact of EW corrections slightly smaller than Tevatron case
  - Suppression of $q\bar{q}$ channel
  - Enhancement of gluon-induced reactions

Scale variation band:

- W: roughly (2.5% − 3%) at LL and (1%) at NLL+NLO
- Z: roughly (2.5% − 3%) at LL and (1.5%) at NLL+NLO



- $\bar{R}(q_T)$ scale variation band: roughly 1.5 %
- At LL central-scale, EW contributions almost vanishing
- Decrease of EW impact also at NLL+NLO

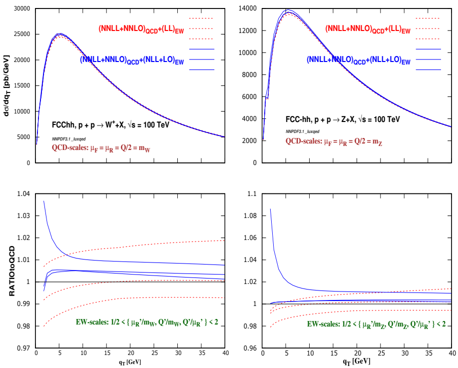
Summary & Outlook

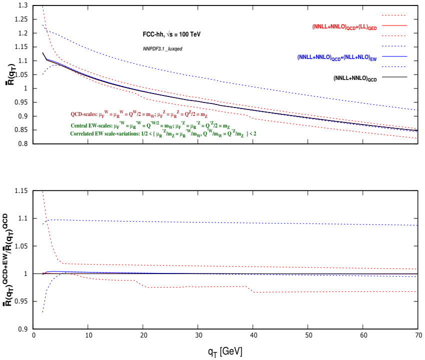
- To fully exploit the potential of LHC measurements accurate theoretical predictions are required $\to$ precise determination of SM parameters (m_W)
- We considered EW corrections to resummation formalism in QCD, to properly include photon radiation, focusing on on-shell W boson production, which has to be treat carefully due to a charged final state
- Final state radiation is fully included by extending the resummation formalism for a coloured final state ($t\bar{t}$)(coloured final state)
- Expansion at small q_T of the real inclusive cross section has confirmed the validity of the replacement $t\bar{t} \to W$
- Through the use of the numerical code DYQT we presented numerical predictions at $(NNLL + NNLO)_{QCD} + (NLL)_{QED} + (NLO)_{EW}$, finding EW effects up to per cent level
- We considered also the ratio distribution $p_T(W)/p_T(Z)$, in the direction of m_W extraction $\to$ a reduction of scale-variation band is observed
- A natural extension of this work is the inclusion of the decay of weak boson and the radiation from leptonic final state (QED parton shower programme, e.g. PHOTOS)

Thank you for the attention!!

BACKUP SLIDE

FCC-hh, $\sqrt{s} = 100$TeV





- enhacement of gluon PDF
- Larger suppression of EW effects (Tevatron, LHC at 13 TeV)
- Large uncertainties in $\bar{R}(q_T)$ distribution

at FCC-hh PDF extrapolation is challenging (out of experimental accessible range)

Matching procedure

- For intemerdiate q_T values, the resummed component should be properly combined with fixed order expansion $\rightarrow$ matching procedure:
  - recover fixed-order series for $q_T \lesssim M_V$ where $\left[\frac{d\sigma_{ab}}{dq_T^2}\right]_{res.} \rightarrow 0$
  - avoid double counting of logarithmic terms $\rightarrow$ counterterm $\left[\frac{d\sigma_{ab}}{dq_T^2}\right]_{asym}$:
- The finite part is $\left[\frac{d\sigma_{ab}}{dq_T^2}\right]_{fin.} = \left[\frac{d\sigma_{ab}}{dq_T^2}\right]_{f.o.} - \left[\frac{d\sigma_{ab}^{res}}{dq_T^2}\right]_{f.o.} = \left[\frac{d\sigma_{ab}}{dq_T^2}\right]_{f.o.} - \left[\frac{d\sigma_{ab}}{dq_T^2}\right]_{asym}$
- The counterterm, in impact parameter and Mellin space, is given by the expansion:

$$\left[\mathcal{H}^V_{a_1 a_2,N} \times \exp\left(\mathcal{G}_{a_1 a_2,N}\right)\right] \approx \sigma^{(0)}_{c\bar{c},V}(\alpha_S, M)\left[\delta_{ca_1}\delta_{ca_2}\delta(1-z)\right]$$

$$+\sum_k \left(\frac{\alpha_S}{\pi}\right)^k \Sigma^{V,(k)}_{c\bar{c}\leftarrow a_1 a_2}(z, L; M^2/\mu_R^2, M^2/\mu_F^2 mM^2/Q^2)$$

$$+\sum_k \left(\frac{\alpha_S}{\pi}\right)^k \mathcal{H}_{c\bar{c}\leftarrow a_1 a_2}(z; M^2/\mu_R^2, M^2/\mu_F^2, M^2/Q^2)$$

Coloured (charged) final state S. Catani, M. Grazzini, A. Torre, [1408.4564 [hep-ph]]

- Drell-Yan final state is coulourless
- In case of charged final state (e.g. $t\bar{t}$) production the hadronic resummed contribution is generalized according to:

$$W_N^V(b, M) = \sum_{ca_1 a_2} \sigma^{(0)}_{c\bar{c},V}(\alpha_S(M^2)) f_{a_1/h_1,N}(b_0^2/b^2) f_{a_2/h_2,N}(b_0^2/b^2)$$

$$\times S_c(M, b) \times [(\mathbf{H}^V \mathbf{\Delta} C_1 C_2)]_{c\bar{c},a_1,a_2;N}(M^2, b_0^2/b^2)]$$

  - $[(\mathbf{H}^V \mathbf{\Delta} C_1 C_2)]$ hard factor; S_c Sudakov form factor
  - $\mathbf{\Delta}$ related to soft wide-angle radiation from final state ($\mathbf{\Delta} = 1$ for colourless final states)
- We extend this formalism for electrically charged emission in the proceeding of the talk

- L. Cieri (a,b) , X. Chen (b) , T. Gehrmann (b) , E.W.N. Glover (c) and A. Huss 1807.11501[hep-ph]
- the central prediction at N^3LO almost coincides with the upper edge of the band

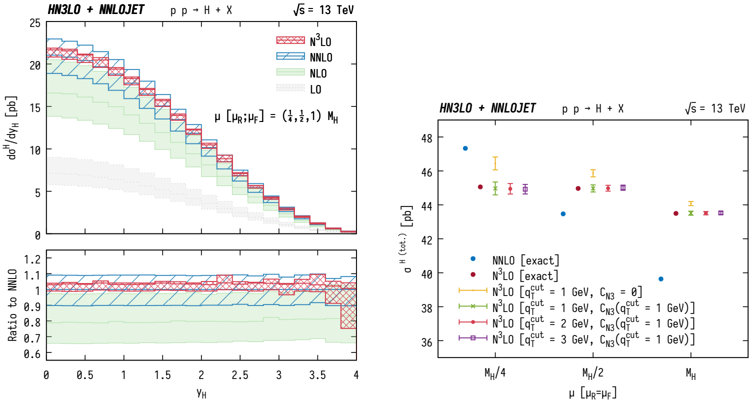


Figure: Rapidity distribution of the Higgs boson computed using the q_T subtraction formalism up to N^3LO (left panel) and the total cross section of the same process.