Two-loop mixed QCD-electroweak amplitudes for Z+jet production

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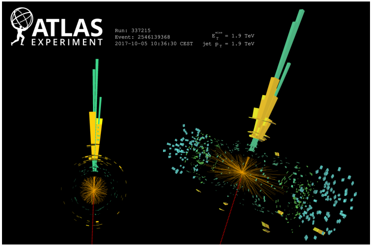
Presentation plan

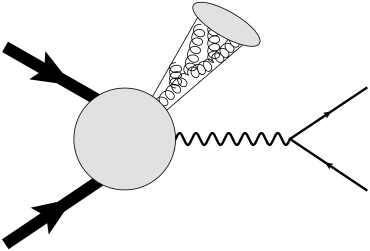
Motivation

Dark Matter searches at the LHC

experiment
monojet + **missing** transverse energy



theory
$p\,p \to \text{jet} + V\,(\to \nu\bar{\nu}/\chi\bar{\chi})$



Introduction

- V channels : γ, W, **Z**
- statistical uncertainty : few % for $p_T \in (200, 2000)$ GeV at $\sqrt{s_H} = 13$ TeV
- systematic improvement : perturbative corrections

$$\sigma = \sigma^0 \left(1 + \alpha_s\,\delta^{(1,0)} + \alpha_s^2\,\delta^{(2,0)} + \alpha\,\delta^{(0,1)} + \alpha_s\,\alpha\,\delta^{\mathbf{(1,1)}} + \mathcal{O}(\alpha_s^2, \alpha^2)\right)$$

- Sudakov enhancement : $\alpha \quad \rightarrow \quad \frac{\alpha}{4\pi s_w^2} \log^2(\frac{s}{m_Z^2}) \sim 10\% \sim \alpha_s$

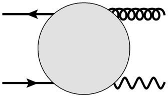
  $\Rightarrow$ **mixed QCD−EWK** corrections important : $\delta^{\mathbf{(1,1)}} \sim \delta^{(2,0)} \sim$ few %
  [*Lindert et al. arXiv:1705.04664*]
  see also [*talk by Pagani, Autieri, Signorile-Signorile*]
- lower order corrections :
  - NLO QCD [*Giele et al. arXiv:9302225*]
  - NLO EWK [*Denner et al. arXiv:1103.0914*]
  - NNLO QCD [*Gehrmann-De Ridder arXiv:1507.02850*]

  t
- cross section $\sim$ **scattering amplitude** $\otimes$ subtraction scheme
- on-shell Z approximation :

$$\mathcal{M}(p\,p \rightarrow Z(\rightarrow \nu\bar{\nu}) + \text{jet}) \approx \mathcal{A}_\mu(p\,p \rightarrow Z\,j)\,\frac{1}{s - m_Z^2 + i\Gamma_Z m_Z}\,\mathcal{L}^\mu(Z \rightarrow \nu\bar{\nu})$$

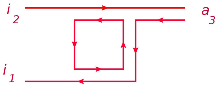
Amplitude structure

the process $u(p_1) + \bar{u}(p_2) \to g(-p_3) + Z(-p_4)$

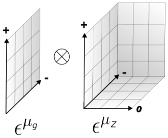


$$A_2 = g_s^2\, e^2\, g_{L/R}\, T^{a_3}_{i_1 i_2} \int \frac{d^d k_1}{(2\pi)^d} \frac{d^d k_2}{(2\pi)^d} \frac{\bar{u}(p_2)(\not{k}_1 \not{\epsilon}_3 \not{p}_3 \not{\epsilon}_4 \not{k}_2 + \ldots)u(p_1)}{\mathcal{D}_1 \ldots \mathcal{D}_7}$$

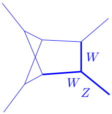
color



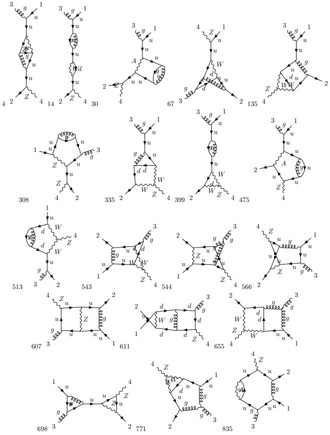
tensors



integrals



Complexity



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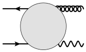
$$u\,\bar{u} \to g\,Z$$



	0L	1L QCD	1L EWK	2L
# diagrams	2	13	35	900
# families	0	1	4	18
# integrals	0	105	275	60968
# Master Integrals	0	7	26	1269

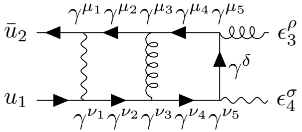
complexity summary

Tensors in d=4−2ϵ dimensions

for further steps, scalar integrals required

$$A_2 \sim \int \frac{d^d k_1}{(2\pi)^d} \frac{d^d k_2}{(2\pi)^d} \frac{\bar{u}(p_2)(\not{k}_1 \not{\epsilon}_3 \not{p}_3 \not{\epsilon}_4 \not{k}_2 + \ldots) u(p_1)}{\mathcal{D}_1 \ldots \mathcal{D}_7} = \sum_{i=1}^{?} \mathcal{F}_i T_i$$

for example, consider vector current



Σ(diagrams) # Lorentz indices > # all invariant structures

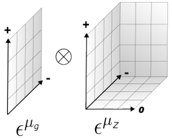
$\#T_i = 17$ (Lorentz invariant tensors) − 4 (by transversality $\epsilon_3 \cdot p_3 = 0$)
− 6 (by gauge fixing $\epsilon_i \cdot p_{i-1} = 0$)
= 7 (independent in d dimensions)
= ? (independent in 4 dimensions)

$$T_i = \epsilon_{3,\mu}(p_3)\, \epsilon_{4,\nu}(p_4)\, \bar{u}(p_2) \quad (p_1^\nu \gamma^\mu\, , p_1^\mu p_1^\nu \not{p}_3\, , \; p_2^\nu \gamma^\mu \; , \; p_1^\mu \gamma^\nu\, , \; p_1^\mu p_2^\nu \not{p}_3\, , \; g^{\mu\nu} \not{p}_3\, , \; \not{\gamma}^\mu \not{p}_3 \not{\gamma}^\nu) \quad u(p_1)$$

Tensors in 4 dimensions

recent loop-univeral **claim** in the 'tHV scheme [*Peraro, Tancredi arXiv:2012.00820*] :
tensors indpt in 4-dim = # indpt helicity states (here $= 3 \times 2^2/2 = 6$)

$$A = \sum_{i=1}^{7} \mathcal{F}_i T_i = \sum_{i=1}^{6} \overline{\mathcal{F}}_i \overline{T}_i$$



orthogonalization : projects out $\overline{T}_7$ from the **physical** 4-dim subspace

$$\sum_{pol} \overline{T}_i^\dagger \overline{T}_j = \left(\begin{array}{c|c} 6\times 6 \text{ (4-dim)} & 0 \\ \hline 0 & 1\times 1 \ (-2\epsilon\text{-dim}) \end{array}\right)$$

gain : 1-1 **correspondence** between form factors and helicity amplitudes

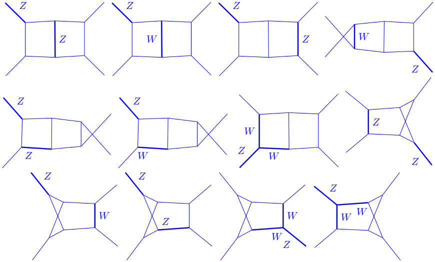
$$\overline{\mathcal{F}}_i \Longleftrightarrow A_{\vec{\lambda}_i}$$

$\Rightarrow$ unphysical information removed

Feynman integrals

$$A_{2,\overline{T}_i} = \sum_{f\in\text{fam}} \sum_{\vec{n}\in\text{int}} \int \frac{d^dk_1}{(2\pi)^d} \frac{d^dk_2}{(2\pi)^d} \frac{c_{f,\vec{n}}(d,m_k,s_{ij})}{\mathcal{D}_{f,1}^{n_1}...\mathcal{D}_{f,9}^{n_9}}$$

example integral **families**

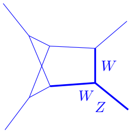


- multiple **scales** : $\{s_{23}, s_{13}, m_Z, m_W, (m_t, m_H)\}$
- usual approach : Integration By Parts reduction ($6\times10^4 \to 1\times10^3$)

$$\mathcal{I} = \sum_n \text{rat}(d,m_k,s_{ij})_n \,\text{MI}(d,m_k,s_{ij})_n$$

IBP ineffective

- reduction with `kira-2.2`
- most involved topology : 2-loop non-planar with the W^+W^-Z vertex
- number of integrals to reduce : 1181 , $(\text{ISP})^4$
- number of master integrals : 95



$$\text{amplitude}_{(\text{NPL},\text{WWZ})} = \text{integrand (5.4MB) /. IBPs (640MB) = simpler ?}$$

physical pole motivation ⇒ **partial fraction** coefficients of Master Integrals

- algebraic geometry ⇒ **Groebner basis**
- number of denominator factors $\mathcal{P}(d, m_k, s_{ij})$: 131
- `Singular` ineffective

⇓⇓⇓

either choose better Master basis [*Bonetti et al. arXiv:2203.17202*]
or evaluate integrals numerically without IBP

our strategy : **numerical** because $2 \to 2$ process easy to grid for phenomenology

on the importance of numerical methods see also [*talk by Lange, Sotnikov*]

Auxiliary Mass Flow method

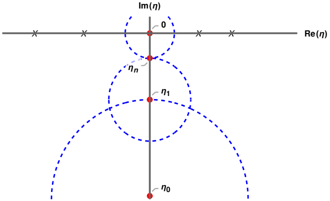
numerical evaluation

- auxiliary mass : $\frac{1}{\mathcal{D}_k + i0^+} \to \frac{1}{\mathcal{D}_k - \eta}$

  [***Xiao Liu*** *et al.* *arXiv:1711.09572*]

- differential equations : $\frac{\partial}{\partial\eta}\mathcal{I}(\eta) = A(\eta)\,\mathcal{I}(\eta)$
  easy to solve

- boundary conditions at $\eta = \infty$:
  expansion by regions



$$\frac{1}{((l+p)^2 - m^2 - \eta)^\nu} \simeq \frac{1}{(l^2-\eta)^\nu} \sum_{i=0}^{N} \frac{\Gamma(\nu+i)}{i!\,\Gamma(\nu)} \left(-\frac{2l.p + p^2 - m^2}{l^2 - \eta}\right)^i$$

- iterative strategy : reduction to **vacuum** bubbles
- **analytic continuation** : path $\{i\infty, \eta_0, \eta_1, ..., \eta_n, 0\}$

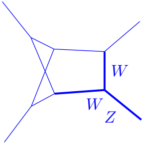
$\Downarrow\Downarrow\Downarrow$

full control on precision with N and n

AMFlow example

- implemented in `AMFlow.m`
- analytic IBP reduction at fixed mass values vastly reduces evaluation time
- example evaluation with at least 27 digits **precision** (at $\frac{m_W^2}{m_Z^2} = \frac{7}{9}$)

$$
\begin{aligned}
&- \frac{0.011955742709286810865609312312731412704773730415683}{\epsilon^4} \\
&- \frac{0.51400897331764447006253493086301699478672740697984 - 0.075120621550692172138625545495563380609786946233307i}{\epsilon^3} \\
&- \frac{11.186966902486393935524816449503039557052361456965 2 - 3.2296095711745512165046978207303473280648508875869i}{\epsilon^2} \\
&- \frac{163.88156956771300428552763557770868752429557895669 - 71.278291806483337832573970278082443477424019905 87i}{\epsilon} \\
&+ (-1813.1703568796044594193394262418828425021433496769 + 1072.1983766480299427459157788856947923510195402455i) =
\end{aligned}
$$



advantage : evaluate numerically the whole amplitude
$\Rightarrow$ no large cancellations in a family

$$A = \sum_{f \in \mathrm{fam}} \#_f$$

Preliminary results

- some kinematic point :
  $s_{12} = \frac{12637286281}{119254}\,, s_{23} = \frac{-4212389009}{875622495}\,, s_{13} = \frac{-185568373013477}{1751244990}$
  $m_Z = 91.1876\,\text{GeV}\,, m_W^2 = \frac{7}{9} m_Z^2\,, \mu^2 = s_{12}$

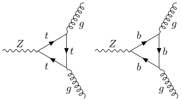
- numerical value :

$$A^{(2,\text{fin})}_{\overline{T}_1,L,n_f=0} = e^2\, g_s^2\, g_L\, T^{a_g}_{i,\bar{i}}\; p_1\cdot\epsilon_4\; \bar{u}(p_2)\not{\epsilon}_3 u(p_1) \times$$

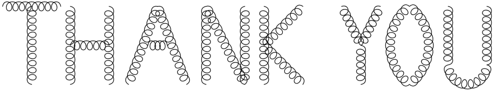
$$\Big(\frac{-1.7763568394002505\cdot 10^{-15}}{\epsilon^4} + \frac{-7.105427357601002\cdot 10^{-15} i}{\epsilon^3} + \frac{5.9117155615240335\cdot 10^{-12} + 5.684341886080802\cdot 10^{-14} i}{\epsilon^2} + \frac{-6.730260793119669\cdot 10^{-11} + 1.3642420526593924\cdot 10^{-11} i}{\epsilon^1} + \mathbf{3.978271611227941}\cdot 10^4 + \mathbf{2.089705739454954}\cdot 10^4 i \Big)$$

Outlook

- discussed methods generalizable for the **top** loop
- careful treatment of anomalies



- **hadronic** cross section



Introduction

- precision measurement of the BSM signal $p\,p \to \text{jet} + V\,(\to \chi\bar{\chi})$ requires control over the SM background $p\,p \to \text{jet} + V\,(\to \nu\bar{\nu})$
- V channels in decreasing contribution order (overall $\frac{d\sigma}{dp_T}$ drop $\sim \mathcal{O}(10^1)$) : γ , $W \to l\nu$, $Z \to \nu\bar{\nu}$, $Z \to l\bar{l}$
- statistical uncertainty : few % for $p_T < 2$ TeV and 10 % above 2.5 TeV

perturbative corretions :

$$\sigma = \sigma^0 \left(1 + \alpha_s\,\delta^{(1,0)} + \alpha_s^2\,\delta^{(2,0)} + \alpha\,\delta^{(0,1)} + \alpha_s\,\alpha\,\delta^{(\textbf{QCD-EWK})} + \mathcal{O}(\alpha_s^2, \alpha^2)\right)$$

$$\delta^{(QCD-EWK)} = \delta_{QCD} \times \delta_{EWK} + \delta_{NF}$$

- factorizable : $\alpha \sim 1\%$ but Sudakov log **enhancement** $\frac{\alpha}{4\pi s_w^2}\log^2(\frac{s}{m_Z^2}) \sim 10\%$
- **nonfactorizable** : $\delta_{NF} \gtrsim \delta^{(0,1)}_{Z+2\,\text{jets}} - \delta^{(0,1)}_{Z+1\,\text{jet}}$ [*Lindert et al. arXiv:1705.04664*]

QCD-EWK dominates at high $p_T \Rightarrow$ expect $\delta^{(QCD-EWK)} \sim \mathcal{O}(\delta^{(2,0)}) \sim$ few %

on the importance of QCD-EWK corrections

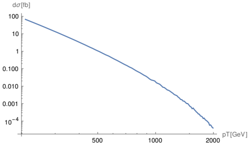
see also [*talk by Pagani, Autieri, Signorile-Signorile*]

Introduction

- for definiteness, consider the neutral Z current :

$$p\,p \to Z(\xrightarrow{\Gamma} l\bar{l}) + \text{jet}$$

- on-shell Z : enough to describe the decay at high p_T
- high p_T range : (200, 2000) GeV at $\sqrt{s_H} = 13$ TeV
- number of events : $(10^7, 10^1)$ at $\mathcal{L} = 300\ \text{fb}^{-1}$



timeline of lower orders

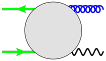
t

- NLO QCD [*Giele et al. arXiv:9302225*]
- NLO EWK [*Denner et al. arXiv:1103.0914*]
- NNLO QCD [*Gehrmann-De Ridder arXiv:1507.02850*]

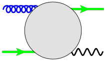
challenge = **amplitude** + subtraction

partonic channels :

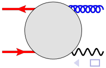
$u\,\bar{u} \to g\,Z$



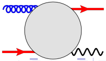
$u\,g \to u\,Z$



$d\,\bar{d} \to g\,Z$



$d\,g \to d\,Z$



UV and IR structure

$$A_b^{(2)} = \frac{c_{4,\mathrm{IR}}}{\epsilon^4} + \frac{c_{3,\mathrm{IR}}}{\epsilon^3} + \frac{c_{2,\mathrm{IR}} + c_{2,\mathrm{UV}}}{\epsilon^2} + \frac{c_{1,\mathrm{IR}} + c_{1,\mathrm{UV}}}{\epsilon^1} + \mathbf{fin}$$

extract physical finite part and check universal pole structure

UV

QCD : $\overline{\mathrm{MS}}$ scheme

$$A^{(1,QCD)} = (1 + g_s^2 \beta_0)\, A_b^{(1,QCD)}$$

EWK : on-shell G_μ scheme [*Denner arXiv:0709.1075*]

$$A^{(1,EWK)} = (1 + e^2 \delta_Z^{(1)})\, A_b^{(1,EWK)}$$

mixed :

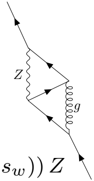
$$A = (1 + g_s^2 \beta_0 + e^2 \delta_Z^{(1)} + e^2 g_s^2 \delta_Z^{(2)})\, A_b$$

IR

$$A^{(2)} = \mathcal{I}_2\, A^{(0)} + \mathcal{I}_{1,QCD}\, A^{(1,EWK,\mathrm{fin})} + \mathcal{I}_{1,EWK}\, A^{(1,QCD,\mathrm{fin})} + A^{(2,\mathrm{fin})}$$

[*Buccioni et al. arXiv:2203.11237*]

Details of the G_μ scheme



- input parameters : $\{G_\mu, m_Z, m_W\}$
- $\alpha(0)\,(1+\Delta r) = \frac{\sqrt{2} G_\mu m_W^2}{\pi}\left(1 - \frac{m_W^2}{m_Z^2}\right)$
- amplitude renormalization

$$A(g_{s,0}, g_{L/R,0}(e_0, c_{w,0}, s_{w,0}))\sqrt{Z_u Z_{\bar{u}} Z_g Z_Z} = A(g_s, g_{L/R}(e, c_w, s_w))\, Z$$

$$Z = 1 + \delta_Z = 1 + \frac{1}{2}(\delta_{Z_g} + \delta_{Z_u} + \delta_{Z_{\bar{u}}} + \delta_{Z_{ZZ}} - \frac{Q_f}{g_{L/R}}\delta_{Z_{AZ}}) + \delta_{g_{L/R}} + \delta_{g_s}$$

all SM particles contribute $\Rightarrow$ much more involved then $\overline{\text{MS}}$

Catani operator

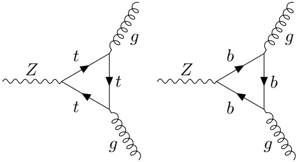
$$
\begin{aligned}
\frac{2\Gamma(1-\epsilon)}{e^{\gamma_E \epsilon}}\mathcal{I}_{1,QCD}(\epsilon) &= \left(-\frac{\mu^2}{s}\right)^{\epsilon}(C_A - 2C_F)\left(\frac{1}{\epsilon^2}+\frac{3}{2\epsilon}\right) \\
&- \left(\left(-\frac{\mu^2}{u}\right)^{\epsilon}+\left(-\frac{\mu^2}{t}\right)^{\epsilon}\right)\left(C_A\left(\frac{1}{\epsilon^2}+\frac{3}{4\epsilon}\right)+\frac{\beta_0}{4\epsilon}\right), \\
-\frac{\Gamma(1-\epsilon)}{e^{\gamma_E \epsilon}}\mathcal{I}_{1,EWK}(\epsilon) &= \left(-\frac{\mu^2}{s}\right)^{\epsilon} S_\epsilon Q_{up}^2\left(\frac{1}{\epsilon^2}+\frac{3}{2\epsilon}\right), \\
\mathcal{I}_2(\epsilon) - \mathcal{I}_{1,QCD}(\epsilon)\,\mathcal{I}_{1,EWK}(\epsilon) &= \frac{e^{\gamma_E \epsilon}}{\Gamma(1-\epsilon)}\left(-\frac{\mu^2}{s}\right)^{2\epsilon}\frac{1}{\epsilon}S_\epsilon C_F Q_{up}^2\left(\frac{\pi^2}{2}-6\zeta_3-\frac{3}{8}\right)
\end{aligned}
$$

The top loop

- closed chiral fermion loops
- new tensors $\overline{T}_i$ with $\gamma_\mu \gamma_5$
- Larin's prescription

  [*Larin arXiv:9302240*]

$$\gamma_\mu \gamma_5 = \frac{i}{3!} \epsilon_{\mu\mu_1\mu_2\mu_3} \gamma^{\mu_1} \gamma^{\mu_2} \gamma^{\mu_3}$$



- massive top quark

  $\Rightarrow$ "anomaly" noncancellation :

$$\sim f(m_t)\, \mathcal{O}(\epsilon^0)$$