Soft gluon resummation for the production of four top quarks at the LHC

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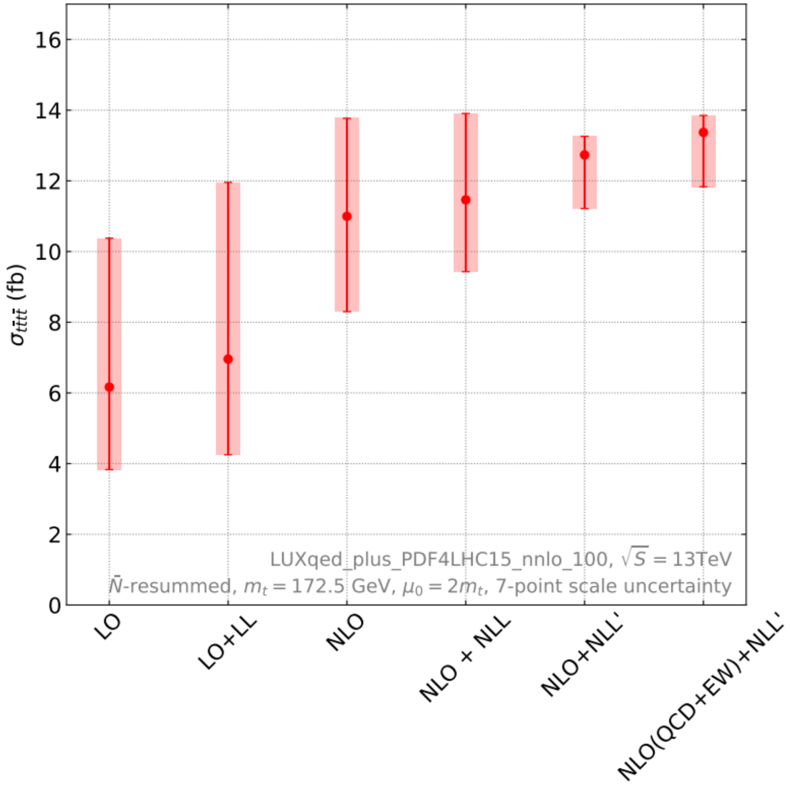

QCD@LHC IJCLab Orsay 28/11/2022 - 2/12/2022

WHY 4 TOP?

- Four coloured massive particles in the final state

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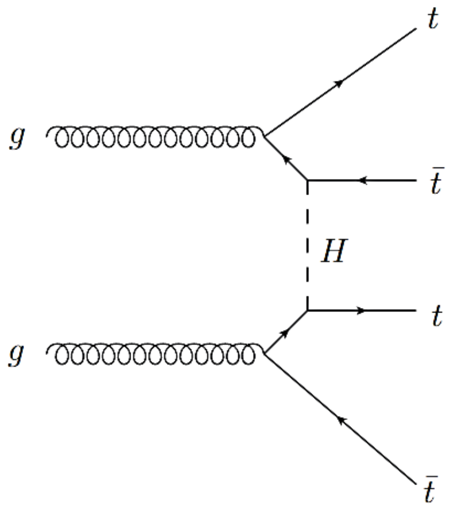
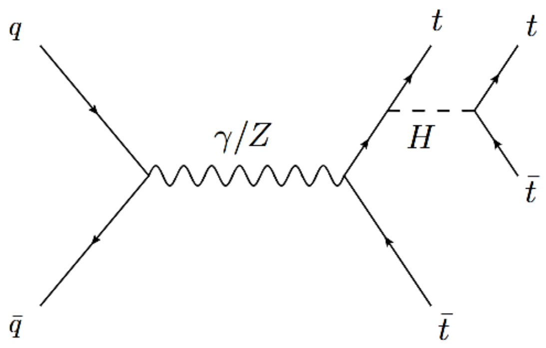
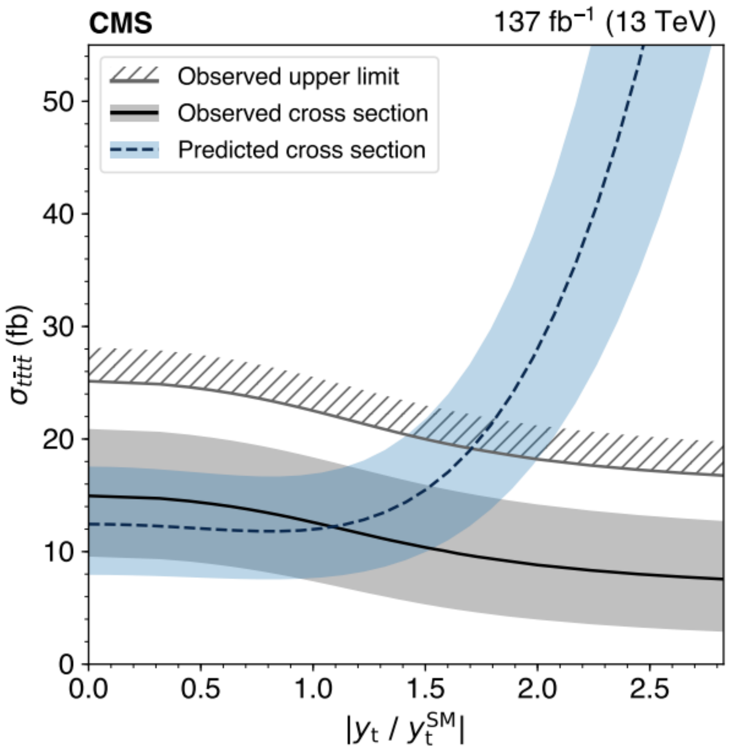
- Sensitive to the Yukawa coupling







[Eur. Phys. J. C (2020) 80, 75]

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- Sensitive to the Yukawa coupling
- Sensitive to new physics (gluinos, scalar gluons, heavy scalar bosons, ...)
- Constrains SMEFT coefficients: e.g. four-fermion operator

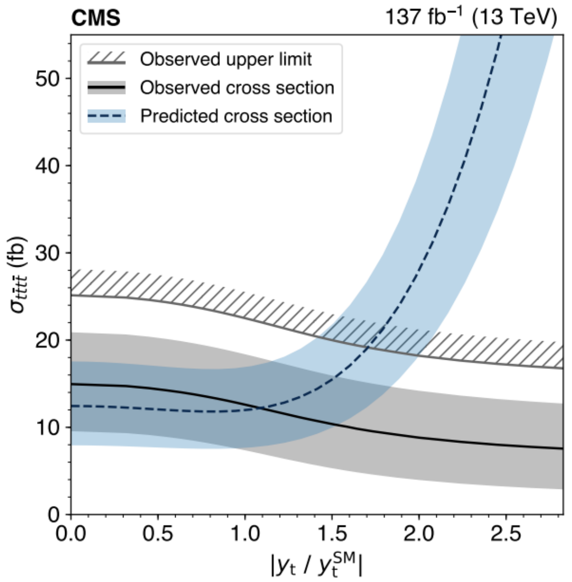
- Measured at the LHC
  - ATLAS [Eur. Phys. J. C (2020) 80, 1085; JHEP 11 (2021), 118; Phys. Rev. D 99, 052009 (2019)]
  - CMS [Eur. Phys. J. C (2020) 80, 75; JHEP 11 (2019), 082]

$$\sigma_{t\bar{t}t\bar{t}}^{\text{SM, NLO (QCD+EW)}} = 12.0 \pm 2.4 \text{ fb}$$

[Frederix, Pagani, Zaro (2017)]

CMS

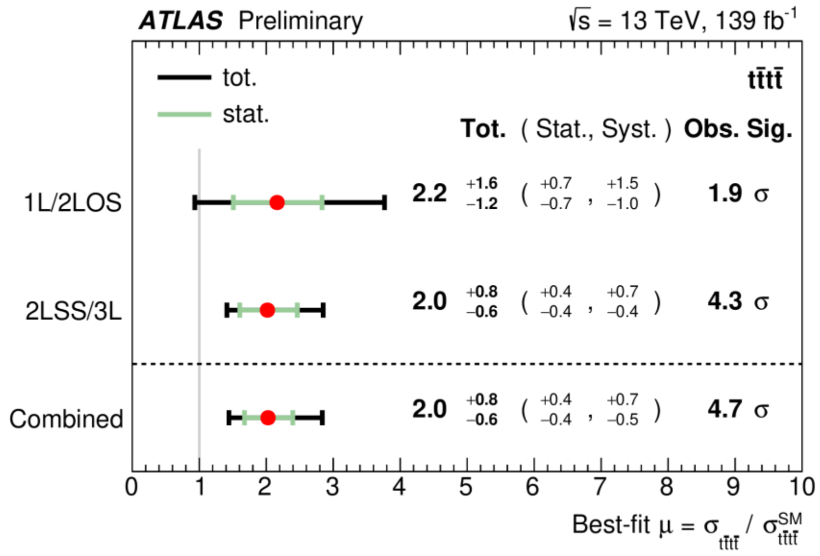
[Eur. Phys. J. C (2020) 80, 75]



$$\sigma_{t\bar{t}t\bar{t}}^{\text{CMS}} = 12.6^{+5.8}_{-5.2} \text{ fb}$$

ATLAS

[JHEP 11 (2021), 118]



$$\sigma_{t\bar{t}t\bar{t}}^{\text{ATLAS}} = 24 \pm 5(\text{stat.})^{+5}_{-4}(\text{syst.}) \text{ fb}$$

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- Theoretical predictions for total rate: NLO (QCD + EW) available

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GOAL: extend the precision of theoretical predictions beyond NLO for $pp \to t\bar{t}t\bar{t}$ by means of **resummation** techniques

- Perturbative expansion cross-section

$$\sigma = \sum_n c_n \alpha_s^n = c_0 + c_1\alpha_s + c_2\alpha_s^2 + \ldots \quad \text{with} \quad c_n = f_n + \sum_{k=0}^{2n-1} d_{nk} L^k \ , \quad L^k = \left[\frac{\ln^k(1-M^2/s)}{1-M^2/s}\right]_+$$

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  - Systematic treatment to all orders: resummation
  - Relies on ME and Phase space factorization → Mellin space $L := \left[\frac{\ln(1 - M^2/s)}{1 - M^2/s} \right]_+ \quad \to \quad \tilde{L} := \ln N$

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$$\hat{\sigma}^{\text{res}}(N) \sim \mathcal{F}(\alpha_s) \exp \left[\tilde{L}\, g_1(\alpha_s \tilde{L}) + g_2(\alpha_s \tilde{L}) + \ldots \right]$$

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LL

$\alpha_s^n \ln^{2n} N$

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LL: $\alpha_s^n \ln^{2n} N$

NLL: $\alpha_s^n \ln^{2n-1} N$

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Finite contributions ($\mathcal{F}(\alpha_s)$)

LL ($\tilde{L}\, g_1(\alpha_s \tilde{L})$): $\alpha_s^n \ln^{2n} N$

NLL ($g_2(\alpha_s \tilde{L})$): $\alpha_s^n \ln^{2n-1} N$

- Resummed partonic cross section for $pp \to t\bar{t}t\bar{t}$ in Mellin space

$$\hat{\sigma}^{\text{res}}_{ij\to t\bar{t}t\bar{t}} = \text{Tr}\left[\ \mathbf{H}_{ij\to t\bar{t}t\bar{t}}\ \mathbf{S}_{ij\to t\bar{t}t\bar{t}}\ \right] \Delta_i\, \Delta_j$$

$$= \text{Tr}\left[\ \mathbf{H}_{ij\to t\bar{t}t\bar{t}}\ \bar{\mathbf{U}}_{ij\to t\bar{t}t\bar{t}}\, \tilde{\mathbf{S}}_{ij\to t\bar{t}t\bar{t}}\, \mathbf{U}_{ij\to t\bar{t}t\bar{t}}\ \right] \Delta_i\, \Delta_j$$

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Incoming jet funcions

(soft-)collinear enhancements

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constant contributions as $N \to \infty$

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Soft function

soft wide-angle enhancements

$$\mathbf{U}_{ij\to t\bar{t}t\bar{t}}(N, M^2, \mu_F^2, \mu_R^2) = \text{Pexp}\left[\int_{\mu}^{M/\bar{N}} \frac{dq}{q}\,\mathbf{\Gamma}_{ij\to t\bar{t}t\bar{t}}\left(\alpha_s(q^2)\right)\right]$$

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Soft anomalous dimension (SAD) matrix

- One-loop SAD matrix needed at NLL

$$\mathbf{U}_{ij\to t\bar{t}t\bar{t}}(N, M^2, \mu_F^2, \mu_R^2) = \mathrm{Pexp}\left[\int_{\mu}^{M/\bar{N}} \frac{dq}{q}\, \mathbf{\Gamma}_{ij\to t\bar{t}t\bar{t}}\left(\alpha_s(q^2)\right)\right]$$

$$\mathbf{\Gamma}_{ij\to t\bar{t}t\bar{t}}\left(\alpha_s\right) = \left(\frac{\alpha_s}{\pi}\right)\mathbf{\Gamma}^{(1)}_{ij\to t\bar{t}t\bar{t}} + \left(\frac{\alpha_s}{\pi}\right)^2 \mathbf{\Gamma}^{(2)}_{ij\to t\bar{t}t\bar{t}} + \ldots$$

- $q\bar{q} \to t\bar{t}t\bar{t}:\ \mathbf{3}\otimes\bar{\mathbf{3}} = \mathbf{3}\otimes\bar{\mathbf{3}}\otimes\mathbf{3}\otimes\bar{\mathbf{3}} \longrightarrow \mathbf{1}\oplus\mathbf{8} = \mathbf{0}\oplus(2\times\mathbf{1})\oplus(2\times\mathbf{8})\oplus\mathbf{8}_S\oplus\mathbf{8}_A\oplus\mathbf{10}\oplus\overline{\mathbf{10}}\oplus\mathbf{27}$

6-dimensional colour space

- $gg \to t\bar{t}t\bar{t}:\ \mathbf{8}\otimes\mathbf{8} = \mathbf{3}\otimes\bar{\mathbf{3}}\otimes\mathbf{3}\otimes\bar{\mathbf{3}} \longrightarrow \mathbf{0}\oplus\mathbf{1}\oplus\mathbf{8}_S\oplus\mathbf{8}_A\oplus\mathbf{10}\oplus\overline{\mathbf{10}}\oplus\mathbf{27} = \mathbf{0}\oplus(2\times\mathbf{1})\oplus(2\times\mathbf{8})\oplus\mathbf{8}_S\oplus\mathbf{8}_A\oplus\mathbf{10}\oplus\overline{\mathbf{10}}\oplus\mathbf{27}$

14-dimensional colour space

COLOUR STRUCTURE

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Diagonal SAD matrix

$$\bar{\mathbf{U}}_R\,\tilde{\mathbf{S}}_R\,\mathbf{U}_R = \tilde{\mathbf{S}}_R\,\exp\left[\frac{2\,\mathrm{Re}\left(\mathbf{\Gamma}_R^{(1)}\right)}{2\pi b_0}\ln(1-2\lambda)\right]$$

with $\lambda = \alpha_s b_0 \ln(\bar{N})$

- $q\bar{q} \to t\bar{t}t\bar{t}: \ \mathbf{3}\otimes\bar{\mathbf{3}} = \mathbf{3}\otimes\bar{\mathbf{3}}\otimes\mathbf{3}\otimes\bar{\mathbf{3}} \rightarrow \mathbf{1}\oplus\mathbf{8} = \mathbf{0}\oplus(2\times\mathbf{1})\oplus(2\times\mathbf{8})\oplus\mathbf{8}_S\oplus\mathbf{8}_A\oplus\mathbf{10}\oplus\overline{\mathbf{10}}\oplus\mathbf{27}$

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$$c_1^{q\bar{q}} = \frac{1}{\sqrt{N_c^3}} \delta_{c_1 c_3} \delta_{c_2 c_4} \delta_{c_6 c_8}$$

$$c_2^{q\bar{q}} = \frac{1}{T_R \sqrt{N_c(N_c^2 - 1)}} \delta_{c_1 c_3} t^{a_1}_{c_2 c_4} t^{a_1}_{c_6 c_8}$$

$$c_3^{q\bar{q}} = \frac{1}{T_R \sqrt{N_c(N_c^2 - 1)}} t^{a_1}_{c_1 c_3} \delta_{c_2 c_4} t^{a_1}_{c_6 c_8}$$

$$c_4^{q\bar{q}} = \frac{1}{T_R \sqrt{N_c(N_c^2 - 1)}} t^{a_1}_{c_1 c_3} t^{a_1}_{c_2 c_4} \delta_{c_6 c_8}$$

$$c_5^{q\bar{q}} = \frac{\sqrt{N_c}}{T_R^2 \sqrt{2(N_c^4 - 5N_c^2 + 4)}} t^{a_1}_{c_1 c_3} d^{a_1 a_2 b_3} t^{a_2}_{c_2 c_4} t^{b_3}_{c_6 c_8}$$

$$c_6^{q\bar{q}} = \frac{1}{T_R^2 \sqrt{2N_c(N_c^2 - 1)}} t^{a_1}_{c_1 c_3} i f^{a_1 a_2 b_3} t^{a_2}_{c_2 c_4} t^{b_3}_{c_6 c_8} ,$$

[Keppeler, Sjodahl (2012)]

6x6 SAD matrix

$$\bar{\mathbf{\Gamma}}^{(1)}_{q\bar{q}\to 4\text{top},1\,1} = -C_F \left(L_{\beta_{34}} + L_{\beta_{56}} + 2\right)$$

$$\bar{\mathbf{\Gamma}}^{(1)}_{q\bar{q}\to 4\text{top},1\,2} = \frac{\sqrt{N_c^2-1}}{2N_c} \left(L_{\beta_{35}} + L_{\beta_{46}} - L_{\beta_{36}} - L_{\beta_{45}}\right)$$

$$\bar{\mathbf{\Gamma}}^{(1)}_{q\bar{q}\to 4\text{top},1\,3} = \frac{\sqrt{N_c^2-1}}{2N_c} \left(T_{15} - T_{16} - T_{25} + T_{26}\right)$$

$$\bar{\mathbf{\Gamma}}^{(1)}_{q\bar{q}\to 4\text{top},1\,4} = \frac{\sqrt{N_c^2-1}}{2N_c} \left(T_{13} - T_{14} - T_{23} + T_{24}\right)$$

$$\bar{\mathbf{\Gamma}}^{(1)}_{q\bar{q}\to 4\text{top},1\,5} = 0$$

$$\bar{\mathbf{\Gamma}}^{(1)}_{q\bar{q}\to 4\text{top},1\,6} = 0$$

$$\bar{\mathbf{\Gamma}}^{(1)}_{q\bar{q}\to 4\text{top},2\,2} = -2C_F + \frac{1}{2N_c} \left(L_{\beta_{34}} + L_{\beta_{56}}\right) - \frac{1}{N_c} \left(L_{\beta_{35}} + L_{\beta_{46}}\right) - \frac{N_c^2-2}{2N_c} \left(L_{\beta_{36}} + L_{\beta_{45}}\right)$$

$$\bar{\mathbf{\Gamma}}^{(1)}_{q\bar{q}\to 4\text{top},2\,3} = \frac{1}{2N_c} \left(T_{13} - T_{14} - T_{23} + T_{24}\right)$$

$$\bar{\mathbf{\Gamma}}^{(1)}_{q\bar{q}\to 4\text{top},2\,4} = \frac{1}{2N_c} \left(T_{15} - T_{16} - T_{25} + T_{26}\right)$$

$$\bar{\mathbf{\Gamma}}^{(1)}_{q\bar{q}\to 4\text{top},2\,5} = \frac{\sqrt{N_c^2-4}}{2\sqrt{2}\,N_c} \left(T_{13} - T_{14} + T_{15} - T_{16} - T_{23} + T_{24} - T_{25} + T_{26}\right)$$

$$\bar{\mathbf{\Gamma}}^{(1)}_{q\bar{q}\to 4\text{top},2\,6} = \frac{1}{2\sqrt{2}} \left(-T_{13} - T_{14} + T_{15} + T_{16} + T_{23} + T_{24} - T_{25} - T_{26}\right)$$

$$\bar{\mathbf{\Gamma}}^{(1)}_{q\bar{q}\to 4\text{top},3\,3} = -\frac{N_c^2-2}{2N_c} - C_F L_{\beta_{34}} + \frac{1}{2N_c} L_{\beta_{56}} + \frac{N_c^2-2}{2N_c} T_{15} + \frac{1}{N_c} T_{16} + \frac{N_c^2-2}{2N_c} T_{26} + \frac{1}{N_c} T_{25}$$

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$$\bar{\mathbf{\Gamma}}^{(1)}_{q\bar{q}\to 4\text{top},4\,4} = -C_F + \frac{1}{2N_c} - C_F L_{\beta_{56}} + \frac{1}{2N_c} L_{\beta_{34}} + \frac{N_c^2-2}{2N_c} \left(T_{13} + T_{24}\right) + \frac{1}{N_c} \left(T_{14} + T_{23}\right)$$

$$\bar{\mathbf{\Gamma}}^{(1)}_{q\bar{q}\to 4\text{top},4\,5} = \frac{\sqrt{N_c^2-4}}{2\sqrt{2}\,N_c} \left(L_{\beta_{35}} - L_{\beta_{36}} - L_{\beta_{45}} + L_{\beta_{46}} + T_{15} - T_{16} - T_{25} + T_{26}\right)$$

$$\bar{\mathbf{\Gamma}}^{(1)}_{q\bar{q}\to 4\text{top},4\,6} = \frac{1}{2\sqrt{2}} \left(L_{\beta_{35}} - L_{\beta_{36}} + L_{\beta_{45}} - L_{\beta_{46}} + T_{15} - T_{16} + T_{25} - T_{26}\right)$$

$$\bar{\mathbf{\Gamma}}^{(1)}_{q\bar{q}\to 4\text{top},5\,5} = -C_F + \frac{1}{2N_c} + \frac{1}{2N_c} \left(L_{\beta_{34}} - 3L_{\beta_{35}} - 3L_{\beta_{46}} + L_{\beta_{56}} - \frac{N_c^2-6}{2} L_{\beta_{36}} - \frac{N_c^2-6}{2} L_{\beta_{45}}\right) + \frac{3}{2N_c} \left(T_{14} + T_{16} + T_{23} + T_{25}\right) + \frac{N_c^2-6}{4N_c} \left(T_{13} + T_{15} + T_{24} + T_{26}\right)$$

$$\bar{\mathbf{\Gamma}}^{(1)}_{q\bar{q}\to 4\text{top},5\,6} = \frac{\sqrt{N_c^2-4}}{4} \left(-L_{\beta_{36}} + L_{\beta_{45}} - T_{13} + T_{15} + T_{24} - T_{26}\right)$$

$$\bar{\mathbf{\Gamma}}^{(1)}_{q\bar{q}\to 4\text{top},6\,6} = -C_F + \frac{1}{2N_c} - \frac{N_c^2-2}{4N_c} \left(L_{\beta_{36}} + L_{\beta_{45}}\right) + \frac{1}{2N_c} \left(L_{\beta_{34}} - L_{\beta_{35}} - L_{\beta_{46}} + L_{\beta_{56}}\right) + \frac{N_c^2-2}{4N_c} \left(T_{13} + T_{15} + T_{24} + T_{26}\right) + \frac{1}{2N_c} \left(T_{14} + T_{16} + T_{23} + T_{25}\right)$$

- $q\bar{q} \to t\bar{t}t\bar{t}:\ \mathbf{3} \otimes \bar{\mathbf{3}} = \mathbf{3} \otimes \bar{\mathbf{3}} \otimes \mathbf{3} \otimes \bar{\mathbf{3}} \longrightarrow \mathbf{1} \oplus \mathbf{8} = \mathbf{0} \oplus (2 \times \mathbf{1}) \oplus (2 \times \mathbf{8}) \oplus \mathbf{8}_S \oplus \mathbf{8}_A \oplus \mathbf{10} \oplus \overline{\mathbf{10}} \oplus \mathbf{27}$

$$c_1^{q\bar{q}} = \frac{1}{\sqrt{N_c^3}} \delta_{c_1 c_3} \delta_{c_2 c_4} \delta_{c_6 c_8}$$

$$c_2^{q\bar{q}} = \frac{1}{T_R \sqrt{N_c(N_c^2-1)}} \delta_{c_1 c_3} t^{a_1}_{c_2 c_4} t^{a_1}_{c_6 c_8}$$

$$c_3^{q\bar{q}} = \frac{1}{T_R \sqrt{N_c(N_c^2-1)}} t^{a_1}_{c_1 c_3} \delta_{c_2 c_4} t^{a_1}_{c_6 c_8}$$

$$c_4^{q\bar{q}} = \frac{1}{T_R \sqrt{N_c(N_c^2-1)}} t^{a_1}_{c_1 c_3} t^{a_1}_{c_2 c_4} \delta_{c_6 c_8}$$

$$c_5^{q\bar{q}} = \frac{\sqrt{N_c}}{T_R^2 \sqrt{2(N_c^4 - 5N_c^2 + 4)}} t^{a_1}_{c_1 c_3} d^{a_1 a_2 b_3} t^{a_2}_{c_2 c_4} t^{b_3}_{c_6 c_8}$$

$$c_6^{q\bar{q}} = \frac{1}{T_R^2 \sqrt{2 N_c (N_c^2-1)}} t^{a_1}_{c_1 c_3} i f^{a_1 a_2 b_3} t^{a_2}_{c_2 c_4} t^{b_3}_{c_6 c_8} \,,$$

[Keppeler, Sjodahl (2012)]

6x6 SAD matrix

$$\bar{\Gamma}^{(1)}_{q\bar{q}\to 4\mathrm{top},1\,1} = -\,C_F\,(L_{\beta_{34}} + L_{\beta_{56}} + 2)$$

$$\bar{\Gamma}^{(1)}_{q\bar{q}\to 4\mathrm{top},1\,2} = \frac{\sqrt{N_c^2-1}}{2N_c}(L_{\beta_{35}} + L_{\beta_{46}} - L_{\beta_{36}} - L_{\beta_{45}})$$

$$\bar{\Gamma}^{(1)}_{q\bar{q}\to 4\mathrm{top},1\,3} = \frac{\sqrt{N_c^2-1}}{2N_c}(T_{15} - T_{16} - T_{25} + T_{26})$$

$$\bar{\Gamma}^{(1)}_{q\bar{q}\to 4\mathrm{top},1\,4} = \frac{\sqrt{N_c^2-1}}{2N_c}(T_{13} - T_{14} - T_{23} + T_{24})$$

$$\bar{\Gamma}^{(1)}_{q\bar{q}\to 4\mathrm{top},1\,5} = 0$$

$$\bar{\Gamma}^{(1)}_{q\bar{q}\to 4\mathrm{top},1\,6} = 0$$

$$\bar{\Gamma}^{(1)}_{q\bar{q}\to 4\mathrm{top},2\,2} = -\,2C_F + \frac{1}{2N_c}(L_{\beta_{34}} + L_{\beta_{56}}) - \frac{1}{N_c}(L_{\beta_{35}} + L_{\beta_{46}}) - \frac{N_c^2-2}{2N_c}(L_{\beta_{36}} + L_{\beta_{45}})$$

$$\bar{\Gamma}^{(1)}_{q\bar{q}\to 4\mathrm{top},2\,3} = \frac{1}{2N_c}(T_{13} - T_{14} - T_{23} + T_{24})$$

$$\bar{\Gamma}^{(1)}_{q\bar{q}\to 4\mathrm{top},2\,4} = \frac{1}{2N_c}(T_{15} - T_{16} - T_{25} + T_{26})$$

$$\bar{\Gamma}^{(1)}_{q\bar{q}\to 4\mathrm{top},2\,5} = \frac{\sqrt{N_c^2-4}}{2\sqrt{2}\,N_c}(T_{13} - T_{14} + T_{15} - T_{16} - T_{23} + T_{24} - T_{25} + T_{26})$$

$$\bar{\Gamma}^{(1)}_{q\bar{q}\to 4\mathrm{top},2\,6} = \frac{1}{2\sqrt{2}}(-T_{13} - T_{14} + T_{15} + T_{16} + T_{23} + T_{24} - T_{25} - T_{26})$$

$$\bar{\Gamma}^{(1)}_{q\bar{q}\to 4\mathrm{top},3\,3} = -\,\frac{N_c^2-2}{2N_c} - C_F L_{\beta_{34}} + \frac{1}{2N_c} L_{\beta_{56}} + \frac{N_c^2-2}{2N_c} T_{15} + \frac{1}{N_c} T_{16} + \frac{N_c^2-2}{2N_c} T_{26} + \frac{1}{N_c} T_{25}$$

$$\bar{\Gamma}^{(1)}_{q\bar{q}\to 4\mathrm{top},3\,4} = \frac{1}{2N_c}(L_{\beta_{35}} + L_{\beta_{46}} - L_{\beta_{36}} - L_{\beta_{45}})$$

$$\bar{\Gamma}^{(1)}_{q\bar{q}\to 4\mathrm{top},3\,5} = \frac{\sqrt{N_c^2-4}}{2\sqrt{2}\,N_c}(L_{\beta_{35}} - L_{\beta_{36}} - L_{\beta_{45}} + L_{\beta_{46}} + T_{13} - T_{14} - T_{23} + T_{24})$$

$$\bar{\Gamma}^{(1)}_{q\bar{q}\to 4\mathrm{top},3\,6} = \frac{1}{2\sqrt{2}}(-L_{\beta_{35}} - L_{\beta_{36}} + L_{\beta_{45}} + L_{\beta_{46}} - T_{13} + T_{14} - T_{23} + T_{24})$$

$$\bar{\Gamma}^{(1)}_{q\bar{q}\to 4\mathrm{top},4\,4} = -\,C_F + \frac{1}{2N_c} - C_F L_{\beta_{56}} + \frac{1}{2N_c} L_{\beta_{34}} + \frac{N_c^2-2}{2N_c}(T_{13} + T_{24}) + \frac{1}{N_c}(T_{14} + T_{23})$$

$$\bar{\Gamma}^{(1)}_{q\bar{q}\to 4\mathrm{top},4\,5} = \frac{\sqrt{N_c^2-4}}{2\sqrt{2}\,N_c}(L_{\beta_{35}} - L_{\beta_{36}} - L_{\beta_{45}} + L_{\beta_{46}} + T_{15} - T_{16} - T_{25} + T_{26})$$

$$\bar{\Gamma}^{(1)}_{q\bar{q}\to 4\mathrm{top},4\,6} = \frac{1}{2\sqrt{2}}(L_{\beta_{35}} - L_{\beta_{36}} + L_{\beta_{45}} - L_{\beta_{46}} + T_{15} - T_{16} + T_{25} - T_{26})$$

$$\bar{\Gamma}^{(1)}_{q\bar{q}\to 4\mathrm{top},5\,5} = -\,C_F + \frac{1}{2N_c} + \frac{1}{2N_c}\left(L_{\beta_{34}} - 3L_{\beta_{35}} - 3L_{\beta_{46}} + L_{\beta_{56}} - \frac{N_c^2-6}{2} L_{\beta_{36}} - \frac{N_c^2-6}{2} L_{\beta_{45}}\right) + \frac{3}{2N_c}(T_{14} + T_{16} + T_{23} + T_{25}) + \frac{N_c^2-6}{4N_c}(T_{13} + T_{15} + T_{24} + T_{26})$$

$$\bar{\Gamma}^{(1)}_{q\bar{q}\to 4\mathrm{top},5\,6} = \frac{\sqrt{N_c^2-4}}{4}(-L_{\beta_{36}} + L_{\beta_{45}} - T_{13} + T_{15} + T_{24} - T_{26})$$

$$\bar{\Gamma}^{(1)}_{q\bar{q}\to 4\mathrm{top},6\,6} = -\,C_F + \frac{1}{2N_c} - \frac{N_c^2-2}{4N_c}(L_{\beta_{36}} + L_{\beta_{45}}) + \frac{1}{2N_c}(L_{\beta_{34}} - L_{\beta_{35}} - L_{\beta_{46}} + L_{\beta_{56}}) + \frac{N_c^2-2}{4N_c}(T_{13} + T_{15} + T_{24} + T_{26}) + \frac{1}{2N_c}(T_{14} + T_{16} + T_{23} + T_{25})$$

DIAGONAL IN THE ABSOLUTE MASS THRESHOLD LIMIT

- $gg \to t\bar{t}t\bar{t}$: $\mathbf{8} \otimes \mathbf{8} = \mathbf{3} \otimes \bar{\mathbf{3}} \otimes \mathbf{3} \otimes \bar{\mathbf{3}} \longrightarrow \mathbf{0} \oplus \mathbf{1} \oplus \mathbf{8}_S \oplus \mathbf{8}_A \oplus \mathbf{10} \oplus \overline{\mathbf{10}} \oplus \mathbf{27} = \boxed{\mathbf{0} \oplus (2 \times \mathbf{1}) \oplus (2 \times \mathbf{8}) \oplus \mathbf{8}_S \oplus \mathbf{8}_A \oplus \mathbf{10} \oplus \overline{\mathbf{10}} \oplus \mathbf{27}}$

$$c_1^{gg} = \frac{1}{T_R}\frac{1}{N_c^2-1} t^{a_1}_{c_2c_4} t^{a_2}_{c_6c_8}\,, \qquad c_2^{gg} = \frac{1}{Nc\sqrt{N_c^2-1}} \delta_{a_1a_2}\delta_{c_2c_4}\delta_{c_6c_8}\,,$$

$$c_3^{gg} = \frac{1}{T_R}\frac{1}{\sqrt{2(N_c^4-5N_c^2+4)}} \delta_{c_2c_4} d_{a_1a_2b_1} t^{b_1}_{c_6c_8}\,, \qquad c_4^{gg} = \frac{1}{T_R}\frac{1}{N_c\sqrt{2(N_c^2-1)}} \delta_{c_2c_4} if_{a_1a_2b_1} t^{b_1}_{c_6c_8}\,,$$

$$c_5^{gg} = \frac{1}{T_R}\frac{1}{\sqrt{2(N_c^4-5N_c^2+4)}} t^{b_1}_{c_2c_4} d_{b_1a_1a_2}\delta_{c_6c_8}\,, \qquad c_6^{gg} = \frac{1}{T_R^2}\frac{N_c}{2(N_c^2-4)\sqrt{N_c^2-1}} t^{b_1}_{c_2c_4} d_{b_1a_1b_2} d_{b_2a_2b_3} t^{b_3}_{c_6c_8}\,,$$

$$c_7^{gg} = \frac{1}{T_R^2}\frac{1}{2\sqrt{N_c^4-5N_c^2+4}} t^{b_1}_{c_2c_4} d_{b_1a_1b_2} if_{b_2a_2b_3} t^{b_3}_{c_6c_8}\,, \qquad c_8^{gg} = \frac{1}{T_R}\frac{1}{N_c\sqrt{2(N_c^2-1)}} t^{b_1}_{c_2c_4} if_{b_1a_1a_2}\delta_{c_6c_8}\,,$$

$$c_9^{gg} = \frac{1}{T_R^2}\frac{1}{2\sqrt{N_c^4-5N_c^2+4}} t^{b_1}_{c_2c_4} if_{b_1a_1b_2} d_{b_2a_2b_3} t^{b_3}_{c_6c_8}\,, \qquad c_{10}^{gg} = \frac{1}{T_R^2}\frac{1}{2N_c\sqrt{N_c^2-1}} t^{b_1}_{c_2c_4} if_{b_1a_1b_2} if_{b_2a_2b_3} t^{b_3}_{c_6c_8}\,,$$

$$c_{11}^{gg} = \frac{1}{T_R}\frac{2}{\sqrt{N_c^4-5N_c^2+4}} t^{b_1}_{c_2c_4} \mathbf{P}^{10}_{a_1b_1a_2b_2} t^{b_2}_{c_6c_8}\,, \qquad c_{12}^{gg} = \frac{1}{T_R}\frac{2}{\sqrt{N_c^4-5N_c^2+4}} t^{b_1}_{c_2c_4} \mathbf{P}^{\overline{10}}_{a_1b_1a_2b_2} t^{b_2}_{c_6c_8}\,,$$

$$c_{13}^{gg} = \frac{1}{T_R}\frac{2}{N_c\sqrt{N_c^2+2N_c-3}} t^{b_1}_{c_2c_4} \mathbf{P}^{27}_{a_1b_1a_2b_2} t^{b_2}_{c_6c_8}\,, \qquad c_{14}^{gg} = \frac{1}{T_R}\frac{2}{N_c\sqrt{N_c^2-2N_c-3}} t^{b_1}_{c_2c_4} \mathbf{P}^{0}_{a_1b_1a_2b_2} t^{b_2}_{c_6c_8}\,.$$

[Keppeler, Sjodahl (2012)]

- $gg \to t\bar{t}t\bar{t}$: $\mathbf{8} \otimes \mathbf{8} = \mathbf{3} \otimes \bar{\mathbf{3}} \otimes \mathbf{3} \otimes \bar{\mathbf{3}} \longrightarrow \mathbf{0} \oplus \mathbf{1} \oplus \mathbf{8}_S \oplus \mathbf{8}_A \oplus \mathbf{10} \oplus \overline{\mathbf{10}} \oplus \mathbf{27} = \boxed{\mathbf{0} \oplus (2 \times \mathbf{1}) \oplus (2 \times \mathbf{8}) \oplus \mathbf{8}_S \oplus \mathbf{8}_A \oplus \mathbf{10} \oplus \overline{\mathbf{10}} \oplus \mathbf{27}}$

14x14 SAD matrix

$$\tilde{\Gamma}^{(1)}_{gg\to 4\text{top},1\,1} = \frac{1}{N_c} + \frac{1}{2N_c}\left(L_{\beta_{34}} + L_{\beta34}\right) + \frac{N_c}{2}\left(T_{13} + T_{14} + T_{25} + T_{26}\right)$$

$$\tilde{\Gamma}^{(1)}_{gg\to 4\text{top},1\,2} = \frac{1}{2N_c\sqrt{N_c^2-1}}\left(L_{\beta35} - L_{\beta_{36}} - L_{\beta45} + L_{\beta46}\right)$$

$$\tilde{\Gamma}^{(1)}_{gg\to 4\text{top},1\,3} = \frac{1}{4N_c}\sqrt{\frac{2(N_c^2-4)}{N_c^2-1}}\left(L_{\beta35} - L_{\beta_{36}} - L_{\beta45} + L_{\beta46}\right)$$

$$\tilde{\Gamma}^{(1)}_{gg\to 4\text{top},1\,4} = \frac{1}{2\sqrt{2(N_c^2-1)}}\left(L_{\beta_{35}} + L_{\beta_{36}} - L_{\beta_{45}} - L_{\beta_{46}} + 2T_{23} - 2T_{24}\right)$$

$$\tilde{\Gamma}^{(1)}_{gg\to 4\text{top},1\,5} = \frac{1}{4N_c}\sqrt{\frac{2(N_c^2-4)}{N_c^2-1}}\left(L_{\beta35} - L_{\beta_{36}} - L_{\beta45} + L_{\beta46}\right)$$

$$\tilde{\Gamma}^{(1)}_{gg\to 4\text{top},1\,6} = \frac{N_c^2-4}{4N_c\sqrt{N_c^2-1}}\left(L_{\beta35} - L_{\beta_{36}} - L_{\beta45} + L_{\beta46}\right)$$

$$\tilde{\Gamma}^{(1)}_{gg\to 4\text{top},1\,7} = \frac{1}{4}\sqrt{\frac{N_c^2-4}{N_c^2-1}}\left(L_{\beta_{35}} + L_{\beta_{36}} - L_{\beta_{45}} - L_{\beta_{46}} + 2T_{23} - 2T_{24}\right)$$

$$\tilde{\Gamma}^{(1)}_{gg\to 4\text{top},1\,8} = \frac{1}{2\sqrt{2(N_c^2-1)}}\left(-L_{\beta_{35}} + L_{\beta_{36}} - L_{\beta_{45}} + L_{\beta_{46}} - 2T_{15} + 2T_{16}\right)$$

$$\tilde{\Gamma}^{(1)}_{gg\to 4\text{top},1\,9} = -\frac{1}{4}\sqrt{\frac{N_c^2-4}{N_c^2-1}}\left(L_{\beta_{35}} - L_{\beta_{36}} + L_{\beta_{45}} - L_{\beta_{46}} + 2T_{15} - 2T_{16}\right)$$

$$\tilde{\Gamma}^{(1)}_{gg\to 4\text{top},1\,10} = -\frac{N_c}{4\sqrt{N_c^2-1}}\Big(4 + L_{\beta_{35}} + L_{\beta_{36}} + L_{\beta_{45}} + L_{\beta_{46}} + 2T_{15} + 2T_{16} + 2T_{23} + 2T_{24}\Big)$$

$$\tilde{\Gamma}^{(1)}_{gg\to 4\text{top},1\,11} = 0$$

$$\tilde{\Gamma}^{(1)}_{gg\to 4\text{top},1\,12} = 0$$

$$\tilde{\Gamma}^{(1)}_{gg\to 4\text{top},1\,13} = 0$$

$$\tilde{\Gamma}^{(1)}_{gg\to 4\text{top},1\,14} = 0$$

$$\tilde{\Gamma}^{(1)}_{gg\to 4\text{top},2\,2} = -2C_F - C_F\left(L_{\beta_{34}} + L_{\beta_{56}}\right)$$

$$\tilde{\Gamma}^{(1)}_{gg\to 4\text{top},2\,3} = 0$$

$$\tilde{\Gamma}^{(1)}_{gg\to 4\text{top},2\,4} = \frac{1}{\sqrt{2}}\left(T_{15} - T_{16} - T_{25} + T_{26}\right)$$

$$\tilde{\Gamma}^{(1)}_{gg\to 4\text{top},2\,5} = 0$$

$$\tilde{\Gamma}^{(1)}_{gg\to 4\text{top},2\,6} = \frac{1}{2N_c}\left(L_{\beta_{35}} - L_{\beta_{36}} - L_{\beta_{45}} + L_{\beta_{46}}\right)$$

$$\tilde{\Gamma}^{(1)}_{gg\to 4\text{top},2\,7} = 0$$

$$\tilde{\Gamma}^{(1)}_{gg\to 4\text{top},2\,8} = \frac{1}{\sqrt{2}}\left(T_{13} - T_{14} - T_{23} + T_{24}\right)$$

$$\tilde{\Gamma}^{(1)}_{gg\to 4\text{top},2\,9} = 0$$

$$\tilde{\Gamma}^{(1)}_{gg\to 4\text{top},2\,10} = \frac{1}{2N_c}\left(L_{\beta_{35}} - L_{\beta_{36}} - L_{\beta_{45}} + L_{\beta_{46}}\right)$$

$$\tilde{\Gamma}^{(1)}_{gg\to 4\text{top},2\,11} = \frac{\sqrt{N_c^2-4}}{4N_c}\left(L_{\beta_{35}} - L_{\beta_{36}} - L_{\beta_{45}} + L_{\beta_{46}}\right)$$

$$\tilde{\Gamma}^{(1)}_{gg\to 4\text{top},2\,12} = \frac{\sqrt{N_c^2-4}}{4N_c}\left(L_{\beta_{35}} - L_{\beta_{36}} - L_{\beta_{45}} + L_{\beta_{46}}\right)$$

$$\tilde{\Gamma}^{(1)}_{gg\to 4\text{top},2\,13} = \frac{1}{4}\sqrt{\frac{N_c+3}{N_c+1}}\left(L_{\beta_{35}} - L_{\beta_{36}} - L_{\beta_{45}} + L_{\beta_{46}}\right)$$

$$\tilde{\Gamma}^{(1)}_{gg\to 4\text{top},2\,14} = 0$$

$$\tilde{\Gamma}^{(1)}_{gg\to 4\text{top},3\,3} = -\frac{N_c^2-2}{2N_c} - C_F L_{\beta_{34}} + \frac{1}{2N_c}L_{\beta_{56}} + \frac{N_c}{4}\left(T_{15} + T_{16} + T_{25} + T_{26}\right)$$

$$\tilde{\Gamma}^{(1)}_{gg\to 4\text{top},3\,4} = \frac{\sqrt{N_c^2-4}}{4}\left(T_{15} - T_{16} - T_{25} + T_{26}\right)$$

$$\tilde{\Gamma}^{(1)}_{gg\to 4\text{top},3\,5} = \frac{1}{2N_c}\left(L_{\beta_{35}} - L_{\beta_{36}} - L_{\beta_{45}} + L_{\beta_{46}}\right)$$

$$\tilde{\Gamma}^{(1)}_{gg\to 4\text{top},3\,6} = \frac{N_c^2-12}{4N_c\sqrt{2(N_c^2-4)}}\left(L_{\beta_{35}} - L_{\beta_{36}} - L_{\beta_{45}} + L_{\beta_{46}}\right)$$

$$\tilde{\Gamma}^{(1)}_{gg\to 4\text{top},3\,7} = \frac{1}{4\sqrt{2}}\left(L_{\beta_{35}} + L_{\beta_{36}} - L_{\beta_{45}} - L_{\beta_{46}} + 2T_{23} - 2T_{24}\right)$$

$$\tilde{\Gamma}^{(1)}_{gg\to 4\text{top},3\,8} = 0$$

$$\tilde{\Gamma}^{(1)}_{gg\to 4\text{top},3\,9} = \frac{1}{4\sqrt{2}}\left(L_{\beta_{35}} + L_{\beta_{36}} - L_{\beta_{45}} - L_{\beta_{46}} + 4T_{13} - 4T_{14} - 2T_{23} + 2T_{24}\right)$$

$$\tilde{\Gamma}^{(1)}_{gg\to 4\text{top},3\,10} = \frac{\sqrt{2(N_c^2-4)}}{8N_c}\left(L_{\beta_{35}} - L_{\beta_{36}} - L_{\beta_{45}} + L_{\beta_{46}}\right)$$

$$\tilde{\Gamma}^{(1)}_{gg\to 4\text{top},3\,11} = \frac{N_c-2}{4N_c\sqrt{2}}\left(L_{\beta_{35}} - L_{\beta_{45}}\right) + \frac{N_c+2}{4N_c\sqrt{2}}\left(L_{\beta_{36}} - L_{\beta_{46}}\right) + \frac{1}{2\sqrt{2}}\left(T_{23} - T_{24}\right)$$

$$\tilde{\Gamma}^{(1)}_{gg\to 4\text{top},3\,12} = \frac{N_c-2}{4N_c\sqrt{2}}\left(L_{\beta_{46}} - L_{\beta_{36}}\right) + \frac{N_c+2}{4N_c\sqrt{2}}\left(L_{\beta_{45}} - L_{\beta_{35}}\right) + \frac{1}{2\sqrt{2}}\left(T_{24} - T_{23}\right)$$

$$\tilde{\Gamma}^{(1)}_{gg\to 4\text{top},3\,13} = \frac{1}{4\sqrt{2}}\sqrt{\frac{(N_c-2)(N_c+3)}{(N_c+1)(N_c+2)}}\left(L_{\beta_{35}} - L_{\beta_{36}} - L_{\beta_{45}} + L_{\beta_{46}}\right)$$

$$\tilde{\Gamma}^{(1)}_{gg\to 4\text{top},3\,14} = 0$$

$$\tilde{\Gamma}^{(1)}_{gg\to 4\text{top},4\,4} = -\frac{N_c^2-2}{2N_c} - C_F L_{\beta_{34}} + \frac{1}{2N_c}L_{\beta_{56}} + \frac{N_c}{4}\left(T_{15} + T_{16} + T_{25} + T_{26}\right)$$

+ 65 more components!

COLOUR STRUCTURE

- $gg \to t\bar{t}t\bar{t}: \ \mathbf{8} \otimes \mathbf{8} = \mathbf{3} \otimes \bar{\mathbf{3}} \otimes \mathbf{3} \otimes \bar{\mathbf{3}} \longrightarrow \mathbf{0} \oplus \mathbf{1} \oplus \mathbf{8}_S \oplus \mathbf{8}_A \oplus \mathbf{10} \oplus \overline{\mathbf{10}} \oplus \mathbf{27} = \mathbf{0} \oplus (2 \times \mathbf{1}) \oplus (2 \times \mathbf{8}) \oplus \mathbf{8}_S \oplus \mathbf{8}_A \oplus \mathbf{10} \oplus \overline{\mathbf{10}} \oplus \mathbf{27}$

14x14 SAD matrix

$$\tilde{\Gamma}^{(1)}_{gg\to 4\mathrm{top},1\,1} = \frac{1}{N_c} + \frac{1}{2N_c}\left(L_{\beta_{34}} + L_{\beta 34}\right) + \frac{N_c}{2}\left(T_{13} + T_{14} + T_{25} + T_{26}\right)$$

$$\tilde{\Gamma}^{(1)}_{gg\to 4\mathrm{top},1\,2} = \frac{1}{2N_c\sqrt{N_c^2-1}}\left(L_{\beta 35} - L_{\beta_{36}} - L_{\beta 45} + L_{\beta 46}\right)$$

$$\tilde{\Gamma}^{(1)}_{gg\to 4\mathrm{top},1\,3} = \frac{1}{4N_c}\sqrt{\frac{2(N_c^2-4)}{N_c^2-1}}\left(L_{\beta 35} - L_{\beta_{36}} - L_{\beta 45} + L_{\beta 46}\right)$$

$$\tilde{\Gamma}^{(1)}_{gg\to 4\mathrm{top},1\,4} = \frac{1}{2\sqrt{2(N_c^2-1)}}\left(L_{\beta_{35}} + L_{\beta_{36}} - L_{\beta_{45}} - L_{\beta_{46}} + 2T_{23} - 2T_{24}\right)$$

$$\tilde{\Gamma}^{(1)}_{gg\to 4\mathrm{top},1\,5} = \frac{1}{4N_c}\sqrt{\frac{2(N_c^2-4)}{N_c^2-1}}\left(L_{\beta 35} - L_{\beta_{36}} - L_{\beta 45} + L_{\beta 46}\right)$$

$$\tilde{\Gamma}^{(1)}_{gg\to 4\mathrm{top},1\,6} = \frac{N_c^2-4}{4N_c\sqrt{N_c^2-1}}\left(L_{\beta 35} - L_{\beta_{36}} - L_{\beta 45} + L_{\beta 46}\right)$$

$$\tilde{\Gamma}^{(1)}_{gg\to 4\mathrm{top},1\,7} = \frac{1}{4}\sqrt{\frac{N_c^2-4}{N_c^2-1}}\left(L_{\beta_{35}} + L_{\beta_{36}} - L_{\beta_{45}} - L_{\beta_{46}} + 2T_{23} - 2T_{24}\right)$$

$$\tilde{\Gamma}^{(1)}_{gg\to 4\mathrm{top},1\,8} = \frac{1}{2\sqrt{2(N_c^2-1)}}\left(-L_{\beta_{35}} + L_{\beta_{36}} - L_{\beta_{45}} + L_{\beta_{46}} - 2T_{15} + 2T_{16}\right)$$

$$\tilde{\Gamma}^{(1)}_{gg\to 4\mathrm{top},1\,9} = -\frac{1}{4}\sqrt{\frac{N_c^2-4}{N_c^2-1}}\left(L_{\beta_{35}} - L_{\beta_{36}} + L_{\beta_{45}} - L_{\beta_{46}} + 2T_{15} - 2T_{16}\right)$$

$$\tilde{\Gamma}^{(1)}_{gg\to 4\mathrm{top},1\,10} = -\frac{N_c}{4\sqrt{N_c^2-1}}\left(4 + L_{\beta_{35}} + L_{\beta_{36}} + L_{\beta_{45}} + L_{\beta_{46}} + 2T_{15} + 2T_{16} + 2T_{23} + 2T_{24}\right)$$

$$\tilde{\Gamma}^{(1)}_{gg\to 4\mathrm{top},1\,11} = 0$$

$$\tilde{\Gamma}^{(1)}_{gg\to 4\mathrm{top},1\,12} = 0$$

$$\tilde{\Gamma}^{(1)}_{gg\to 4\mathrm{top},1\,13} = 0$$

$$\tilde{\Gamma}^{(1)}_{gg\to 4\mathrm{top},1\,14} = 0$$

$$\tilde{\Gamma}^{(1)}_{gg\to 4\mathrm{top},2\,2} = -2C_F - C_F\left(L \ldots L_{\beta_{56}}\right)$$

$$\tilde{\Gamma}^{(1)}_{gg\to 4\mathrm{top},2\,3} = 0$$

$$\tilde{\Gamma}^{(1)}_{gg\to 4\mathrm{top},2\,4} = \frac{1}{\sqrt{2}}\left(T_1 \ldots - T_{25} + T_{26}\right)$$

$$\tilde{\Gamma}^{(1)}_{gg\to 4\mathrm{top},2\,5} = 0$$

$$\tilde{\Gamma}^{(1)}_{gg\to 4\mathrm{top},2\,6} = \frac{1}{2N_c}\left(L_{\beta_{35}} \ldots L_{\beta_{45}} + L_{\beta_{46}}\right)$$

$$\tilde{\Gamma}^{(1)}_{gg\to 4\mathrm{top},2\,7} = 0$$

$$\tilde{\Gamma}^{(1)}_{gg\to 4\mathrm{top},2\,8} = \frac{1}{\sqrt{2}}\left(T_{1} \ldots T_{14} \ldots T_{24}\right)$$

$$\tilde{\Gamma}^{(1)}_{gg\to 4\mathrm{top},2\,9} = 0$$

$$\tilde{\Gamma}^{(1)}_{gg\to 4\mathrm{top},2\,10} = \frac{1}{2N_c}\left(L \ldots L_{\beta_{45}} + L_{\beta_{46}}\right)$$

$$\tilde{\Gamma}^{(1)}_{gg\to 4\mathrm{top},2\,11} = \frac{\sqrt{N_c^2-4}}{4N_c}\left(L_{\beta_{35}} - L_{\beta_{36}} - L_{\beta_{45}} + L_{\beta_{46}}\right)$$

$$\tilde{\Gamma}^{(1)}_{gg\to 4\mathrm{top},2\,12} = \frac{\sqrt{N_c^2-4}}{4N_c}\left(L_{\beta_{35}} - L_{\beta_{36}} - L_{\beta_{45}} + L_{\beta_{46}}\right)$$

$$\tilde{\Gamma}^{(1)}_{gg\to 4\mathrm{top},2\,13} = \frac{1}{4}\sqrt{\frac{N_c+3}{N_c+1}}\left(L_{\beta_{35}} - L_{\beta_{36}} - L_{\beta_{45}} + L_{\beta_{46}}\right)$$

$$\tilde{\Gamma}^{(1)}_{gg\to 4\mathrm{top},2\,14} = 0$$

NOT DIAGONAL IN THE ABSOLUTE MASS THRESHOLD LIMIT

$$\tilde{\Gamma}^{(1)}_{gg\to 4\mathrm{top},3\,3} = -\frac{N_c^2-2}{2N_c} - C_F L_{\beta_{34}} + \frac{1}{2N_c}L_{\beta_{56}} + \frac{N_c}{4}\left(T_{15} + T_{16} + T_{25} + T_{26}\right)$$

$$\tilde{\Gamma}^{(1)}_{gg\to 4\mathrm{top},3\,4} = \frac{\sqrt{N_c^2-4}}{4}\left(T_{15} - T_{16} - T_{25} + T_{26}\right)$$

$$\tilde{\Gamma}^{(1)}_{gg\to 4\mathrm{top},3\,5} = \frac{1}{2N_c}\left(L_{\beta_{35}} - L_{\beta_{36}} - L_{\beta_{45}} + L_{\beta_{46}}\right)$$

$$\tilde{\Gamma}^{(1)}_{gg\to 4\mathrm{top},3\,6} = \frac{N_c^2-12}{4N_c\sqrt{2(N_c^2-4)}}\left(L_{\beta_{35}} - L_{\beta_{36}} - L_{\beta_{45}} + L_{\beta_{46}}\right)$$

$$\tilde{\Gamma}^{(1)}_{gg\to 4\mathrm{top},3\,7} = \frac{1}{4\sqrt{2}}\left(L_{\beta_{35}} + L_{\beta_{36}} - L_{\beta_{45}} - L_{\beta_{46}} + 2T_{23} - 2T_{24}\right)$$

$$\tilde{\Gamma}^{(1)}_{gg\to 4\mathrm{top},3\,8} = 0$$

$$\tilde{\Gamma}^{(1)}_{gg\to 4\mathrm{top},3\,9} = \frac{1}{4\sqrt{2}}\left(L_{\beta_{35}} + L_{\beta_{36}} - L_{\beta_{45}} - L_{\beta_{46}} + 4T_{13} - 4T_{14} - 2T_{23} + 2T_{24}\right)$$

$$\tilde{\Gamma}^{(1)}_{gg\to 4\mathrm{top},3\,10} = \frac{\sqrt{2(N_c^2-4)}}{8N_c}\left(L_{\beta_{35}} - L_{\beta_{36}} - L_{\beta_{45}} + L_{\beta_{46}}\right)$$

$$\tilde{\Gamma}^{(1)}_{gg\to 4\mathrm{top},3\,11} = \frac{N_c-2}{4N_c\sqrt{2}}\left(L_{\beta_{35}} - L_{\beta_{45}}\right) + \frac{N_c+2}{4N_c\sqrt{2}}\left(L_{\beta_{36}} - L_{\beta_{46}}\right) + \frac{1}{2\sqrt{2}}\left(T_{23} - T_{24}\right)$$

$$\tilde{\Gamma}^{(1)}_{gg\to 4\mathrm{top},3\,12} = \frac{N_c-2}{4N_c\sqrt{2}}\left(L_{\beta_{46}} - L_{\beta_{36}}\right) + \frac{N_c+2}{4N_c\sqrt{2}}\left(L_{\beta_{45}} - L_{\beta_{35}}\right) + \frac{1}{2\sqrt{2}}\left(T_{24} - T_{23}\right)$$

$$\tilde{\Gamma}^{(1)}_{gg\to 4\mathrm{top},3\,13} = \frac{1}{4\sqrt{2}}\sqrt{\frac{(N_c-2)(N_c+3)}{(N_c+1)(N_c+2)}}\left(L_{\beta_{35}} - L_{\beta_{36}} - L_{\beta_{45}} + L_{\beta_{46}}\right)$$

$$\tilde{\Gamma}^{(1)}_{gg\to 4\mathrm{top},3\,14} = 0$$

$$\tilde{\Gamma}^{(1)}_{gg\to 4\mathrm{top},4\,4} = -\frac{N_c^2-2}{2N_c} - C_F L_{\beta_{34}} + \frac{1}{2N_c}L_{\beta_{56}} + \frac{N_c}{4}\left(T_{15} + T_{16} + T_{25} + T_{26}\right)$$

+ 65 more components!

- $gg \to t\bar{t}t\bar{t}$: $\mathbf{8} \otimes \mathbf{8} = \mathbf{3} \otimes \bar{\mathbf{3}} \otimes \mathbf{3} \otimes \bar{\mathbf{3}} \longrightarrow \mathbf{0} \oplus \mathbf{1} \oplus \mathbf{8}_S \oplus \mathbf{8}_A \oplus \mathbf{10} \oplus \overline{\mathbf{10}} \oplus \mathbf{27} = \mathbf{0} \oplus (2 \times \mathbf{1}) \oplus (2 \times \mathbf{8}) \oplus \mathbf{8}_S \oplus \mathbf{8}_A \oplus \mathbf{10} \oplus \overline{\mathbf{10}} \oplus \mathbf{27}$

Solution:

$$
\begin{aligned}
\bar{c}_1^{gg} &= \frac{3\sqrt{3}}{8} c_1^{gg} + \frac{3}{10}\sqrt{\frac{3}{2}} c_6^{gg} - \frac{1}{2}\sqrt{\frac{3}{2}} c_{10}^{gg} - \frac{1}{4}\sqrt{\frac{3}{10}} c_{11}^{gg} - \frac{1}{4}\sqrt{\frac{3}{10}} c_{12}^{gg} + \frac{7}{40} c_{13}^{gg} \,, \\
\bar{c}_2^{gg} &= -\frac{\sqrt{5}}{4} c_1^{gg} + \sqrt{\frac{2}{5}} c_6^{gg} - \frac{1}{2\sqrt{2}} c_{11}^{gg} - \frac{1}{2\sqrt{2}} c_{12}^{gg} + \frac{1}{4}\sqrt{\frac{3}{5}} c_{13}^{gg} \,, \\
\bar{c}_3^{gg} &= -\frac{1}{\sqrt{2}} c_7^{gg} + \frac{1}{\sqrt{2}} c_9^{gg} \,, \\
\bar{c}_5^{gg} &= -\frac{1}{2\sqrt{2}} c_1^{gg} - \frac{1}{2} c_6^{gg} - \frac{1}{2} c_{10}^{gg} + \frac{1}{2}\sqrt{\frac{3}{2}} c_{13}^{gg} \,, \\
\bar{c}_6^{gg} &= -\frac{1}{2}\sqrt{\frac{5}{14}} c_1^{gg} + \frac{3}{2\sqrt{35}} c_6^{gg} - \frac{1}{2}\sqrt{\frac{5}{7}} c_{10}^{gg} + \frac{2}{\sqrt{7}} c_{12}^{gg} - \frac{3}{2}\sqrt{\frac{3}{70}} c_{13}^{gg} \,, \\
\bar{c}_7^{gg} &= -\frac{1}{2\sqrt{7}} c_1^{gg} + \frac{3}{5\sqrt{14}} c_6^{gg} - \frac{1}{\sqrt{14}} c_{10}^{gg} + \sqrt{\frac{7}{10}} c_{11}^{gg} - \frac{3}{\sqrt{70}} c_{12}^{gg} - \frac{3}{10}\sqrt{\frac{3}{7}} c_{13}^{gg} \,, \\
\bar{c}_8^{gg} &= \frac{1}{\sqrt{2}} c_7^{gg} + \frac{1}{\sqrt{2}} c_9^{gg} \,, \\
\bar{c}_{13}^{gg} &= \frac{1}{8} c_1^{gg} + \frac{1}{2\sqrt{2}} c_6^{gg} + \frac{1}{2\sqrt{2}} c_{10}^{gg} + \frac{1}{4}\sqrt{\frac{5}{2}} c_{11}^{gg} + \frac{1}{4}\sqrt{\frac{5}{2}} c_{12}^{gg} + \frac{3\sqrt{3}}{8} c_{13}^{gg} \,, \\
\bar{c}_4^{gg} &= c_{14}^{gg} \,, \quad \bar{c}_9^{gg} = c_8^{gg} \,, \quad \bar{c}_{10}^{gg} = c_5^{gg} \,, \quad \bar{c}_{11}^{gg} = c_4^{gg} \,, \quad \bar{c}_{12}^{gg} = c_3^{gg} \,, \quad \bar{c}_{14}^{gg} = c_2^{gg} \,.
\end{aligned}
$$

SAD MATRIX

- One-loop SAD matrix needed at NLL

$$\mathbf{U}_{ij\to t\bar{t}t\bar{t}}(N, M^2, \mu_F^2, \mu_R^2) = \mathrm{Pexp}\left[\int_{\mu}^{M/\bar{N}} \frac{dq}{q}\, \mathbf{\Gamma}_{ij\to t\bar{t}t\bar{t}}\left(\alpha_s(q^2)\right)\right]$$

$$\mathbf{\Gamma}_{ij\to t\bar{t}t\bar{t}}(\alpha_s) = \left(\frac{\alpha_s}{\pi}\right)\mathbf{\Gamma}^{(1)}_{ij\to t\bar{t}t\bar{t}} + \left(\frac{\alpha_s}{\pi}\right)^2 \mathbf{\Gamma}^{(2)}_{ij\to t\bar{t}t\bar{t}} + \ldots$$

Diagonal SAD matrix

$$\bar{\mathbf{U}}_R\, \tilde{\mathbf{S}}_R\, \mathbf{U}_R = \tilde{\mathbf{S}}_R \exp\left[\frac{2\,\mathrm{Re}\left(\mathbf{\Gamma}_R^{(1)}\right)}{2\pi b_0}\ln(1-2\lambda)\right]$$

with $\lambda = \alpha_s b_0 \ln(\bar{N})$

- $q\bar{q} \to t\bar{t}t\bar{t}:\ \mathbf{3}\otimes\bar{\mathbf{3}} = \mathbf{3}\otimes\bar{\mathbf{3}}\otimes\mathbf{3}\otimes\bar{\mathbf{3}} \to \mathbf{1}\oplus\mathbf{8} = \mathbf{0}\oplus(2\times\mathbf{1})\oplus(2\times\mathbf{8})\oplus\mathbf{8}_S\oplus\mathbf{8}_A\oplus\mathbf{10}\oplus\overline{\mathbf{10}}\oplus\mathbf{27}$

- $gg \to t\bar{t}t\bar{t}:\ \mathbf{8}\otimes\mathbf{8} = \mathbf{3}\otimes\bar{\mathbf{3}}\otimes\mathbf{3}\otimes\bar{\mathbf{3}} \to \mathbf{0}\oplus\mathbf{1}\oplus\mathbf{8}_S\oplus\mathbf{8}_A\oplus\mathbf{10}\oplus\overline{\mathbf{10}}\oplus\mathbf{27} = \mathbf{0}\oplus(2\times\mathbf{1})\oplus(2\times\mathbf{8})\oplus\mathbf{8}_S\oplus\mathbf{8}_A\oplus\mathbf{10}\oplus\overline{\mathbf{10}}\oplus\mathbf{27}$

- One-loop SAD matrix needed at NLL

$$\mathbf{U}_{ij\to t\bar{t}t\bar{t}}(N, M^2, \mu_F^2, \mu_R^2) = \text{Pexp}\left[\int_{\mu}^{M/\bar{N}} \frac{dq}{q}\,\mathbf{\Gamma}_{ij\to t\bar{t}t\bar{t}}\left(\alpha_s(q^2)\right)\right]$$

$$\mathbf{\Gamma}_{ij\to t\bar{t}t\bar{t}}\left(\alpha_s\right) = \left(\frac{\alpha_s}{\pi}\right)\mathbf{\Gamma}^{(1)}_{ij\to t\bar{t}t\bar{t}} + \left(\frac{\alpha_s}{\pi}\right)^2 \mathbf{\Gamma}^{(2)}_{ij\to t\bar{t}t\bar{t}} + \ldots$$

Diagonal SAD matrix

$$\bar{\mathbf{U}}_R\,\tilde{\mathbf{S}}_R\,\mathbf{U}_R = \tilde{\mathbf{S}}_R\,\exp\left[\frac{2\,\text{Re}\left(\mathbf{\Gamma}^{(1)}_R\right)}{2\pi b_0}\ln(1-2\lambda)\right]$$

with $\lambda = \alpha_s b_0 \ln(\bar{N})$

- $q\bar{q} \to t\bar{t}t\bar{t}:\ \mathbf{3}\otimes\bar{\mathbf{3}} = \mathbf{3}\otimes\bar{\mathbf{3}}\otimes\mathbf{3}\otimes\bar{\mathbf{3}} \to \mathbf{1}\oplus\mathbf{8} = \mathbf{0}\oplus(2\times\mathbf{1})\oplus(2\times\mathbf{8})\oplus\mathbf{8}_S\oplus\mathbf{8}_A\oplus\mathbf{10}\oplus\overline{\mathbf{10}}\oplus\mathbf{27}$

$$2\text{Re}\left[\mathbf{\Gamma}^{(1)}_{R,q\bar{q}\to t\bar{t}t\bar{t}}\right] = \text{diag}\Big(0, 0, -3, -3, -3, -3\Big)$$

- $gg \to t\bar{t}t\bar{t}:\ \mathbf{8}\otimes\mathbf{8} = \mathbf{3}\otimes\bar{\mathbf{3}}\otimes\mathbf{3}\otimes\bar{\mathbf{3}} \to \mathbf{0}\oplus\mathbf{1}\oplus\mathbf{8}_S\oplus\mathbf{8}_A\oplus\mathbf{10}\oplus\overline{\mathbf{10}}\oplus\mathbf{27} = \mathbf{0}\oplus(2\times\mathbf{1})\oplus(2\times\mathbf{8})\oplus\mathbf{8}_S\oplus\mathbf{8}_A\oplus\mathbf{10}\oplus\overline{\mathbf{10}}\oplus\mathbf{27}$

$$2\text{Re}\left[\mathbf{\Gamma}^{(1)}_{R,gg\to t\bar{t}t\bar{t}}\right] = \text{diag}\Big(-8, -6, -6, -4, -3, -3, -3, -3, -3, -3, -3, -3, 0, 0\Big)$$

- Quadratic Casimir Invariants $N_c = 3$

$$
\begin{aligned}
C_2(\mathbf{1}) &= 0\\
C_2(\mathbf{8}_{(S/A)}) &= 3\\
C_2(\mathbf{10},\overline{\mathbf{10}}) &= 6\\
C_2(\mathbf{27}) &= 8\\
C_2(\mathbf{0}) &= 4
\end{aligned}
$$

- $q\bar{q} \to t\bar{t}t\bar{t}$: $\mathbf{3}\otimes\bar{\mathbf{3}} = \mathbf{3}\otimes\bar{\mathbf{3}}\otimes\mathbf{3}\otimes\bar{\mathbf{3}} \longrightarrow \mathbf{1}\oplus\mathbf{8} = \mathbf{0}\oplus \boxed{(2\times\mathbf{1})\oplus(2\times\mathbf{8})\oplus\mathbf{8}_S\oplus\mathbf{8}_A}\oplus\mathbf{10}\oplus\overline{\mathbf{10}}\oplus\mathbf{27}$

$$2\mathrm{Re}\left[\mathbf{\Gamma}^{(1)}_{R,q\bar{q}\to t\bar{t}t\bar{t}}\right] = \mathrm{diag}\Big(0,0,-3,-3,-3,-3\Big)$$

- $gg \to t\bar{t}t\bar{t}$: $\mathbf{8}\otimes\mathbf{8} = \mathbf{3}\otimes\bar{\mathbf{3}}\otimes\mathbf{3}\otimes\bar{\mathbf{3}} \longrightarrow \mathbf{0}\oplus\mathbf{1}\oplus\mathbf{8}_S\oplus\mathbf{8}_A\oplus\mathbf{10}\oplus\overline{\mathbf{10}}\oplus\mathbf{27} = \boxed{\mathbf{0}\oplus(2\times\mathbf{1})\oplus(2\times\mathbf{8})\oplus\mathbf{8}_S\oplus\mathbf{8}_A\oplus\mathbf{10}\oplus\overline{\mathbf{10}}\oplus\mathbf{27}}$

$$2\mathrm{Re}\left[\mathbf{\Gamma}^{(1)}_{R,gg\to t\bar{t}t\bar{t}}\right] = \mathrm{diag}\Big(-8,-6,-6,-4,-3,-3,-3,-3,-3,-3,-3,-3,0,0\Big)$$

RESUMMATION: NLL and NLL’

$$\mathbf{H} = \mathbf{H}^{(0)} + \frac{\alpha_s(\mu_R)}{\pi}\mathbf{H}^{(1)} + \dots$$

$$\tilde{\mathbf{S}} = \tilde{\mathbf{S}}^{(0)} + \frac{\alpha_s(\mu_R)}{\pi}\tilde{\mathbf{S}}^{(1)} + \dots$$

- **NLL** accuracy: exponential functions at NLL together with

$$\mathrm{Tr}\left[\mathbf{H}\tilde{\mathbf{S}}\right] = \mathrm{Tr}\left[\mathbf{H}^{(0)}\tilde{\mathbf{S}}^{(0)}\right]$$

- **NLL’** accuracy: exponential functions at NLL together with

$$\mathrm{Tr}\left[\mathbf{H}\tilde{\mathbf{S}}\right] = \mathrm{Tr}\left[\mathbf{H}^{(0)}\tilde{\mathbf{S}}^{(0)} + \frac{\alpha_s(\mu_R)}{\pi}\mathbf{H}^{(1)}\tilde{\mathbf{S}}^{(0)} + \frac{\alpha_s(\mu_R)}{\pi}\mathbf{H}^{(0)}\tilde{\mathbf{S}}^{(1)}\right]$$

$$\sigma^{\mathrm{NLL}(')}_{t\bar{t}t\bar{t}}(\tau) = \int_{\mathcal{C}} \frac{\mathrm{d}N}{2\pi i}\, \tau^{-N} f_i(N+1,\mu_F^2)\, f_j(N+1,\mu_F^2)\, \hat{\sigma}^{\mathrm{res}}_{ij\to t\bar{t}t\bar{t}}(N)$$

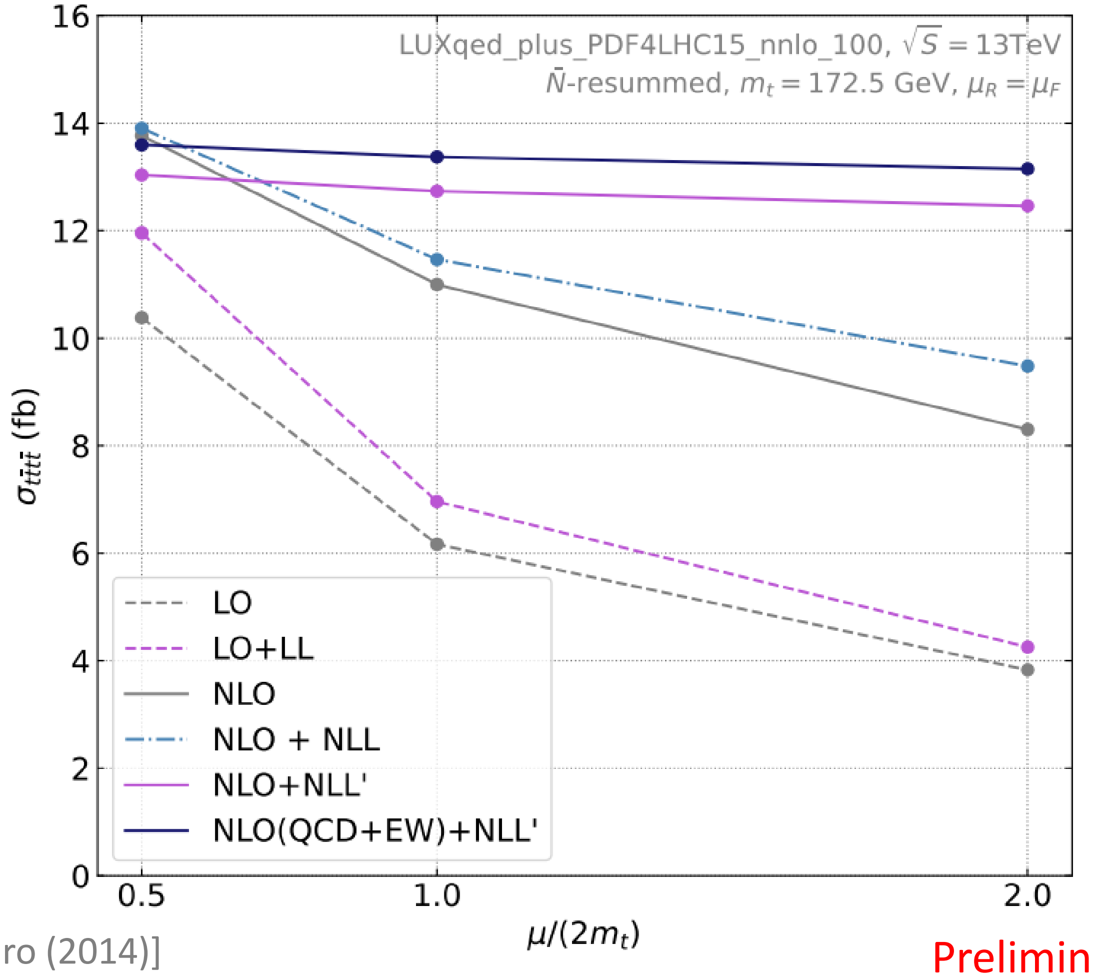
- Combination fixed-order + resummation → **Matching**

$$\sigma^{\mathrm{NLO+NLL}(')}_{t\bar{t}t\bar{t}}(\tau) = \sigma^{\mathrm{NLO}}_{t\bar{t}t\bar{t}}(\tau) + \int_{\mathcal{C}} \frac{\mathrm{d}N}{2\pi i}\, \tau^{-N} f_i(N+1,\mu_F^2)\, f_j(N+1,\mu_F^2) \times \underbrace{\left[\hat{\sigma}^{\mathrm{res}}_{ij\to t\bar{t}t\bar{t}}(N) - \hat{\sigma}^{\mathrm{res}}_{ij\to t\bar{t}t\bar{t}}(N)\big|_{\mathrm{NLO}}\right]}_{\text{avoid double counting with NLO!}}$$

QCD-only NLO and
QCD + EW NLO

- $\sqrt{S} = 13$ TeV
- $\mu_R = \mu_F$
- Big impact of resummation

NLO (QCD+EW): EW corrections up to $\mathcal{O}(\alpha^2)$



Preliminary

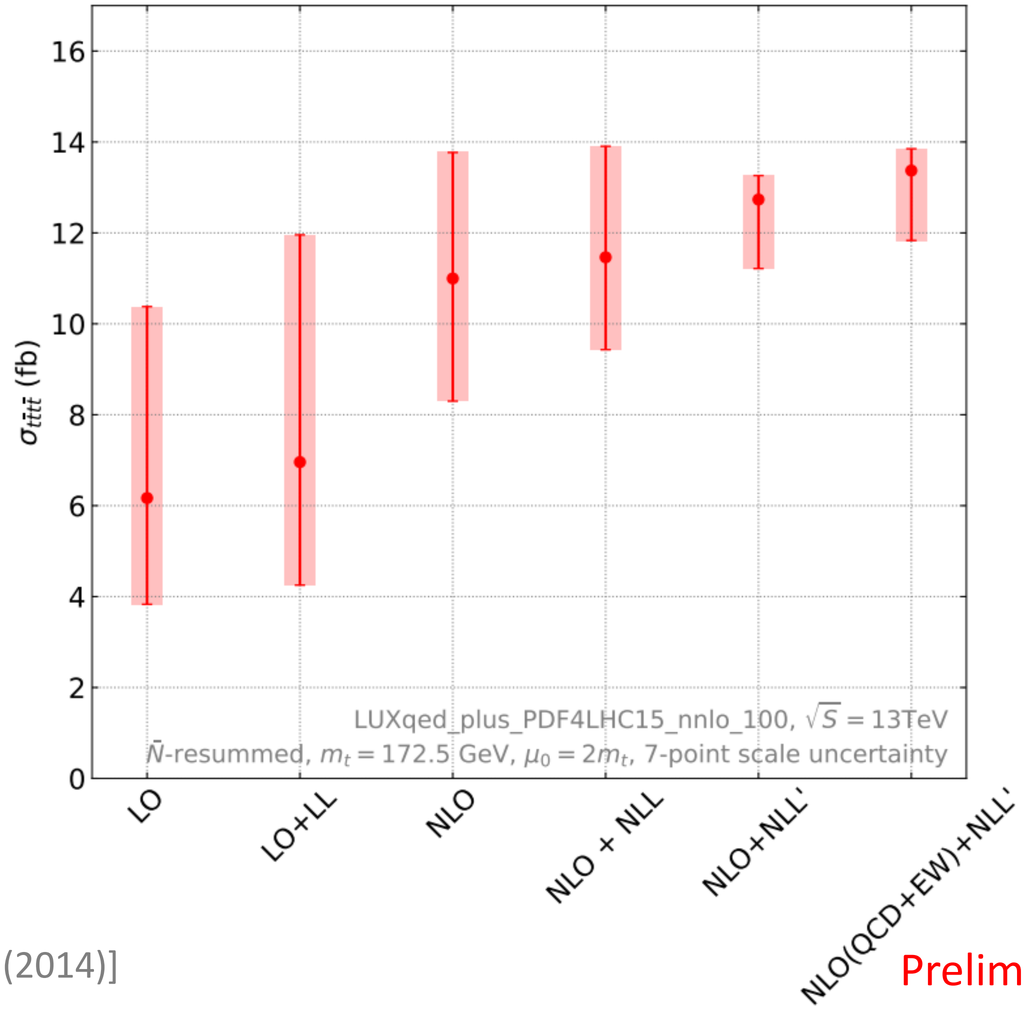
Fixed-order results obtained with
[Frederix, Frixione, Hirschi, Pagani, Shao, Zaro (2018)]
[Alwall, Frederix, Frixione, Hirschi, Maltoni, Mattelaer, Shao, Stelzer, Torrielli, Zaro (2014)]

- 7-point scale variation

	$\sigma_{t\bar{t}t\bar{t}}$ [fb]	K-factor
NLO	$11.00(2)^{+25.2\%}_{-24.5\%}$	
NLO+NLL	$11.46(2)^{+21.3\%}_{-17.7\%}$	1.04
NLO+NLL′	$12.73(2)^{+4.1\%}_{-11.8\%}$	1.16
NLO (QCD+EW)	$11.64(2)^{+23.2\%}_{-22.8\%}$	
NLO (QCD+EW)+NLL′	$13.37(2)^{+3.6\%}_{-11.4\%}$	1.15

NLO (QCD+EW): EW corrections up to $\mathcal{O}(\alpha^2)$

PDF error: $\pm$ 6.9%



Preliminary

Fixed-order results obtained with
[Frederix, Frixione, Hirschi, Pagani, Shao, Zaro (2018)]
[Alwall, Frederix, Frixione, Hirschi, Maltoni, Mattelaer, Shao, Stelzer, Torrielli, Zaro (2014)]

RESULTS for $\sqrt{S} = 13.6$ TeV

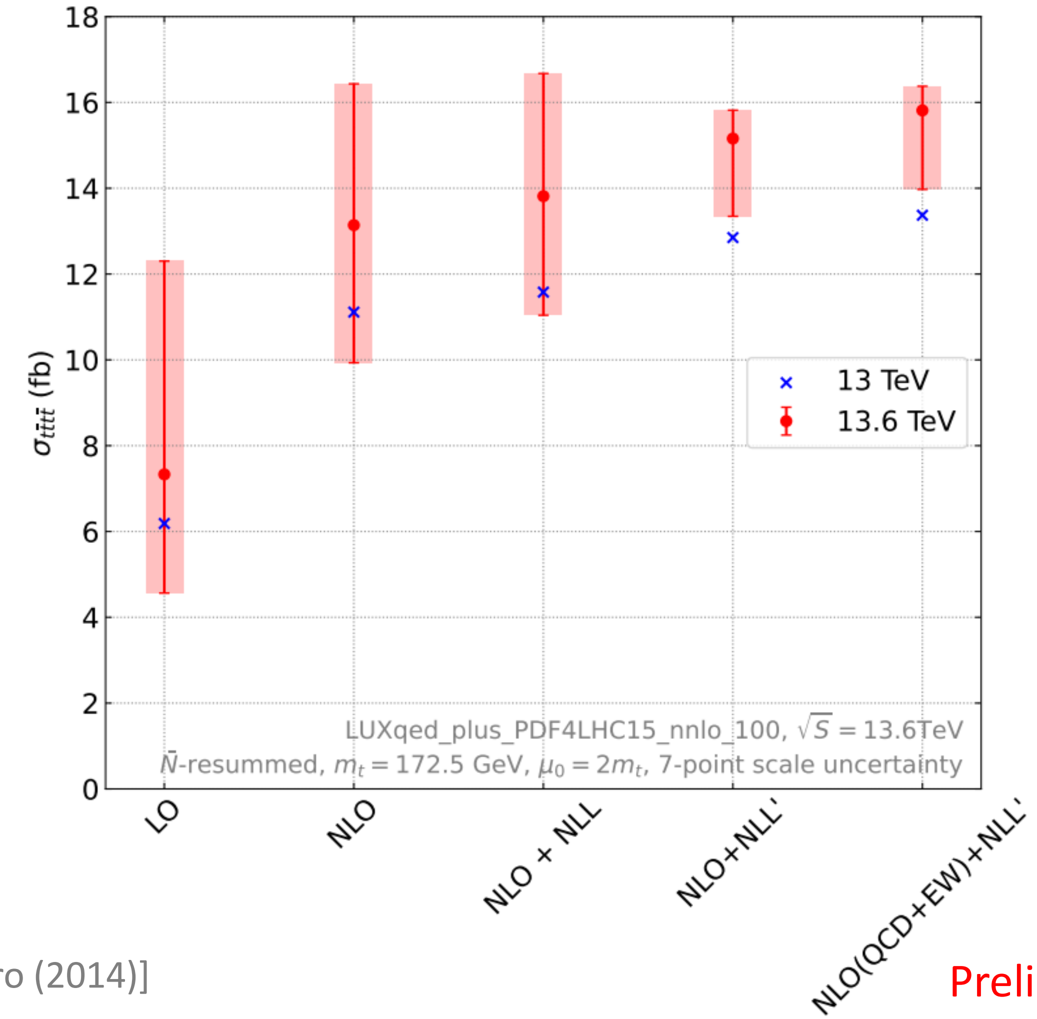
- 7-point scale variation

	$\sigma_{t\bar{t}t\bar{t}}$ [fb]	K-factor
NLO	$13.14(2)^{+25.1\%}_{-24.4\%}$	
NLO+NLL	$13.81(2)^{+20.7\%}_{-20.1\%}$	1.05
NLO+NLL′	$15.16(2)^{+4.3\%}_{-11.9\%}$	1.15
NLO (QCD+EW)	$13.80(2)^{+22.9\%}_{-22.6\%}$	
NLO (QCD+EW)+NLL′	$15.81(2)^{+3.6\%}_{-11.6\%}$	1.14

NLO (QCD+EW): EW corrections up to $\mathcal{O}(\alpha^2)$

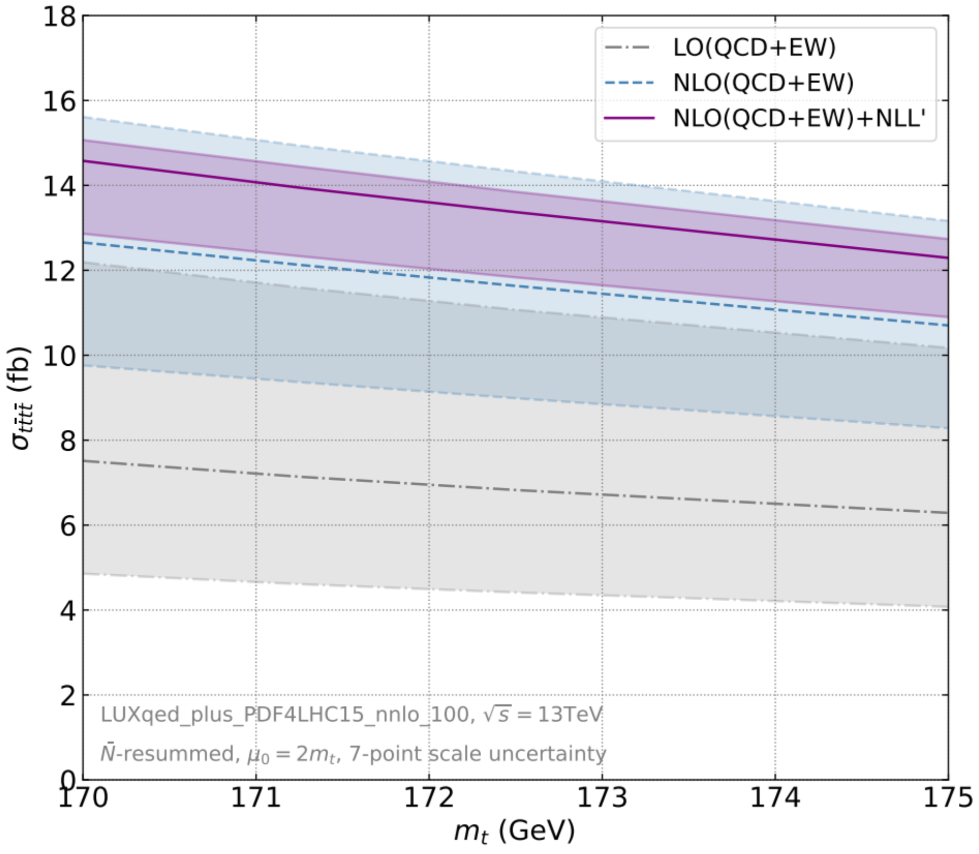
PDF error: $\pm$ 6.7%

Fixed-order results obtained with
[Frederix, Frixione, Hirschi, Pagani, Shao, Zaro (2018)]
[Alwall, Frederix, Frixione, Hirschi, Maltoni, Mattelaer, Shao, Stelzer, Torrielli, Zaro (2014)]



Preliminary

- Top mass $\in$ [170,175] GeV

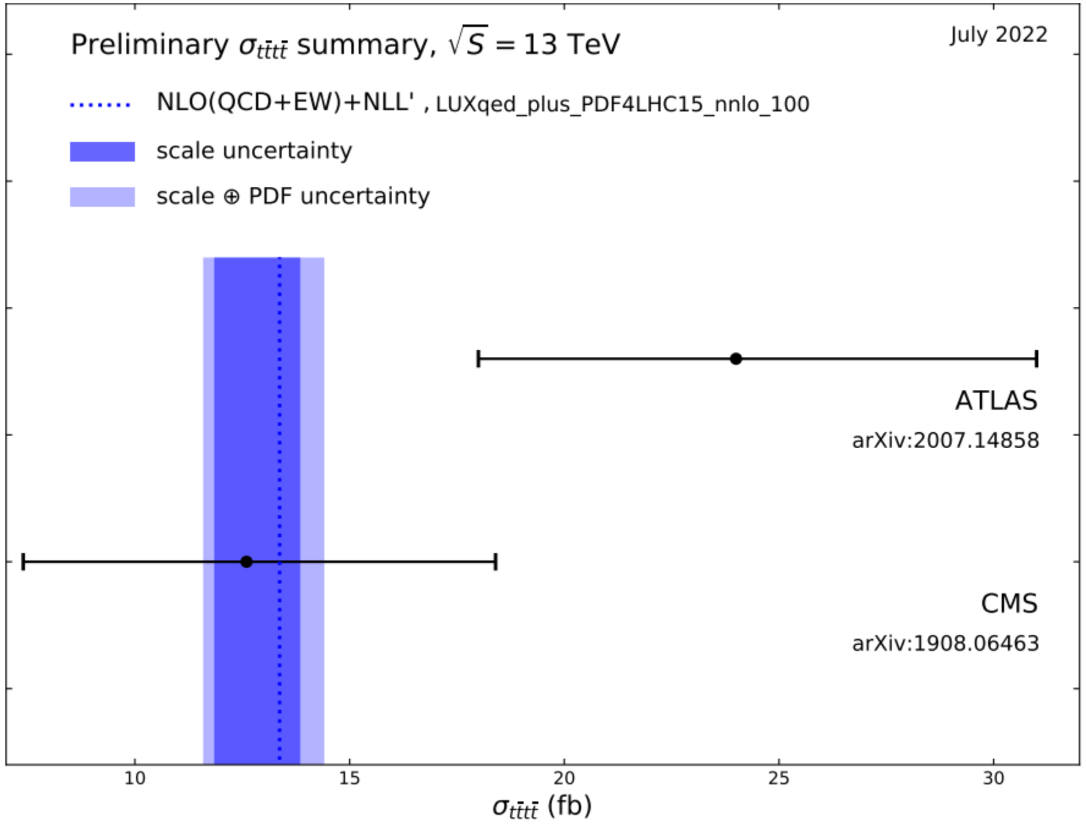


Error bands: 7-point scale uncertainty

NLO (QCD+EW): EW corrections up to $\mathcal{O}(\alpha^2)$

Fixed-order results obtained with
[Frederix, Frixione, Hirschi, Pagani, Shao, Zaro (2018)]
[Alwall, Frederix, Frixione, Hirschi, Maltoni, Mattelaer, Shao, Stelzer, Torrielli, Zaro (2014)]

Preliminary



- Soft gluon resummation at NLL' accuracy
- Significant reduction of the total scale uncertainty

NLO (QCD + EW) + NLL'

$$13.37\,(2)\,{}^{+3.6\%}_{-11.4\%}\,{}^{+6.9\%}_{-6.9\%}\ \text{fb}$$

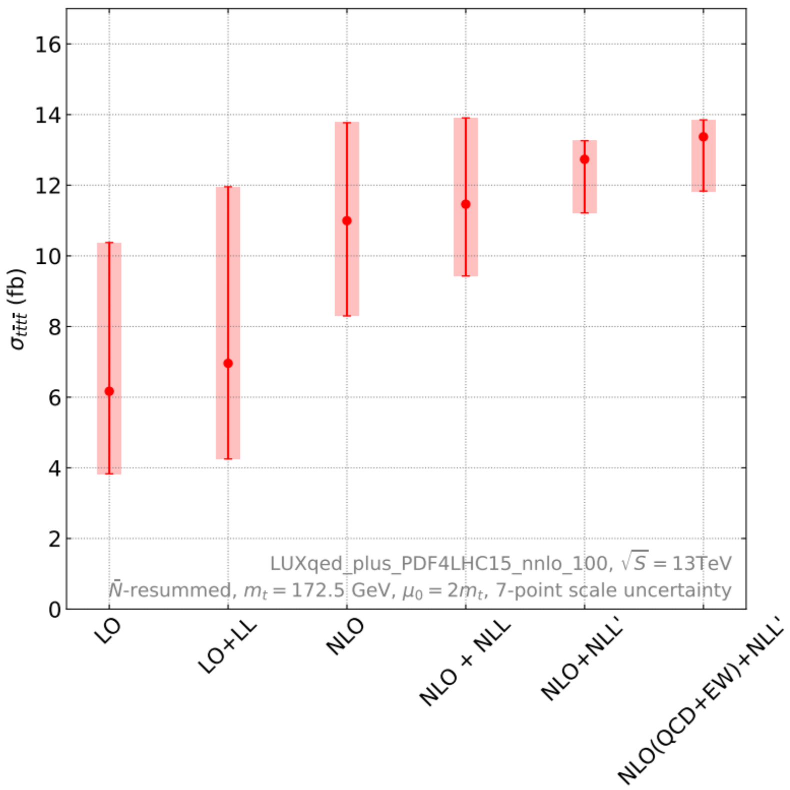
Preliminary

Soft gluon resummation for the production of four top quarks at the LHC

Laura Moreno Valero

in collaboration with Melissa van Beekveld and Anna Kulesza

Institute for Theoretical Physics, University of Münster



WWU MÜNSTER

Graduiertenkolleg 2149 Research Training Group

QCD@LHC IJCLab Orsay 28/11/2022 - 2/12/2022