Soft gluon resummation for the production of four top quarks at the LHC

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Four coloured massive particles in the final state
 WHY 4 TOP?

- Four coloured massive particles in the final state
- Sensitive to the Yukawa coupling

[Diagram showing the interaction of four top quarks, a soft gluon, and other particles like Higgs bosons and W bosons, with references to Eur. Phys. J. C (2020) 80, 75].

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- Sensitive to new physics (gluinos, scalar gluons, heavy scalar bosons, ...)

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WHY 4 TOP?

- Four coloured massive particles in the final state
- Sensitive to the Yukawa coupling
- Sensitive to new physics (gluinos, scalar gluons, heavy scalar bosons, ...)
- Constrains SMEFT coefficients: e.g. four-fermion operator
 WHY 4 TOP?

▶ Measured at the LHC


WHY 4 TOP?

CMS

\[ \sigma_{tttt}^{\text{CMS}, \text{NLO (QCD+EW)}} = 12.0 \pm 2.4 \text{ fb} \]


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ATLAS

\[ \sigma_{tttt}^{\text{ATLAS}} = 24 \pm 5(\text{stat.}) +_{4}^{-5}(\text{syst.}) \text{ fb} \]

[Frederix, Pagani, Zaro (2017)]

[JHEP 11 (2021), 118]
WHY 4 TOP?

- Measured at the LHC

- Theoretical predictions for total rate: NLO (QCD + EW) available
  [Bevilacqua, Worek (2012)], [Frederix, Pagani, Zaro (2017)], [Ježo, Kraus (2021)]
WHY 4 TOP?

- Measured at the LHC

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**GOAL**: extend the precision of theoretical predictions beyond NLO for \( pp \rightarrow t\bar{t}t\bar{t} \) by means of **resummation** techniques
Perturbative expansion cross-section

\[ \sigma = \sum_{n} c_n \alpha_s^n = c_0 + c_1 \alpha_s + c_2 \alpha_s^2 + \ldots \quad \text{with} \quad c_n = f_n + \sum_{k=0}^{2n-1} d_{nk} L^k, \quad L^k = \left[ \frac{\ln^k (1 - M^2/s)}{1 - M^2/s} \right] + \]
Resummation

Perturbative expansion cross-section

\[ \sigma = \sum_n c_n \alpha_s^n = c_0 + c_1 \alpha_s + c_2 \alpha_s^2 + \ldots \quad \text{with} \quad c_n = f_n + \sum_{k=0}^{2n-1} d_{nk} L^k, \quad L^k = \left[ \frac{\ln^k (1 - M^2/s)}{1 - M^2/s} \right] + \]

Large logs \( L \) can spoil predictive power, \( L^k \to \infty \) for \( M^2 \approx s \)
Perturbative expansion cross-section

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- Large logs \( L \) can spoil predictive power, \( L^k \to \infty \) for \( M^2 \approx s \)

**Soft Gluon Resummation**

- Systematic treatment to all orders: resummation
- Relies on ME and Phase space factorization \( \rightarrow \) Mellin space

\[ L := \left[ \ln \left( \frac{1 - M^2/s}{1 - M^2/s} \right) \right]_+ \quad \rightarrow \quad \tilde{L} := \ln N \]
Perturbative expansion cross-section

\[ \sigma = \sum_n c_n \alpha_s^n = c_0 + c_1 \alpha_s + c_2 \alpha_s^2 + \ldots \quad \text{with} \quad c_n = f_n + \sum_{k=0}^{2n-1} d_{nk} L^k, \quad L^k = \left[ \frac{\ln^k(1 - M^2/s)}{1 - M^2/s} \right]_+ \]

Large logs $L$ can spoil predictive power, $L^k \to \infty$ for $M^2 \approx s$

**Soft Gluon Resummation**

- Systematic treatment to all orders: resummation
- Relies on ME and Phase space factorization $\rightarrow$ Mellin space

\[ L := \left[ \frac{\ln(1 - M^2/s)}{1 - M^2/s} \right]_+ \quad \rightarrow \quad \tilde{L} := \ln N \]

\[ \hat{\sigma}^{\text{res}}(N) \sim \mathcal{F}(\alpha_s) \exp \left[ \tilde{L} g_1(\alpha_s \tilde{L}) + g_2(\alpha_s \tilde{L}) + \ldots \right] \]
Perturbative expansion cross-section

\[ \sigma = \sum_{n} c_n \alpha_s^n = c_0 + c_1 \alpha_s + c_2 \alpha_s^2 + \ldots \quad \text{with} \quad c_n = f_n + \sum_{k=0}^{2n-1} d_{nk} L_k^k \, , \quad L_k^k = \left[ \ln^k \frac{1 - M^2/s}{1 - M^2/s} \right]_+ \]

Large logs \( L \) can spoil predictive power, \( L_k^k \to \infty \) for \( M^2 \approx s \)

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- Systematic treatment to all orders: resummation
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\[ L := \left[ \ln \frac{1 - M^2/s}{1 - M^2/s} \right]_+ \quad \text{→} \quad \tilde{L} := \ln N \]

\[ \hat{\sigma}^{\text{res}}(N) \sim F(\alpha_s) \exp \left[ \tilde{L} g_1(\alpha_s \tilde{L}) + g_2(\alpha_s \tilde{L}) + \ldots \right] \]

\[ \alpha_n \ln^{2n} N \]
Perturbative expansion cross-section

\[
\sigma = \sum_n c_n \alpha_s^n = c_0 + c_1 \alpha_s + c_2 \alpha_s^2 + \ldots \quad \text{with} \quad c_n = f_n + \sum_{k=0}^{2n-1} d_{nk} L^k , \quad L^k = \left[ \frac{\ln^k (1 - M^2/s)}{1 - M^2/s} \right]_+ 
\]

Large logs \( L \) can spoil predictive power, \( L^k \to \infty \) for \( M^2 \approx s \)

Soft Gluon Resummation

- Systematic treatment to all orders: resummation
- Relies on ME and Phase space factorization \( \to \) Mellin space

\[
\hat{\sigma}^{\text{res}}(N) \sim \mathcal{F}(\alpha_s) \exp \left( \tilde{L} g_1(\alpha_s \tilde{L}) + g_2(\alpha_s \tilde{L}) + \ldots \right)
\]

\( \tilde{L} := \left[ \frac{\ln(1 - M^2/s)}{1 - M^2/s} \right]_+ \quad \text{\( \to \) } \quad \tilde{L} := \ln N \)
Perturbative expansion cross-section

\[ \sigma = \sum_n c_n \alpha_s^n = c_0 + c_1 \alpha_s + c_2 \alpha_s^2 + \ldots \quad \text{with} \quad c_n = f_n + \sum_{k=0}^{2n-1} d_{nk} L^k, \quad L^k = \left[ \frac{\ln^k(1-M^2/s)}{1-M^2/s} \right]_+ \]

Large logs \( L \) can spoil predictive power, \( L^k \to \infty \) for \( M^2 \simeq s \)

Soft Gluon Resummation

Systematic treatment to all orders: resummation

Relies on ME and Phase space factorization \( \to \) Mellin space

Finite contributions

LL

NLL

\[ \hat{\sigma}^{\text{res}}(N) \sim \mathcal{F}(\alpha_s) \exp \left[ \tilde{L} g_1(\alpha_s \tilde{L}) + g_2(\alpha_s \tilde{L}) + \ldots \right] \]

\[ L := \left[ \frac{\ln(1-M^2/s)}{1-M^2/s} \right]_+ \quad \rightarrow \quad \tilde{L} := \ln N \]
Resummed partonic cross section for $pp \to t\bar{t}t\bar{t}$ in Mellin space

$$\hat{\sigma}_{ij \to t\bar{t}t\bar{t}}^{\text{res}} = \text{Tr} \left[ H_{ij \to t\bar{t}t\bar{t}} S_{ij \to t\bar{t}t\bar{t}} \right] \Delta_i \Delta_j$$

$$= \text{Tr} \left[ H_{ij \to t\bar{t}t\bar{t}} \tilde{U}_{ij \to t\bar{t}t\bar{t}} \tilde{S}_{ij \to t\bar{t}t\bar{t}} U_{ij \to t\bar{t}t\bar{t}} \right] \Delta_i \Delta_j$$
Resummed partonic cross section for $pp \to t\bar{t}t\bar{t}$ in Mellin space

$$\hat{\sigma}_{ij \to t\bar{t}t\bar{t}}^{\text{res}} = \text{Tr} \left[ H_{ij \to t\bar{t}t\bar{t}} \ S_{ij \to t\bar{t}t\bar{t}} \right] \Delta_i \Delta_j$$

Incoming jet functions

(soft-)collinear enhancements

$$= \text{Tr} \left[ H_{ij \to t\bar{t}t\bar{t}} \ U_{ij \to t\bar{t}t\bar{t}} \ S_{ij \to t\bar{t}t\bar{t}} \ U_{ij \to t\bar{t}t\bar{t}} \right] \Delta_i \Delta_j$$
Resummed partonic cross section for $pp \rightarrow t\bar{t}t\bar{t}$ in Mellin space

\[
\hat{\sigma}_{ij \rightarrow t\bar{t}t\bar{t}}^{\text{res}} = \text{Tr} \left[ H_{ij \rightarrow t\bar{t}t\bar{t}} S_{ij \rightarrow t\bar{t}t\bar{t}} \right] \Delta_i \Delta_j
\]

Incoming jet functions

(soft-)collinear enhancements

Hard function

constant contributions as $N \rightarrow \infty$
Resummed partonic cross section for $pp \to t\bar{t}t\bar{t}$ in Mellin space

$$\hat{\sigma}_{ij \to t\bar{t}t\bar{t}}^{\text{res}} = \text{Tr} \left[ H_{ij \to t\bar{t}t\bar{t}} S_{ij \to t\bar{t}t\bar{t}} \Delta_i \Delta_j \right]$$

Incoming jet functions
(soft-)collinear enhancements

Hard function
constant contributions as $N \to \infty$

Soft function
soft wide-angle enhancements

$$U_{ij \to t\bar{t}t\bar{t}}(N, M^2, \mu_F^2, \mu_R^2) = \text{Pexp} \left[ \int_{\mu}^{M/N} \frac{dq}{q} \Gamma_{ij \to t\bar{t}t\bar{t}}(\alpha_s(q^2)) \right]$$
Resummed partonic cross section for $pp \to t\bar{t}t\bar{t}$ in Mellin space

\[
\hat{\sigma}_{ij \rightarrow t\bar{t}t\bar{t}}^{\text{res}} = \text{Tr} \left[ \begin{array}{c|c}
H_{ij \rightarrow t\bar{t}t\bar{t}} & S_{ij \rightarrow t\bar{t}t\bar{t}} \\
\hline
\tilde{U}_{ij \rightarrow t\bar{t}t\bar{t}} & \tilde{S}_{ij \rightarrow t\bar{t}t\bar{t}} \\
\end{array} \right] \Delta_i \Delta_j
\]

- **Incoming jet functions**
- **(soft-)collinear enhancements**

**Hard function**
- constant contributions as $N \to \infty$

**Soft function**
- soft wide-angle enhancements

**Soft anomalous dimension (SAD) matrix**

\[
U_{ij \rightarrow t\bar{t}t\bar{t}}(N, M^2, \mu_F^2, \mu_R^2) = \text{Pexp} \left[ \int_{\mu}^{M/\bar{N}} \frac{dq}{q} \Gamma_{ij \rightarrow t\bar{t}t\bar{t}}(\alpha_s(q^2)) \right]
\]
One-loop SAD matrix needed at NLL

\[ U_{ij \to t\bar{t}t\bar{t}}(N, M^2, \mu_F^2, \mu_R^2) = \text{Pexp} \left[ \int_{\mu}^{M/N} \frac{dq}{q} \Gamma_{ij \to t\bar{t}t\bar{t}}(\alpha_s(q^2)) \right] \]

\[ \Gamma_{ij \to t\bar{t}t\bar{t}}(\alpha_s) = \left( \frac{\alpha_s}{\pi} \right) \Gamma_{ij \to t\bar{t}t\bar{t}}^{(1)} + \left( \frac{\alpha_s}{\pi} \right)^2 \Gamma_{ij \to t\bar{t}t\bar{t}}^{(2)} + \ldots \]

\[ q\bar{q} \to t\bar{t}t\bar{t} : \quad 3 \otimes \bar{3} = 3 \otimes \bar{3} \otimes 3 \otimes \bar{3} \rightarrow 1 \oplus 8 = 0 \oplus (2 \times 1) \oplus (2 \times 8) \oplus 8_S \oplus 8_A \oplus 10 \oplus \bar{10} \oplus 27 \]

6-dimensional colour space

\[ gg \to t\bar{t}t\bar{t} : \quad 8 \otimes 8 = 3 \otimes \bar{3} \otimes 3 \otimes \bar{3} \rightarrow 0 \oplus 1 \oplus 8_S \oplus 8_A \oplus 10 \oplus \bar{10} \oplus 27 = 0 \oplus (2 \times 1) \oplus (2 \times 8) \oplus 8_S \oplus 8_A \oplus 10 \oplus \bar{10} \oplus 27 \]

14-dimensional colour space
One-loop SAD matrix needed at NLL

\[ U_{ij \rightarrow \bar{t} \bar{t} \bar{t} \bar{t}}(N, M^2, \mu_F^2, \mu_R^2) = \text{Pexp} \left[ \int_{\mu}^{M/\tilde{N}} \frac{dq}{q} \Gamma_{ij \rightarrow \bar{t} \bar{t} \bar{t} \bar{t}}(\alpha_s(q^2)) \right] \]

\[ \Gamma_{ij \rightarrow \bar{t} \bar{t} \bar{t} \bar{t}}(\alpha_s) = \left( \frac{\alpha_s}{\pi} \right) \Gamma_{ij \rightarrow \bar{t} \bar{t} \bar{t} \bar{t}}^{(1)} + \left( \frac{\alpha_s}{\pi} \right)^2 \Gamma_{ij \rightarrow \bar{t} \bar{t} \bar{t} \bar{t}}^{(2)} + \ldots \]

Diagonal SAD matrix

\[ \bar{U}_R \bar{S}_R \bar{U}_R = \bar{S}_R \exp \left[ \frac{2 \text{Re} \left( \Gamma_R^{(1)} \right)}{2\pi b_0} \ln(1 - 2\lambda) \right] \]

with \( \lambda = \alpha_s b_0 \ln(\tilde{N}) \)

\[ q\bar{q} \rightarrow \bar{t} \bar{t} \bar{t} \bar{t} : \quad 3 \otimes 3 = 3 \otimes 3 \otimes 3 \otimes 3 \rightarrow 1 \oplus 8 = 0 \oplus (2 \times 1) \oplus (2 \times 8) \oplus 8_S \oplus 8_A \oplus 10 \oplus \bar{10} \oplus 27 \]

6-dimensional colour space

\[ gg \rightarrow \bar{t} \bar{t} \bar{t} \bar{t} : \quad 8 \otimes 8 = 3 \otimes 3 \otimes 3 \otimes 3 \rightarrow 0 \oplus 1 \oplus 8_S \oplus 8_A \oplus 10 \oplus \bar{10} \oplus 27 = 0 \oplus (2 \times 1) \oplus (2 \times 8) \oplus 8_S \oplus 8_A \oplus 10 \oplus \bar{10} \oplus 27 \]

14-dimensional colour space
\[ q \bar{q} \rightarrow t \bar{t} t \bar{t} : \quad 3 \otimes \bar{3} = 3 \otimes \bar{3} \otimes 3 \otimes \bar{3} \rightarrow 1 \oplus 8 = 0 \oplus (2 \times 1) \oplus (2 \times 8) \oplus 8_s \oplus 8_d \oplus 10 \oplus \bar{10} \oplus 27 \]

[Keppeler, Sjodahl (2012)]
\[ q\bar{q} \rightarrow t\bar{t}t\bar{t} : \ 3 \otimes \bar{3} = 3 \otimes \bar{3} \otimes 3 \otimes \bar{3} \rightarrow 1 \oplus 8 = 0 \oplus (2 \times 1) \oplus (2 \times 8) \oplus 8_S \oplus 8_A \oplus 10 \oplus \bar{1}0 \oplus 27 \]

\[ c_{q\bar{q}}^j = \frac{1}{\sqrt{N_c^3}} \delta_{c_1c_3} \delta_{c_2c_4} \delta_{c_6c_8} \]

\[ c_{q\bar{q}}^2 = \frac{1}{T_R \sqrt{N_c(N_c^2 - 1)}} \delta_{c_1c_3} \delta_{c_2c_4} \delta_{c_6c_8} \]

\[ c_{q\bar{q}}^3 = \frac{1}{T_R \sqrt{N_c(N_c^2 - 1)}} \delta_{c_1c_3} \delta_{c_2c_4} \delta_{c_6c_8} \]

\[ c_{q\bar{q}}^4 = \frac{1}{T_R \sqrt{N_c(N_c^2 - 1)}} \delta_{c_1c_3} \delta_{c_2c_4} \delta_{c_6c_8} \]

\[ c_{q\bar{q}}^5 = \frac{1}{T_R \sqrt{2(N_c^2 - 5N_c^2 + 4)}} \delta_{c_1c_3} \delta_{c_2c_4} \delta_{c_6c_8} \]

\[ c_{q\bar{q}}^6 = \frac{1}{T_R \sqrt{2N_c(N_c^2 - 1)}} \delta_{c_1c_3} \delta_{c_2c_4} \delta_{c_6c_8} \]

6x6 SAD matrix

\[ \Gamma^{(0)}_{q\bar{q} \rightarrow t\bar{t}t} = -C_F(L_{th} + L_{ts} + 2) \]

\[ \Gamma^{(2)}_{q\bar{q} \rightarrow t\bar{t}t} = \sqrt{N_c^2 - 1} \left( L_{th} + L_{ts} - L_{th} - L_{ts} \right) \]

\[ \Gamma^{(4)}_{q\bar{q} \rightarrow t\bar{t}t} = \frac{N_c^2 - 1}{2N_c} \left( -L_{th} + L_{ts} - L_{th} - L_{ts} \right) \]

\[ \Gamma^{(6)}_{q\bar{q} \rightarrow t\bar{t}t} \]

\[ \Gamma^{(8)}_{q\bar{q} \rightarrow t\bar{t}t} \]

\[ \Gamma^{(10)}_{q\bar{q} \rightarrow t\bar{t}t} \]

\[ \Gamma^{(27)}_{q\bar{q} \rightarrow t\bar{t}t} \]

[Keppeler, Sjodahl (2012)]

Soft gluon resummation for four top quarks
\[
gg \rightarrow t\bar{t}t\bar{t} : 8 \otimes 8 = 3 \otimes \bar{3} \otimes 3 \otimes \bar{3} \rightarrow 0 \oplus 1 \oplus 8_S \oplus 8_A \oplus 10 \oplus 10 \oplus 27 = 0 \oplus (2 \times 1) \oplus (2 \times 8) \oplus 8_S \oplus 8_A \oplus 10 \oplus 10 \oplus 27
\]

\[
\begin{align*}
c_{1g} &= \frac{1}{T_R N_c^2 - 1} t^{a_1}_{C_4} t^{a_2}_{C_8}, \\
c_{3g} &= \frac{1}{T_R \sqrt{2(N_c^2 - 5N_c^2 + 4)}} \delta_{c_2c_4} d_{a_1a_2b_1} t^{b_1}_{C_8}, \\
c_{5g} &= \frac{1}{T_R \sqrt{2(N_c^2 - 5N_c^2 + 4)}} \psi_{c_2c_4} d_{b_1a_1a_2} \delta_{c_8c_8}, \\
c_{7g} &= \frac{1}{T_R^2 2(N_c^2 - 5N_c^2 + 4)} \phi_{c_2c_4} d_{b_1a_1a_2} f_{b_2a_2b_3} t^{b_3}_{C_8}, \\
c_{9g} &= \frac{1}{T_R^2 2(N_c^2 - 5N_c^2 + 4)} \phi_{c_2c_4} f_{b_1a_1a_2} d_{b_2a_2b_3} t^{b_3}_{C_8}, \\
c_{11g} &= \frac{2}{T_R \sqrt{N_c^2 - 2N_c - 3}} \psi_{c_2c_4} P_{a_1a_2b_2} t^{b_3}_{C_8}, \\
c_{13g} &= \frac{2}{T_R N_c \sqrt{N_c^2 - 2N_c - 3}} \psi_{c_2c_4} P_{a_1a_2b_2} t^{b_3}_{C_8},
\end{align*}
\]

\[
\begin{align*}
c_{2g} &= \frac{1}{N_c \sqrt{N_c^2 - 1}} \delta_{a_1a_2} \delta_{c_2c_4} \delta_{c_8c_8}, \\
c_{4g} &= \frac{1}{T_R N_c \sqrt{2(N_c^2 - 1)}} \delta_{c_2c_4} f_{a_1a_2b_1} t^{b_1}_{C_8}, \\
c_{6g} &= \frac{1}{T_R^2 2(N_c^2 - 4) \sqrt{N_c^2 - 1}} \phi_{c_2c_4} d_{b_1a_1a_2} d_{b_2a_2b_3} t^{b_3}_{C_8}, \\
c_{8g} &= \frac{1}{T_R N_c \sqrt{2(N_c^2 - 1)}} \phi_{c_2c_4} f_{b_1a_1a_2} \delta_{c_8c_8}, \\
c_{10g} &= \frac{1}{T_R^2 2 N_c \sqrt{N_c^2 - 1}} \phi_{c_2c_4} f_{b_1a_1a_2} f_{b_2a_2b_3} t^{b_3}_{C_8}, \\
c_{12g} &= \frac{1}{T_R N_c \sqrt{N_c^2 - 2N_c - 3}} \phi_{c_2c_4} P_{a_1a_2b_2} t^{b_3}_{C_8}, \\
c_{14g} &= \frac{1}{T_R N_c \sqrt{N_c^2 - 2N_c - 3}} \phi_{c_2c_4} P_{a_1a_2b_2} t^{b_3}_{C_8},
\end{align*}
\]

[Keppeler, Sjodahl (2012)]
\[ gg \rightarrow t\bar{t}t\bar{t} : 8 \otimes 8 = 3 \otimes \bar{3} \otimes 3 \otimes \bar{3} \rightarrow 0 + 1 \otimes 8_S \otimes 8_A \otimes 10 + 10 + 27 = 0 + (2 \times 1) + (2 \times 8) + 8_S + 8_A + 10 + 10 + 27 \]

14x14 SAD matrix

\[
\begin{align*}
\tilde{f}_{gg\rightarrow t\bar{t}t\bar{t},11} & = \frac{N_c^2 - 2}{4N_c} C_F L_{13} + \frac{N_c + 2}{4N_c} (T_{13} + T_{14} + T_{25} + T_{26}) \\
\tilde{f}_{gg\rightarrow t\bar{t}t\bar{t},12} & = 0 \\
\tilde{f}_{gg\rightarrow t\bar{t}t\bar{t},13} & = 0 \\
\tilde{f}_{gg\rightarrow t\bar{t}t\bar{t},14} & = 0 \\
\tilde{f}_{gg\rightarrow t\bar{t}t\bar{t},22} & = -2C_F C_P (L_{13} + L_{14}) \\
\tilde{f}_{gg\rightarrow t\bar{t}t\bar{t},23} & = 0 \\
\tilde{f}_{gg\rightarrow t\bar{t}t\bar{t},24} & = \frac{1}{\sqrt{2}} (T_{15} - T_{16} + T_{25} + T_{26}) \\
\tilde{f}_{gg\rightarrow t\bar{t}t\bar{t},25} & = \frac{1}{\sqrt{2}} (T_{15} - T_{16} + T_{25} + T_{26}) \\
\tilde{f}_{gg\rightarrow t\bar{t}t\bar{t},26} & = \frac{1}{\sqrt{2}} (T_{15} - T_{16} + T_{25} + T_{26}) \\
\tilde{f}_{gg\rightarrow t\bar{t}t\bar{t},27} & = 0 \\
\tilde{f}_{gg\rightarrow t\bar{t}t\bar{t},28} & = 0 \\
\tilde{f}_{gg\rightarrow t\bar{t}t\bar{t},29} & = 0 \\
\tilde{f}_{gg\rightarrow t\bar{t}t\bar{t},30} & = \frac{1}{4N_c^2} (L_{13} + L_{14} - L_{15}^2 - L_{16} + 4T_{15} + 4T_{16} + 4T_{25} + 4T_{26}) \\
\tilde{f}_{gg\rightarrow t\bar{t}t\bar{t},31} & = \frac{1}{4N_c^2} (L_{13} + L_{14} - L_{15}^2 - L_{16} + 4T_{15} + 4T_{16} + 4T_{25} + 4T_{26}) \\
\tilde{f}_{gg\rightarrow t\bar{t}t\bar{t},32} & = \frac{1}{4N_c^2} (L_{13} + L_{14} - L_{15}^2 - L_{16} + 4T_{15} + 4T_{16} + 4T_{25} + 4T_{26}) \\
\tilde{f}_{gg\rightarrow t\bar{t}t\bar{t},33} & = \frac{1}{4N_c^2} (L_{13} + L_{14} - L_{15}^2 - L_{16} + 4T_{15} + 4T_{16} + 4T_{25} + 4T_{26}) \\
\tilde{f}_{gg\rightarrow t\bar{t}t\bar{t},34} & = 0 \\
\tilde{f}_{gg\rightarrow t\bar{t}t\bar{t},35} & = 0 \\
\tilde{f}_{gg\rightarrow t\bar{t}t\bar{t},36} & = 0 \\
\tilde{f}_{gg\rightarrow t\bar{t}t\bar{t},37} & = \frac{1}{4N_c^2} (L_{13} + L_{14} - L_{15}^2 - L_{16} + 4T_{15} + 4T_{16} + 4T_{25} + 4T_{26}) \\
\tilde{f}_{gg\rightarrow t\bar{t}t\bar{t},38} & = 0 \\
\tilde{f}_{gg\rightarrow t\bar{t}t\bar{t},39} & = 0 \\
\tilde{f}_{gg\rightarrow t\bar{t}t\bar{t},40} & = 0 \\
\tilde{f}_{gg\rightarrow t\bar{t}t\bar{t},41} & = \frac{1}{4N_c^2} (L_{13} + L_{14} - L_{15}^2 - L_{16} + 4T_{15} + 4T_{16} + 4T_{25} + 4T_{26}) \\
\tilde{f}_{gg\rightarrow t\bar{t}t\bar{t},42} & = 0 \\
\tilde{f}_{gg\rightarrow t\bar{t}t\bar{t},43} & = 0 \\
\tilde{f}_{gg\rightarrow t\bar{t}t\bar{t},44} & = 0 \\
\end{align*}

+ 65 more components!
\[ gg \rightarrow t\overline{t}t\overline{t} : \begin{array}{cccc} 8 \times 8 = \begin{array}{ccc} 3 & \bar{3} & 3 \\ \bar{3} & 3 & \bar{3} \end{array} \end{array} \rightarrow 0 + 1 + S_S + S_A + 10 + 10 + 27 = 0 + (2 \times 1) + (2 \times 8) + S_S + S_A + 10 + 10 + 27 \]
\[
\begin{align*}
\mathcal{O}(gg) & \rightarrow t\bar{t}t\bar{t} : \ 8 \otimes 8 = 3 \otimes \bar{3} \otimes 3 \otimes \bar{3} \\
& \rightarrow 0 \oplus 1 \oplus 8_S \oplus 8_A \oplus 10 \oplus 10 \oplus 27 = 0 \oplus (2 \times 1) \oplus (2 \times 8) \oplus 8_S \oplus 8_A \oplus 10 \oplus 10 \oplus 27
\end{align*}
\]

\[
\begin{align*}
\tilde{c}_1^{gg} &= \frac{3\sqrt{3}}{8} c_1^{gg} + \frac{3}{10} \sqrt{\frac{3}{2}} c_6^{gg} - \frac{1}{2} \sqrt{\frac{3}{2}} c_{10}^{gg} - \frac{1}{4} \sqrt{\frac{3}{10}} c_{11}^{gg} - \frac{1}{4} \sqrt{\frac{3}{10}} c_{12}^{gg} + \frac{7}{40} c_{13}^{gg}, \\
\tilde{c}_2^{gg} &= -\frac{\sqrt{5}}{4} c_1^{gg} + \sqrt{\frac{2}{5}} c_6^{gg} - \frac{1}{2\sqrt{2}} c_{11}^{gg} - \frac{1}{4} \sqrt{\frac{3}{5}} c_{13}^{gg}, \\
\tilde{c}_3^{gg} &= -\frac{1}{\sqrt{2}} c_7^{gg} + \frac{1}{\sqrt{2}} c_9^{gg}, \\
\tilde{c}_5^{gg} &= -\frac{1}{2\sqrt{2}} c_1^{gg} - \frac{1}{2} c_6^{gg} - \frac{1}{2} c_{10}^{gg} + \frac{1}{2} \sqrt{\frac{3}{2}} c_{13}^{gg}, \\
\tilde{c}_6^{gg} &= -\frac{1}{2} \sqrt{\frac{5}{14}} c_1^{gg} + \frac{3}{2\sqrt{35}} c_6^{gg} - \frac{1}{2} \sqrt{7} c_{10}^{gg} - \frac{2}{\sqrt{7}} c_{12}^{gg} - \frac{3}{2} \sqrt{\frac{3}{70}} c_{13}^{gg}, \\
\tilde{c}_7^{gg} &= -\frac{1}{2\sqrt{7}} c_1^{gg} + \frac{3}{5\sqrt{14}} c_6^{gg} - \frac{1}{\sqrt{14}} c_{10}^{gg} + \sqrt{\frac{7}{10}} c_{11}^{gg} - \frac{3}{\sqrt{10}} c_{12}^{gg} - \frac{3}{10} \sqrt{\frac{3}{7}} c_{13}^{gg}, \\
\tilde{c}_8^{gg} &= \frac{1}{\sqrt{2}} c_7^{gg} + \frac{1}{\sqrt{2}} c_9^{gg}, \\
\tilde{c}_{13}^{gg} &= \frac{1}{8} c_1^{gg} + \frac{1}{2\sqrt{2}} c_6^{gg} + \frac{1}{2\sqrt{2}} c_{10}^{gg} + \frac{1}{4} \sqrt{\frac{5}{2}} c_{11}^{gg} + \frac{1}{4} \sqrt{\frac{5}{2}} c_{12}^{gg} + \frac{3\sqrt{3}}{8} c_{13}^{gg}, \\
\tilde{c}_4^{gg} &= c_4^{gg}, \quad \tilde{c}_5^{gg} = c_5^{gg}, \quad \tilde{c}_{10}^{gg} = c_{10}^{gg}, \quad \tilde{c}_{11}^{gg} = c_{11}^{gg}, \quad \tilde{c}_{12}^{gg} = c_{12}^{gg}, \quad \tilde{c}_{13}^{gg} = c_{13}^{gg}, \quad \tilde{c}_{14}^{gg} = c_{14}^{gg}.
\end{align*}
\]
One-loop SAD matrix needed at NLL

\[ U_{ij \rightarrow t\bar{t}t\bar{t}}(N, M^2, \mu_F^2, \mu_R^2) = \text{Pexp} \left[ \int_{\mu}^{M/\tilde{N}} \frac{dq}{q} \Gamma_{ij \rightarrow t\bar{t}t\bar{t}}(\alpha_s(q^2)) \right] \]

\[ \Gamma_{ij \rightarrow t\bar{t}t\bar{t}}(\alpha_s) = \left( \frac{\alpha_s}{\pi} \right) \Gamma_{ij \rightarrow t\bar{t}t\bar{t}}^{(1)} + \left( \frac{\alpha_s}{\pi} \right)^2 \Gamma_{ij \rightarrow t\bar{t}t\bar{t}}^{(2)} + \ldots \]

Diagonal SAD matrix

\[ \tilde{U}_R \tilde{S}_R \ U_R = \tilde{S}_R \ \text{exp} \left[ \frac{2 \text{Re} \left( \Gamma_R^{(1)} \right)}{2\pi b_0} \ln(1 - 2\lambda) \right] \]

with \( \lambda = \alpha_s b_0 \ln(\tilde{N}) \)

\( q\bar{q} \rightarrow t\bar{t}t\bar{t} : \ 3 \otimes \bar{3} = 3 \otimes \bar{3} \otimes 3 \otimes \bar{3} \rightarrow 1 \oplus 8 = 0 \oplus (2 \times 1) \oplus (2 \times 8) \oplus 8_S \oplus 8_A \oplus 10 \oplus \bar{10} \oplus 27 \)

\( gg \rightarrow t\bar{t}t\bar{t} : \ 8 \otimes 8 = 3 \otimes \bar{3} \otimes 3 \otimes \bar{3} \rightarrow 0 \oplus 1 \oplus 8_S \oplus 8_A \oplus 10 \oplus \bar{10} \oplus 27 = 0 \oplus (2 \times 1) \oplus (2 \times 8) \oplus 8_S \oplus 8_A \oplus 10 \oplus \bar{10} \oplus 27 \)
One-loop SAD matrix needed at NLL

\[ U_{ij\rightarrow t\bar{t}t\bar{t}}(N, M^2, \mu_F^2, \mu_R^2) = \text{Pexp} \left[ \int_{\mu}^{\tilde{N}/\tilde{\mu}} \frac{dq}{q} \Gamma_{ij\rightarrow t\bar{t}t\bar{t}}(\alpha_s(q^2)) \right] \]

\[ \Gamma_{ij\rightarrow t\bar{t}t\bar{t}}(\alpha_s) = \left( \frac{\alpha_s}{\pi} \right) \Gamma^{(1)}_{ij\rightarrow t\bar{t}t\bar{t}} + \left( \frac{\alpha_s}{\pi} \right)^2 \Gamma^{(2)}_{ij\rightarrow t\bar{t}t\bar{t}} + \ldots \]

Diagonal SAD matrix

\[ \bar{U}_R \bar{S}_R U_R = \bar{S}_R \exp \left[ \frac{2 \text{Re} \left( \Gamma^{(1)}_R \right)}{2\pi b_0} \ln(1 - 2\lambda) \right] \]

with \( \lambda = \alpha_s b_0 \ln(\tilde{N}) \)

\[ q\bar{q} \rightarrow t\bar{t}t\bar{t} : \quad 3 \otimes \bar{3} = 3 \otimes \bar{3} \otimes 3 \otimes \bar{3} \rightarrow \mathbf{1} \oplus \mathbf{8} = \mathbf{0} \oplus (2 \times 1) \oplus (2 \times 8) \oplus \mathbf{8}_S \oplus \mathbf{8}_A \oplus \mathbf{10} \oplus \mathbf{10} \oplus \mathbf{27} \]

\[ 2\text{Re} \left[ \Gamma^{(1)}_{R,q\bar{q}\rightarrow t\bar{t}t\bar{t}} \right] = \text{diag} \left( 0, 0, -3, -3, -3, -3 \right) \]

\[ gg \rightarrow t\bar{t}t\bar{t} : \quad 8 \otimes \bar{8} = 3 \otimes \bar{3} \otimes 3 \otimes \bar{3} \rightarrow \mathbf{0} \oplus \mathbf{1} \oplus \mathbf{8}_S \oplus \mathbf{8}_A \oplus \mathbf{10} \oplus \mathbf{10} \oplus \mathbf{27} = \mathbf{0} \oplus (2 \times 1) \oplus (2 \times 8) \oplus \mathbf{8}_S \oplus \mathbf{8}_A \oplus \mathbf{10} \oplus \mathbf{10} \oplus \mathbf{27} \]

\[ 2\text{Re} \left[ \Gamma^{(1)}_{R,gg\rightarrow t\bar{t}t\bar{t}} \right] = \text{diag} \left( -8, -6, -6, -4, -3, -3, -3, -3, -3, -3, -3, 0, 0 \right) \]
Quadratic Casimir Invariants $N_c = 3$

- $q\bar{q} \to t\bar{t}t\bar{t} : \quad 3 \otimes \bar{3} = 3 \otimes \bar{3} \otimes 3 \otimes \bar{3} \rightarrow 1 \oplus 8 = 0 \oplus (2 \times 1) \oplus (2 \times 8) \oplus 8_S \oplus 8_A \oplus 10 \oplus \bar{10} \oplus 27$

$$2\text{Re} \left[ \Gamma_{R,q\bar{q}\to t\bar{t}t}^{(1)} \right] = \text{diag} \left( 0, 0, -3, -3, -3, -3 \right)$$

- $gg \to t\bar{t}t\bar{t} : \quad 8 \otimes 8 = 3 \otimes \bar{3} \otimes 3 \otimes \bar{3} \rightarrow 0 \oplus 1 \oplus 8_S \oplus 8_A \oplus 10 \oplus \bar{10} \oplus 27 = 0 \oplus (2 \times 1) \oplus (2 \times 8) \oplus 8_S \oplus 8_A \oplus 10 \oplus \bar{10} \oplus 27$

$$2\text{Re} \left[ \Gamma_{R,gg\to t\bar{t}t}^{(1)} \right] = \text{diag} \left( -8, -6, -6, -4, -3, -3, -3, -3, -3, -3, -3, -3, 0, 0 \right)$$

$C_2(1) = 0$

$C_2(8_{(S/A)}) = 3$

$C_2(10, \bar{10}) = 6$

$C_2(27) = 8$

$C_2(0) = 4$
\[ H = H^{(0)} + \frac{\alpha_s(\mu_R)}{\pi} H^{(1)} + \ldots \]
\[ \tilde{S} = \tilde{S}^{(0)} + \frac{\alpha_s(\mu_R)}{\pi} \tilde{S}^{(1)} + \ldots \]

- **NLL** accuracy: exponential functions at NLL together with

\[ \text{Tr} \left[ H \tilde{S} \right] = \text{Tr} \left[ H^{(0)} \tilde{S}^{(0)} \right] \]

- **NLL’** accuracy: exponential functions at NLL together with

\[ \text{Tr} \left[ H \tilde{S} \right] = \text{Tr} \left[ H^{(0)} \tilde{S}^{(0)} + \frac{\alpha_s(\mu_R)}{\pi} H^{(1)} \tilde{S}^{(0)} + \frac{\alpha_s(\mu_R)}{\pi} H^{(0)} \tilde{S}^{(1)} \right] \]
\[ \sigma_{tttt}^{\text{NLO} + \text{NLL}'}(\tau) = \int_C \frac{dN}{2\pi i} \tau^{-N} f_i(N + 1, \mu_F^2) f_j(N + 1, \mu_F^2) \hat{\sigma}_{ij \to t\bar{t}t\bar{t}}^{\text{res}}(N) \]

**Combination fixed-order + resummation → Matching**

\[ \sigma_{tttt}^{\text{NLO} + \text{NLL}'}(\tau) = \sigma_{tttt}^{\text{NLO}}(\tau) + \int_C \frac{dN}{2\pi i} \tau^{-N} f_i(N + 1, \mu_F^2) f_j(N + 1, \mu_F^2) \times \left[ \hat{\sigma}_{ij \to t\bar{t}t\bar{t}}^{\text{res}}(N) - \hat{\sigma}_{ij \to t\bar{t}t\bar{t}}^{\text{res}}(N)|_{\text{NLO}} \right] \]

QCD-only NLO and QCD + EW NLO

avoid double counting with NLO!
RESULTS

- $\sqrt{S} = 13$ TeV
- $\mu_R = \mu_F$
- Big impact of resummation

NLO (QCD+EW): EW corrections up to $\mathcal{O}(\alpha^2)$

Fixed-order results obtained with
[Frederix, Frixione, Hirschi, Pagani, Shao, Zaro (2018)]
[Alwall, Frederix, Frixione, Hirschi, Maltoni, Mattelaer, Shao, Stelzer, Torrielli, Zaro (2014)]
### 7-point scale variation

<table>
<thead>
<tr>
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<th>$\sigma_{titi}$ [fb]</th>
<th>K-factor</th>
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<tbody>
<tr>
<td>NLO</td>
<td>$11.00(2)^{+25.2%}_{-24.5%}$</td>
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<tr>
<td>NLO+NLL</td>
<td>$11.46(2)^{+21.3%}_{-17.7%}$</td>
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<tr>
<td>NLO+NLL'</td>
<td>$12.73(2)^{+4.1%}_{-11.8%}$</td>
<td>1.16</td>
</tr>
<tr>
<td>NLO (QCD+EW)</td>
<td>$11.64(2)^{+23.2%}_{-22.8%}$</td>
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<tr>
<td>NLO (QCD+EW)+NLL'</td>
<td>$13.37(2)^{+3.6%}_{-11.4%}$</td>
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</tbody>
</table>

NLO (QCD+EW): EW corrections up to $O(\alpha^2)$

PDF error: $\pm 6.9\%$

Fixed-order results obtained with
[Frederix, Frixione, Hirschi, Pagani, Shao, Zaro (2018)]
[Alwall, Frederix, Frixione, Hirschi, Maltoni, Mattelaer, Shao, Stelzer, Torrielli, Zaro (2014)]
### RESULTS for $\sqrt{S} = 13.6$ TeV

#### 7-point scale variation

<table>
<thead>
<tr>
<th></th>
<th>$\sigma_{t\bar{t}t\bar{t}}$ [fb]</th>
<th>$K$-factor</th>
</tr>
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<tbody>
<tr>
<td>NLO</td>
<td>$13.14(2)^{+25.1%}_{-24.4%}$</td>
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<tr>
<td>NLO+NLL</td>
<td>$13.81(2)^{+20.7%}_{-20.1%}$</td>
<td>1.05</td>
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<tr>
<td>NLO+NLL'</td>
<td>$15.16(2)^{+4.3%}_{-11.9%}$</td>
<td>1.15</td>
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<tr>
<td>NLO (QCD+EW)</td>
<td>$13.80(2)^{+22.9%}_{-22.6%}$</td>
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<tr>
<td>NLO (QCD+EW)+NLL'</td>
<td>$15.81(2)^{+3.6%}_{-11.6%}$</td>
<td>1.14</td>
</tr>
</tbody>
</table>

NLO (QCD+EW): EW corrections up to $\mathcal{O}(\alpha^2)$

PDF error: $\pm$ 6.7%

Fixed-order results obtained with
[Frederix, Frixione, Hirschi, Pagani, Shao, Zaro (2018)]
[Alwall, Frederix, Frixione, Hirschi, Maltoni, Mattelaer, Shao, Stelzer, Torrielli, Zaro (2014)]
Top mass $\in [170,175]$ GeV

Error bands: 7-point scale uncertainty

NLO (QCD+EW): EW corrections up to $\mathcal{O}(\alpha^2)$

Fixed-order results obtained with
[Frederix, Frixione, Hirschi, Pagani, Shao, Zaro (2018)]
[Alwall, Frederix, Frixione, Hirschi, Maltoni, Mattelaer, Shao, Stelzer, Torrielli, Zaro (2014)]
CONCLUSIONS

> Soft gluon resummation at NLL’ accuracy

> Significant reduction of the total scale uncertainty

**Preliminary**

Laura Moreno Valero

**Soft gluon resummation for four top quarks**
Soft gluon resummation for the production of four top quarks at the LHC

Laura Moreno Valero

in collaboration with Melissa van Beekveld and Anna Kulesza

Institute for Theoretical Physics, University of Münster

QCD@LHC  IJCLab Orsay  28/11/2022 - 2/12/2022