

Higher-order electroweak (EW) and mixed QCD-QED resummation effects for W and Z boson production at hadron colliders

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Introduction & Motivations

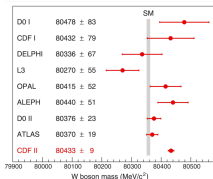
- Drell-Yan mechanism is a **benchmark process** at hadron colliders (SM and BSM studies, standard candle for detector calibration, m_W determination, ...) and is measured with **high experimental precision**

⇒ Need of **accurate theoretical predictions** ⇒ **higher-order radiative corrections**

- Distribution in q_T , in the framework of **RESUMMATION FORMALISM** (G. Bozzi, S. Catani, D. de Florian, M. Grazzini [hep-ph/0508068]), are known at the high-precision level of **N3LL+N3LO** in QCD (S. Camarda, L. Cieri, G. Ferrera [hep-ph/2103.04974 [hep-ph]]) ⇒

EW corrections must be taken into account : $\alpha \sim \alpha_S^2$

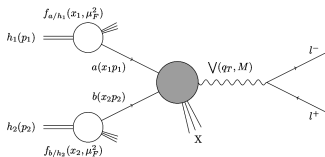
- QED correction NLL+NLO was computed only for on-shell Z production (L. Cieri, G. Ferrera, G.F.R. Sborlini [hep-ph/1805.11948 [hep-ph]]) → **We** extend the formalism to reach **$NLL_{QED} + NLO_{EW}$** accuracy for both **charged** and **neutral** current on-shell Drell-Yan processes
- The last extremely precise extraction of M_W at CDFII detector shows a substantial tension with **SM prediction** and **past m_W measurements** (ATLAS, DELPHI, OPAL, ...)



- CDF collaboration: High-precision measurement of the W boson mass with the CDF II detector 10.1126/science.abk1781
- theoretical studies should be directed to relevant kinematic distribution (e.g. $P_T(W/Z)$, ...)
- EW corrections could give $\mathcal{O}(\text{MeV})$ shift of m_W value extraction
- understand deeply the tension (**underestimation of uncertainties**, BSM effects, extension of the theory...)

Drell-Yan q_T distribution

$$\frac{d\sigma_V}{dq_T^2}(q_T, M, s) \stackrel{\text{factorization theorem}}{=} \sum_{a;b} \int_0^1 dx_1 \int_0^1 dx_2 f_{a=h_1}(x_1, \mu_F^2) f_{b=h_2}(x_2, \mu_F^2) \times \frac{d\hat{\sigma}_{V_{ab}}}{dq_T^2}\left(q_T, M, \hat{s}, \alpha_S(\mu_R^2), \mu_R^2, \mu_F^2\right)$$



- In the region $q_T \gtrsim M_V$ the perturbative fixed-order expansion is reliable:

$$\frac{d\hat{\sigma}_{V_{ab}}}{dq_T^2} = \frac{d\hat{\sigma}_{V_{ab}}^{(0)}}{dq_T^2} + \frac{\alpha_S}{\pi} \frac{d\hat{\sigma}_{V_{ab}}^{(1)}}{dq_T^2} + \left(\frac{\alpha_S}{\pi}\right)^2 \frac{d\hat{\sigma}_{V_{ab}}^{(2)}}{dq_T^2} + \mathcal{O}\left[\left(\frac{\alpha_S}{\pi}\right)^3\right]$$

- In the region $q_T \ll m_V$ (bulk of the events) **large logarithmic corrections** of the type $\alpha_S^n \ln^m(M_V^2/q_T^2)$, due to soft and/or collinear parton radiations, spoils the convergence

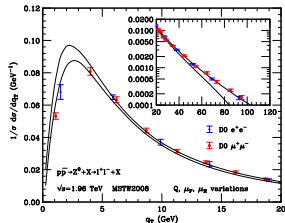
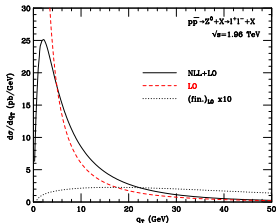
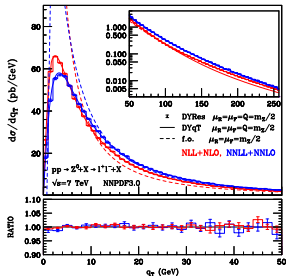


Resummation at all perturbative orders is mandatory:

$$\frac{d\hat{\sigma}_{V_{ab}}}{dq_T^2} = \frac{\hat{\sigma}_V^{(0)}}{q_T^2} \sum_{n=1}^{+\infty} \sum_{m=0}^{2n-1} A_{n;m}^V \ln^m\left(\frac{M^2}{q_T^2}\right) \alpha_S^n(M^2), \quad \alpha_S^n \ln^m(M_V^2/q_T^2) \gg 1$$

Resummation formalism

G. Bozzi, S. Catani, D. de Florian, M. Grazzini hep-ph/0508068



G. Bozzi, S. Catani, G. Ferrera, D. de Florian, M. Grazzini [1007.2351[hep-ph]], [1507.06937[hep-ph]]

- Partonic cross section is explicitly splitted as:

$$\frac{d\hat{\sigma}}{dq_T^2} = \frac{d\hat{\sigma}^{res}}{dq_T^2} + \frac{d\hat{\sigma}^{fin}}{dq_T^2}, \quad \text{with} \quad \lim_{q_T \rightarrow 0} \int_0^{q_T} dq_T^2 \frac{d\hat{\sigma}^{fin}}{dq_T^2} = 0$$

- Resummation is performed in impact parameter (b) space

$$\frac{d\hat{\sigma}_{a_1 a_2}^{res:V}}{dq_T^2}(q_T; M, \hat{s}; \alpha_S(\mu_R^2), \mu_F^2, \mu_R^2) = \frac{M^2}{\hat{s}} \int_0^1 db \frac{b}{2} J_0(bq_T) \mathcal{W}_{a_1 a_2}^V(b; M, \hat{s}, \alpha_S(\mu_R^2), \mu_F^2, \mu_R^2).$$

- The resummed form factor $\mathcal{W}_{a_1 a_2}^V$ can be expressed in an **exponential** and **factorized** form in the Mellin space $\rightarrow z = M_V^2/\hat{s}$, $f_N = \int_0^1 dz z^{N-1} f(z)$:

$$\mathcal{W}_{N;a_1 a_2}^V = \mathcal{H}_N(M_W, \alpha_S(\mu_R^2)) \times \exp \mathcal{G}_N(\alpha_S(\mu_R^2), L; M^2/\mu_R^2, M^2/Q^2),$$

$$L = \ln \frac{Q^2 b^2}{b_0^2} + 1 ; b_0 = 2 \exp(-E); E = 0.5772:::$$

Hard collinear coefficient function $\mathcal{H}_{N;a_1 a_2}^V$

- includes **finite** terms (e.g. virtual emission)
- process dependent

$$\mathcal{H}_{N;a_1 a_2} \left(M, \alpha_S, \frac{M^2}{\mu_R^2}, \frac{M^2}{\mu_F^2}, \frac{M^2}{Q^2} \right) = \hat{\sigma}_{0;a_1 a_2}^V(M) \left[1 + \sum_{n=1}^l \left(\frac{\alpha_S}{\pi} \right)^n \mathcal{H}_N^{(n)} \left(\frac{M^2}{\mu_R^2}, \frac{M^2}{\mu_F^2}, \frac{M^2}{Q^2} \right) \right]$$

Sudakov form factor \mathcal{G}_N

- contains the **logarithmic enhanced** contributions
- universal structure

$$\begin{aligned} \mathcal{G}_{N;a_1 a_2}^V \left(\alpha_S(\mu_R^2), L; \frac{M^2}{\mu_R^2}, \frac{M^2}{Q^2} \right) &= L g_N^{(1)}(\alpha_S(\mu_R^2)L) + g_N^{(2)} \left(\alpha_S(\mu_R^2)L; \frac{M^2}{\mu_R^2}, \frac{M^2}{Q^2} \right) + \\ &+ \frac{\alpha_S(\mu_R^2)}{\pi} g_N^{(3)} \left(\alpha_S(\mu_R^2); \frac{M^2}{\mu_R^2}, \frac{M^2}{Q^2} \right) + \sum_{n=4}^l \left(\frac{\alpha_S}{\pi} \right)^n g_N^{(n)} \left(\alpha_S(\mu_R^2)L; \frac{M^2}{\mu_R^2}, \frac{M^2}{Q^2} \right). \end{aligned}$$

+

Perturbative structure of the resummed component \rightarrow **LL** accuracy ($\sim \alpha_S^n L^{n+1}$): $g_N^{(1)}$; **NLL** accuracy ($\sim \alpha_S^N L^n$): $g_N^{(2)}$, $\mathcal{H}_N^{(1)}$; **NNLL** accuracy: ($\sim \alpha_S^N L^{n-1}$): $g_N^{(3)}$, $\mathcal{H}_N^{(2)}$; **N3LL** accuracy ($\sim \alpha_S^N L^{n-2}$): $g_N^{(4)}$, $\mathcal{H}_N^{(3)}$

On-shell Z boson production

- QED corrections at NLL+NLO known
 - [L. Cieri, G. Ferrera, G. F. R. Sborlini 1805.11948[hep-ph]]
- We incorporated also weak corrections at one loop within the hard factor \mathcal{H}
 - We consider one-loop renormalized form factor
 - Modifications only in the hard factor H (massive loop corrections)

On-shell W boson production (NEW)

- Charged final state \rightarrow a "naive abelianization" of QCD formulas for DY process is not suitable
 - We use the formalism of $t\bar{t}$ production
- 1 Replacement: $t\bar{t} \rightarrow W$ (colour charged \rightarrow electrically charged)
 - 2 Abelianization of Casimir operators: $\mathbf{T}_i \rightarrow e_i$ (S. Catani, S.Dittmaier, Zoltan Trocsany [9802439[hep-ph]], [0011222[hep-ph]])

- We start from the formalism of $t\bar{t}$ resummation programme (S. Catani, M. Grazzini, A. Torre 1408.4564[hep-ph]):

$$W_N^V(b; M) = \sum_{c a_1 a_2}^{(0)}_{c\bar{c}, V} (s(M^2)) f_{a_1/h_1, N}(b_0^2=b^2) f_{a_2/h_2, N}(b_0^2=b^2) S_c(M; b) [(\mathbf{H}^V \Delta C_1 C_2)]_{c\bar{c}, a_1, a_2; N}(M^2; b_0^2=b^2)$$

- $[(\mathbf{H}^V C_1 C_2)]$ hard factor; S_c Sudakov form factor
- related to soft wide-angle radiation from final state (= 1 for neutral final states)
- $(\mathbf{b}, M, \gamma, y_{34}, \phi_3) = \mathbf{V}^Y(\mathbf{b}, M, \gamma, y_{34}, \phi_3) \mathbf{D}(\alpha_S(b_0^2/b^2); y_{34}, \phi_3) \mathbf{V}(\mathbf{b}, M, \gamma, y_{34}, \phi_3)$
 - \mathbf{V} evolution operator
 - \mathbf{D} color operator: resums logarithms from final-state emission
 - explicit expressions: 1408.4564[hep-ph]
- We can drop charge correlations involving initial and final state (abelian limit) $\rightarrow \mathbf{V}$ is diagonal \rightarrow commutation
- We are only interested on \mathbf{D} at the lowest order

$$\mathbf{D}(\alpha_S; y_{34}, \phi_3) = 1 + \frac{\alpha_S}{\pi} \mathbf{D}^{(1)}(y_{34}, \phi_3) + \mathcal{O}(\alpha_S^2)$$

$$\mathbf{D}^{(1)}(y_{34}, \phi_3) = (\mathbf{T}_3^2 + \mathbf{T}_4^2) d_a(y_{34}, \phi_3) - (\mathbf{T}_3^2 + \mathbf{T}_4^2)^2 d_b(y_{34}, \phi_3) + \frac{1}{2V} \mathbf{T}_3 \cdot \mathbf{T}_4 d_c(y_{34}, \phi_3), \text{ for } t\bar{t}$$

- Replacements and simplifications: $\mathbf{T}_3 \rightarrow e_W$, $\alpha_S \rightarrow \alpha$; $y_{34} = \phi_3 = 0 = \mathbf{T}_4$, $d_b(0, 0) = \frac{1}{2}$,
- $\mathbf{D} \rightarrow D = 1 + -\frac{e_W}{2} + \mathcal{O}(\alpha^2)$ (diagonal)

Sudakov form factor

- The coefficient D can be absorbed in the colourless Sudakov form factor
- In mixed QCD-EW, we finally obtain the following generalization:

$$\tilde{S}_{cc'}(M, b) = \exp \left[- \int_{b_0^2}^{b^2} \frac{dq^2}{q^2} \left(A_{c;c'}(\alpha_S(q^2), \alpha(q^2)) \log \left(\frac{M^2}{Q^2} \right) + B_{c;c'}(\alpha_S(q^2), \alpha(q^2)) + D_W(\alpha_S(q^2), \alpha(q^2)) \right) \right]$$

- A and B are obtainable from the QCD analogous of 0508068[hep-ph] by making an abelianization of Casimir operators, which leads to

$$2 C_F \rightarrow (e_q^2 + e_{q'}^2)$$

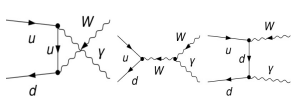
- D is instead an additional term, due to a charged final state, **characteristic of final-state massive radiation**
 - it generates (only) single-log terms (emission from massive leg: only-soft singularity!)

$$D^{(1)} = -\frac{eW^2}{2}$$

- Observation: the additional resummed contribution **implies** the replacement $B_1 \rightarrow B_1 + D_1$ in all the parts of the original formalism ($\Sigma, \mathcal{H}, \tilde{S}$)

Hard collinear coefficient function

- We started from $t\bar{t}$ subtraction operator of 1408.4564[hep-ph], transforming it properly
- The one-loop virtual renormalized form factor was included in \mathcal{H}



- This calculation reproduces the logarithmic structure of the fixed-order expansion of the Sudakov form factor

- Cross-check of our formulas
- Confirm the validity of the replacement $t\bar{t} \rightarrow W$ and abelianization procedures

$$\text{Hadronic cross section : } \sigma = \sum_{ab} \tau \int^1 \frac{dz}{z} \mathcal{L}_{ab} \left(\frac{\tau}{z} \right) \frac{1}{z} \int dq_T^2 \frac{d\hat{\sigma}_{ab}(q_T, z)}{dq_T^2}$$

$$\text{Partonic inclusive cross section : } \hat{\sigma}_{ab}(z) = \int_{(q_T^{\text{cut}})^2}^{(q_T^{\text{max}})^2} dq_T^2 \frac{d\hat{\sigma}_{ab}(q_T, z)}{dq_T^2},$$

Inclusive hadronic cross section, introducing $f(a) = 2\sqrt{a}(\sqrt{1+a} - \sqrt{a})$, $a \equiv \frac{q_T^2}{Q^2}$:

$$\sigma_{u\bar{d}}^{>(1)} = \tau \int_0^1 \frac{dz}{z} \mathcal{L}_{u\bar{d}} \left(\frac{\tau}{z} \right) \frac{1}{z} \hat{\sigma}_{u\bar{d}}^{(1)}(z) = \tau \int^1 \frac{dz}{z} \mathcal{L}_{u\bar{d}} \left(\frac{\tau}{z} \right) \hat{\sigma}^{(0)} \hat{G}_{u\bar{d}}^{(1)}(z), \text{ with :}$$

$$\hat{G}_{u\bar{d}} = \sum_{m;r} \log^m(a) a^{\frac{r}{2}} \hat{G}_{u\bar{d}}^{(1;m;r)}(z), \text{ power series in the cutoff}$$

- Final expression obtained:

$$\hat{G}_{ud}^1 = \mathbf{log}(a) \left(\frac{3}{2} \delta(1-z) \frac{(e_D^2 + e_U^2)}{2} - \frac{1}{2} \left((P^{QED})_{dd} + e_W^2 \delta(1-z) - (P^{QED})_{uu} \right) \right) +$$

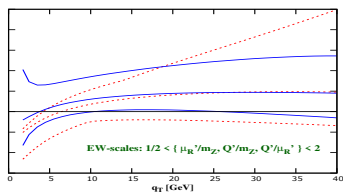
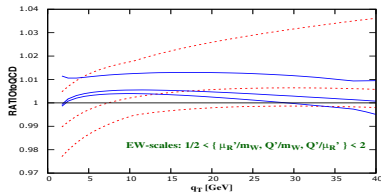
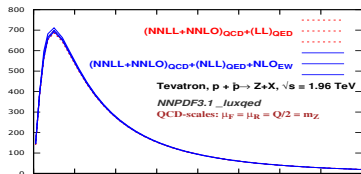
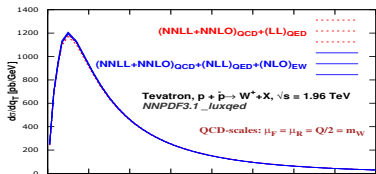
$$+ \frac{1}{2} \mathbf{log}^2(a) \delta(1-z) \frac{(e_D^2 + e_U^2)}{2} + \sqrt{a} \frac{1}{2} e_W^2 (2\pi \delta^\theta(1-z) - 3\pi \delta(1-z))$$

+ *finite terms* + *higher order terms*

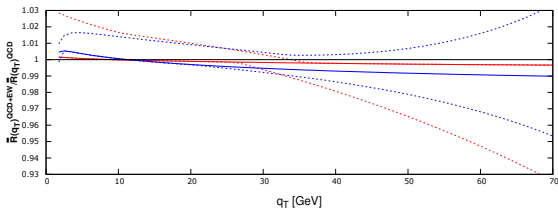
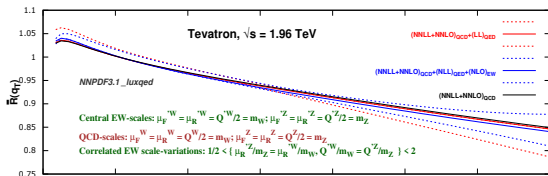
- $P_{q\bar{q}}^{QED}$ AP splitting functions in QED (D. de Florian, G. Rodrigo, G. F. R. Sborlini: 1611.04785[hep-ph], 1512.00612[hep-ph], 1606.02887[hep-ph])
- We reproduce the known A and B perturbative coefficient of the QCD resummation formalism, modulo $C_F \rightarrow \frac{e_U^2 + e_D^2}{2}$
- Additional logarithmic divergence from the charged final state $\sim D_1 \log(a)$, $D_1 = \frac{e_W^2}{2}$
- A linear power correction in the cutoff (\sqrt{a}) and proportional to the charged final state is present
 - Accordingly with L. Buonocore, M. Grazzini, F. Tramontano: 1911.10166 [hep-ph] (massive leg emission ! linear power correction)

Numerical Results at hadron colliders

Code: DYqT G. Bozzi, S. Catani, G. Ferrera, D. de Florian, M. Grazzini, [1007.2351 [hep-ph]], [1007.2351 [hep-ph]], [0812.2862 [hep-ph]]



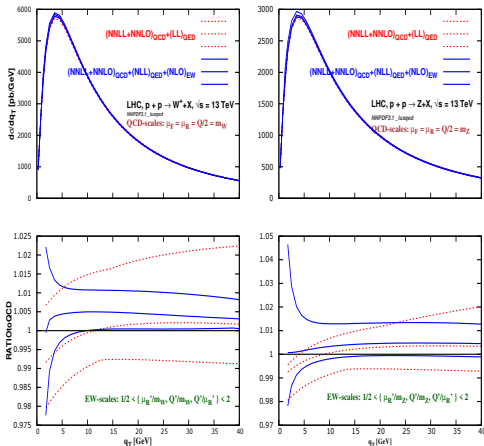
- EW corrections: effects at percent and per-thousand level (slightly greater in Z case)
- Scale variation band:
 - W : roughly (2:5% 4%) at LL and (1% 1:5%) at NLL+NLO
 - Z : roughly (1% 5:5%) at LL and (2%) at NLL+NLO
 - Inclusion of NLL+NLO reduces scale variation band (roughly 1:5% 3%)
 - W boson: small band-sizes (also at LL)
- Different q_T dependence at NLL+NLO (W , Z)
 - radiation from final state



$$\bullet \bar{R}(q_T) = \frac{d\sigma^W}{dq_T^2} \frac{1}{\sigma^W};$$

$$\frac{d\sigma^Z}{dq_T^2} \frac{1}{\sigma^Z};$$

- **LL**: EW contributions simplify in the ratio
 - similar structure of leading logs terms
- **NLL+NLO**: EW effects up to $O(1\%)$ (modification of sub-leading logs from massive charged legs)
- Scale variation band reduced at roughly 1% in low-intermediate q_T
 - Simplifications of common uncertainties in the ratio (especially at **LL**) (contribution from final-state radiation at **NLL+NLO**)

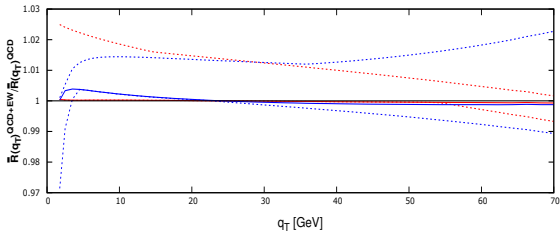
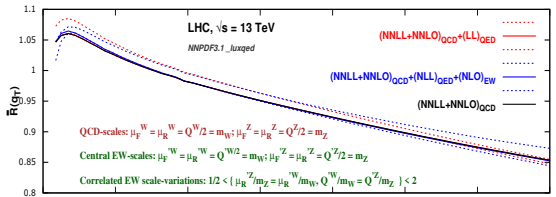


- Impact of EW corrections slightly smaller than Tevatron case

- Suppression of $q\bar{q}$ channel
- Enhancement of gluon-induced reactions

Scale variation band:

- W : roughly (2:5% 3%) at LL and (1%) at NLL+NLO
- Z : roughly (2:5% 3%) at LL and (1:5%) at NLL+NLO



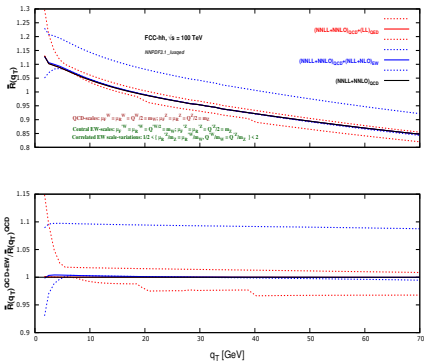
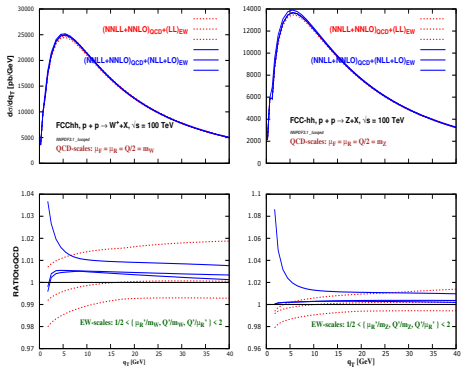
- $\bar{R}(q_T)$ scale variation band: roughly 1.5 %
- At LL central-scale, EW contributions almost vanishing
- Decrease of EW impact also at NLL+NLO

- To fully exploit the potential of LHC measurements accurate theoretical predictions are required \rightarrow precise determination of SM parameters (m_W)
- We considered EW corrections to resummation formalism in QCD, to properly include photon radiation, focusing on on-shell W boson production, which has to be treated carefully due to a charged final state
- Final state radiation is fully included by extending the resummation formalism for a coloured final state ($t\bar{t}$) (coloured final state)
- Expansion at small q_T of the real inclusive cross section has confirmed the validity of the replacement $t\bar{t} \rightarrow W$
- Through the use of the numerical code **DYqT** we presented numerical predictions at $(NNLL + NNLO)_{QCD} + (NLL)_{QED} + (NLO)_{EW}$, finding EW effects up to **per cent level**
- We considered also the ratio distribution $p_T(W)/p_T(Z)$, in the direction of m_W extraction \rightarrow a reduction of scale-variation band is observed
- A natural extension of this work is the inclusion of the decay of weak boson and the radiation from leptonic final state (QED parton shower programme, e.g. **photos**)

Thank you for the attention!!

BACKUP SLIDE

FCC-hh, $\sqrt{s} = 100\text{TeV}$



- enhancement of gluon PDF
- Larger suppression of EW effects (Tevatron, LHC at 13 TeV)
- Large uncertainties in $\bar{R}(q_T)$ distribution

at FCC-hh PDF extrapolation is challenging (out of experimental accessible range)

Matching procedure

- For intermediate q_T values, the resummed component should be properly combined with fixed order expansion \rightarrow **matching procedure**:

- recover fixed-order series for $q_T < M_V$ where $\frac{d\sigma_{ab}}{dq_T^2} \Big|_{res.} \neq 0$

- avoid double counting of logarithmic terms ! counterterm $\frac{d\sigma_{ab}}{dq_T^2} \Big|_{asym}$:

- The finite part is $\left[\frac{d}{dq_T^2} \right]_{fin.} = \left[\frac{d}{dq_T^2} \right]_{f:o.} - \left[\frac{d}{dq_T^2} \right]_{f:o.}^{res} = \left[\frac{d}{dq_T^2} \right]_{f:o.} - \left[\frac{d}{dq_T^2} \right]_{asym}$

- The counterterm, in impact parameter and Mellin space, is given by the expansion:

$$\begin{aligned} \left[\mathcal{H}_{a_1 a_2; N}^V \times \exp(\mathcal{G}_{a_1 a_2; N}) \right] &\approx \sigma_{c\bar{c}; V}^{(0)}(\alpha_S, M) \left[\delta_{ca_1} \delta_{c\bar{a}_2} \delta(1-z) \right] \\ &+ \sum_k \left(\frac{\alpha_S}{\pi} \right)^k \Sigma_{c\bar{c}}^{V;(k)}{}_{a_1 a_2}(z, L; M^2/\mu_R^2, M^2/\mu_F^2, m^2/Q^2) \\ &+ \sum_k \left(\frac{\alpha_S}{\pi} \right)^k \mathcal{H}_{c\bar{c}}{}_{a_1 a_2}(z; M^2/\mu_R^2, M^2/\mu_F^2, M^2/Q^2) \end{aligned}$$

Coloured (charged) final state S. Catani, M. Grazzini, A. Torre, [1408.4564 [hep-ph]]

- Drell-Yan final state is colourless
- In case of charged final state (e.g. $t\bar{t}$) production the hadronic resummed contribution is generalized according to:

$$W_N^V(b, M) = \sum_{ca_1 a_2} \sigma_{c\bar{c};V}^{(0)}(\alpha_S(M^2)) f_{a_1=h_1;N}(b_0^2/b^2) f_{a_2=h_2;N}(b_0^2/b^2) \\ \times S_c(M, b) \times [(\mathbf{H}^V \quad C_1 C_2)]_{c\bar{c};a_1;a_2;N}(M^2, b_0^2/b^2)]$$

- $[(\mathbf{H}^V \quad C_1 C_2)]$ hard factor; S_c Sudakov form factor
- related to soft wide-angle radiation from final state ($\beta = 1$ for colourless final states)
- **We extend** this formalism for electrically charged emission in the proceeding of the talk

Higgs production at the LHC using q_T subtraction formalism at N^3LO QCD

- L. Cieri (a,b) , X. Chen (b) , T. Gehrmann (b) , E.W.N. Glover (c) and A. Huss 1807.11501[hep-ph]
- the central prediction at N^3LO almost coincides with the upper edge of the band

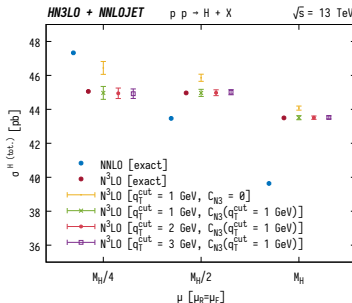
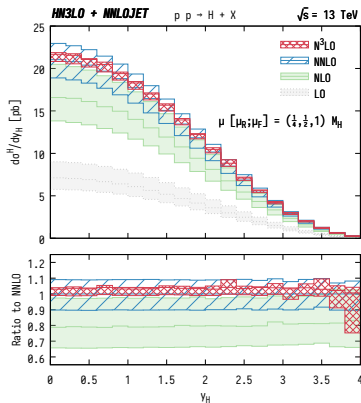


Figure: Rapidity distribution of the Higgs boson computed using the q_T subtraction formalism up to N^3LO (left panel) and the total cross section of the same process.