

Weighing the Top with energy correlators

J. Holguin, I. Moul, A. Pathak, and M. Procura

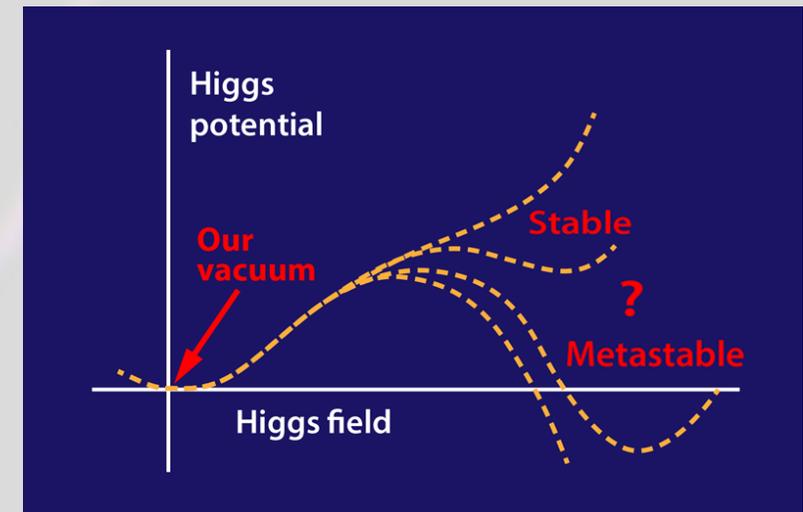
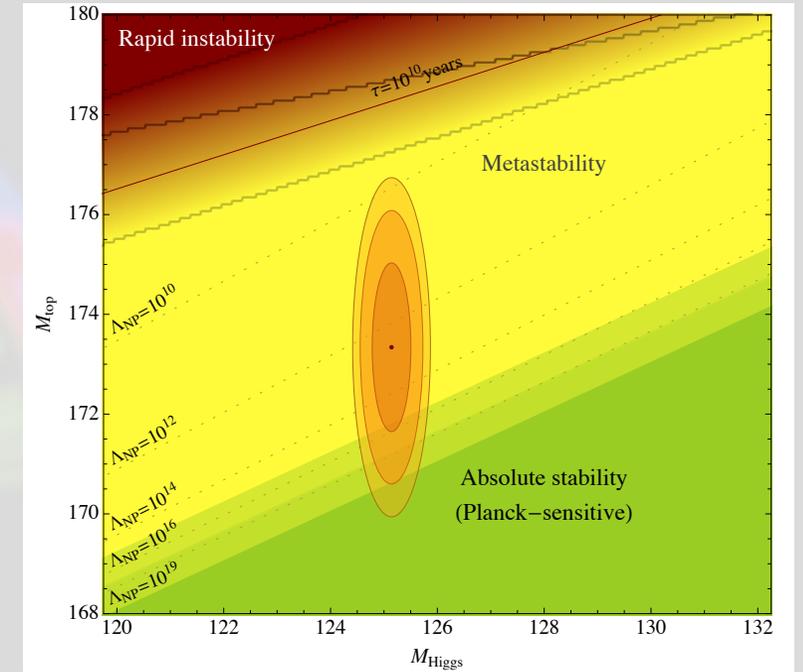
arXiv:2201.08393

cnrs



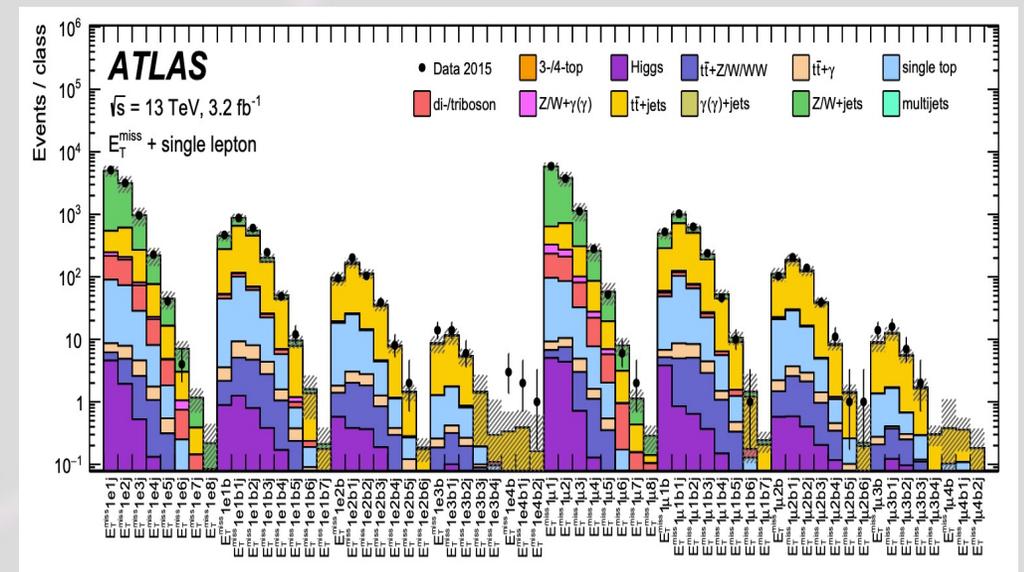
Why the top mass?

- The Top is very interesting!
 - Largest Yukawa coupling – sensitive to new physics.
 - The Top and the Higgs masses determine the stability of the electroweak vacuum. [1307.3536](#), [1408.0292](#)



Why the top mass?

- The Top is very interesting!
 - Largest Yukawa coupling – sensitive to new physics.
 - The Top and the Higgs masses determine the stability of the electroweak vacuum.
- We are in an unprecedented era of high statistics collider physics!
 - Top measurements have transitioned from discovery to precision.



Current status of top mass measurements

- Current world average (HL-LHC projection ~ 200 MeV)

- $m_t = 172.76 \pm 0.30$ GeV [10.1093/ptep/ptaa104](https://arxiv.org/abs/10.1093/ptep/ptaa104)

- An impressive uncertainty ~ 0.2 %!

- Some of the numbers that enter this world average:

- $m_t = 172.67 \pm 0.48$ GeV ATLAS, 1810.01772

- $m_t = 172.26 \pm 0.61$ GeV CMS, 1812.06489

- $m_t = 174.34 \pm 0.64$ GeV Tevatron, 1407.2682

- $m_t = 170.5 \pm 0.8$ GeV CMS, 1904.05237

The only quark with **three masses in PDG**:

Mass (direct measurements) $m = 172.76 \pm 0.30$ GeV ^[a,b] (S = 1.2)

Mass (from cross-section measurements) $m = 162.5^{+2.1}_{-1.5}$ GeV ^[a]

Mass (Pole from cross-section measurements) $m = 172.5 \pm 0.7$ GeV

An orthogonal approach?

A completely orthogonal measurement would be useful to help resolve discrepancies/debates.

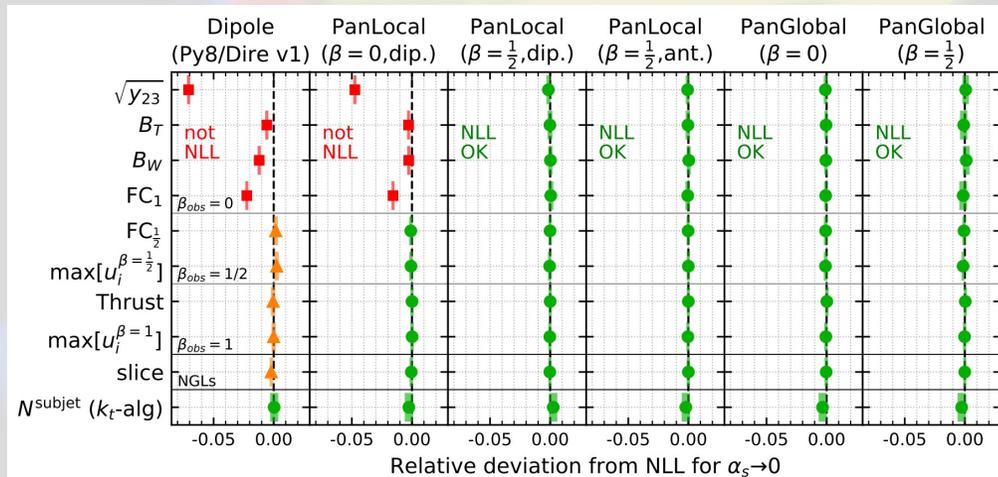
What do we have to work with?

- A (partial*) kinematic breakdown of the particles in each jet.
- A lot of statistics!
 - The HL-LHC will be a top factory.
 - It is forecast that 3Billion ttbar events and 800Million mono-top events will be measured. [1902.04070](#)

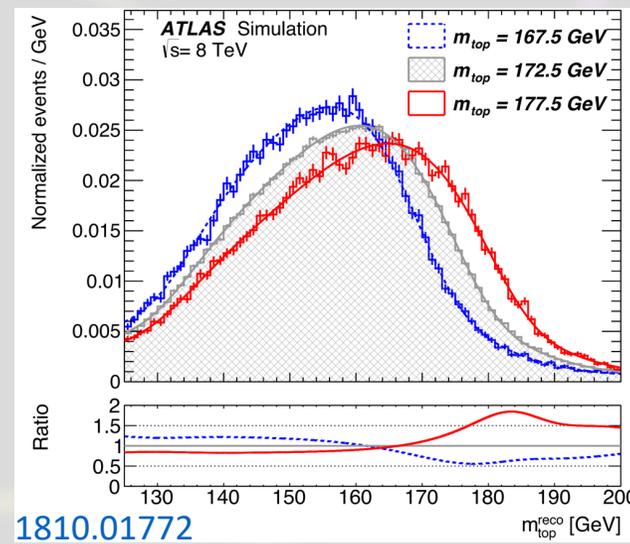
What observables do other fields of physics use when trying to extract simple properties from complicated environments?

The current approach

There has been an exceptionally successful program of precision QCD with event shape observables and algorithmic substructure. Current direct measurements all use this paradigm.



Panscales [arXiv:2002.11114](https://arxiv.org/abs/2002.11114)



[1810.01772](https://arxiv.org/abs/1810.01772)

PDG 2021 global average of direct measurements
 $m_t = 172.76 \pm 0.30 \text{ GeV}$

What else is there?

However, there is an interesting disconnect between much of the developments in precision QCD phenomenology and formal theory.

arXiv:1309.0769v2 [hep-th] 1 May 2014

arXiv:0803.1467v3 [hep-th] 4 May 2008

Conformal collider physics: Energy and charge correlations

Diego M. Hofman^a and Juan Maldacena^b

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^b School of Natural Sciences, Institute for Advanced Study
Princeton, NJ 08540, USA

We study observables in a conformal field theory which are used to describe hadronic events at colliders. We focus on the energies deposited on calorimeters placed at a large angle. We consider initial states produced by an operator insertion and study the properties of the energy correlation functions for conformal field theories. We discuss the small angle singularities of energy correlation functions.

CERN-PH-TH/2013-211
IPhT-T13-210
LAPTh-047/13

From correlation functions to event shapes

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^f Department of Physics, Princeton University
Princeton, NJ 08544, USA

Abstract

We present a new approach to computing event shape distributions or, more precisely, charge flow correlations in a generic conformal field theory (CFT). These infrared finite observables are familiar from collider physics studies and describe the angular distribution of global charges in outgoing radiation created from the vacuum by some source. The charge flow correlations can be expressed in terms of Wightman correlation functions in a certain limit. We explain how to compute these quantities starting from their Euclidean analogues by means of a non-trivial analytic continuation which, in the framework of CFT, can be performed elegantly in Mellin

arXiv:0505088v3 [hep-th] 26 Aug 2019

Light-ray operators in conformal field theory

Petr Kravchuk and David Simmons-Duffin

Walter Burke Institute for Theoretical Physics, Caltech, Pasadena, California 91125, USA

ABSTRACT: We argue that every CFT contains light-ray operators labeled by a continuous spin J . When J is a positive integer, light-ray operators become integrals of local operators over a null line. However for non-integer J , light-ray operators are nominally nonlocal and give the analytic continuation of our construction. We generalize the shape of the integral of a d-dimensional operator of Caron-Huot's L-point functions. CFT correlators, The average null

CALTECH

[hep-th] 25 May 2016

Modular Hamiltonians for Deformed Half-Spaces and the Averaged Null Energy Condition

Thomas Faulkner, Robert G. Leigh, Onkar Parrikar and Huajia Wang

Department of Physics, University of Illinois, 1110 W. Green St., Urbana IL 61801-3080, U.S.A.

Abstract

We study modular Hamiltonians corresponding to the vacuum state for deformed half-spaces in relativistic quantum field theories on $\mathbb{R}^{1,d-1}$. We show that in addition to

The light-ray OPE and conformal colliders

Murat Koloğlu^a, Petr Kravchuk^b, David Simmons

^a Walter Burke Institute for Theoretical Physics, Caltech

^b School of Natural Sciences, Institute for Advanced Study

^c CERN, Theoretical Physics Department, 1211 Geneva

ABSTRACT: We derive a nonperturbative, convergent null-integrated operators on the same null plane in expansion are light-ray operators, whose matrix elements are Lorentzian inversion formula. For example, a p -operator has an expansion in the light-ray operators. An important application is to collider event shapes. We derive a new expansion for event shapes in special functions. We apply the celestial block expansion to Yang-Mills theory. Using known OPE data, we find both at weak and strong coupling, and make new predictions for loops (NNLO).

arXiv:1610.05308v1 [hep-th] 17 Oct 2016

Averaged Null Energy Condition from Causality

Thomas Hartman, Sandipan Kundu, and Amirhossein Tajdini

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Abstract

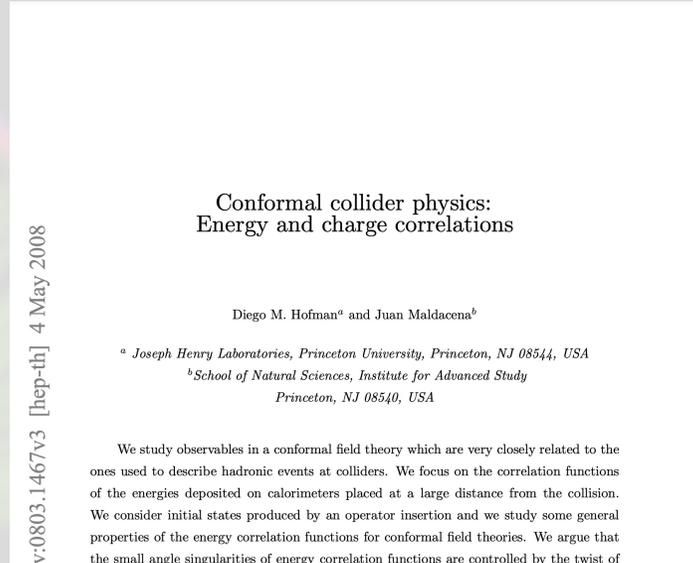
Unitary, Lorentz-invariant quantum field theories in flat spacetime obey microcausality: commutators vanish at spacelike separation. For interacting theories in more than two dimensions, we show that this implies that the averaged null energy, $\int du T_{uu}$, must be positive. This non-local operator appears in the operator product expansion of local operators in the lightcone limit, and therefore contributes to n -point functions. We derive a sum rule that isolates this contribution and is manifestly positive. The argument also applies to certain higher spin operators other than the stress tensor, generating an infinite family of new constraints of the form $\int du X_{uv} \dots \geq 0$. These lead to new inequalities for the coupling constants of spinning operators in conformal field theory, which include as special cases (but are generally stronger than) the existing constraints from the lightcone bootstrap, deep inelastic scattering, conformal collider methods, and relative entropy. We also comment on the relation to the recent derivation of the averaged null energy condition from relative entropy, and suggest a more

The middle ground

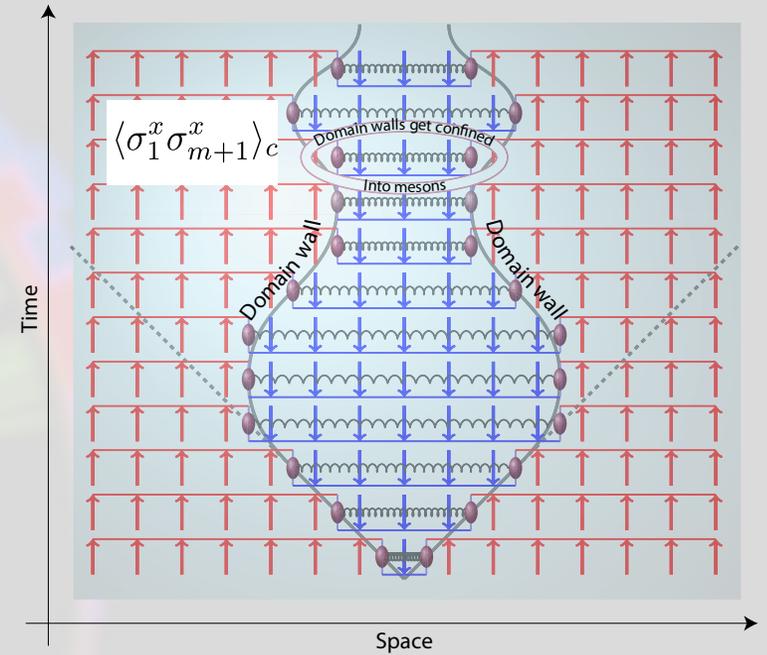
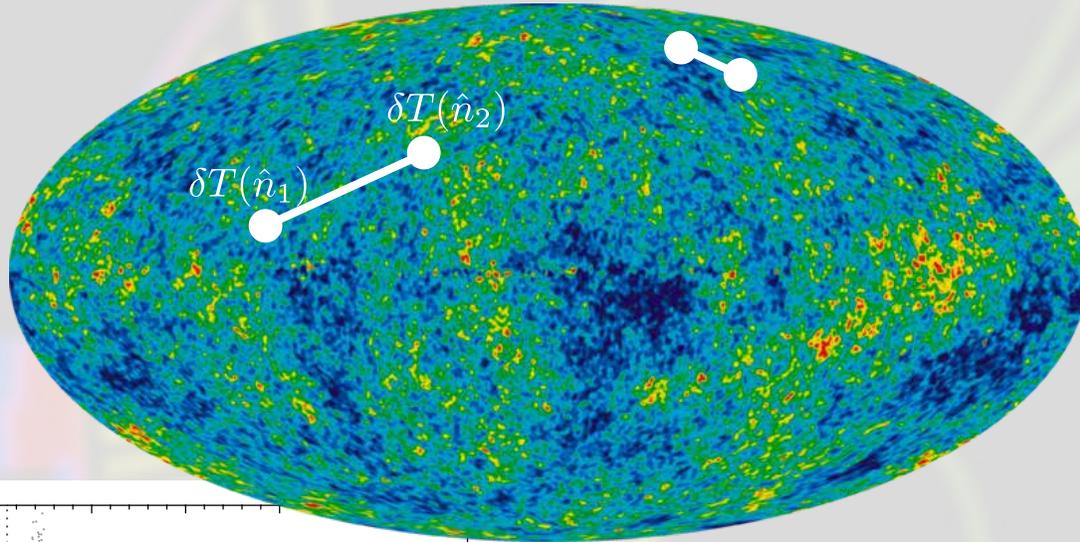
$$\mathcal{E}(\vec{n}) = \lim_{r \rightarrow \infty} \int_0^{\infty} dt r^2 n^i T_{0i}(t, r\vec{n})$$

The energy flow operator is an ANEC operator, it can be computed directly in CFTs admitting many new computational techniques.

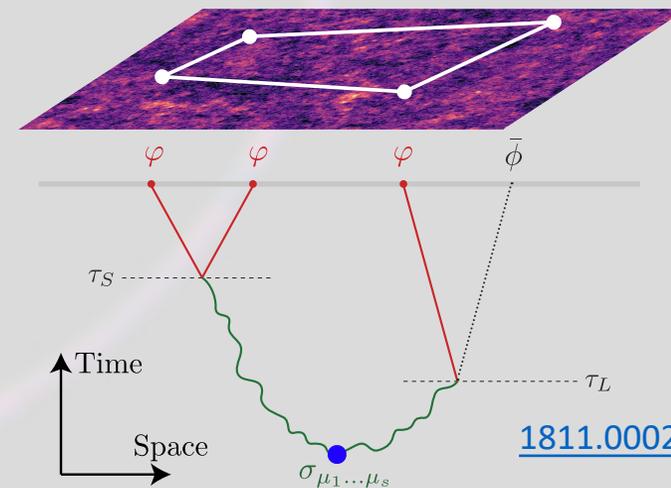
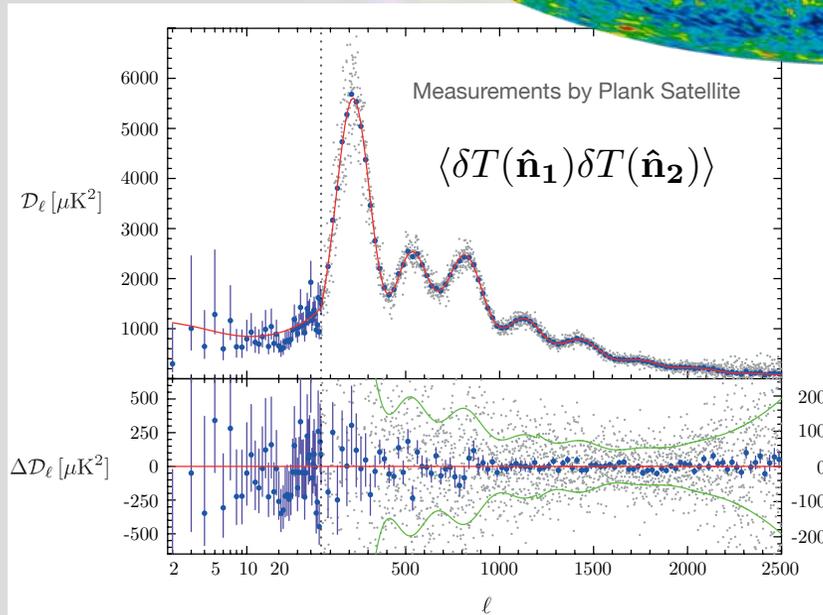
Its correlation functions are also directly a collider observables – Energy Correlators.



Correlation Functions



[1604.03571](#)



[1811.00024](#), [1503.08043](#)

Correlation Functions

Recap

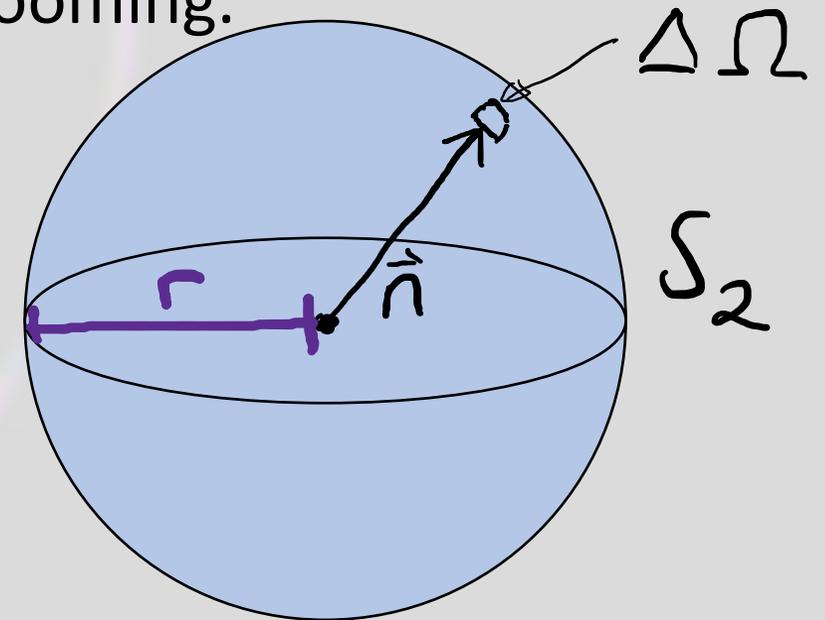
- Correlation functions in statistics:
 - $\text{Corr}_2(X, Y) = \langle XY \rangle - \langle X \rangle \langle Y \rangle$ (also just the covariance)
 - $\text{Corr}_3(X, Y, Z) = \langle XYZ \rangle - \langle X \rangle (\langle Y \rangle \langle Z \rangle - \text{Corr}_2(Y, Z))$
 - ...
- In physics we usually refer to $\langle X_1 \dots X_n \rangle$ as an n point correlator. This is just conventional and has origins in that often $\langle X_i \rangle = 0$.
- Time ordered QFT correlators (propogators) relate back to these statistical correlators through the path integral and statistical mechanics...

Energy Correlators

- Generally one can define correlators of any quantum charge or conserved quantity.
- For QCD, correlators of energy flux are usually of most interest – these naturally remove soft physics without grooming.

$$\mathcal{E}(\vec{n}) = \lim_{r \rightarrow \infty} \int_0^{\infty} dt r^2 n^i T_{0i}(t, r\vec{n})$$

$$\mathcal{E}(\vec{n}) \approx \int_0^{\infty} dt E_{\text{flux through } \Delta\Omega}(t)$$

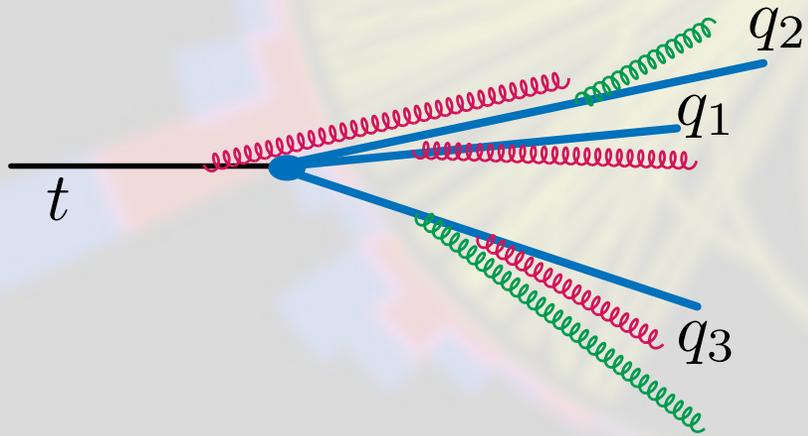


Energy Correlators

$$\mathcal{E}(\vec{n}) = \lim_{r \rightarrow \infty} \int_0^\infty dt r^2 n^i T_{0i}(t, r\vec{n})$$

$$\langle \mathcal{E}(\vec{n}_1) \mathcal{E}(\vec{n}_2) \rangle = \sum_{ij} \int \frac{d\sigma_{ij}}{d^2\vec{n}_i d^2\vec{n}_j} E_i E_j \delta^2(\vec{n}_1 - \vec{n}_i) \delta^2(\vec{n}_2 - \vec{n}_j)$$

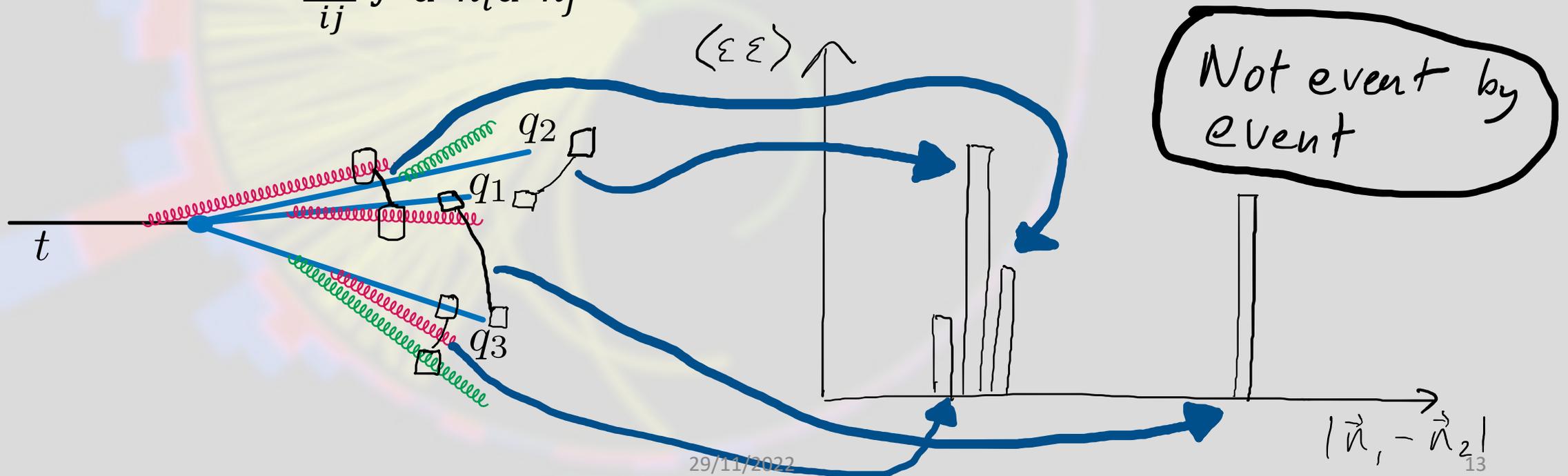
inclusive cross section to produce two particles, ij , and anything else!



Energy Correlators

$$\mathcal{E}(\vec{n}) = \lim_{r \rightarrow \infty} \int_0^\infty dt r^2 n^i T_{0i}(t, r\vec{n})$$

$$\langle \mathcal{E}(\vec{n}_1) \mathcal{E}(\vec{n}_2) \rangle = \sum_{ij} \int \frac{d\sigma_{ij}}{d^2\vec{n}_i d^2\vec{n}_j} E_i E_j \delta^2(\vec{n}_1 - \vec{n}_i) \delta^2(\vec{n}_2 - \vec{n}_j)$$



Energy Correlators

Pros:

- Defined on inclusive cross-sections and can be made insensitive to soft radiation. Textbook example of where pp CSS factorisation can be used without any violation. [2109.03665](#)

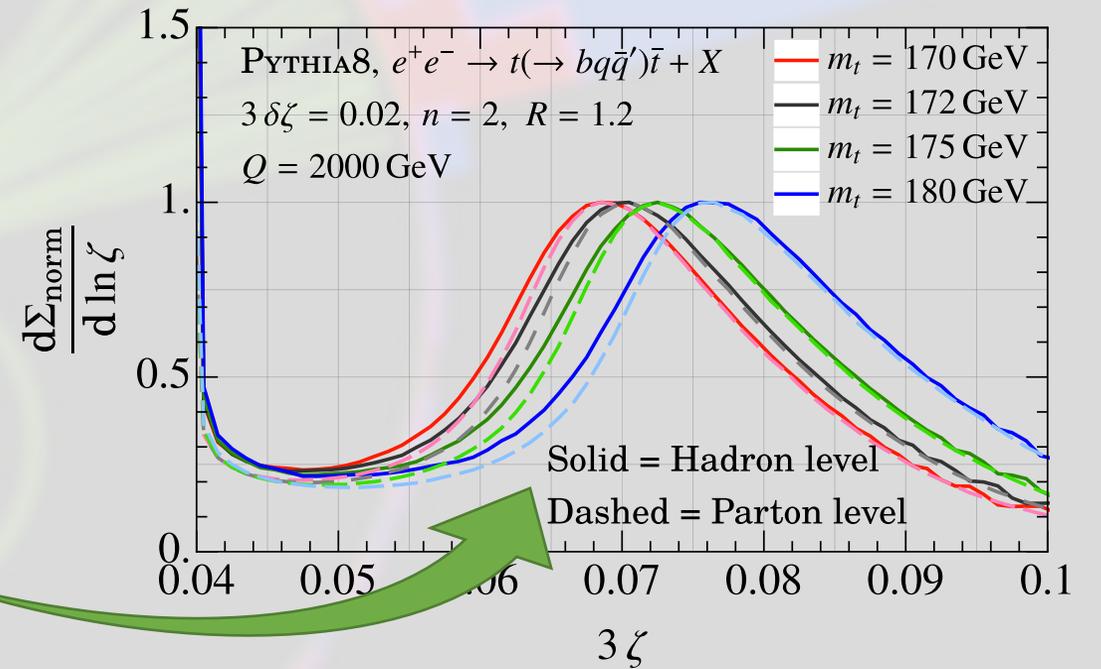
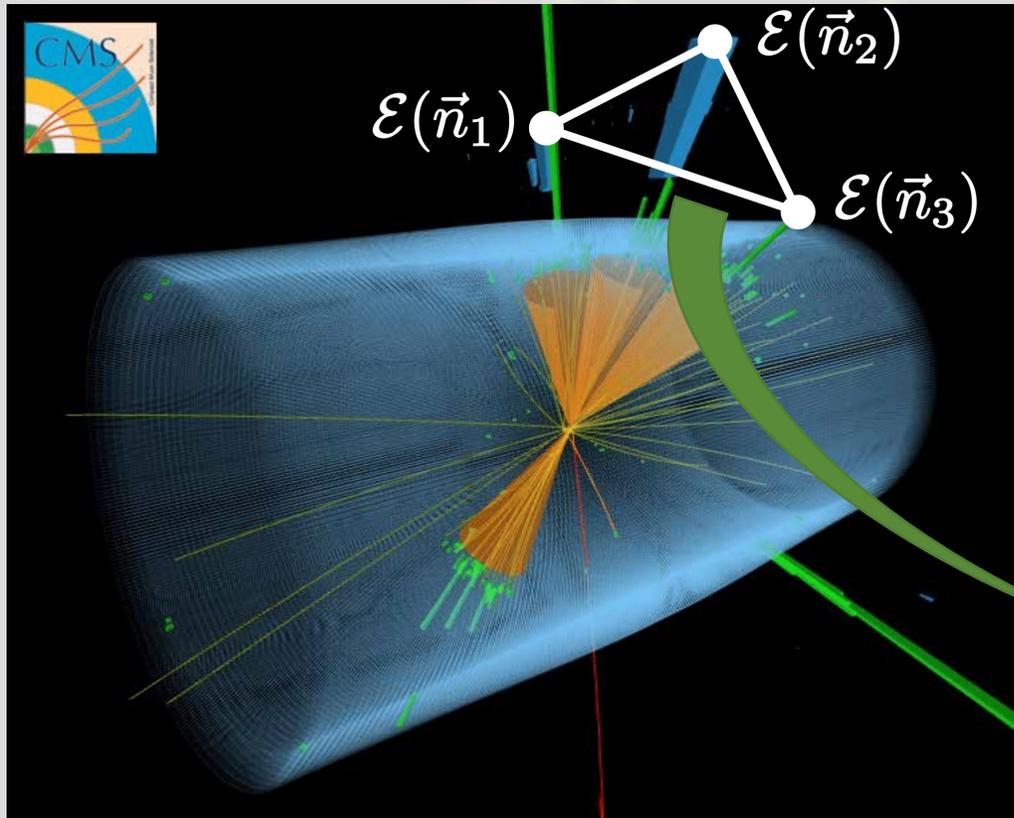
$$\frac{d\sigma}{d\zeta} = \int dE_J E_J^2 H(E_J) J_{\text{EEC}}(\zeta, E_J) + \text{power corrections},$$

- Well studied by CFT community. Powerful techniques exist for calculations: light-ray OPE, celestial Blocks, lorentzian inversion. [2202.04085](#)

Cons:

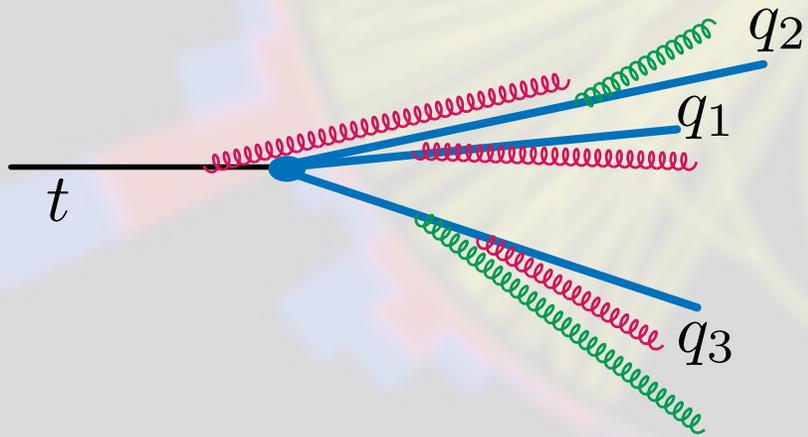
- Reliant on high stats. A precision tool, not a typical discovery tool.
- Not event-by-event so cannot be directly used to tag.

Part 3: Energy Correlators for Tops



Energy Correlators for Tops

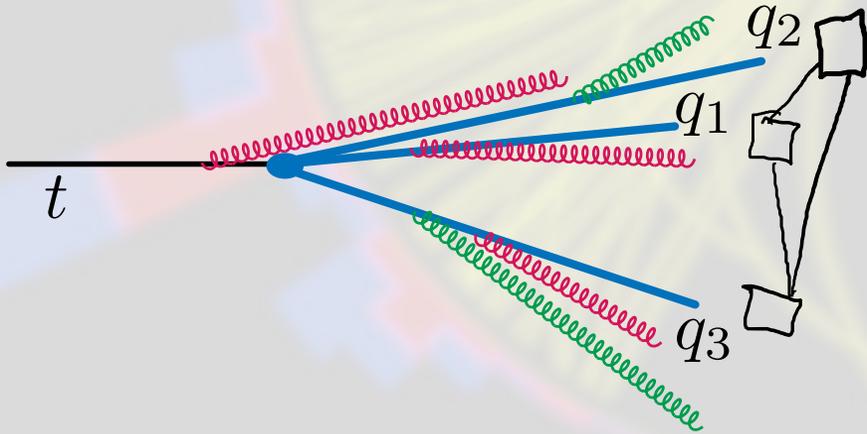
Which correlator will well characterise the top decay?



Energy Correlators for Tops

Which correlator will well characterise the top decay?

$$\langle \mathcal{E}(\vec{n}_1) \mathcal{E}(\vec{n}_2) \mathcal{E}(\vec{n}_3) \rangle = \sum_{ij} \int \frac{d\sigma_{ijk}}{d^2\vec{n}_i d^2\vec{n}_j d^2\vec{n}_k} E_i E_j E_k \delta^2(\vec{n}_1 - \vec{n}_i) \delta^2(\vec{n}_2 - \vec{n}_j) \delta^2(\vec{n}_2 - \vec{n}_k)$$



Energy Correlators for Tops

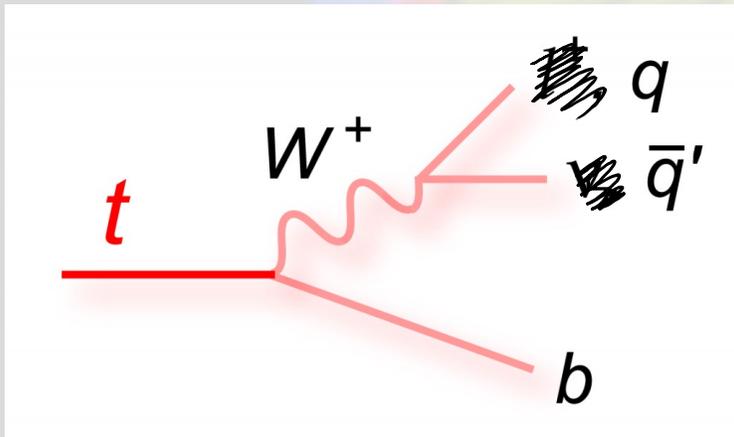
The correlator is sensitive to the angles between the decay products. What angles do we expect to see at fixed order?

I'm going to sketch a 'back of the envelope' calculation which gives intuition for the observable.

3-body kinematics

The correlator is sensitive to the angles between the decay products. What angles do we expect to see at fixed order?

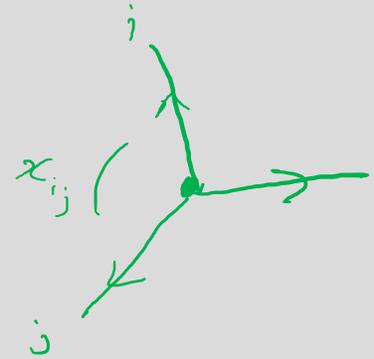
$$\{ ij = \frac{p_i \cdot p_j}{2} = \frac{(1 - \cos \theta_{ij})}{2} \approx \frac{\theta_{ij}^2}{4}$$



3-body kinematics

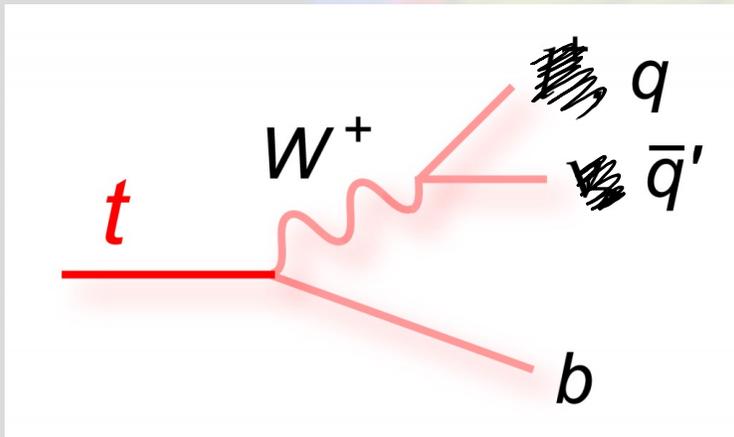
The correlator is sensitive to the angles between the decay products. What angles do we expect to see at fixed order?

$$\{_{ij} = \frac{p_i \cdot p_j}{2} = \frac{(1 - \cos \theta_{ij})}{2} \approx \frac{\theta_{ij}^2}{4}$$



In the Top rest frame

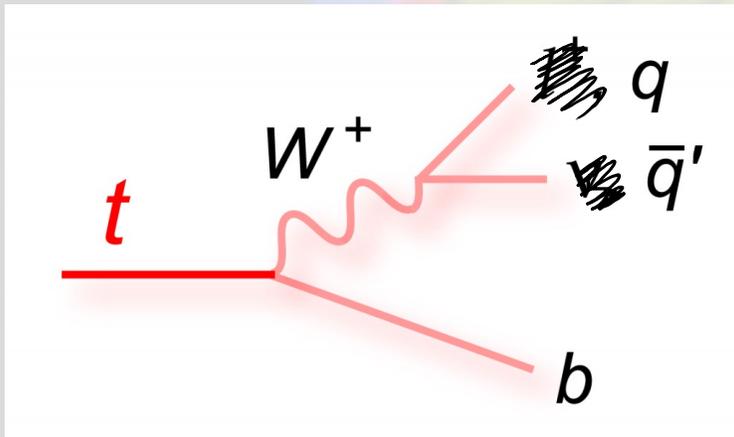
$$\{_{12} + \{_{23} + \{_{31} \in [2, 2.25]$$



3-body kinematics

The correlator is sensitive to the angles between the decay products. What angles do we expect to see at fixed order?

One can find the boost from the Top rest frame to the lab frame where we measure angles $\tilde{\zeta}_{ij}$. We find that



$$\sum_{ij} \tilde{\zeta}_{ij} \approx \left(\frac{m_t}{Q}\right)^2 \sum_{ij} \zeta_{ij}$$

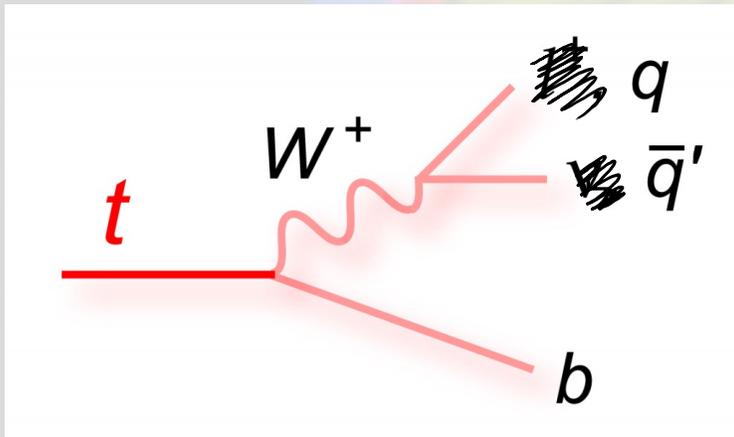
Building the observable

The correlator is sensitive to the angles between the decay products. What do we expect to see at fixed order?

Fixed order teaches us that look at

$$G(\{ \}) = \int d^2 \vec{n}_1 d^2 \vec{n}_2 d^2 \vec{n}_3 \langle \mathcal{E}(\vec{n}_1) \mathcal{E}(\vec{n}_2) \mathcal{E}(\vec{n}_3) \rangle$$

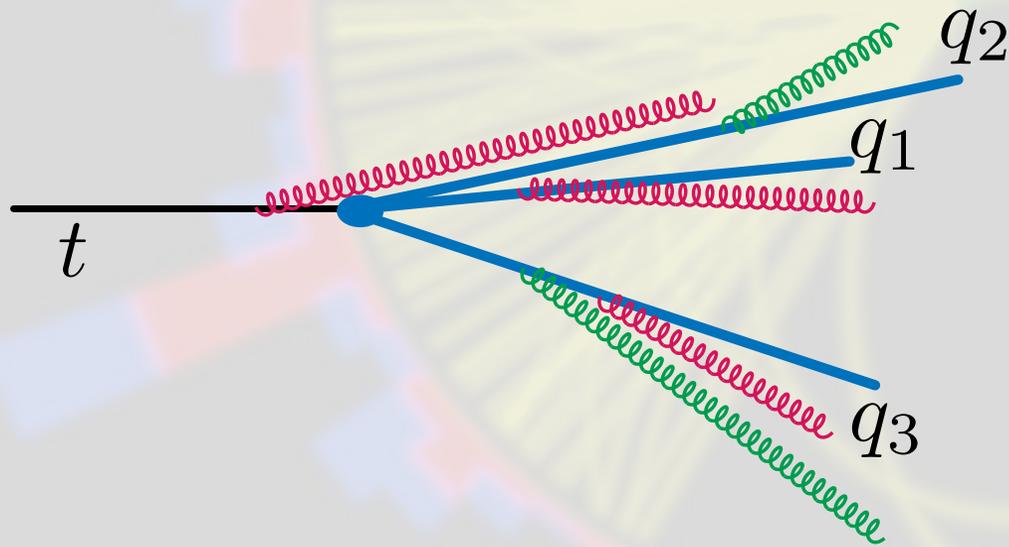
$$\times \delta(\{ \} - |\vec{n}_1 - \vec{n}_2| - |\vec{n}_2 - \vec{n}_3| - |\vec{n}_3 - \vec{n}_1|).$$



Doing the average over \vec{n}_i
we find $\langle \{ \} \rangle \approx 3 \frac{m_c^2}{Q^2}$

Building the observable

What about higher order perturbative corrections?

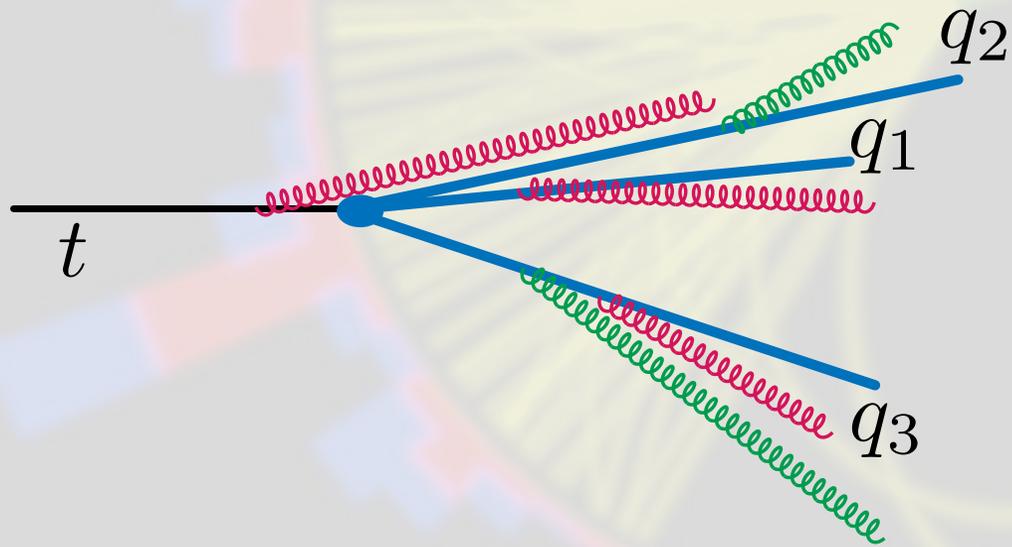


We want to preserve
the $\langle Z \rangle = 3 \frac{m_t^2}{Q^2}$ dependence!

Energy correlators not
sensitive to soft physics,
but will pick up collinear.
How to minimise?

Building the observable

What about higher order perturbative corrections?



Solution :

require $|\vec{n}_1 - \vec{n}_2| \approx |\vec{n}_2 - \vec{n}_3|$
 $\approx |\vec{n}_3 - \vec{n}_1|$

that way we never have
a small angle in the sum
over i, j around $\xi = \frac{3m^2}{Q^2}$.

Building the observable

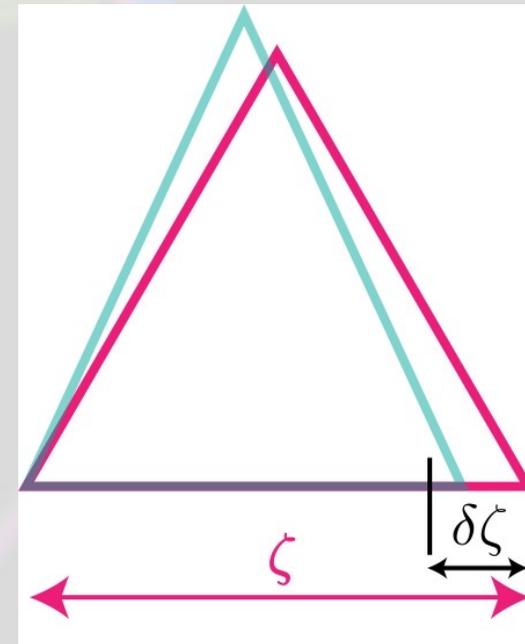
In all, we have...

$$\frac{d\Sigma(\delta\zeta)}{dQd\zeta} = \int d\zeta_{12}d\zeta_{23}d\zeta_{31} \int d\sigma \widehat{\mathcal{M}}_{\Delta}^{(n)}(\zeta_{12}, \zeta_{23}, \zeta_{31}, \zeta, \delta\zeta), \quad (4)$$

where the measurement operator $\widehat{\mathcal{M}}_{\Delta}^{(n)}$ is

$$\begin{aligned} \widehat{\mathcal{M}}_{\Delta}^{(n)}(\zeta_{12}, \zeta_{23}, \zeta_{31}, \zeta, \delta\zeta) &= \widehat{\mathcal{M}}^{(n)}(\zeta_{12}, \zeta_{23}, \zeta_{31}) \quad (5) \\ &\times \delta(3\zeta - \zeta_{12} - \zeta_{23} - \zeta_{31}) \prod_{l,m,n \in \{1,2,3\}} \Theta(\delta\zeta - |\zeta_{lm} - \zeta_{mn}|). \end{aligned}$$

$$\begin{aligned} \widehat{\mathcal{M}}^{(n)}(\zeta_{12}, \zeta_{23}, \zeta_{31}) &= \quad (2) \\ &\sum_{i,j,k} \frac{E_i^n E_j^n E_k^n}{Q^{3n}} \delta(\zeta_{12} - \hat{\zeta}_{ij}) \delta(\zeta_{23} - \hat{\zeta}_{ik}) \delta(\zeta_{31} - \hat{\zeta}_{jk}). \end{aligned}$$

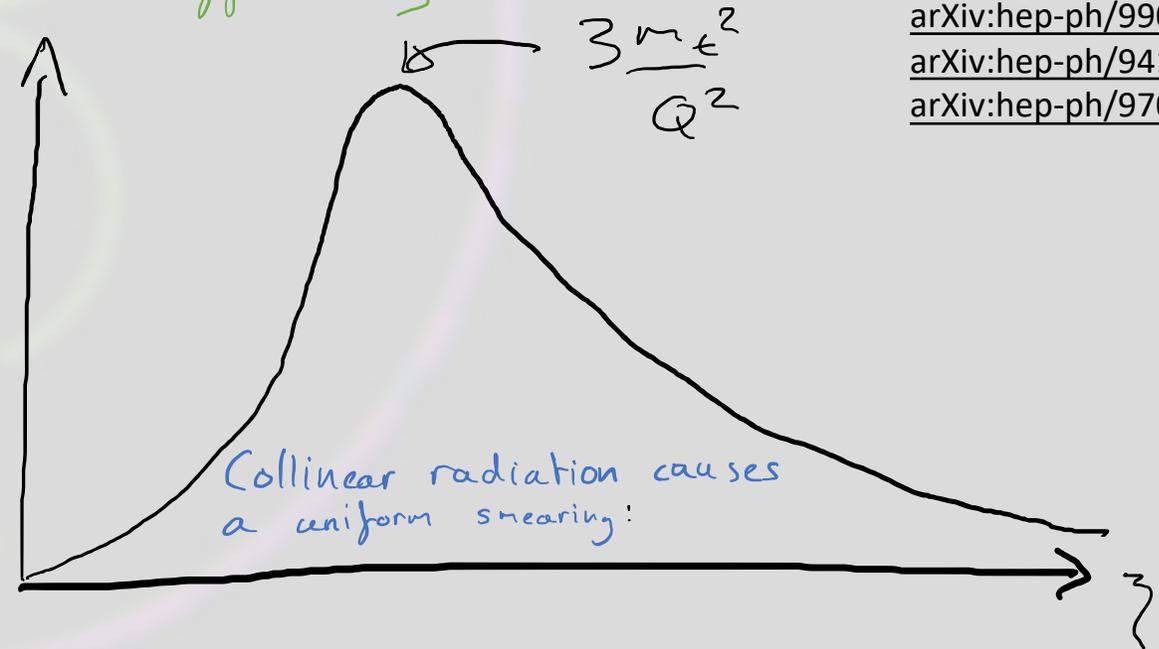
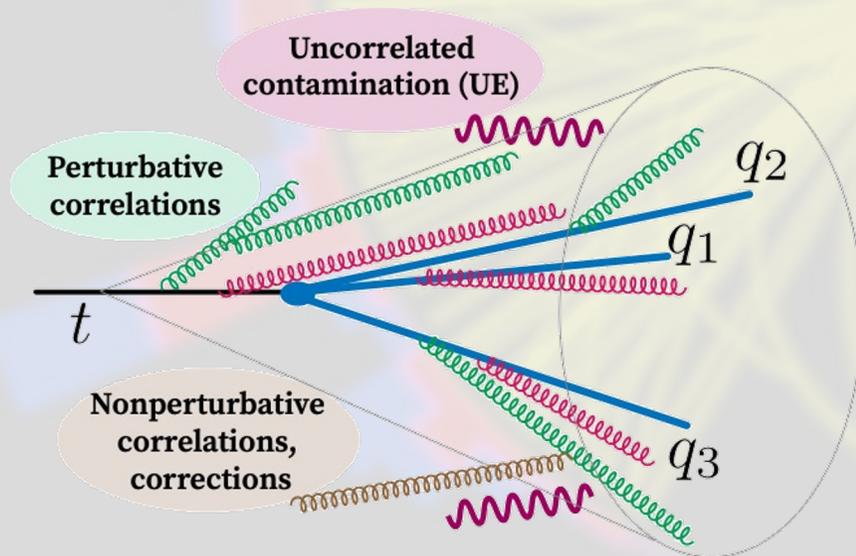


Understanding the distribution

What will the distribution look like with N.P. corrections?

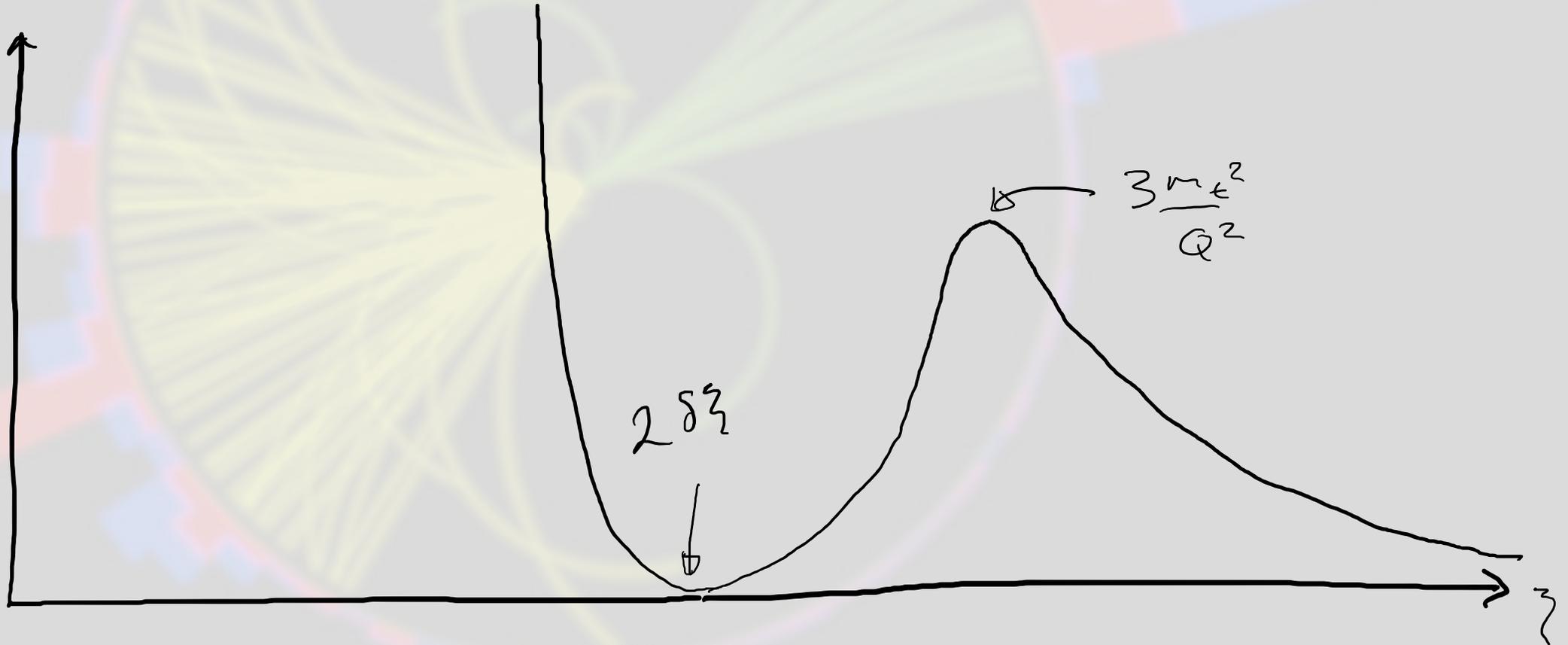
We know from other studies of energy correlators that N.P. corrections are an additive power law. Effectively a change in normalisation.

[arXiv:hep-ph/9902341](https://arxiv.org/abs/hep-ph/9902341)
[arXiv:hep-ph/9411211](https://arxiv.org/abs/hep-ph/9411211)
[arXiv:hep-ph/9708346](https://arxiv.org/abs/hep-ph/9708346)



Understanding the distribution

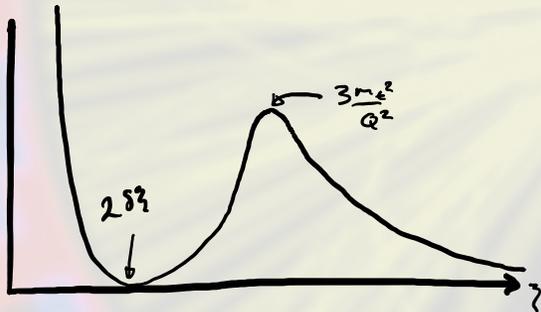
What is the effect of the asymmetry in the triangle?



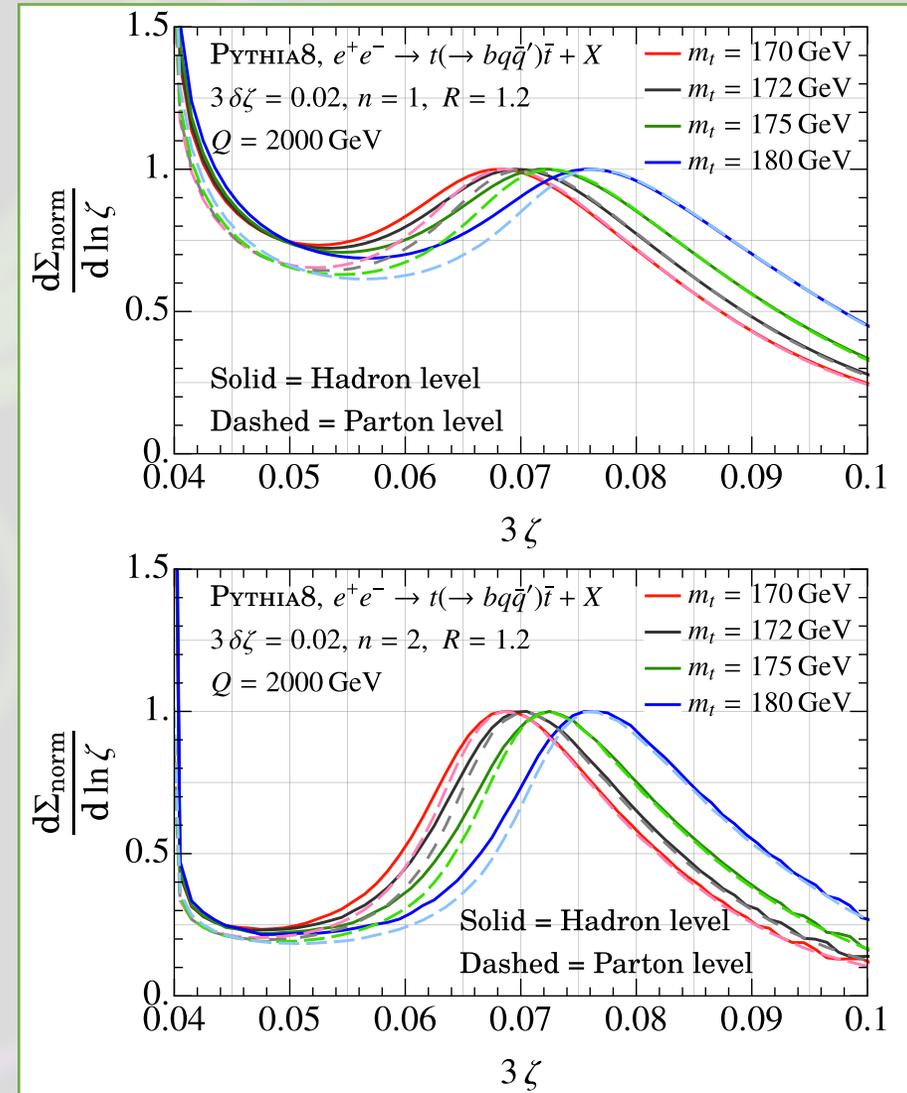
Simulation in Pythia8

We did a complete simulated pp analysis, however for this talk consider e^+e^- where the hard scale Q is just half the CoM Energy.

- Key features exactly as expected.



- Peak is sensitive to Top mass.
- Very low sensitivity to hadronisation. The shift is equivalent to $\Delta m_t = 150 \pm 50 \text{ MeV}$.



Simulation in Pythia8

Now consider hadron colliders. Must now use boost invariant quantities.

- Q is now (initially) the partonic Top p_T . This is not a measurable quantity. Instead we have the p_T of the Top jet. This adds a little complexity.

$$\frac{d\Sigma(\delta\zeta)}{dp_{T,\text{jet}}d\zeta} = \frac{d\Sigma(\delta\zeta)}{dp_{T,t}d\zeta} \frac{dp_{T,t}}{dp_{T,\text{jet}}}$$

- Now must consider underlying event.
- Can we measure on tracks? (Yes)

$$\widehat{\mathcal{M}}^{(n)}(\zeta_{12}, \zeta_{23}, \zeta_{31}) = \sum_{i,j,k} \frac{E_i^n E_j^n E_k^n}{Q^{3n}} \delta(\zeta_{12} - \hat{\zeta}_{ij}) \delta(\zeta_{23} - \hat{\zeta}_{ik}) \delta(\zeta_{31} - \hat{\zeta}_{jk}). \quad (2)$$



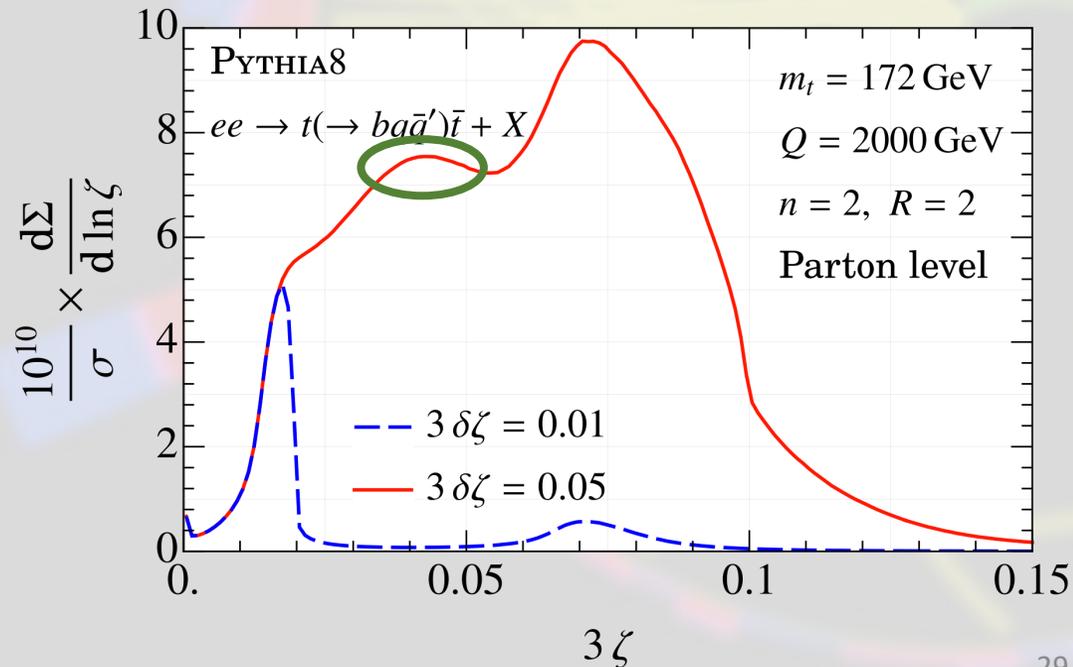
$$\widehat{\mathcal{M}}_{(pp)}^{(n)}(\zeta_{12}, \zeta_{23}, \zeta_{31}) = \sum_{i,j,k \in \text{jet}} \frac{(p_{T,i})^n (p_{T,j})^n (p_{T,k})^n}{(p_{T,\text{jet}})^{3n}} \times \delta(\zeta_{12} - \hat{\zeta}_{ij}^{(pp)}) \delta(\zeta_{23} - \hat{\zeta}_{ik}^{(pp)}) \delta(\zeta_{31} - \hat{\zeta}_{jk}^{(pp)}), \quad (7)$$

where $\hat{\zeta}_{ij}^{(pp)} = \Delta R_{ij}^2 = \sqrt{\Delta\eta_{ij}^2 + \Delta\phi_{ij}^2}$, with η, ϕ the standard rapidity, azimuth coordinates.

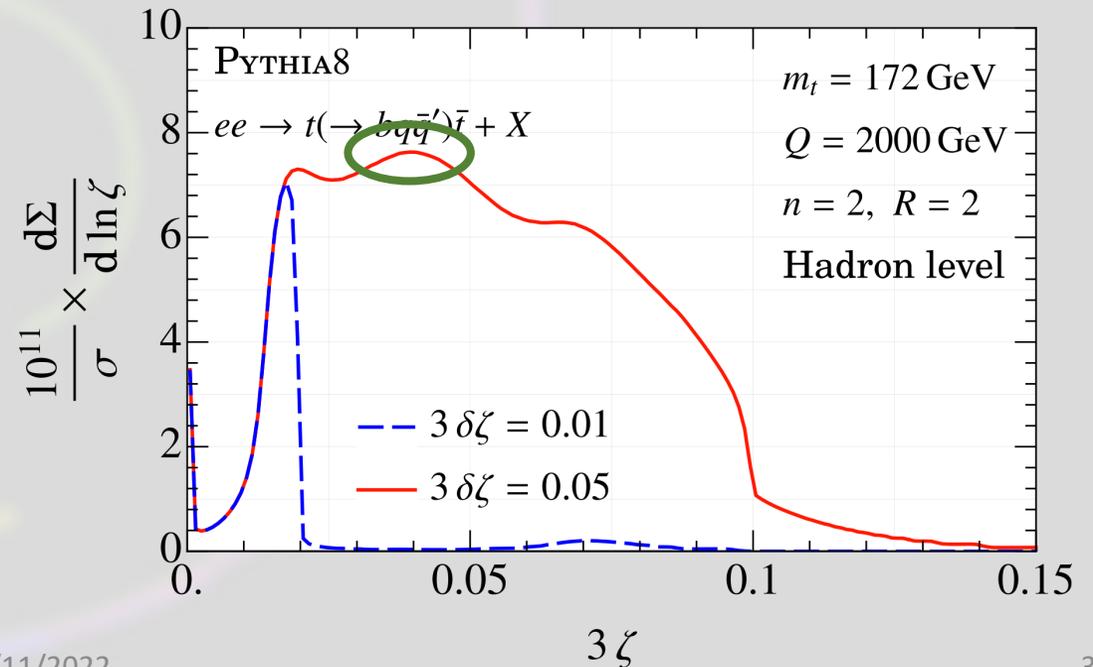
Further improved?

Using the equatorial configuration we projected onto the top peak. However the W also imprints on the correlator in a different part of the parameter space.

The distribution $\frac{d\Sigma(\delta\zeta)}{d\zeta_W d\zeta}$ is independent of the pt distribution, gives $m_t(m_W)$.



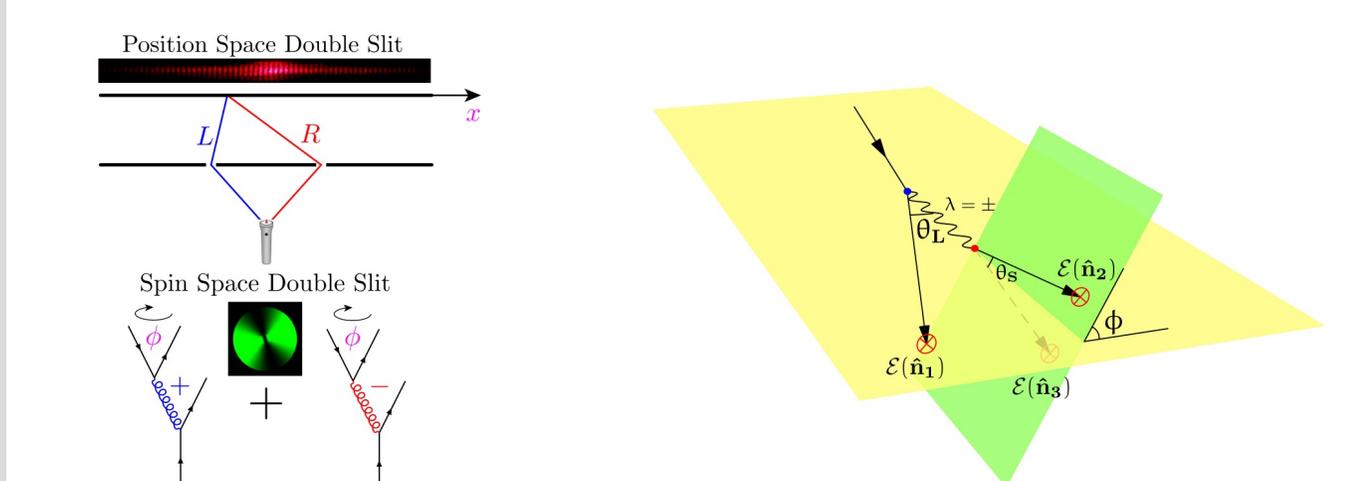
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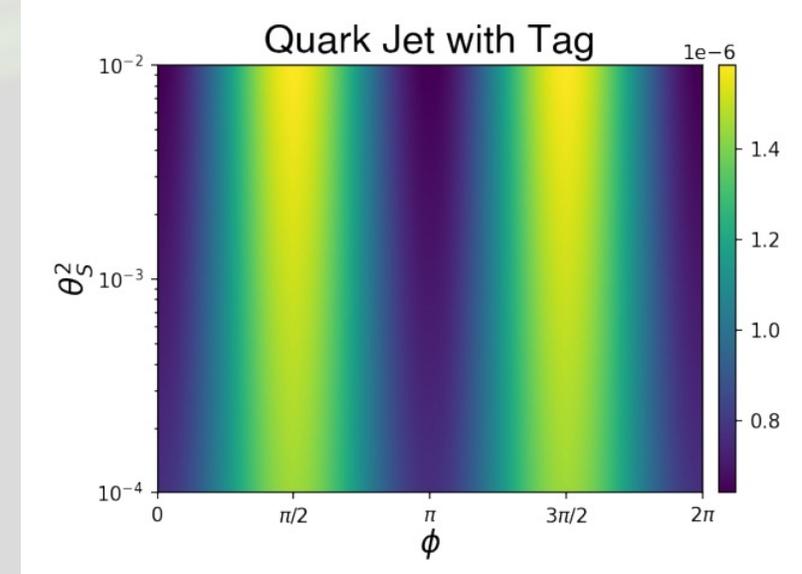
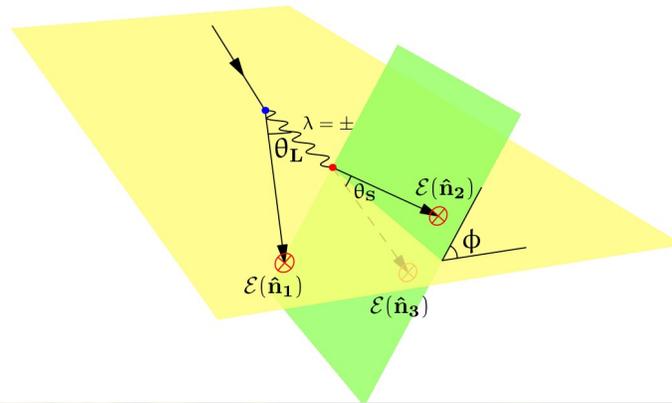
30

More than just mass?

- 3-point correlator is sensitive to spin.



Chen et al. arXiv:2011.02492



Outlook

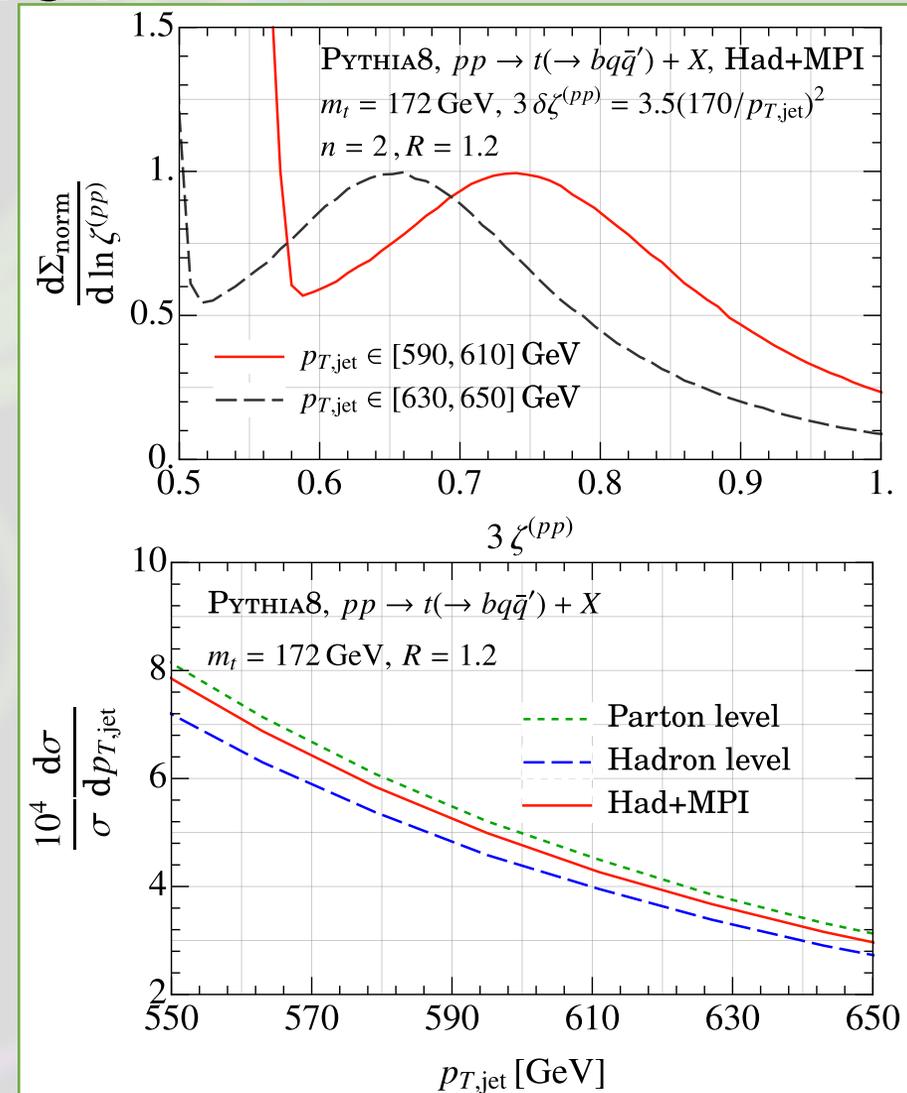
- The three-point energy correlator shows promise as a top mass sensitive observable with theoretical control comparable to the current precision of direct measurements.
- So far studies have been proof-of-concept. Experimental feasibility study is underway, as are precision calculations.
 - Much of the ingredients needed for a precision calculation already exist. Missing pieces are the EEEC jet function and a broader study of factorisation.
- Other studies possible: spin correlations, new physics?

Supplementary material

Stage 2

Let us study the hadron collider environment in two parts.

1. First study the observable whilst unphysically fixing the partonic Top p_T . This is to answer:
 - Now must consider underlying event.
 - Can we measure on tracks? (Yes)
2. Then study the physical observable and in conjunction with the p_T spectrum.

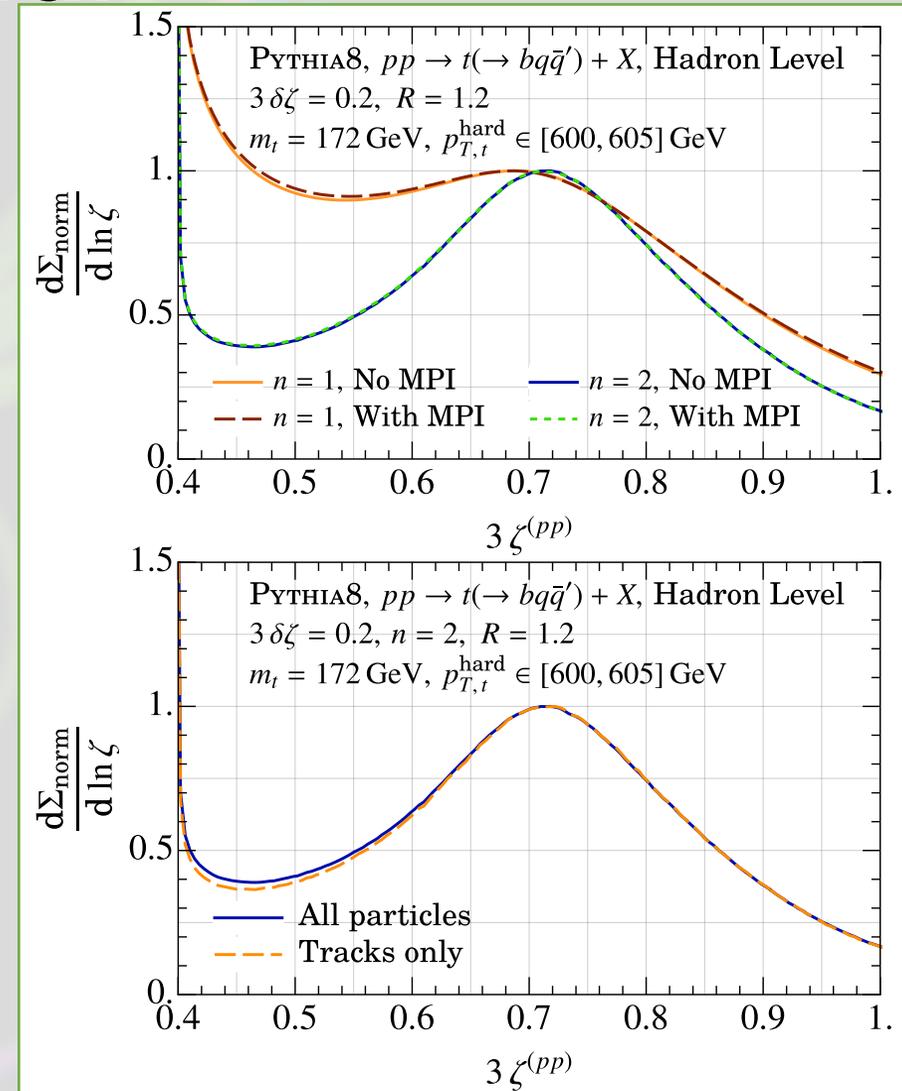


Supplementary material

Stage 1

Let us study the hadron collider environment in two parts.

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Supplementary material

Stage 2

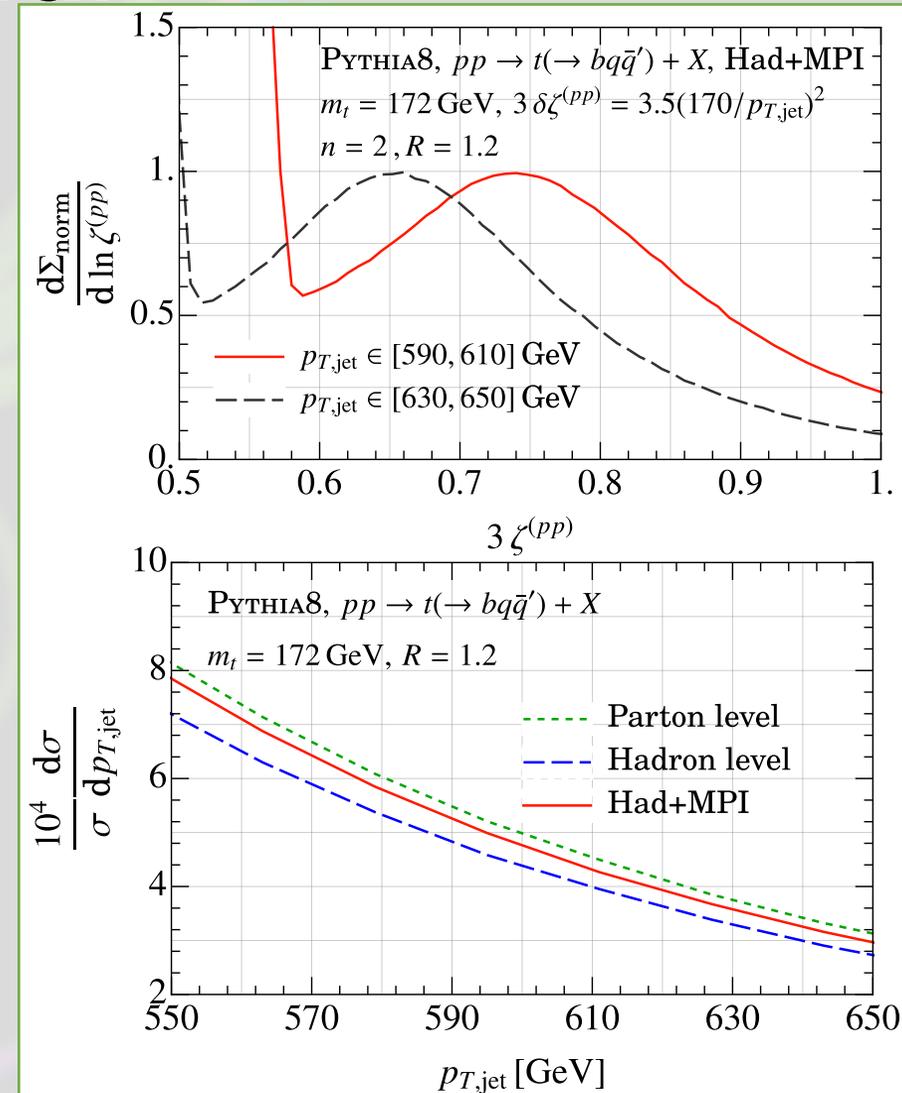
How to handle these p_T shifts? One method:

- Fixed order gives

$$\zeta_{\text{peak}}^{(pp)} \approx 3m_t^2 / p_{T,t}^2$$

- From Factorisation properties of the observable,

$$\zeta_{\text{peak}}^{(pp)} = \frac{3F_{\text{pert}}(m_t, p_{T,\text{jet}}, \alpha_s, R)}{(p_{T,\text{jet}} + \Delta_{\text{NP}}(R) + \Delta_{\text{MPI}}(R))^2}.$$



Supplementary material

Stage 2

How to handle these p_T shifts? One method:

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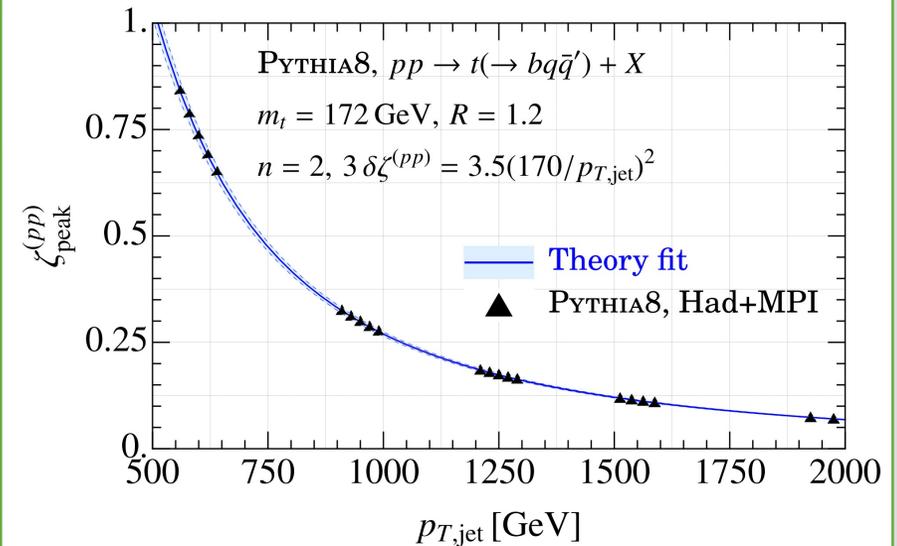
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PYTHIA8 m_t	Parton $\sqrt{F_{\text{pert}}}$	Hadron + MPI $\sqrt{F_{\text{pert}}}$
172 GeV	172.6 ± 0.3 GeV	$172.3 \pm 0.2 \pm 0.4$ GeV
173 GeV	173.5 ± 0.3 GeV	$173.6 \pm 0.2 \pm 0.4$ GeV
175 GeV	175.5 ± 0.4 GeV	$175.1 \pm 0.3 \pm 0.4$ GeV
173 – 172	0.9 ± 0.4 GeV	1.3 ± 0.6 GeV
175 – 172	2.9 ± 0.5 GeV	2.8 ± 0.6 GeV

TABLE I: Values of the effective parameter $F_{\text{pert}}(m_t)$ extracted at parton level, and hadron+MPI level. The consistency of the two approaches provides a measure of our uncertainty due to non-perturbative corrections.



Supplementary material

1. Parameterize the all orders peak position:

$$\zeta_{\text{peak}}^{(pp)} = 3(1 + \mathcal{O}(\alpha_s)) \frac{m_t^2}{f(p_{T,\text{jet}}, m_t, \alpha_s, \Lambda_{\text{QCD}})^2} \equiv 3(1 + \mathcal{O}(\alpha_s)) \frac{m_t^2}{(p_{T,\text{jet}} + \Delta(p_{T,\text{jet}}, m_t, \alpha_s, \Lambda_{\text{QCD}}))^2}$$

2. Work with

$$\rho^2(\zeta_{\text{peak}}^{(pp)v}, p_{T,\text{jet}}^v) = \left(\zeta_{\text{peak}}^{(pp)\text{ref}} - \zeta_{\text{peak}}^{(pp)v} \right) \left(\frac{3(1 + \mathcal{O}(\alpha_s))}{(p_{T,\text{jet}}^v)^2} - \frac{3(1 + \mathcal{O}(\alpha_s))}{(p_{T,\text{jet}}^{\text{ref}})^2} \right)^{-1},$$

3. Define

$$\Delta^{\text{ref}} \equiv \Delta(p_{T,\text{jet}}^{\text{ref}}, m_t, \alpha_s, \Lambda_{\text{QCD}}), \quad \Delta^v(p_{T,\text{jet}}^v - p_{T,\text{jet}}^{\text{ref}}, m_t, \alpha_s, \Lambda_{\text{QCD}}) \equiv \Delta(p_{T,\text{jet}}^v, m_t, \alpha_s, \Lambda_{\text{QCD}}) - \Delta^{\text{ref}}$$

4. Solve for ρ :

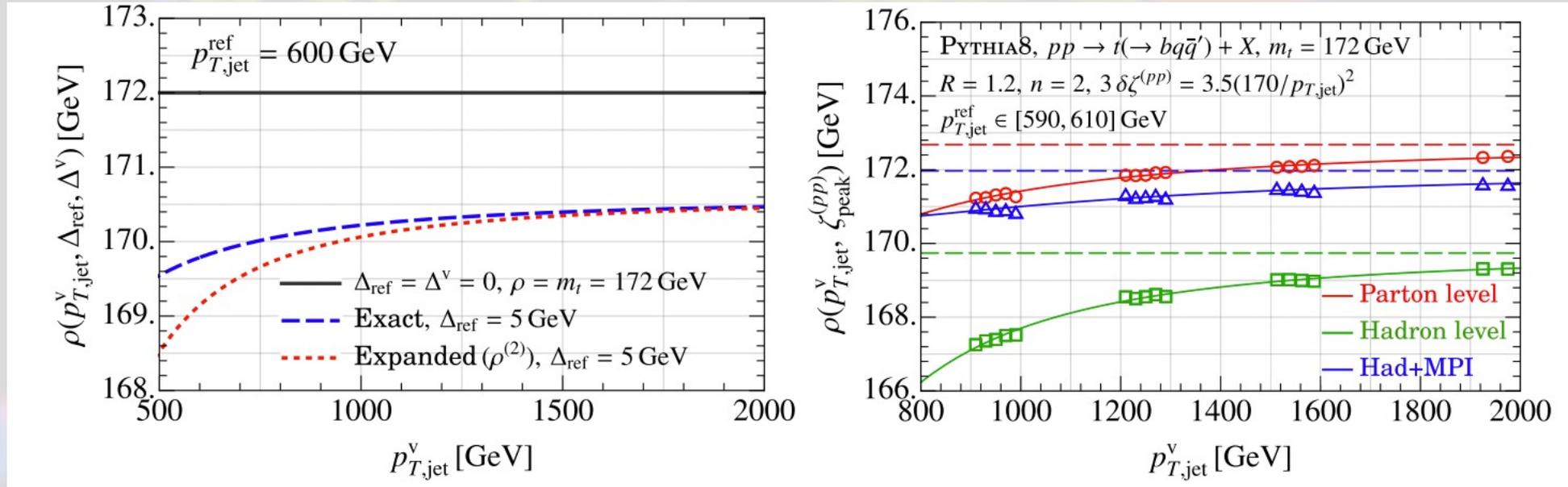
$$\rho(p_{T,\text{jet}}^v, \Delta^{\text{ref}}, \Delta^v) = \sqrt{F_{\text{pert}}} \frac{p_{T,\text{jet}}^{\text{ref}}}{p_{T,\text{jet}}^{\text{ref}} + \Delta^{\text{ref}}} \left(1 - \frac{2p_{T,\text{jet}}^{\text{ref}} \Delta^{\text{ref}} + (\Delta^{\text{ref}})^2}{2(p_{T,\text{jet}}^v)^2} + \frac{(p_{T,\text{jet}}^{\text{ref}} + \Delta^{\text{ref}})^2 (\Delta^{\text{ref}} + \Delta^v)}{8(p_{T,\text{jet}}^v)^3} + \mathcal{O}\left(\frac{1}{(p_{T,\text{jet}}^v)^4}\right) \right)$$

5. The asymptotic value for $p_{T,\text{jet}}^v$ depends only on m_t and Δ^{ref} .

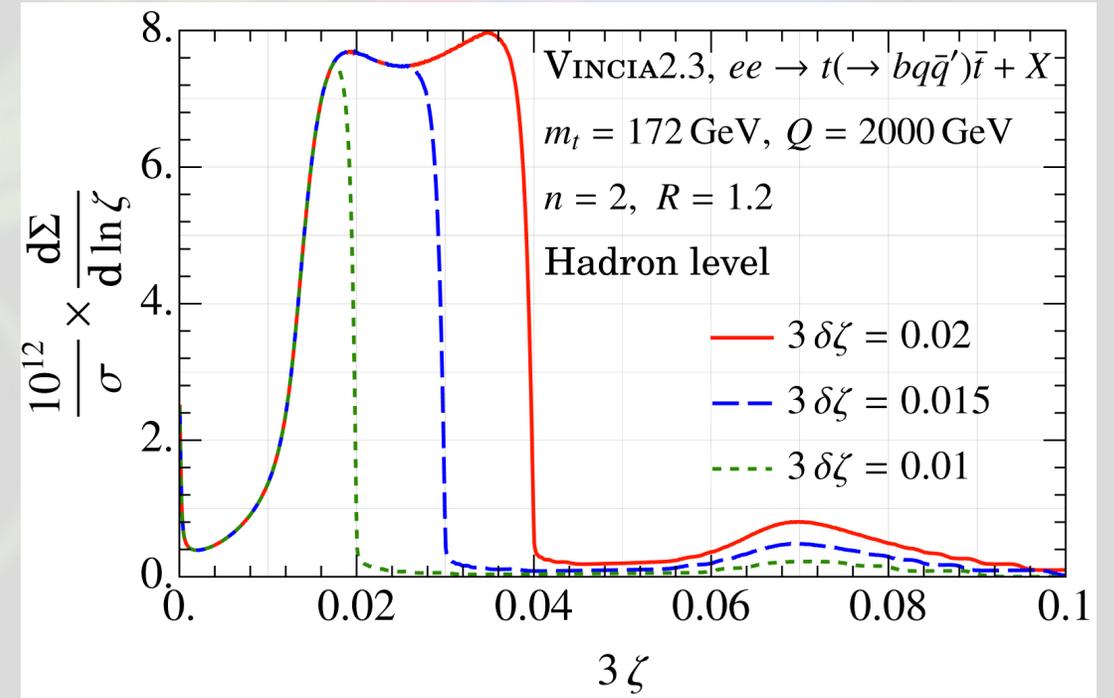
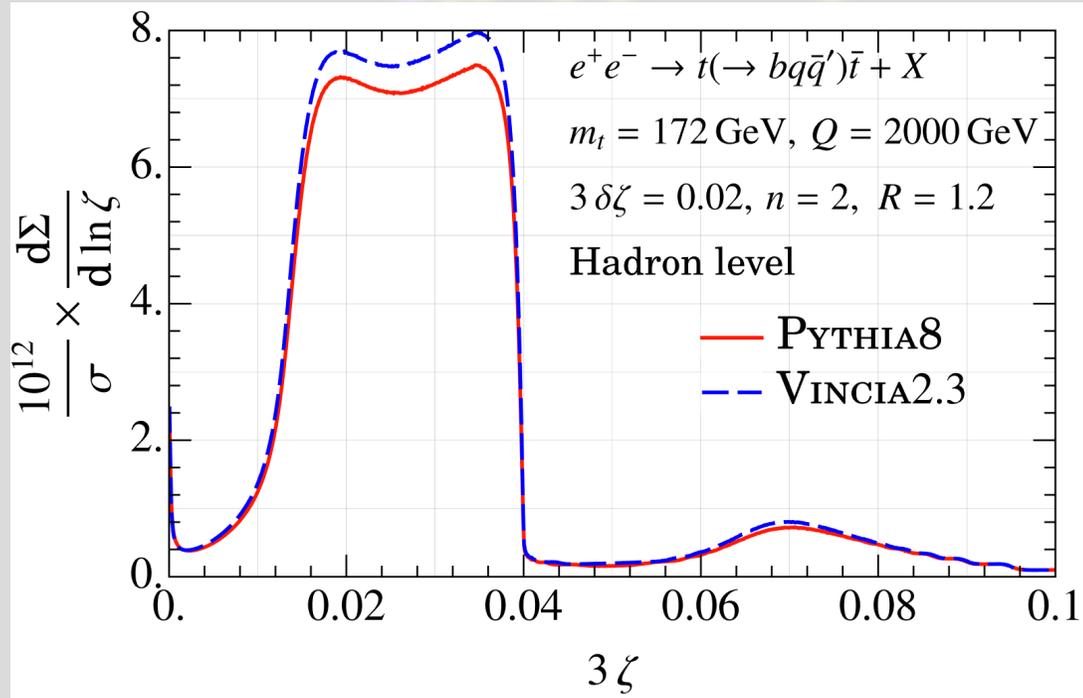
Supplementary material

Fit function:

$$\rho = \rho_{\text{asy}} + c_2(p_{T,\text{jet}}^{\text{v}})^{-2} + c_3(p_{T,\text{jet}}^{\text{v}})^{-3}$$

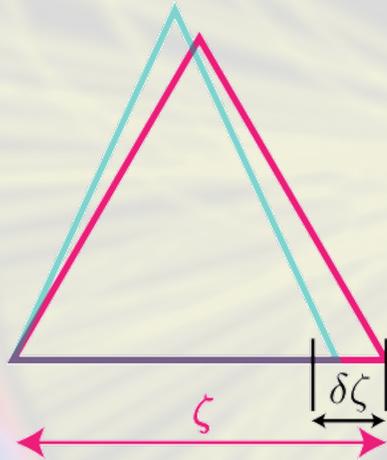


Supplementary material

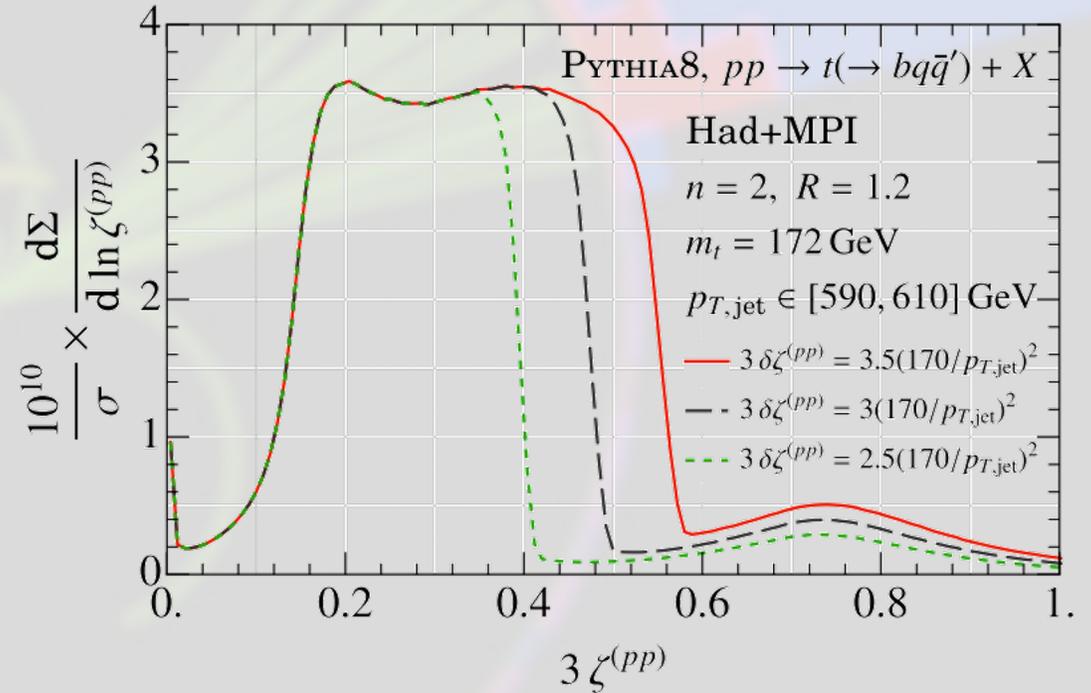


Supplementary material

Asymmetry cut $\delta\zeta$ only constrains triangles with $\zeta > 2\delta\zeta$

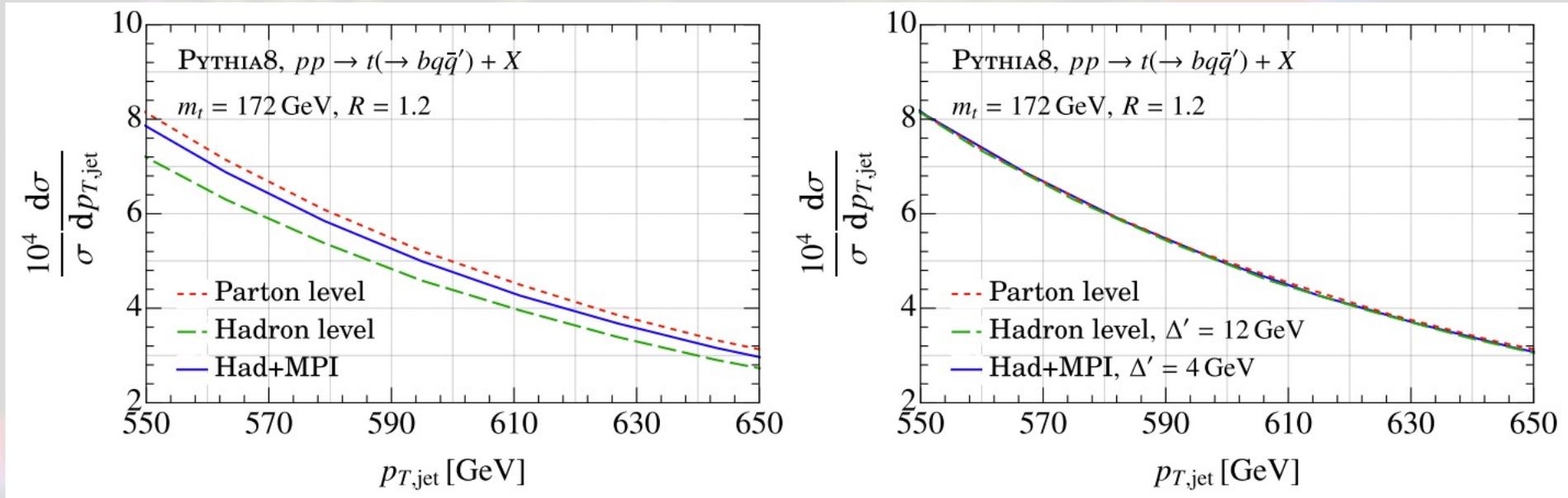


$$\frac{d\Sigma}{d\zeta} \approx 4(\delta\zeta)^2 G^{(n)}(\zeta, \zeta, \zeta; m_t), \quad \delta\zeta \ll \zeta$$

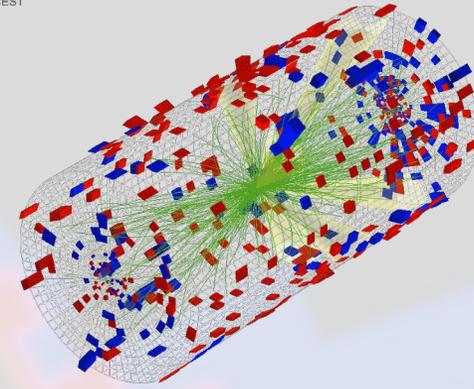


Supplementary material

Here we show $p_{T,\text{jet}}$ shifts relative to parton level:



Correlation Functions



- Case study from study of the QGP in Pb-Pb:

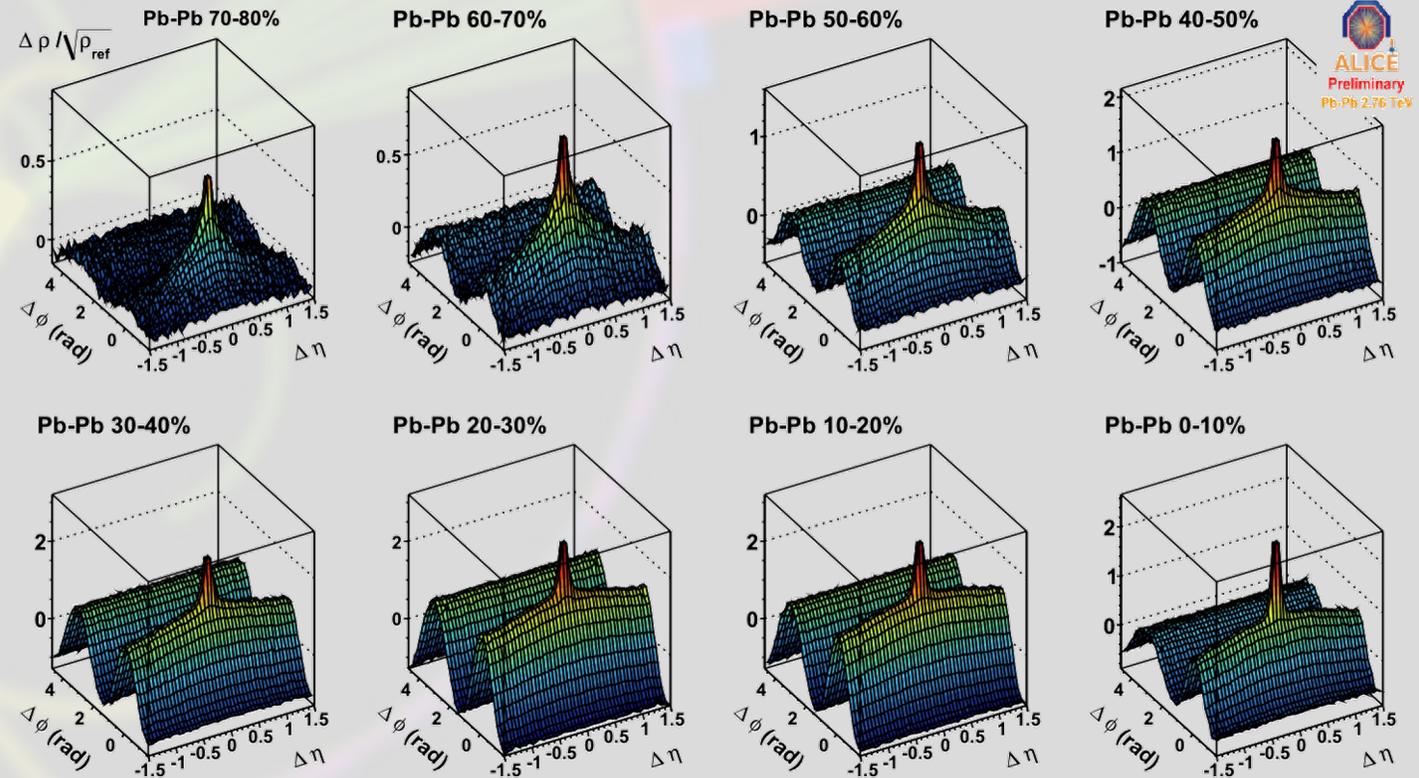
$$\langle N(\eta_1, \phi_1) N(\eta_2, \phi_2) \rangle = N_1 N_2 P(\eta_1, \phi_1, \eta_2, \phi_2)$$

$$\sim \sum_X N_X(\eta_1, \phi_1) N_X(\eta_2, \phi_2) \langle \text{Pb} - \text{Pb} | X \rangle \langle X | \text{Pb} - \text{Pb} \rangle$$

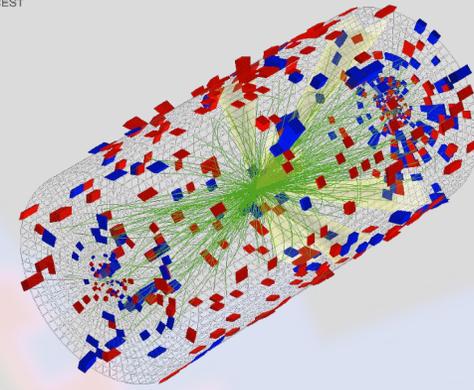
$$= \sum_X \langle \text{Pb} - \text{Pb} | \hat{N}(\eta_1, \phi_1) \hat{N}(\eta_2, \phi_2) | X \rangle \langle X | \text{Pb} - \text{Pb} \rangle$$

$$= \langle \text{Pb} - \text{Pb} | \hat{N}(\eta_1, \phi_1) \hat{N}(\eta_2, \phi_2) | \text{Pb} - \text{Pb} \rangle$$

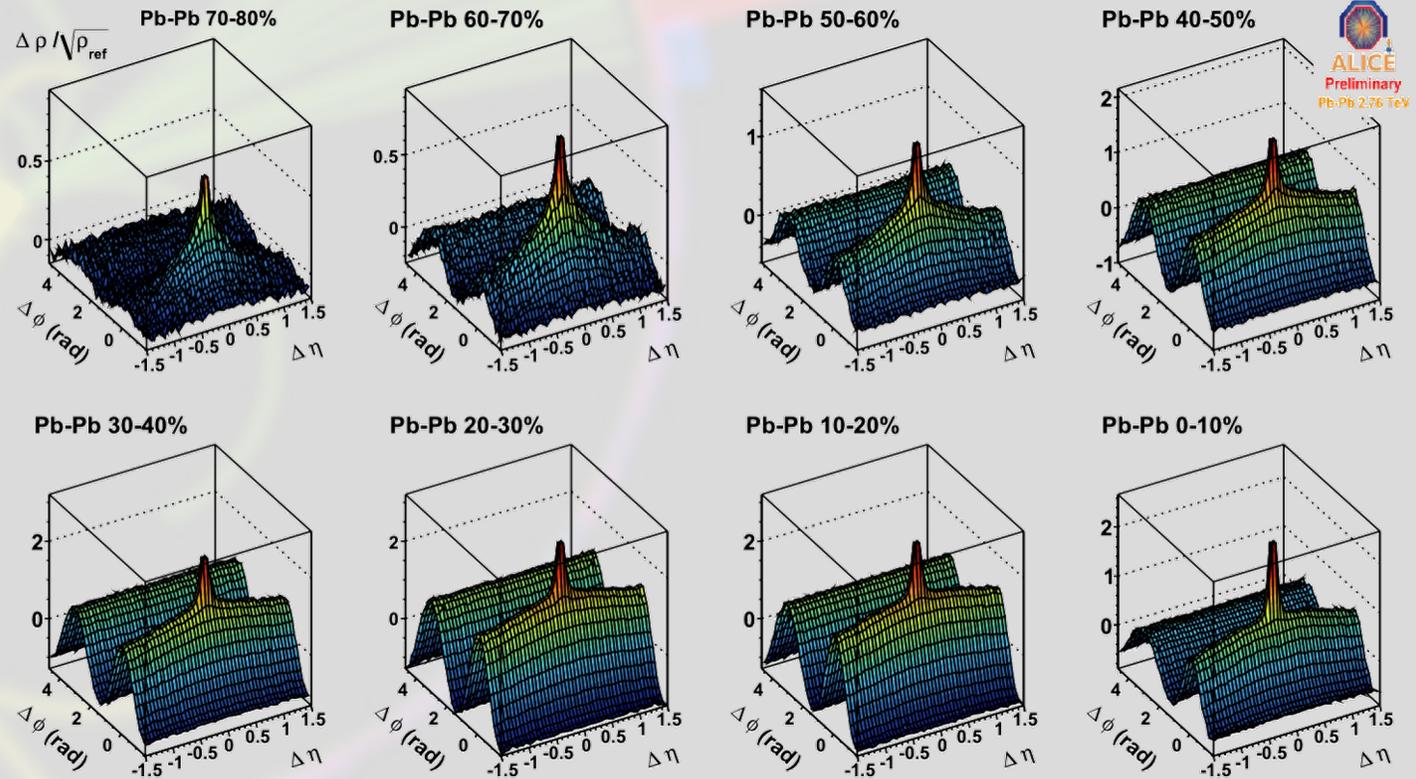
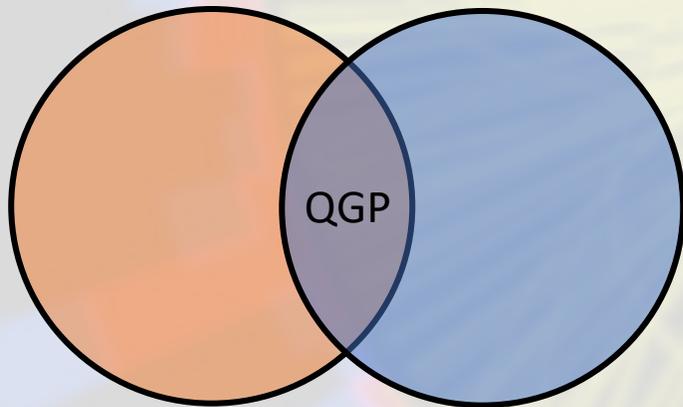
$$\frac{dN}{d^2 p d^2 k d\eta d\xi} = \langle \hat{\sigma}(k) \hat{\sigma}(p) \rangle_{P,T}$$



Correlation Functions

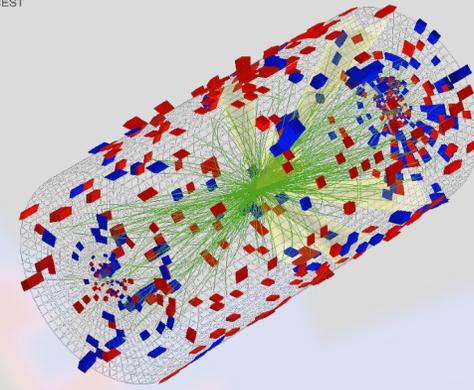


- Case study from study of the QGP in Pb-Pb:

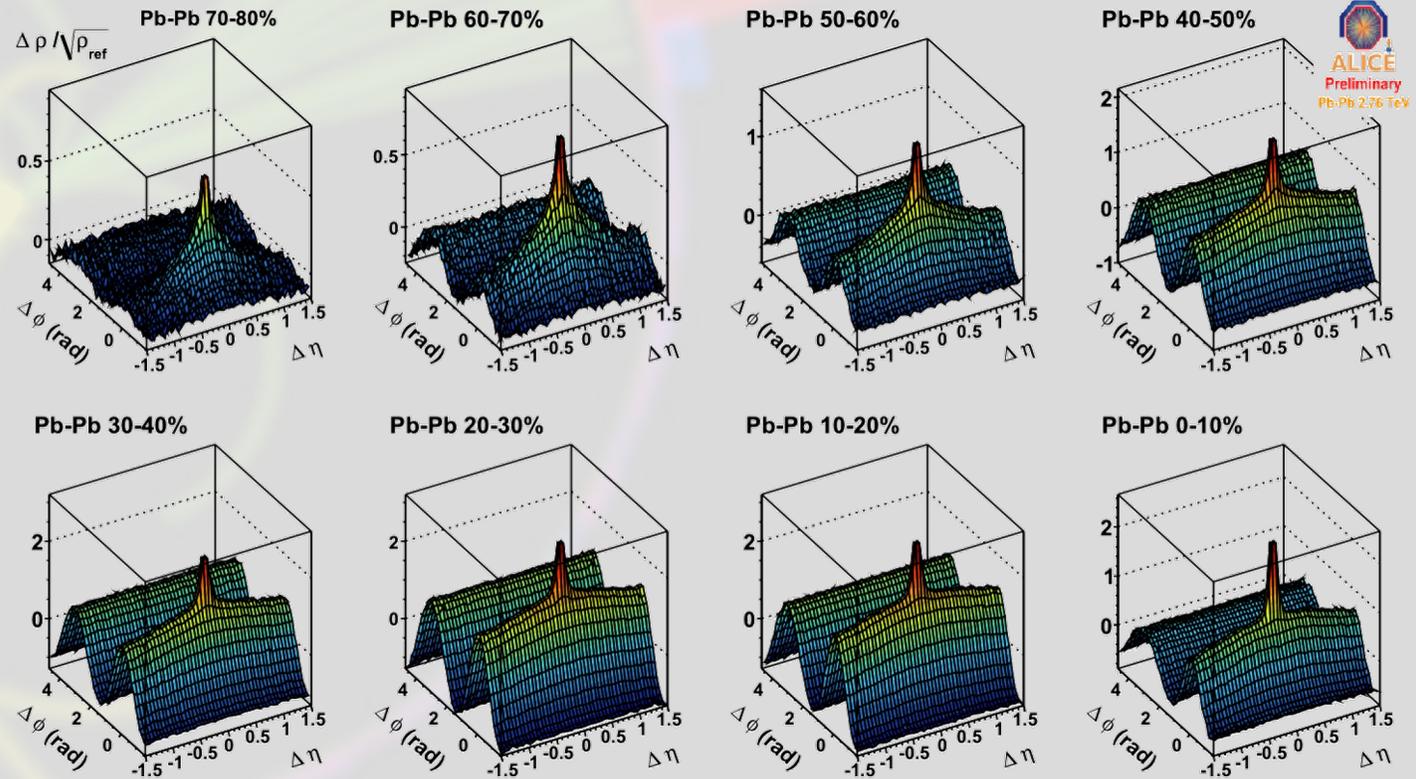
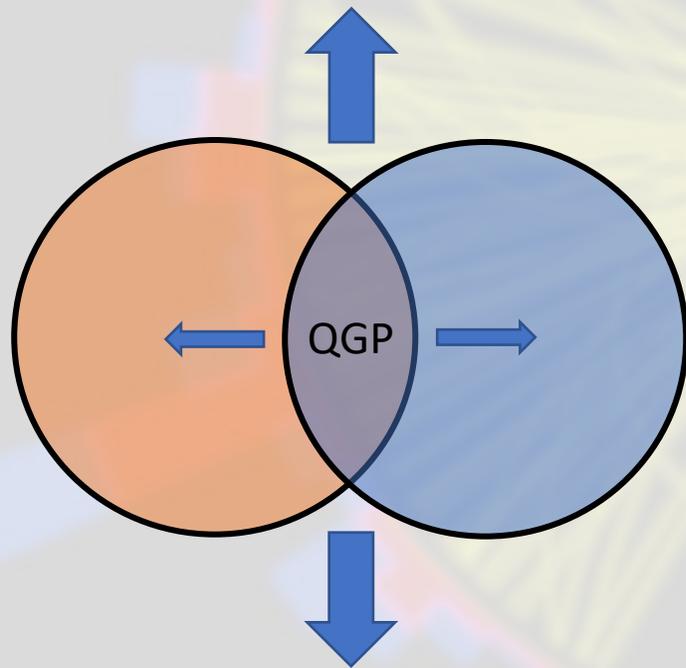


[1106.6057](https://doi.org/10.1106.6057)

Correlation Functions

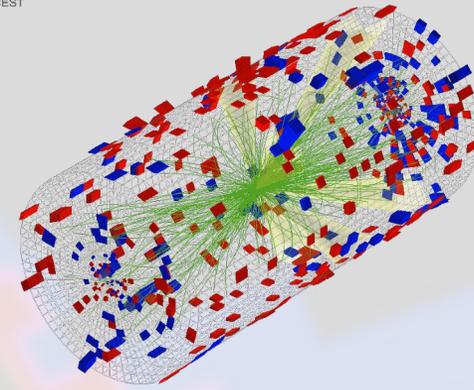


- Case study from study of the QGP in Pb-Pb:

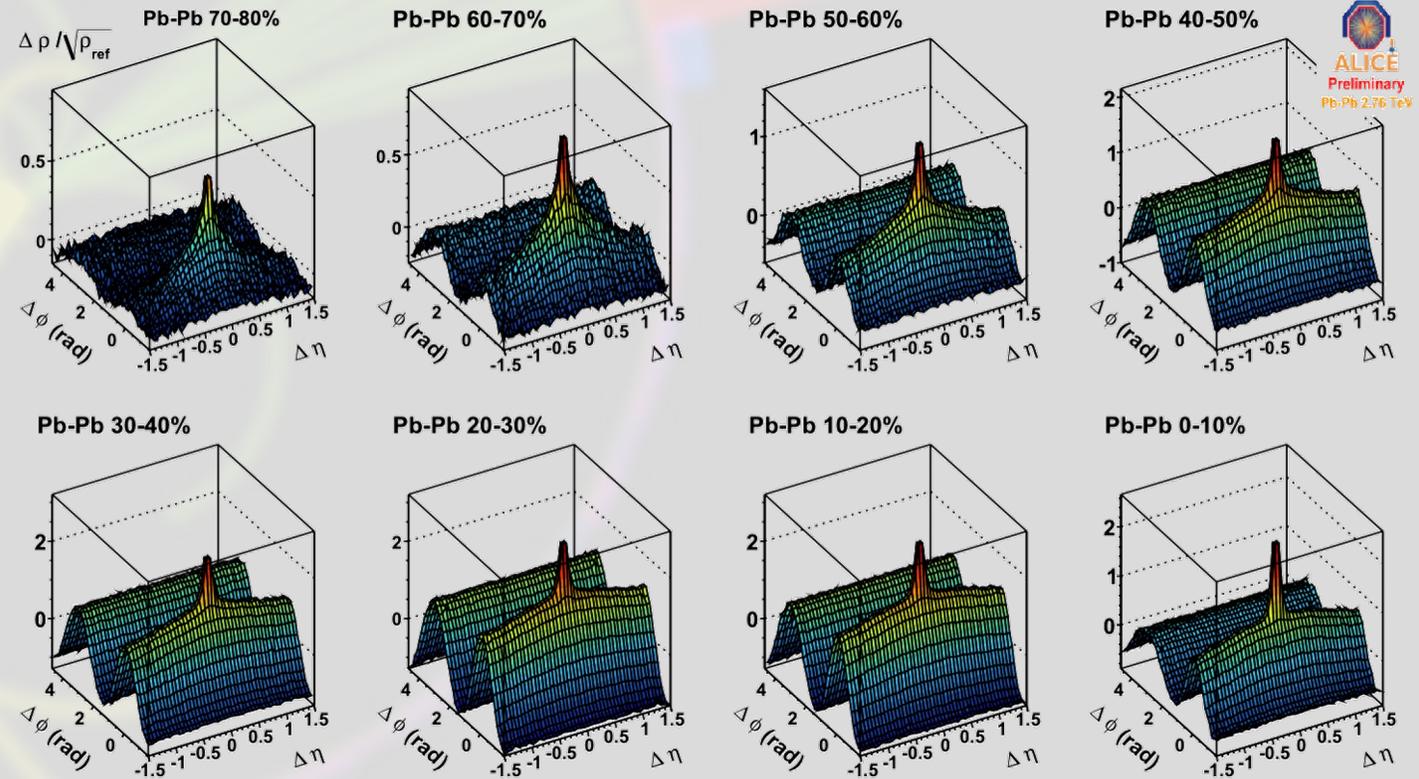
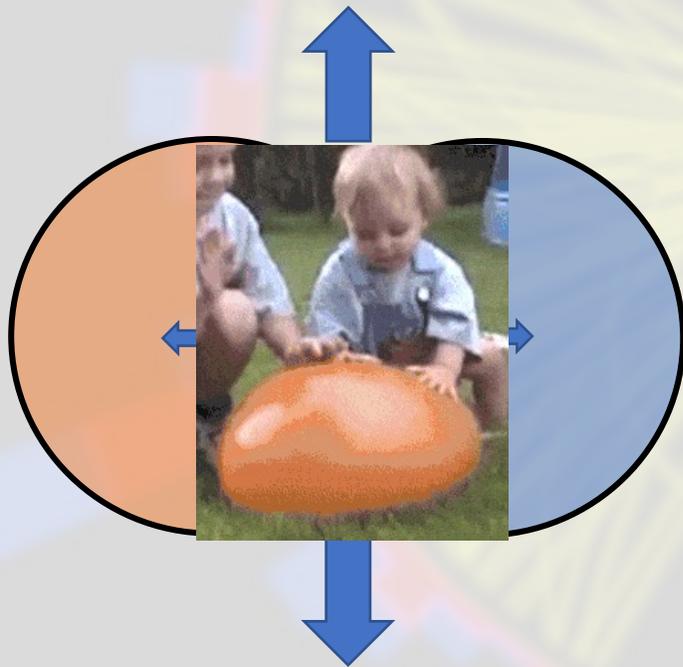


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Correlation Functions

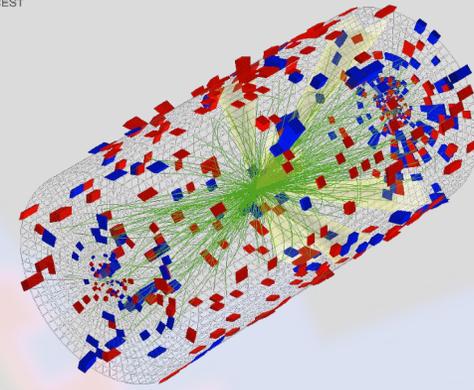


- Case study from study of the QGP in Pb-Pb:

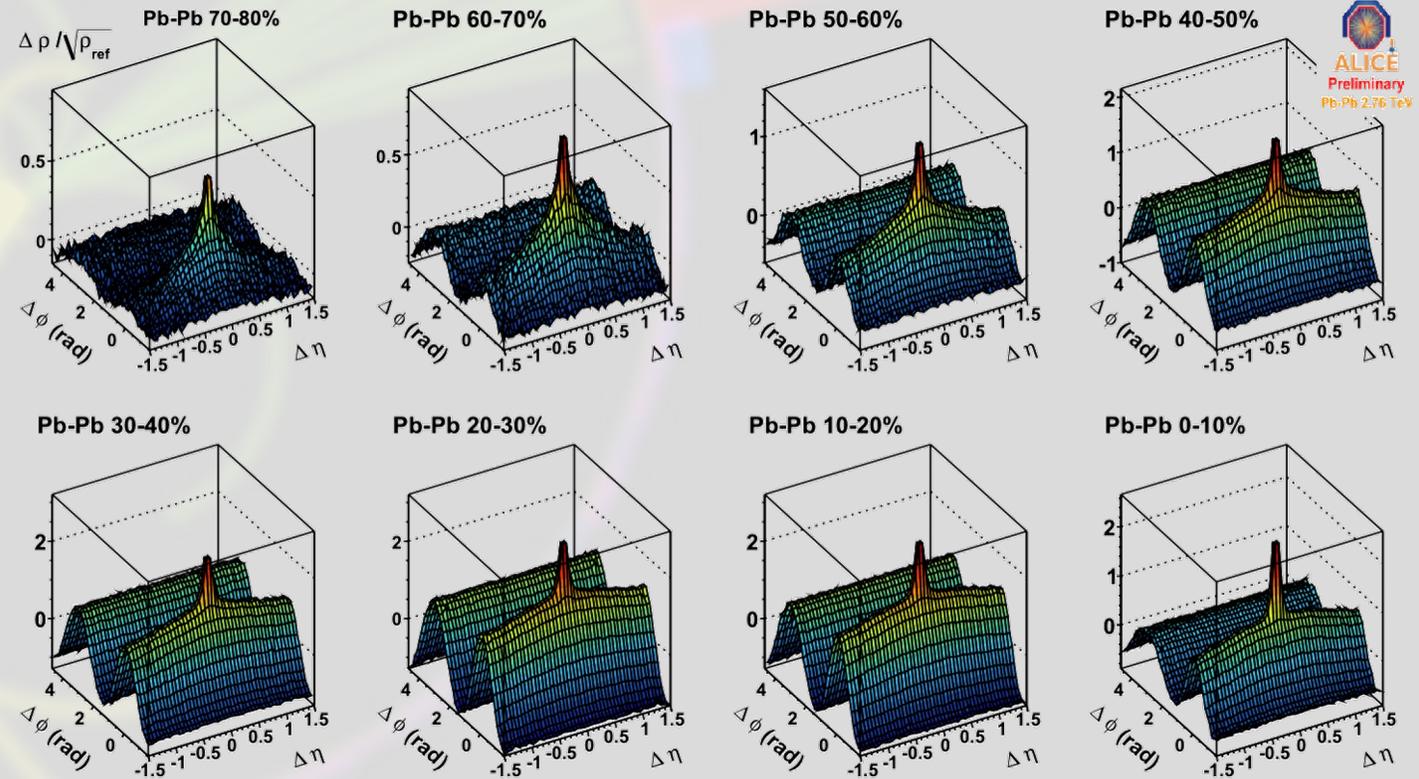
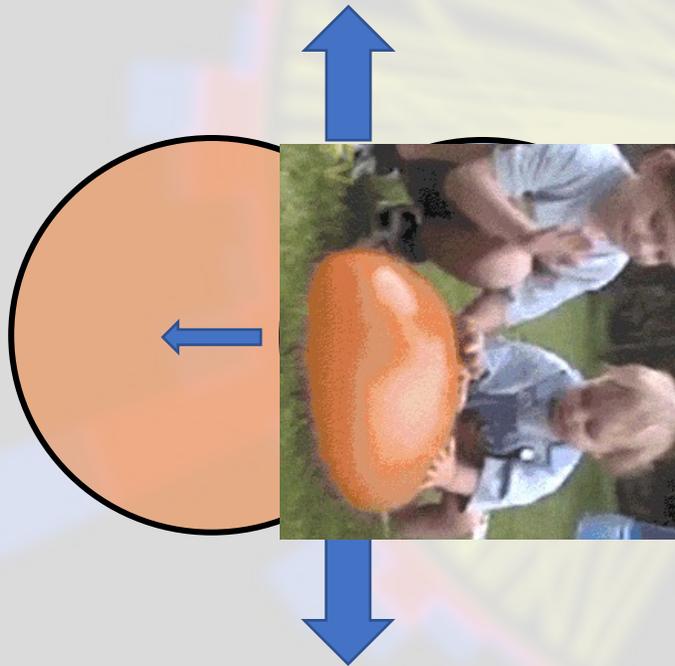


[1106.6057](https://doi.org/10.1106.6057)

Correlation Functions



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[1106.6057](https://doi.org/10.1106.6057)

Correlation Functions

$$\mathcal{E}(\vec{n}) = \lim_{r \rightarrow \infty} \int_0^{\infty} dt r^2 n^i T_{0i}(t, r\vec{n})$$

$\langle \mathcal{E}(\vec{n}_1) \mathcal{E}(\vec{n}_2) \rangle \leftarrow$ Not the QFT VEV.

Quantum mechanical expectation value, just as $\langle \psi | \hat{p} | \psi \rangle = \langle p \rangle$:

i.e.

$$\delta_{i \rightarrow j} = \langle j | i \rangle \quad \text{let } |j\rangle = \mathcal{O} | \Omega \rangle$$

$$\langle \mathcal{E}(\vec{n}_1) \mathcal{E}(\vec{n}_2) \rangle = N \langle \Omega | \mathcal{O}^\dagger \hat{\mathcal{E}}(\vec{n}_1) \hat{\mathcal{E}}(\vec{n}_2) \mathcal{O} | \Omega \rangle$$

How to measure the Top mass*

- What we have access to at a collider is a set of events and a (partial) kinematic breakdown of each event into particles with 4-momenta.
- Methods we might use:
 - Simplest: count the number of tagged top events to find $\sigma(m_t)$.
 - Slightly more sophisticated: measure a distribution differential in the event kinematics.
 - Best precision: direct measurement using Monte Carlo to simulate the top events and compare for a mass extraction.

*please allow me to oversimplify for a little bit

...but what is the Top mass we measure?

- The top quark mass is not a physical observable but a Lagrangian parameter,

$$\text{---} \xrightarrow{p} \text{---} + \text{---} \xrightarrow{p} \text{---} \text{---} + \dots \sim \frac{i}{\not{p} - m_t^0 - \Sigma(p, m_t^0, \mu)}$$

and must be renormalized in a definite *mass scheme*.

- We gain access to this parameter through a sensitive **physical observable**: $\sigma^{\text{exp}}(m_t^X, \Lambda_{\text{QCD}}, Y) = \sigma^{\text{pert}}(m_t^X, \alpha_s, Y, \dots) + \sigma^{\text{NP}}(\Lambda_{\text{QCD}}, Y, \dots)$

$$m_t^{\text{pole}} = m_t^X + \delta m_t^X$$

We will come back to this as it leads to complications!

How to measure the Top mass

- Can measure many different differential distributions.

$$\frac{d\sigma}{dm_t^{\text{reco}}}, \quad \frac{d\sigma}{dM_{bl}}, \quad \frac{d\sigma}{dM_{t\bar{t}}}, \quad \frac{d\sigma}{dM_{t\bar{t}j}}$$

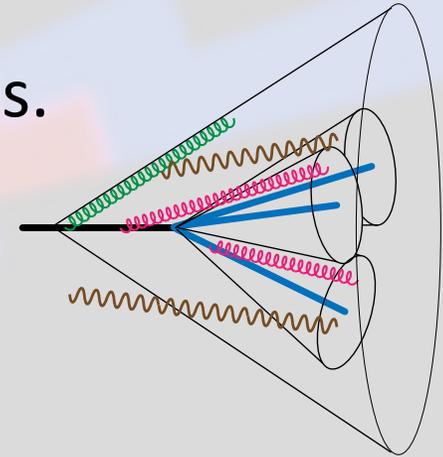
- These can be computed from theory

- Arguably best efforts for rigorous control over m_t use groomed jet masses: i.e.

$$(m_t)^2 = (\sum_i p_i)^2 \quad \text{0711.2079 and references therein} \quad m_t = 172.6 \pm 2.5 \text{ GeV} \quad \text{A recent measurement in CMS 1911.03800}$$

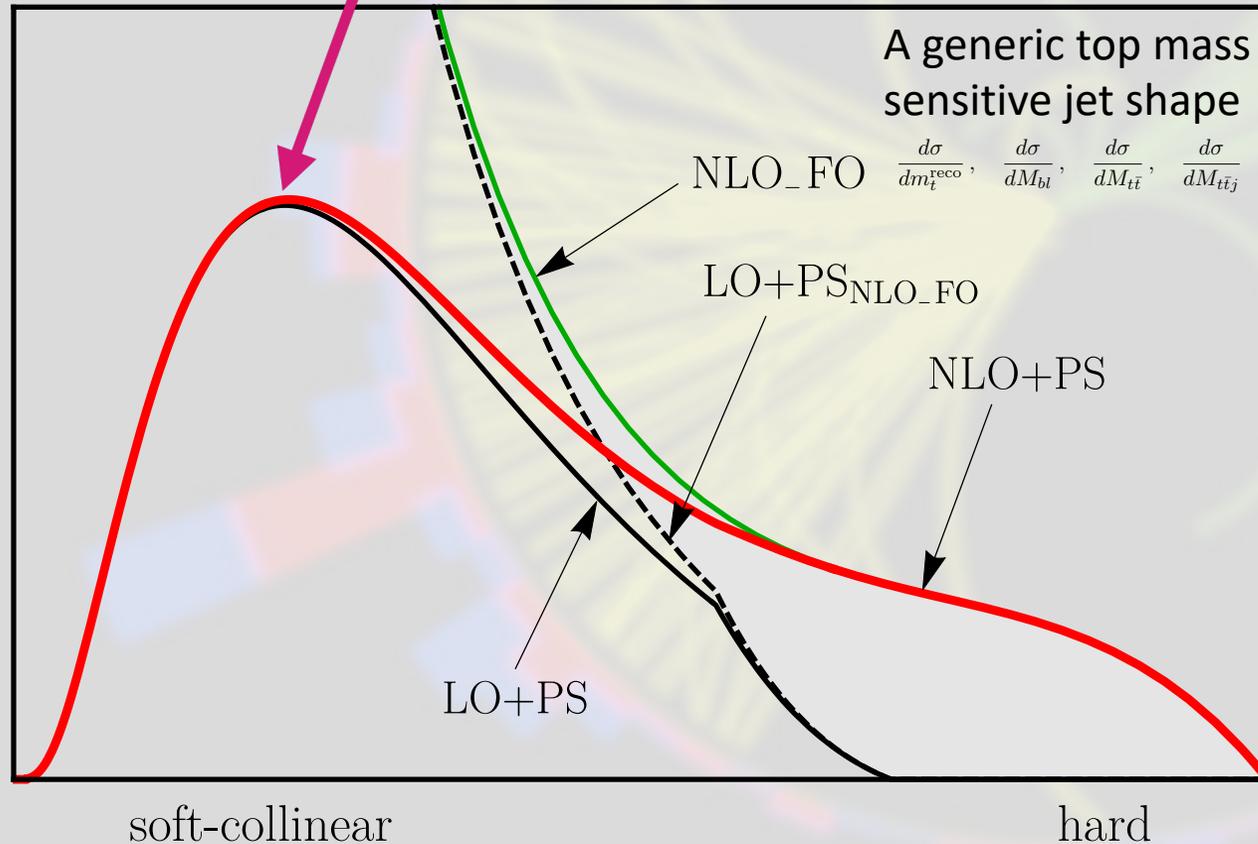
- Or can be computed from Monte Carlo Event generators...

- This is where best experimental precision has been achieved **We will come back to this in 2 slides**



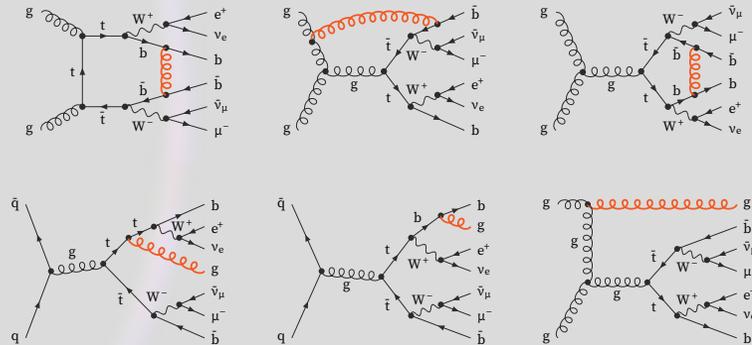
Why is measuring m_t hard theoretically?

Threshold structure sensitive to m_t



Top Production and decay at NLO, NNLO

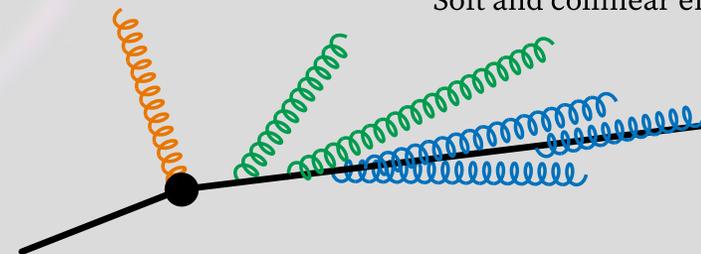
Mazzitelli et al. 2012.14267; Cormier et al. 1810.06493; Frederix et al. 1603.01178; Jezo et al. 1607.04538; Hoeche et al. 1402.6293



1207.5018

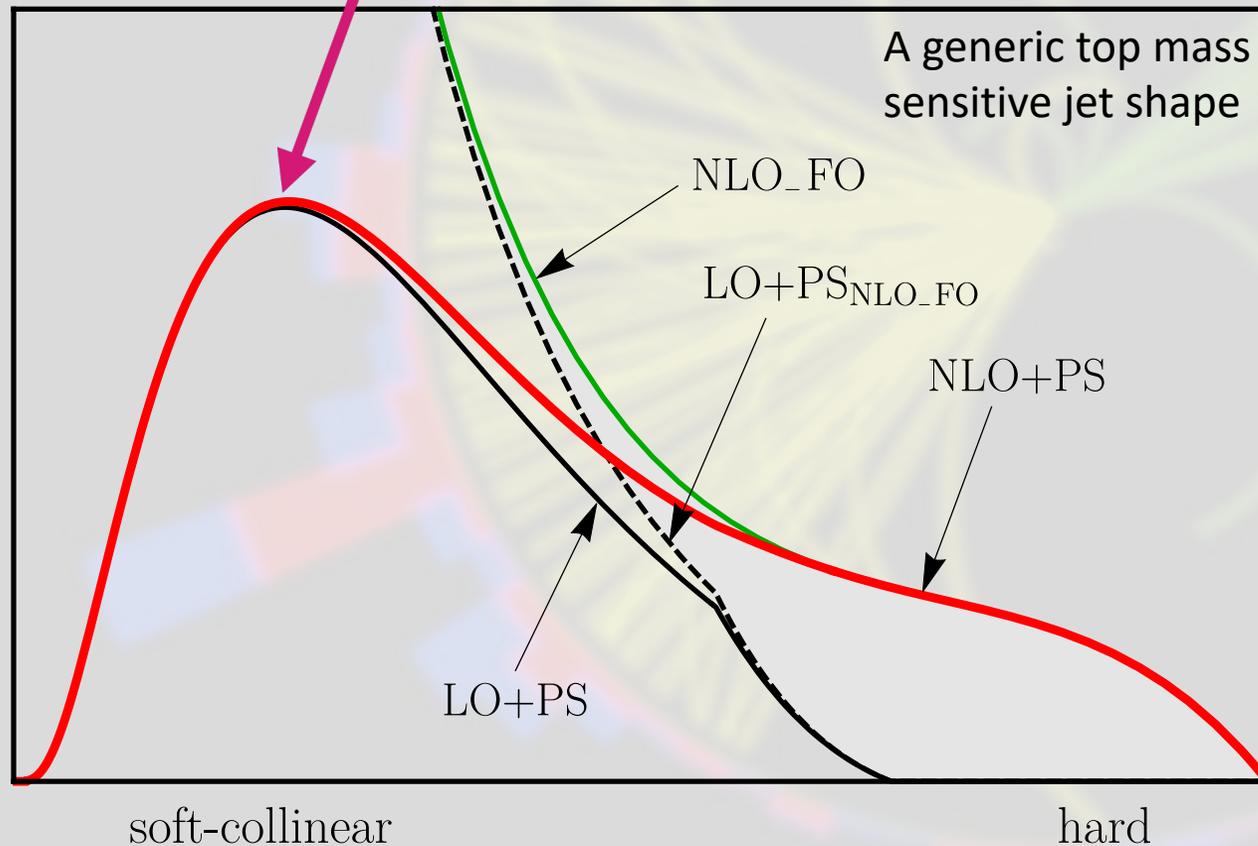
First hard emission

Soft and collinear emissions



Why is measuring m_t hard theoretically?

Threshold structure sensitive to m_t

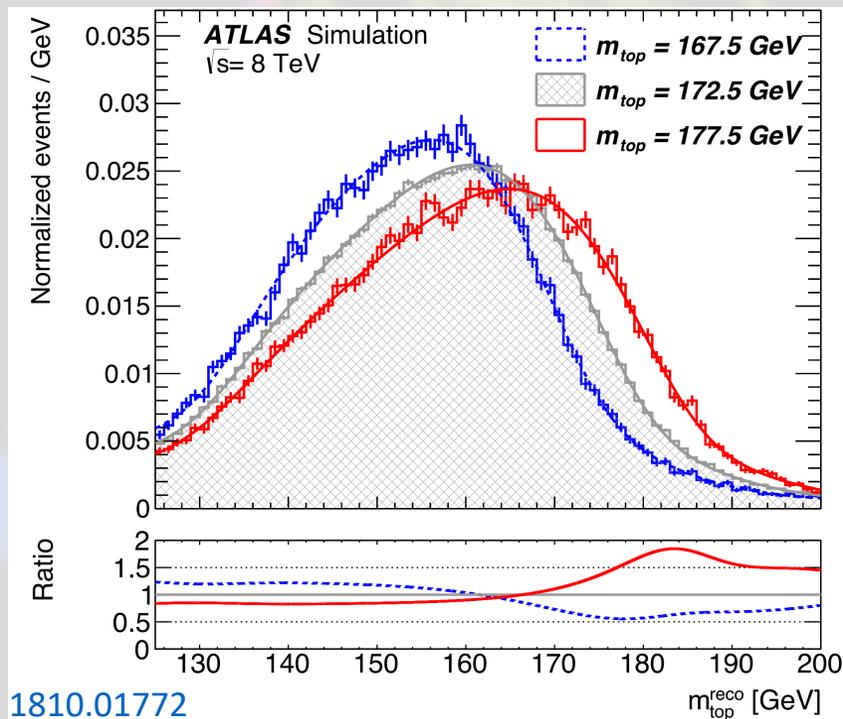


Observations:

- Threshold structure typically appears in the region where shape is dominated by **soft-collinear radiation**.
 - **Almost entirely dependent on parton shower or complicated resummation.**
- NLO corrections make an impact **only in the tail**.
 - High accuracy fixed order computations are only weakly m_t dependent.

Monte Carlo Top mass

- Event generator reconstruction of m_t has yielded the most precise measurements:



[1810.01772](#)

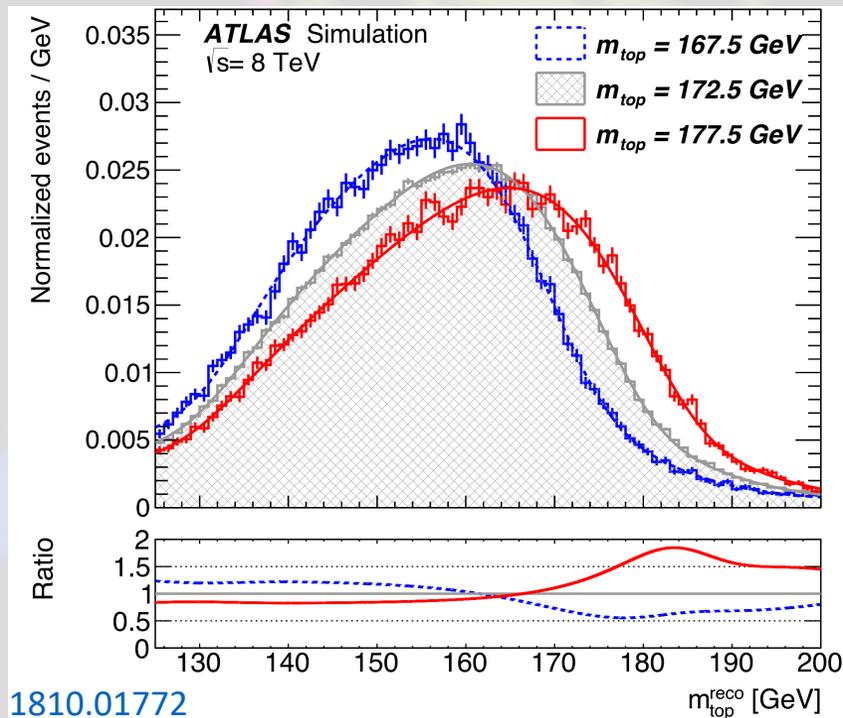
PDG 2021 global average of direct measurements

$$m_t = 172.76 \pm 0.30 \text{ GeV} \quad \text{10.1093/ptep/ptaa104}$$

Order of magnitude better experimental precision than analytically calculable observables.

Monte Carlo Top mass

- Event generator reconstruction of m_t has yielded the most precise measurements:



[1810.01772](#)

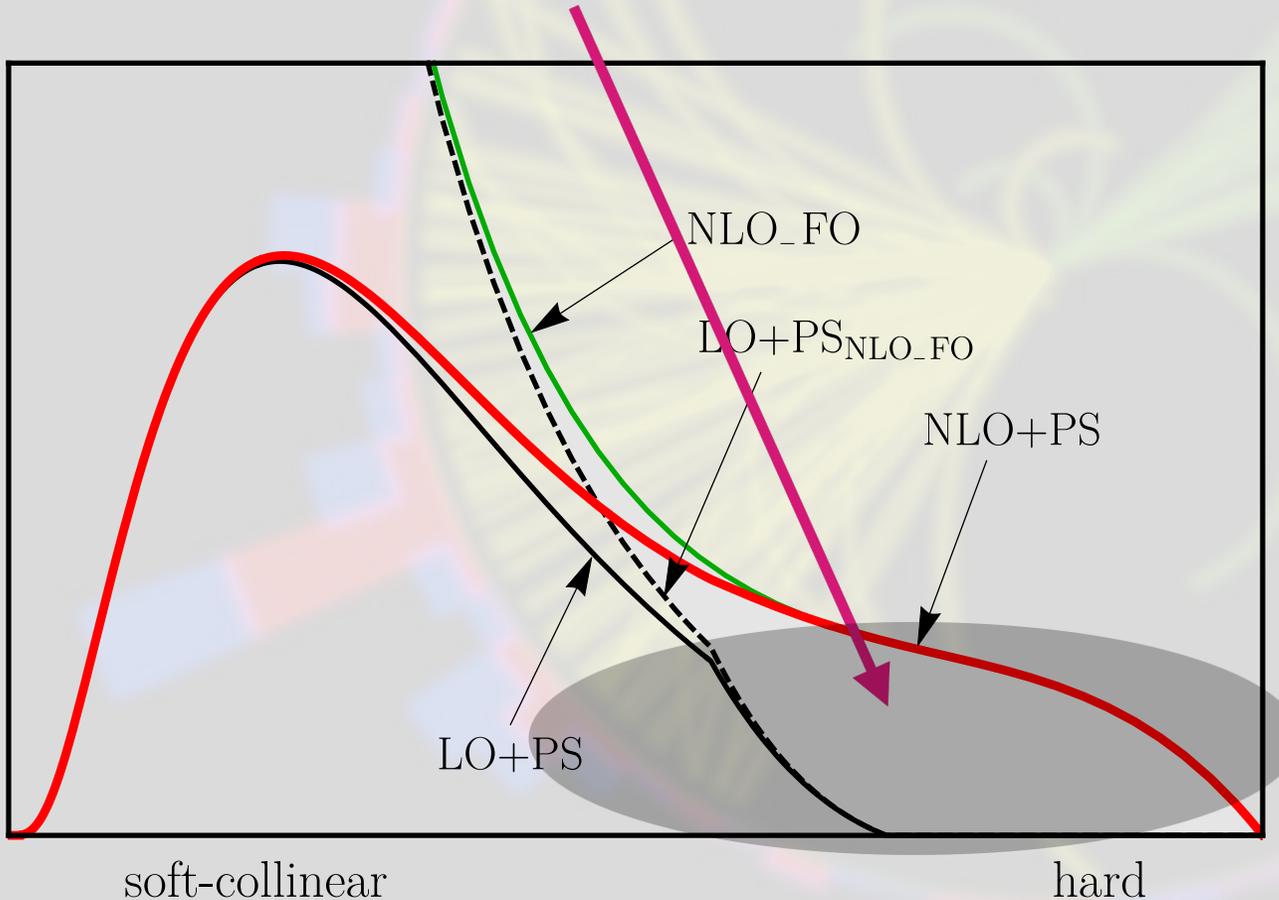
The heavy dependence on Parton Shower Monte Carlo reduces theoretical control on this measurement.

Argued to induce a further $O(1)\text{GeV}$ error on the extracted Top mass in a given theory mass scheme.

[2004.12915](#) and references therein

Summary: measuring m_t is challenging

Focus here for good theoretical control, here PS is not dominant but a well controlled small perturbation.



ATLAS+CMS Preliminary m_{top} from cross-section measurements
LHCtopWG Sep 2019

	total	stat	$m_{top} \pm \text{tot (stat} \pm \text{syst} \pm \text{theo)}$	Ref.
$\sigma(t\bar{t})$ inclusive, NNLO+NNLL				
ATLAS, 7+8 TeV			$172.9^{+2.5}_{-2.6}$	[1]
CMS, 7+8 TeV			$173.8^{+1.7}_{-1.8}$	[2]
CMS, 13 TeV			$169.9^{+1.9}_{-2.1} (0.1 \pm 1.5^{+1.2}_{-1.5})$	[3]
ATLAS, 13 TeV			$173.1^{+2.0}_{-2.1}$	[4]
$\sigma(t\bar{t}+1j)$ differential, NLO				
ATLAS, 7 TeV			$173.7^{+2.3}_{-2.1} (1.5 \pm 1.4^{+1.0}_{-0.5})$	[5]
CMS, 8 TeV			$169.9^{+4.5}_{-3.7} (1.1^{+2.5}_{-3.1} +3.6_{-1.6})$	[6]
ATLAS, 8 TeV			$171.1^{+1.2}_{-1.0} (0.4 \pm 0.9^{+0.7}_{-0.3})$	[7]
$\sigma(t\bar{t})$ n-differential, NLO				
ATLAS, n=1, 8 TeV			$173.2 \pm 1.6 (0.9 \pm 0.8 \pm 1.2)$	[8]
CMS, n=3, 13 TeV			170.9 ± 0.8	[9]
m_{top} from top quark decay				
	CMS, 7+8 TeV comb. [10]			
	ATLAS, 7+8 TeV comb. [11]			

155 160 165 170 175 180 185 190
 m_{top} [GeV]

Unfortunately, **poor sensitivity when not leveraging the threshold**

End

