Resummation of high-energy logarithms in Higgs boson production at the LHC

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Outline

Introduction and motivations

BFKL resummation

NLO impact factors: Higgs case

LO Higgs impact factor

NLO Higgs impact factor: Real corrections

NLO Higgs impact factor: Virtual corrections

Cancellation of divergences

Higgs plus jet

Conclusions and outlook

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Motivation

- Record energies in the center-of-mass reachable by modern and future colliders allow us to study strong interactions in so far unexplored kinematic regions
- **Semi-hard** collision process → stringent *scale hierarchy*

$$s \gg Q^2 \gg \Lambda^2_{\rm QCD}, \qquad Q^2 \text{ a hard scale,}$$

$\nearrow$

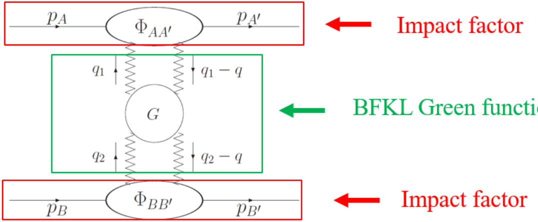
Regge kinematic region

$$\alpha_s(Q^2)\ \ln\left(\frac{s}{Q^2}\right) \sim 1 \implies \text{all-order } \textbf{resummation} \text{ needed}$$

- The Balitskii-Fadin-Kuraev-Lipatov (**BFKL**) approach is the general framework for this *high-energy* resummation
  - Leading-Logarithmic-Approximation (**LLA**): $(\alpha_s \ln s)^n$
  - Next-to-Leading-Logarithmic-Approximation (**NLLA**): $\alpha_s(\alpha_s \ln s)^n$
- Progress on **NNLLA**

BFKL resummation

- Diffusion $A + B \longrightarrow A' + B'$ in the Regge kinematic region
- Gluon Reggeization
- BFKL factorization for $\Im\mathcal{A}_{AB}^{A'B'}$: convolution of a Green function (process independent) with the Impact factors of the colliding particles (process dependent).



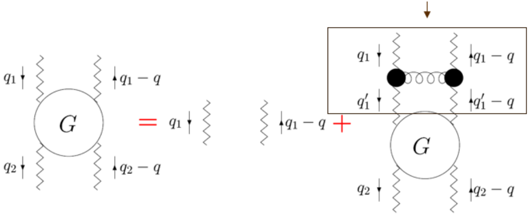
$$\Im\mathcal{A}_{AB}^{A'B'} = \frac{s}{(2\pi)^{D-2}} \int \frac{d^{D-2}q_1}{\vec{q}_1^{\,2}(\vec{q}_1 - \vec{q}\,)^2} \frac{d^{D-2}q_2}{\vec{q}_2^{\,2}(\vec{q}_2 - \vec{q}\,)^2}$$

$$\times \sum_{\nu} \Phi_{A'A}^{(R,\nu)}(\vec{q}_1, \vec{q}, s_0) \int \frac{d\omega}{2\pi i} \left[\left(\frac{s}{s_0} \right)^{\omega} G_{\omega}^{(R)}(\vec{q}_1, \vec{q}_2; \vec{q}\,) \right] \Phi_{B'B}^{(R,\nu)}(-\vec{q}_2, \vec{q}, s_0)$$

BFKL resummation

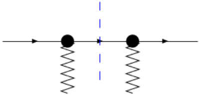
- $G_\omega^{(R)}(\vec{q}_1, \vec{q}_2; \vec{q}\,)$-Mellin transform of the Green function for the Reggeon-Reggeon scattering

$$\omega G_\omega^{(R)}(\vec{q}_1, \vec{q}_2; \vec{q}\,) = \vec{q}_1^{\,2}(\vec{q}_1 - \vec{q}\,)^2 \delta^{(D-2)}(\vec{q}_1 - \vec{q}_2) + \int \frac{d^{D-2}q_1'}{\vec{q}_1^{\,\prime 2}(\vec{q}_1^{\,\prime} - \vec{q}\,)^2} \mathcal{K}^{(R)}(\vec{q}_1, \vec{q}_1^{\,\prime}; \vec{q}\,) G_\omega^{(R)}(\vec{q}_1^{\,\prime}, \vec{q}_2; \vec{q}\,)$$



- $\Phi_{P'P}^{(R,\nu)}$- LO impact factor in the t-channel color state (R, ν)

$$\Phi_{PP'}^{(R,\nu)} = \langle cc'|\hat{\mathcal{P}}|\nu\rangle \sum_{\{f\}} \int \frac{ds_{PR}}{2\pi} d\rho_f \Gamma^c_{\{f\}P} (\Gamma^{c'}_{\{f\}P'})^*$$



BFKL resummation

- BFKL factorization

$$\Im\mathcal{A}_{AB}^{AB} = \frac{s}{(2\pi)^{D-2}} \int d^{D-2}q_1 d^{D-2}q_2$$
$$\times \frac{\Phi_{AA}^{(0)}(\vec{q}_1, s_0)}{\vec{q}_1^{\,2}} \int \frac{d\omega}{2\pi i} \left[\left(\frac{s}{s_0} \right)^{\omega} G_\omega(\vec{q}_1, \vec{q}_2) \right] \frac{\Phi_{BB}^{(0)}(-\vec{q}_2, s_0)}{\vec{q}_2^{\,2}}$$

- BFKL equation

$$\omega G_\omega(\vec{q}_1, \vec{q}_2) = \delta^{(D-2)}(\vec{q}_1 - \vec{q}_2) + \int d^{D-2}q_r\, \mathcal{K}(\vec{q}_1, \vec{q}_r\,) G(\vec{q}_r, \vec{q}_2)$$

[Ya.Ya. Balitskii, V.S. Fadin, E.A. Kuraev, L.N. Lipatov (1975)]

- **NLO definition of impact factors**

$$\Phi_{AA}(\vec{q}_1; s_0) = \left(\frac{s_0}{\vec{q}_1^{\,2}} \right)^{\omega(-\vec{q}_1^{\,2})} \sum_{\{f\}} \int \theta(s_\Lambda - s_{AR}) \frac{ds_{AR}}{2\pi}\, d\rho_f\, \Gamma_{\{f\}A}^{c} \left(\Gamma_{\{f\}A}^{c'} \right)^* \langle cc' | \hat{\mathcal{P}}_0 | 0 \rangle$$

$$- \frac{1}{2} \int d^{D-2}q_2\, \frac{\vec{q}_1^{\,2}}{\vec{q}_2^{\,2}}\, \Phi_{AA}(\vec{q}_2)\, \mathcal{K}_r(\vec{q}_2, \vec{q}_1)\, \ln \left(\frac{s_\Lambda^2}{s_0 (\vec{q}_2 - \vec{q}_1)^2} \right)$$

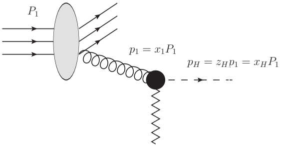
$s_\Lambda \to$ rapidity regulator $\qquad$ $\omega(-\vec{q}_1^{\,2}) \to$ 1-loop Regge trajectory

Factorization scheme for hadronic impact factors

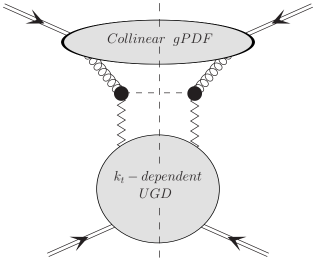
- Infrared safety of impact factor for colorless particle
  [V. S. Fadin, A. D. Martin (1999)]
- Impact factors of colored particles afflicted by *infrared singularities*

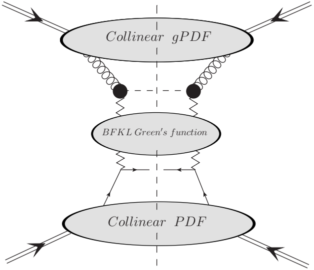
$$p_H = z_H p_1 + \frac{m_H^2 + \vec{p}_H^{\,2}}{z_H s} p_2 + p_{H,\perp}$$

$$\frac{d\Phi_{PP}^{H}}{dx_H d^2\vec{p}_H} = \int_{x_H}^{1} \frac{dz_H}{z_H} f_g\left(\frac{x_H}{z_H}\right) \frac{d\Phi_{gg}^{H}}{dz_H d^2\vec{p}_H}$$



- **Hybrid factorization(s)**



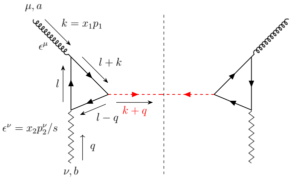


[A. H. Mueller, H. Navelet (1987)]

Outline

LO Higgs impact factor

- Gluon-Reggeon $\longrightarrow$ Higgs (through the top quark loop)
- Off-shell t-channel gluon



- **LO impact factor**

$$\frac{d\Phi_{PP}^{\{H\}(0)}}{dx_H d^2\vec{p}_H} = \frac{\alpha_s^2}{v^2} \frac{\vec{q}^{\,2} |\mathcal{F}(m_t, m_H, \vec{q}^{\,2})|^2}{128\pi^2 \sqrt{2(N^2-1)}} f_g(x_H) \delta^{(2)}(\vec{p}_H - \vec{q})$$

$\downarrow$ Infinite top-mass limit

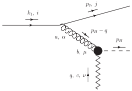
$$\frac{d\Phi_{PP}^{\{H\}(0)}}{dx_H d^2\vec{p}_H} = \frac{g_H^2 \vec{q}^{\,2} f_g(x_H) \delta^{(2)}(\vec{q} - \vec{p}_H)}{8\sqrt{N^2-1}}$$

- The study can be upgraded to **Next-to-Leading Order (NLO)**, in the limit $m_t \to \infty$, by using the effective lagrangian

$$\mathcal{L}_{\mathbf{ggH}} = -\frac{\mathbf{1}}{\mathbf{4}} \mathbf{g_H F^a_{\mu\nu} F^{\mu\nu,a} H} \qquad g_H = \frac{\alpha_s}{3\pi v} + \mathcal{O}(\alpha_s^2)$$

NLO Higgs impact factor: Real corrections

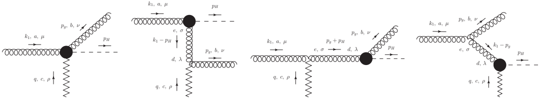
- **Quark initiated contribution**



$$d\Phi_{qq}^{\{Hq\}} \sim \left[\frac{4(1-z_H)\,(\vec{r}\cdot\vec{q}\,)^2 + z_H^2 \vec{q}^{\,2} \vec{r}^2}{z_H\,(\vec{r}^2)^2} \right]$$

Rapidity divergence absent $\implies$ $s_\Lambda \to \infty$ Collinear divergence: $r \equiv (\vec{q} - \vec{p}_H) \to \vec{0}$

- **Gluon initiated contribution**



$$d\Phi_{gg}^{\{Hg\}} \sim \left\{ \frac{\vec{q}^{\,2} z_H}{(1-z_H)\vec{r}^{\,2}} + \frac{\vec{q}^{\,2}}{\vec{r}^{\,2}} \left[z_H(1-z_H) + 2(1-\epsilon) \frac{1-z_H}{z_H} \frac{(\vec{q}\cdot\vec{r})^2}{\vec{q}^{\,2}\vec{r}^{\,2}} \right] \right\}$$

$$\times\, \theta\left(s_\Lambda - \frac{(1-z_H)m_H^2 + \vec{\Delta}^2}{z_H(1-z_H)} \right) + \text{finite}$$

Rapidity divergence $\implies$ s_Λ still present

Collinear divergence: $\vec{r} \to \vec{0}$

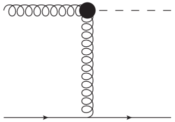
Soft divergence: $z_H \to 1$, $\vec{r} \to \vec{0}$

$\vec{\Delta} = \vec{p}_H - z_H \vec{q}$

- Agreement with calculation within Lipatov effective action framework

[M. Hentschinski, K. Kutak, A. van Hameren (2021)]

NLO Higgs impact factor: Virtual corrections



- 1-loop ggH effective vertex

$$\Gamma^{ac(1)}_{\{H\}g}(q) = \Gamma^{ac(0)}_{\{H\}g}(q)\,[1+\delta_{\rm NLO}]$$

- General strategy: Comparison of a suitable amplitude (in the high-energy limit) with the expected *Regge form*

$$\mathcal{A}^{(8,-)}_{gq\to Hq} = \Gamma^{ac}_{\{H\}g}\frac{s}{t}\left[\left(\frac{s}{-t}\right)^{\omega(t)}+\left(\frac{-s}{-t}\right)^{\omega(t)}\right]\Gamma^{c}_{qq} \approx \Gamma^{ac(0)}_{\{H\}g}\frac{2s}{t}\Gamma^{c(0)}_{qq}$$

$$+\Gamma^{ac(0)}_{\{H\}g}\frac{s}{t}\omega^{(1)}(t)\left[\ln\left(\frac{s}{-t}\right)+\ln\left(\frac{-s}{-t}\right)\right]\Gamma^{c(0)}_{qq}+\Gamma^{ac(0)}_{\{H\}g}\frac{2s}{t}\Gamma^{c(1)}_{qq}+\Gamma^{ac(1)}_{\{H\}g}\frac{2s}{t}\Gamma^{c(0)}_{qq}$$

- **Virtual corrections** to the impact factor

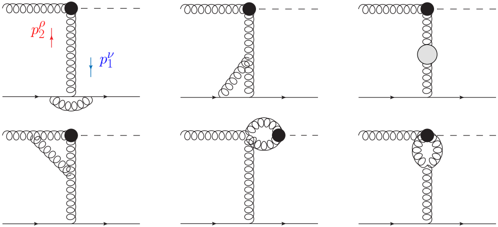
$$\frac{d\Phi^{\{H\}(1)}_{gg}}{dz_H d^2\vec p_H} = \frac{d\Phi^{\{H\}(0)}_{gg}}{dz_H d^2\vec p_H}\,\frac{\bar\alpha_s}{2\pi}\left(\frac{\vec q^{\,2}}{\mu^2}\right)^{-\epsilon}\left[-\frac{C_A}{\epsilon^2}-\frac{C_A}{\epsilon}\ln\left(\frac{\vec q^{\,2}}{s_0}\right)+\frac{\beta_0}{2\epsilon}\right.$$
$$\left.+11-\frac{5n_f}{9}+C_A\left(2\,\Re e\left(\mathrm{Li}_2\left(1+\frac{m_H^2}{\vec q^{\,2}}\right)\right)+\frac{\pi^2}{3}+\frac{67}{18}\right)\right]$$

- Same calculation within Lipatov effective action framework

[M. Nefedov (2019)]

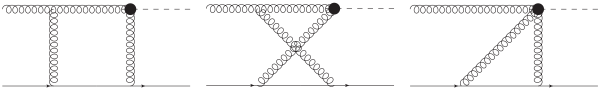
NLO Higgs impact factor: Virtual corrections

- Single gluon in the t-channel diagrams



Gribov's trick: $g^{\rho\nu} = g_{\perp\perp}^{\rho\nu} + 2\frac{p_1^\rho p_2^\nu + p_1^\nu p_2^\rho}{s} \to 2s\,\frac{p_1^\nu}{s}\,\frac{p_2^\rho}{s}$

- Two gluons in the t-channel diagrams



Dimension-5 operator in $\mathcal{L} = \mathcal{L}_{QCD} + \mathcal{L}_{ggH}$ $\to$ **Gribov's trick violation**

$$g^{\rho\nu} = g_{\perp\perp}^{\rho\nu} + 2\frac{p_1^\rho p_2^\nu + p_1^\nu p_2^\rho}{s} \to 2s\,\frac{p_1^\nu}{s}\,\frac{p_2^\rho}{s} + g_{\perp\perp}^{\rho\nu}$$

Showing cancellation of divergences

- **BFKL cross section**

$$\sigma_{AB} = \frac{1}{(2\pi)^{D-2}} \int \frac{d^{D-2} q_1}{\vec{q}_1^{\,2}} \int \frac{d^{D-2} q_2}{\vec{q}_2^{\,2}}$$

$$\times \Phi_{AA}(\vec{q}_1; s_0) \int_{\delta - i\infty}^{\delta + i\infty} \frac{d\omega}{2\pi i} \left[\left(\frac{s}{s_0} \right)^{\omega} G_{\omega}^{(0)}(\vec{q}_1, \vec{q}_2) \right] \Phi_{BB}(-\vec{q}_2; s_0)$$

- Spectral representation of the Green function at LO

$$G_{\omega}^{(0)}(\vec{q}_1, \vec{q}_2) = \sum_{n=-\infty}^{\infty} \int_{-\infty}^{+\infty} d\nu \frac{\phi_{\nu}^{n}(\vec{q}_1^{\,2}) \phi_{\nu}^{n*}(\vec{q}_2^{\,2})}{\omega - \frac{\alpha_s C_A}{\pi} \chi(n, \nu)} \,,$$

- **Projection** onto the eigenfunction of the BFKL kernel

$$\int \frac{d^{2-2\epsilon} q}{\pi \sqrt{2}} (\vec{q}^{\,2})^{i\nu - \frac{3}{2}} e^{in\phi} \Phi_{AA}^{(0)}(\vec{q}\,) \equiv \Phi_{AA}^{(0)}(n, \nu)$$

- LO projected impact factor

$$\frac{d\Phi_{PP}^{\{H\}(0)}(x_H, \vec{p}_H, n, \nu)}{dx_H d^2\vec{p}_H} = \frac{g_H^2}{8(1-\epsilon)\sqrt{N^2-1}} \frac{(\vec{p}_H^{\,2})^{i\nu - \frac{1}{2}} e^{in\phi_H}}{\pi\sqrt{2}} f_g(x_H)$$

- Scheme for *cancellation of divergences*
  - *i.* Rapidity divergence → removed by the BFKL counterterm
  - *ii.* UV divergences → QCD coupling renormalization
  - *iii.* Soft divergence → cancelled in the real plus virtual combination
  - *iiii.* Surviving initial-state IR divergences → gPDF renormalization

Cancellation of divergences in the (n,ν)-space

- UV counterterm $d\Phi_{PP}^{\{H\}}\Big|_{\alpha_s\ c.t.} = d\Phi_{PP}^{\{H\}(0)} \frac{\bar{\alpha}_s}{2\pi}\left[-\frac{\beta_0}{\epsilon}\right] + \text{finite}$
- gPDF counterterm

$$d\Phi_{PP}^{\{H\}}\Big|_{\mathrm{P_{qg}\ c.t.}} = \frac{d\Phi_{PP}^{\{H\}(0)}}{f_g(x_H)}\frac{\bar{\alpha}_s}{2\pi}\left[\frac{1}{\epsilon}P_{gq}\otimes\sum_{a=q\bar{q}}f_a\right] + \text{finite}$$

$$d\Phi_{PP}^{\{H\}}\Big|_{\mathrm{P_{gg}\ c.t.}} = \frac{d\Phi_{PP}^{\{H\}(0)}}{f_g(x_H)}\frac{\bar{\alpha}_s}{2\pi}\left[\frac{1}{\epsilon}\tilde{P}_{gg}\otimes f_g + \frac{1}{2}\frac{\beta_0}{\epsilon}f_g(x_H)\right] + \text{finite}$$

- Real quark contribution

$$d\Phi_{PP}^{\{Hg\}}\Big|_{\text{quark}} = \frac{d\Phi_{PP}^{\{H\}(0)}}{f_g(x_H)}\frac{\bar{\alpha}_s}{2\pi}\left[-\frac{1}{\epsilon}P_{gq}\otimes\sum_{a=q\bar{q}}f_a\right] + \text{finite}$$

- Real gluon contribution (BFKL counterterm subtracted)

$$d\Phi_{PP}^{\{Hq\}}\Big|_{\text{gluon}} = \frac{d\Phi_{PP}^{\{H\}(0)}}{f_g(x_H)}\frac{\bar{\alpha}_s}{2\pi}\left(\frac{\vec{p}_H^{\,2}}{\mu^2}\right)^{-\epsilon}\left[\left(\frac{C_A}{\epsilon^2}+\frac{C_A}{\epsilon}\ln\left(\frac{\vec{p}_H^{\,2}}{s_0}\right)\right)f_g(x_H) - \frac{1}{\epsilon}\tilde{P}_{gg}\otimes f_g\right] + \text{finite}$$

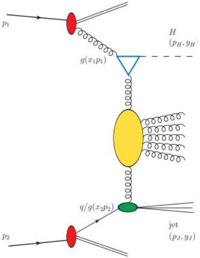
- Virtual corrections contribution

$$d\Phi_{PP}^{\{H\}}\Big|_{\text{virtual}} = d\Phi_{PP}^{\{H\}(0)}\frac{\bar{\alpha}_s}{2\pi}\left(\frac{\vec{p}_H^{\,2}}{\mu^2}\right)^{-\epsilon}\left[-\frac{C_A}{\epsilon^2} - \frac{C_A}{\epsilon}\ln\left(\frac{\vec{p}_H^{\,2}}{s_0}\right) + \frac{1}{\epsilon}\frac{\beta_0}{2}\right] + \text{finite}$$

Outline

Higgs plus jet

- Inclusive **Higgs plus jet** production in proton-proton collision
  - *i.* Full NLL Green function + Partial NLO impact factors (full m_t-dep.)
    [F.G. Celiberto, D. Yu. Ivanov, M.M.A. Mohammed, A. Papa (2021)]
  - *ii.* Same process in HEJ framework (full m_t,m_b-dep.)
    [J. R. Andersen et al. (2022)]



$$\frac{d\sigma_{\rm pp}}{dy_H\, dy_J\, d|\vec{p}_H|\, d|\vec{p}_J|\, d\phi_1\, d\phi_2} = \frac{1}{(2\pi)^2}$$

$$\times \int \frac{d^2\vec{q}_1}{\vec{q}_1^{\,2}} \left(\mathcal{V}_H^{(g)}(\vec{q}_1, s_0, x_1, \vec{p}_H) \otimes f_g(x_1)\right)$$

$$\times \int_{\delta - i\infty}^{\delta + i\infty} \frac{d\omega}{2\pi i} \left(\frac{x_1 x_2 s}{s_0}\right)^{\omega} G_\omega(\vec{q}_1, \vec{q}_2)$$

$$\times \int \frac{d^2\vec{q}_2}{\vec{q}_2^{\,2}} \left(\sum_r \mathcal{V}_J^{(p)}(\vec{q}_2, s_0, x_2, \vec{p}_J) \otimes f_r(x_2)\right)$$

- Hadronic cross section expanded in **azimuthal coefficients**

$$\frac{d\sigma_{\rm pp}}{dy_H\, dy_J\, d|\vec{p}_H|\, d|\vec{p}_J|\, d\phi_1\, d\phi_2} = \frac{1}{(2\pi)^2} \left[\mathcal{C}_0 + 2 \sum_{n=1}^{\infty} \cos(n\phi)\, \mathcal{C}_n\right]$$

Outline

Conclusions and outlook

Conclusions

- **NLO corrections** to the forward Higgs boson impact factor has been obtained both in q_T and (n, ν)-space in the $m_t \to \infty$ limit
- *Gribov's philosophy* for high-energy computations proposed in QCD needs to be *revisited* in the present case.

Outlook and related topics

- **Stability of the BFKL series** under higher-order corrections and scale variations has been observed, with partial NLLA, in the inclusive forward emissions of a Higgs in association with a backward jet
- **Full NLL matched to NLO Higgs plus jet production**
- **Finite top-mass corrections**
- The impact of the high-energy resummation in central inclusive Higgs production at FCC center-of-mass energies is expected to be large

  [M. Bonvini, S. Marzani (2018)]
- Unified formalism to include different kinds of resummations

  (BFKL+Sudakov) [B. Xiao, F. Yuan (2018)]

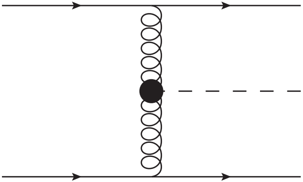
Thank you for the attention !

Backup

Outlook: Extension to the central production at NLO

- LO vertex with full mass corrections

  [R.S. Pasechnik, O.V. Teryaev, A. Szczurek (2006)]
- Computation of real corrections quite straightforward
- Need to extract the vertex at one-loop in the central region of rapidity
- Necessity of a reference NLO two into three particle amplitude, e. g. $\mathcal{A}_{q+q \to q+H+q}$



- Necessary scalar integrals known: I_4^{2m} and I_5^{1m}

  [Z. Bern, L. Dixon, D. A. Kosower (1998)]

BFKL resummation

What is the BFKL resummation?

- The Balitsky-Fadin-Kuraev-Lipatov (BFKL) approach is the general framework for the resummation of energy-type logarithms
  - Leading-Logarithm-Approximation (LLA): $(\alpha_s \ln s)^n$
  - Next-to-Leading-Logarithm-Approximation (NLLA): $\alpha_s(\alpha_s \ln s)^n$

In which contexts can BFKL approach be applied?

- Semi-hard collision processes, featuring the scale hierarchy

$$s \gg Q^2 \gg \Lambda^2_{\rm QCD}, \qquad Q^2 \text{ a hard scale,}$$

$$\alpha_s(Q^2)\ \ln\left(\frac{s}{Q^2}\right) \sim 1 \implies \text{all-order resummation needed}$$

- UGD sector

  The evolution of the Unintegrated gluon density,

$$\mathcal{F}(x,\vec{k}) \quad \text{t.c.} \quad f^g(x,Q^2) = \int \frac{d^2\vec{k}}{\pi\vec{k}^2}\mathcal{F}(x,\vec{k})\theta(Q^2-\vec{k}^2)$$

  as a function of $\ln(1/x) = \ln(s/Q^2)$, is governed by BFKL:

$$\frac{\partial \mathcal{F}}{\partial \ln(1/x)} = \mathcal{F} \otimes \mathcal{K}$$

Before QCD

- Assumptions on S-matrix ($S_{ab} = \langle b_{out} | a_{in} \rangle$):
  - Lorentz invariance:
    It can be expressed as a function of Lorentz invariant scalar product, e.g (s, t) for $2 \to 2$ particle scattering.
  - Analiticity

    Causality $\to$ Analytic function with only those singularity required by unitarity.
  - Unitarity

Cutkosky rules

$$2\Im \mathcal{A}_{ab} = (2\pi)^4 \delta^4 (\sum_a p_a - \sum_b p_b) \sum_c \mathcal{A}_{ac} \mathcal{A}^\dagger_{cb}$$

Optical theorem

$$2\Im \mathcal{A}_{aa}(s, 0) = F \sigma_{tot}$$

- Unitarity $\to$ relates the imaginary parts of amplitudes to sum of products of other amplitudes, **dispersion relations** $\to$ reconstruct the corresponding real parts
- More in general **subtract dispersion relation** $\to$ we must know the asymptotic behaviour of amplitudes $\to$ **Regge theory**

Before QCD

- Asymptotic behavior of amplitudes in the Regge region:

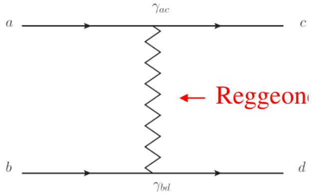
$$\mathcal{A}(s,t) \xrightarrow[s \gg |t|]{} \frac{\eta + e^{-i\pi\alpha(t)}}{2} \beta(t) s^{\alpha(t)}$$

- Definition of **"Reggeization"**

A particle of mass M and spin J is said to "Reggeize" if the amplitude, $\mathcal{A}$, for a process involving the exchange in the t-channel of the quantum numbers of that particle behaves asymptotically in s as

$$\mathcal{A} \propto s^{\alpha(t)}$$

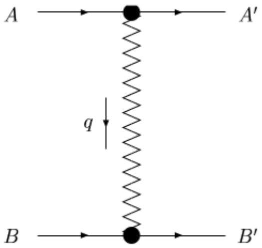
where $\alpha(t)$ is the trajectory and $\alpha(M^2) = J$, so that the particle itself lies on the trajectory.



The Reggeized gluon

Elastic scattering process $A + B \longrightarrow A' + B'$

- Gluon quantum numbers in the t-channel
- Regge limit: $s \simeq -u \to \infty$, t fixed (i.e not growing with s)
- All-order resummation:
  leading logarithmic approximation (LLA): $(\alpha_s \ln s)^n$
  next-to-leading logarithmic approximation (NLA): $\alpha_s(\alpha_s \ln s)^n$



$$(\mathcal{A})_{AB}^{A'B'} = \Gamma^c_{A'A}\left[\left(\frac{-s}{-t}\right)^{j(t)} - \left(\frac{s}{-t}\right)^{j(t)}\right]\Gamma^c_{B'B}$$

$$j(t) = 1 + \omega(t), \qquad j(0) = 1$$

$j(t)$- Reggeized gluon trajectory

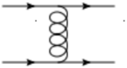
$$\Gamma^c_{A'A} = g\,\langle A'|T^c|A\rangle\,\Gamma_{A'A}$$

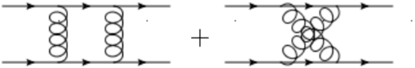
T^c- fundamental(quarks) or adjoint(gluons)

- LLA [Ya. Ya. Balitsky, V.S. Fadin, L.N. Lipatov (1979)]

$$\Gamma^{(0)}_{A'A} = \delta_{\lambda_{A'}\lambda_A}, \quad \omega^{(1)}(t) = \frac{g^2 t}{(2\pi)^{(D-1)}}\frac{N}{2}\int\frac{d^{D-2}k_\perp}{k_\perp^2\,(q-k)_\perp^2} = -g^2\frac{N\Gamma(1-\epsilon)}{(4\pi)^{2+\epsilon}}\frac{\Gamma^2(\epsilon)}{\Gamma(2\epsilon)}(\vec{q}^{\,2})^{\epsilon}$$

The Reggeized gluon

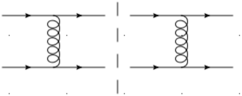




$$\mathcal{A}_0^8 = 8\pi\alpha_s \frac{s}{t}\delta_{\lambda_{A'}\lambda_A}\delta_{\lambda_{B'}\lambda_B} G_0^8 \qquad\qquad \mathcal{A}_1^8 = \mathcal{A}_0^8\,\omega(t)\ln\left(\frac{s}{\vec{q}^{\,2}}\right)$$

$$\mathcal{A}^8 = \mathcal{A}_0^8\left[1 + \omega(t)\ln\left(\frac{s}{\vec{q}^{\,2}}\right) + \frac{1}{2}\left(\omega(t)\ln\left(\frac{s}{\vec{q}^{\,2}}\right)\right)^2 + \ldots\right] \to \mathcal{A}^8 = \mathcal{A}_0^8\left(\frac{s}{\vec{q}^{\,2}}\right)^{\omega(t)}$$

$$\Im\mathcal{A}_1 = \frac{1}{2}\int d(P.S.^2)\mathcal{A}_0^8(k)\mathcal{A}_0^8(k-q)$$

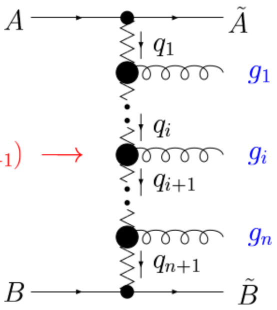


The integration that appears in $\omega(t)$ is the residue of that over the phase space.
The terms in the denominator come from the propagators.

- NLLA [V.S. Fadin, R. Fiore, M.G. Kozlov, A.V. Reznichenko (2006)]

BFKL in LLA

Inelastic scattering process $A + B \longrightarrow \tilde{A} + \tilde{B} + n$ in the LLA



Leading logarithms resummation
↓
Multi-Regge kinematics (MRK)

$$\Re\mathcal{A}^{\tilde{A}\tilde{B}+n}_{AB} = 2s\Gamma^{c_1}_{\tilde{A}A}\left(\prod_{i=1}^{n}\gamma^{P_i}_{c_i c_{i+1}}(q_i, q_{i+1})\left(\frac{s_i}{s_0}\right)^{\omega(t_i)}\frac{1}{t_i}\right)\frac{1}{t_{n+1}}\left(\frac{s_{n+1}}{s_0}\right)^{\omega(t_{n+1})}\Gamma^{c_{n+1}}_{\tilde{B}B}$$

- s_0-energy scale, arbitrary in LLA.
- Terms that contain fermions in intermediate states are suppressed in relation to those that involve the exchange of gluons
- "Vertical" gluons become Reggeized due to radiative corrections ("ladders within ladders")

Multi-Regge kinematics

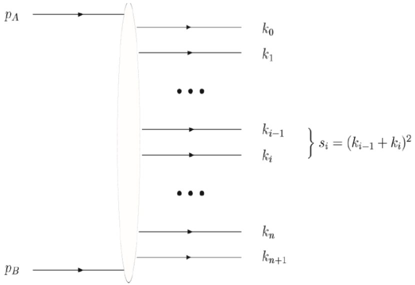
Multi-Regge kinematics

- Sudakov decomposition for the produced particles: $k_i = z_i p_1 + \lambda_i p_2 + k_{i\perp}$

- Transverse momenta of the produced particles are limited
- Their Sudakov variables z_i and λ_i, are strongly ordered:

$$z_0 \gg z_1 \gg \ldots \gg z_n \gg z_{n+1}$$

$$\lambda_{n+1} \gg \lambda_n \gg \ldots \gg \lambda_1 \gg \lambda_0$$

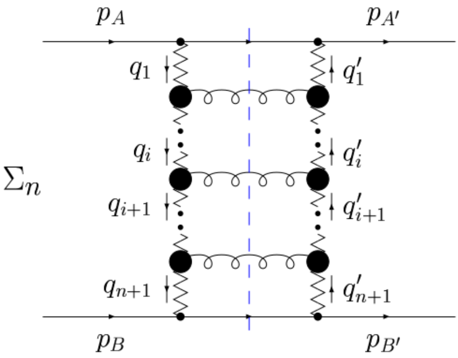


- Leading logarithms come from the integration over the longitudinal momenta of the produced particles
- In the LLA, where each added particle contributes only one $\ln s$, only this kinematics counts

BFKL in LLA

Amplitude $A + B \to A' + B'$ in the LLA via Cutkosky rules

$$\Im \mathcal{A}_{AB}^{A'B'} = \frac{1}{2} \sum_{n=0}^{\infty} \sum_{f} \int \mathcal{A}_{AB}^{\tilde{A}\tilde{B}+n} \left(\mathcal{A}_{A'B'}^{\tilde{A}\tilde{B}+n} \right)^{*} d\Phi_{\tilde{A}\tilde{B}+n}$$



$$\mathcal{A}_{AB}^{A'B'} = \sum_{\mathcal{R}} (\mathcal{A}_R)_{AB}^{A'B'} \qquad \mathcal{R} = 1(\text{singlet}), 8(\text{octect}), \ldots$$

Solution of the BFKL equation

- Let's solve the equation

$$\omega G_\omega(\vec{q}_1, \vec{q}_2) = \delta^{(D-2)}(\vec{q}_1 - \vec{q}_2) + \int d^{D-2}q_r\, \mathcal{K}(\vec{q}_1, \vec{q}_r\,) G(\vec{q}_r, \vec{q}_2)$$

$$\mathcal{K}(\vec{q}_1, \vec{q}_r) = \mathcal{K}^{(R)}(\vec{q}_1, \vec{q}_r) + 2\omega(\vec{q}_1^{\,2})\delta^{(2)}(\vec{q}_1 - \vec{q}_r)$$

- We can see $\mathcal{K}(\vec{k}, \vec{k}')$ as the integral kernel of an operator acting on a space of complex functions (defined on a bi-dimensional vector space)

$$\hat{\mathcal{K}}\left[f(\vec{k})\right] = \int d^2\vec{k}' \mathcal{K}(\vec{k}, \vec{k}')\, f(\vec{k}')$$

- We solve the eigenvalue problem for the Kernel

Eigenvalues $\longrightarrow \omega_n(\nu) = \bar{\alpha}_s \chi_n(\nu)\ , \quad \bar{\alpha}_s = \frac{\alpha_s N}{\pi}$

Eigenfunctions $\longrightarrow \phi_\nu^n(\vec{q}\,) = \frac{1}{\pi\sqrt{2}} \left(\vec{q}^{\,2}\right)^{-\frac{1}{2}+i\nu} e^{in\theta}$

- Then we are able to reconstruct the $G_\omega(\vec{q}_1, \vec{q}_2)$

$$G_\omega(\vec{q}_1, \vec{q}_2) = \sum_{n=0}^{\infty} \int_{-\infty}^{\infty} d\nu \left(\frac{\vec{q}_1^{\,2}}{\vec{q}_2^{\,2}}\right)^{i\nu} \frac{e^{in(\theta_1-\theta_2)}}{2\pi^2 q_1 q_2} \frac{1}{\omega - \bar{\alpha}_s \chi(n,\nu)} \longrightarrow G_s(\vec{q}_1, \vec{q}_2) \sim s^{\omega_0}$$

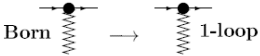
$\omega_0 = 4\bar{\alpha}_s \ln 2 \simeq 0.40$ for $\alpha_s = 0.15$

- Resummation of subleading logarithms means **new kinematics**
  1. Multi-Regge kinematics (MRK)
  2. Quasi multi-Regge kinematics (QMRK)
- Production amplitudes keep the simple factorized form

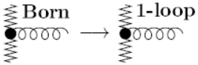
$$\Re\mathcal{A}_{AB}^{\tilde{A}\tilde{B}+n} = 2s\Gamma_{\tilde{A}A}^{c_1}\left(\prod_{i=1}^{n}\gamma_{c_i c_{i+1}}^{P_i}(q_i, q_{i+1})\left(\frac{s_i}{s_0}\right)^{\omega(t_i)}\frac{1}{t_i}\right)\frac{1}{t_{n+1}}\left(\frac{s_{n+1}}{s_0}\right)^{\omega(t_{n+1})}\Gamma_{\tilde{B}B}^{c_{n+1}}$$

- Multi-Regge kinematics $\rightarrow$ previous quantity must be calculated at 1-loop (one α_s more)

- $\omega^{(1)} \longrightarrow \omega^{(2)}$
- $\Gamma_{P'P}^{c\,\text{(Born)}} \longrightarrow \Gamma_{P'P}^{c\,\text{(1-loop)}}$



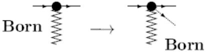
- $\gamma_{c_i c_{i+1}}^{G_i\text{(Born)}} \longrightarrow \gamma_{c_i c_{i+1}}^{G_i\text{(1-loop)}}$



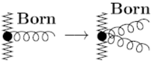
[V.S. Fadin, L.N. Lipatov (1989)]

- Quasi multi-Regge kinematics → A pair of particles, but only one!, may have longitudinal Sudakov variables of the same order (one logarithm less)

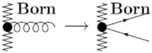
- $\Gamma^{c\,\text{(Born)}}_{P'P} \longrightarrow \Gamma^{c\,\text{(Born)}}_{\{f\}P}$



- $\gamma^{G_i\text{(Born)}}_{c_i c_{i+1}} \longrightarrow \gamma^{Q\overline{Q}\text{(Born)}}_{c_i c_{i+1}}$

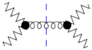


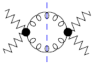
- $\gamma^{G_i\text{(Born)}}_{c_i c_{i+1}} \longrightarrow \gamma^{GG\text{(Born)}}_{c_i c_{i+1}}$

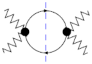


- 3 new contributions to the kernel

$$\mathcal{K} = \mathcal{K}^{Born}_{RRG} + \mathcal{K}^{1-loop}_{RRG} + \mathcal{K}^{Born}_{RRGG} + \mathcal{K}^{Born}_{RR\bar{Q}Q}$$


$\mathcal{K}^{(1)}_{RRG}$


$\mathcal{K}^{(B)}_{RRGG}$


$\mathcal{K}^{(B)}_{RRQ\overline{Q}}$

- NLO definition of the impact factor

$$\Phi_{AA}(\vec{q}_1; s_0) = \left(\frac{s_0}{\vec{q}_1^{\,2}}\right)^{\omega(-\vec{q}_1^{\,2})} \sum_{\{f\}} \int \theta(s_\Lambda - s_{AR}) \frac{ds_{AR}}{2\pi} \, d\rho_f \; \Gamma^c_{\{f\}A} \left(\Gamma^{c'}_{\{f\}A}\right)^* \langle cc'|\hat{\mathcal{P}}_0|0\rangle$$

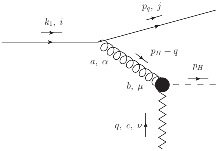
$$-\frac{1}{2}\int d^{D-2}q_2 \; \frac{\vec{q}_1^{\,2}}{\vec{q}_2^{\,2}} \; \Phi^{(0)}_{AA}(\vec{q}_2)\,\mathcal{K}^{(0)}_r(\vec{q}_2,\vec{q}_1) \; \ln\left(\frac{s_\Lambda^2}{s_0(\vec{q}_2-\vec{q}_1)^2}\right)$$

$s_\Lambda \to$ rapidity regulator
$\omega(-\vec{q}_1^{\,2}) \to$ 1-loop Regge trajectory

- Quark initiated contribution

$$\frac{d\Phi^{\{Hq\}}_{qq}(z_H, \vec{p}_H, \vec{q})}{dz_H\, d^2\vec{p}_H} = \frac{g^2 g_H^2 \sqrt{N^2-1}}{16N(2\pi)^{D-1} z_H}\left[\frac{4(1-z_H)\left[(\vec{q}-\vec{p}_H)\cdot\vec{q}\,\right]^2 + z_H^2 \vec{q}^{\,2}(\vec{q}-\vec{p}_H)^2}{[(\vec{q}-\vec{p}_H)^2]^2}\right]$$

- Rapidity divergence absent $\implies$ $s_\Lambda \to \infty$
- Collinear divergence: $(\vec{q}-\vec{p}_H) \to \vec{0}$
- Gauge invariance: $d\Phi^{\{gH\}}_{gg}|_{\vec{q}^{\,2}=0} \longrightarrow 0$

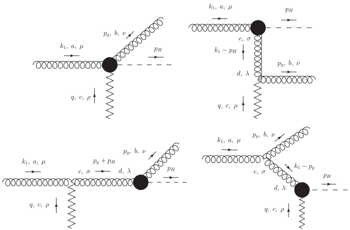


NLO Higgs impact factor: Real corrections

- Gluon initiated contribution

$$
\frac{d\Phi_{gg}^{\{Hg\}}(z_H, \vec{p}_H, \vec{q}; s_0)}{dz_H d^2 p_H} = \frac{g^2 g_H^2 C_A}{8(2\pi)^{D-1}(1-\epsilon)\sqrt{N^2-1}}
$$
$$
\times \Bigg\{ \frac{2}{z_H(1-z_H)} \Bigg[2z_H^2 + \frac{(1-z_H)z_H m_H^2 (\vec{q}\cdot\vec{r})[z_H^2 - 2(1-z_H)\epsilon] + 2z_H^3(\vec{p}_H\cdot\vec{r})(\vec{p}_H\cdot\vec{q})}{\vec{r}^{\,2}[(1-z_H)m_H^2 + \vec{p}_H^{\,2}]} - \frac{2z_H^2(1-z_H)m_H^2}{[(1-z_H)m_H^2 + \vec{p}_H^{\,2}]}
$$
$$
- \frac{(1-z_H)z_H m_H^2(\vec{q}\cdot\vec{r})[z_H^2 - 2(1-z_H)\epsilon] + 2z_H^3(\vec{\Delta}\cdot\vec{r})(\vec{\Delta}\cdot\vec{q})}{\vec{r}^{\,2}\left[(1-z_H)m_H^2 + \vec{\Delta}^2\right]} - \frac{2z_H^2(1-z_H)m_H^2}{\left[(1-z_H)m_H^2 + \vec{\Delta}^2\right]}
$$
$$
+ \frac{(1-\epsilon)z_H^2(1-z_H)^2 m_H^4}{2}\left(\frac{1}{[(1-z_H)m_H^2 + \Delta^2]} + \frac{1}{[(1-z_H)m_H^2 + \vec{p}_H^{\,2}]}\right)^2 - \frac{2z_H^2(\vec{p}_H\cdot\vec{\Delta})^2 - 2\epsilon(1-z_H)^2 z_H^2 m_H^4}{[(1-z_H)m_H^2 + \vec{p}_H^{\,2}][(1-z_H)m_H^2 + \Delta^2]}\Bigg]
$$
$$
+ \frac{2\vec{q}^{\,2}}{\vec{r}^{\,2}}\left[\frac{z_H}{1-z_H} + z_H(1-z_H) + 2(1-\epsilon)\frac{(1-z_H)}{z_H}\frac{(\vec{q}\cdot\vec{r})^2}{\vec{q}^{\,2}\vec{r}^{\,2}}\right]\Bigg\}\,\theta\left(s_\Lambda - \frac{(1-z_H)m_H^2 + \vec{\Delta}^2}{z_H(1-z_H)}\right)
$$

- $\vec{\Delta} = \vec{p}_H - z_H\vec{q} \qquad \vec{r} = \vec{q} - \vec{p}_H$
- Rapidity divergence $\rightarrow$ s_Λ still present
- Soft and Collinear divergences
- Gauge invariance: $d\Phi_{gg}^{\{gH\}}|_{\vec{q}^{\,2}=0} \longrightarrow 0$



Agreement with independent calculation in the Lipatov effective action framework

[M. Hentschinski, K. Kutak, A. van Hameren (2021)]

PDF and α_s counterterms in the (n,ν)-space

- 1-loop α_s running produces the UV-counterterm

$$\left.\frac{d\Phi^{\{H\}}_{PP}(x_H,\vec{p}_H,n,\nu)}{dx_H d^2\vec{p}_H}\right|_{\text{coupling c.t.}} = \frac{d\Phi^{\{H\}(0)}_{PP}(x_H,\vec{p}_H,n,\nu)}{dx_H d^2\vec{p}_H}\frac{\bar{\alpha}_s}{2\pi}\left(\frac{\vec{p}_H^{\,2}}{\mu^2}\right)^{-\epsilon}$$

$$\times\underline{\left(\frac{11C_A}{3}-\frac{2n_f}{3}\right)}\left(-\frac{1}{\epsilon}+\ln\left(\frac{\mu_R^2}{\vec{p}_H^{\,2}}\right)\right)$$

- PDF counter terms produced through DGLAP evolution equations

$$\left.\frac{d\Phi^{\{H\}}_{PP}(x_H,\vec{p}_H,n,\nu)}{dx_H d^2\vec{p}_H}\right|_{\text{P}_{\text{qg}}\text{ c.t.}} = -\frac{1}{f_g(x_H)}\frac{d\Phi^{\{H\}(0)}_{PP}(x_H,\vec{p}_H,n,\nu)}{dx_H d^2\vec{p}_H}\frac{\bar{\alpha}_s}{2\pi}\left(\frac{\vec{p}_H^{\,2}}{\mu^2}\right)^{-\epsilon}$$

$$\times\left(-\frac{1}{\epsilon}+\ln\left(\frac{\mu_F^2}{\vec{p}_H^{\,2}}\right)\right)\int_{x_H}^{1}\frac{dz_H}{z_H}\left[P_{gq}(z_H)\sum_{a=q\bar{q}}f_a\left(\frac{x_H}{z_H},\mu_F\right)\right]$$

$$\left.\frac{d\Phi^{\{H\}}_{PP}(x_H,\vec{p}_H,n,\nu)}{dx_H d^2\vec{p}_H}\right|_{\text{P}_{\text{gg}}\text{ c.t.}} = -\frac{1}{f_g(x_H)}\frac{d\Phi^{\{H\}(0)}_{PP}(x_H,\vec{p}_H,n,\nu)}{dx_H d^2\vec{p}_H}\frac{\bar{\alpha}_s}{2\pi}\left(\frac{\vec{p}_H^{\,2}}{\mu^2}\right)^{-\epsilon}$$

$$\times\left(-\frac{1}{\epsilon}+\ln\left(\frac{\mu_F^2}{\vec{p}_H^{\,2}}\right)\right)\int_{x_H}^{1}\frac{dz_H}{z_H}\left[P_{gg}(z_H)f_g\left(\frac{x_H}{z_H},\mu_F\right)\right]$$

$$P_{gq}(z)=C_F\underline{\frac{1+(1-z)^2}{z}}\,,\qquad P_{gg}(z)=2C_A\underline{\left(\frac{z}{(1-z)_+}+\frac{(1-z)}{z}+z(1-z)\right)}+\underline{\frac{11C_A-2n_f}{6}\delta(1-z)}$$

Real quark and virtual contribution in the (n,ν)-space

- BFKL counterterm + Rapidity divergent part in the real gluon NLO contribution in the (n, ν)-space

$$\frac{d\Phi^{\rm BFKL}_{PP}(x_H,\vec{p}_H,n,\nu)}{dx_H d^2 p_H} = \frac{d\Phi^{\{H\}(0)}_{PP}(x_H,\vec{p}_H,n,\nu)}{dx_H d^2\vec{p}_H}\frac{\bar{\alpha}_s}{2\pi}\left(\frac{\vec{p}_H^{\,2}}{\mu^2}\right)^{-\epsilon}$$

$$\times\left\{\underline{\frac{C_A}{\epsilon^2}+\frac{C_A}{\epsilon}\ln\left(\frac{\vec{p}_H^{\,2}}{s_0}\right)} - \underline{2\frac{C_A}{\epsilon}\ln(1-x_H)} + \mathcal{O}(\epsilon^0)\right\}$$

- Projection of the virtual contribution

$$\frac{d\Phi^{\{H\}(1)}_{PP}(x_H,\vec{p}_H,n,\nu;s_0)}{dx_H d^2\vec{p}_H} = \frac{d\Phi^{\{H\}(0)}_{PP}(x_H,\vec{p}_H,n,\nu)}{dx_H d^2\vec{p}_H}\frac{\bar{\alpha}_s}{2\pi}\left(\frac{\vec{p}_H^{\,2}}{\mu^2}\right)^{-\epsilon}\left[-\underline{\frac{C_A}{\epsilon^2}}\right.$$

$$\left.+\underline{\frac{11C_A-2n_f}{6\epsilon}} - \underline{\frac{C_A}{\epsilon}\ln\left(\frac{\vec{p}_H^{\,2}}{s_0}\right)} - \frac{5n_f}{9} + C_A\left(2\,\Re e\left(\mathrm{Li}_2\left(1+\frac{m_H^2}{\vec{p}_H^{\,2}}\right)\right) + \frac{\pi^2}{3} + \frac{67}{18}\right) + 11\right]$$

- Projection of the real quark contribution

$$\frac{d\Phi^{\{Hq\}}_{PP}(x_H,\vec{p}_H,n,\nu)}{dx_H d^2\vec{p}_H} = \frac{1}{f_g(x_H)}\frac{d\Phi^{\{H\}(0)}_{PP}(x_H,\vec{p}_H,n,\nu)}{dx_H d^2 p_H}\frac{\bar{\alpha}_s}{2\pi}\left(\frac{\vec{p}_H^{\,2}}{\mu^2}\right)^{-\epsilon}$$

$$\times\int_{x_H}^1\frac{dz_H}{z_H}\sum_{a=q\bar{q}}f_a\left(\frac{x_H}{z_H},\mu_F\right)\left\{-\frac{1}{\epsilon}C_F\underline{\left(\frac{1+(1-z_H)^2}{z_H}\right)}+\mathcal{O}(\epsilon^0)\right\}$$

Real gluon contribution in the (n,ν)-space

- "Plus" term

$$\frac{d\Phi_{PP}^{\{Hg\}\mathrm{plus}}(x_H,\vec{p}_H,n,\nu;s_0)}{dx_H\,d^2\vec{p}_H} = -\frac{1}{f_g(x_H)}\frac{d\Phi_{PP}^{\{H\}(0)}(x_H,\vec{p}_H,n,\nu)}{dx_H\,d^2\vec{p}_H}\frac{\bar{\alpha}_s}{2\pi}\left(\frac{\vec{p}_H^{\,2}}{\mu^2}\right)^{-\epsilon}$$

$$\times\int_{x_H}^{1}\frac{dz_H}{z_H}f_g\left(\frac{x_H}{z_H}\right)2C_A\underline{\frac{z_H}{(1-z_H)_+}}\left[\frac{1}{\epsilon}+\mathcal{O}(\epsilon^0)\right]$$

- $(1-x_H)$-term

$$\frac{d\Phi_{PP}^{\{Hg\}(1-x_H)}(x_H,\vec{p}_H,n,\nu)}{dx_H\,d^2\vec{p}_H} = \frac{d\Phi_{PP}^{\{H\}(0)}(x_H,\vec{p}_H,n,\nu)}{dx_H\,d^2\vec{p}_H}\frac{\bar{\alpha}_s}{2\pi}\left(\frac{\vec{p}_H^{\,2}}{\mu^2}\right)^{-\epsilon}$$

$$\times\underline{2C_A\ln(1-x_H)}\left[\frac{1}{\epsilon}+\mathcal{O}(\epsilon^0)\right]$$

- Collinear part of the remaining term

$$\frac{d\Phi_{PP}^{\{Hg\}\mathrm{coll}}(x_H,\vec{p}_H,n,\nu)}{dx_H\,d^2\vec{p}_H} = \frac{1}{f_g(x_H)}\frac{d\Phi_{PP}^{\{H\}(0)}(x_H,\vec{p}_H,n,\nu)}{dx_H\,d^2\vec{p}_H}\frac{\bar{\alpha}_s}{2\pi}\left(\frac{\vec{p}_H^{\,2}}{\mu^2}\right)^{-\epsilon}$$

$$\times\int_{x_H}^{1}\frac{dz_H}{z_H}f_g\left(\frac{x_H}{z_H}\right)\left\{-\frac{1}{\epsilon}2\,C_A\underline{\left(z_H(1-z_H)+\frac{(1-z_H)}{z_H}\right)}+\mathcal{O}(\epsilon^0)\right\}$$

- Complete cancellation of divergences $\to \epsilon = 0$

Finite part of the result in the (n,ν)-space

- The complete finite result is obtained in terms of hypergeometric functions and integrals of hypergeometric functions (with some shrewdness!), i.e,

$$I_2(\gamma_1,n,\nu) = \int \frac{d^{2-2\epsilon}\vec{q}}{\pi\sqrt{2}}(\vec{q}^{\,2})^{i\nu-\frac{3}{2}}e^{in\phi}(\vec{q}^{\,2})^{-\gamma_1}\frac{1}{\left[(\vec{q}-\vec{p}_H)^2\right]\left[(1-z_H)m_H^2+(\vec{p}_H-z_H\vec{q})^2\right]}$$

$$= \frac{(\vec{p}_H^{\,2})^{\frac{n}{2}}e^{in\phi_H}}{z_H^2\sqrt{2}\pi^\epsilon}\left[\frac{\Gamma\left(\frac{5}{2}+\gamma_1+\frac{n}{2}-i\nu+\epsilon\right)\Gamma\left(-\frac{1}{2}-\gamma_1+\frac{n}{2}+i\nu-\epsilon\right)}{\Gamma(1+n-\epsilon)}\right]$$

$$\times\int_0^1 d\Delta\left(\Delta+\frac{(1-\Delta)}{z_H}\right)^n\left[\left(\Delta+\frac{(1-\Delta)}{z_H^2}\right)\vec{p}_H^{\,2}+\frac{(1-\Delta)(1-z_H)m_H^2}{z_H^2}\right]^{-\frac{5}{2}-\gamma_1+i\nu-\frac{n}{2}-\epsilon}$$

$$\times\,{}_2F_1\left(-\frac{1}{2}-\gamma_1+\frac{n}{2}+i\nu-\epsilon,\frac{5}{2}+\gamma_1-i\nu+\frac{n}{2}+\epsilon,1+n-\epsilon,\zeta\right)\ ,\qquad \zeta\xrightarrow{\Delta\to 1}1$$

- Extracting singular part

$$I_{2,\rm as}(\gamma_1,n,\nu) = \frac{(\vec{p}_H^{\,2})^{-\frac{3}{2}-\gamma_1+i\nu-\epsilon}e^{in\phi_H}\Gamma(1+\epsilon)}{(1-z_H)\sqrt{2}\pi^\epsilon}\frac{1}{\left(m_H^2+(1-z_H)\vec{p}_H^{\,2}\right)}\int_0^1 d\Delta(1-\Delta)^{-\epsilon-1}$$

$$= -\frac{1}{\epsilon}\frac{(\vec{p}_H^{\,2})^{-\frac{3}{2}-\gamma_1+i\nu-\epsilon}e^{in\phi_H}\Gamma(1+\epsilon)}{(1-z_H)\sqrt{2}\pi^\epsilon}\frac{1}{\left(m_H^2+(1-z_H)\vec{p}_H^{\,2}\right)}$$

- Replacement: $I_2 = I_{2,as}+(I_2-I_{2,\rm as})\equiv I_{2,as}+I_{2,\rm reg}$

Projection onto the eigenfunctions of the BFKL kernel

- **BFKL cross section**

$$d\sigma_{AB} = \frac{1}{(2\pi)^{D-2}} \int \frac{d^{D-2}q_1}{\vec{q}_1^{\,2}} \int \frac{d^{D-2}q_2}{\vec{q}_2^{\,2}}$$

$$\times d\Phi_{AA}(\vec{q}_1; s_0) \int_{\delta-i\infty}^{\delta+i\infty} \frac{d\omega}{2\pi i} \left[\left(\frac{s}{s_0}\right)^{\omega} G_{\omega}^{(0)}(\vec{q}_1, \vec{q}_2) \right] d\Phi_{BB}(-\vec{q}_2; s_0)$$

- Momentum representation

$$\hat{\vec{p}}\,|\vec{p}_i\rangle = \vec{p}_i\,|\vec{p}_i\rangle \;, \quad \langle A|B\rangle = \langle A|\vec{q}_1\rangle \langle \vec{q}_1|B\rangle = \int d^{D-2}\vec{q}_1\, A(\vec{q}_1)\, B(\vec{q}_1) \;, \quad O(\vec{q}_1, \vec{q}_2) = \langle \vec{q}_1|\hat{O}|\vec{q}_2\rangle \,,$$

$$d\sigma_{AB} = \frac{1}{(2\pi)^{D-2}} \int_{\delta-i\infty}^{\delta+i\infty} \frac{d\omega}{2\pi i} \left(\frac{s}{s_0}\right)^{\omega} \langle \frac{d\Phi_{AA}}{\vec{q}_1^{\,2}} |\hat{G}_{\omega}| \frac{d\Phi_{BB}}{\vec{q}_2^{\,2}} \rangle$$

- BFKL equation

$$\hat{1} = \left(\omega - \hat{K}\right) \hat{G}_{\omega} \implies \hat{G}_{\omega} = \left(\omega - \hat{K}\right)^{-1} .$$

- Perturbative expansion of the Kernel: $\hat{K} = \bar{\alpha}_s \hat{K}^0 + \bar{\alpha}_s^2 \hat{K}^1$

$$\hat{G}_{\omega} \simeq \left(\omega - \bar{\alpha}_s \hat{K}^0\right)^{-1} + \left(\omega - \bar{\alpha}_s \hat{K}^0\right)^{-1} \left(\bar{\alpha}_s^2 \hat{K}^1\right) \left(\omega - \bar{\alpha}_s \hat{K}^0\right)^{-1}$$

- Eigenfunctions of the LO kernel

$$\hat{K}^0\,|n,\nu\rangle = \chi(n,\nu)\,|n,\nu\rangle \;, \qquad \langle \vec{q}\,|n,\nu\rangle = \frac{1}{\pi\sqrt{2}} (\vec{q}^{\,2})^{i\nu-\frac{3}{2}} e^{in\phi}$$

Alternative: NLO eigenfunctions [G. A. Chirilli, Y. V. Kovchegov (2013)]

Showing cancellation of divergences

- BFKL cross-section

$$d\sigma_{AB} = \frac{1}{(2\pi)^{D-2}} \sum_{n,n'} \int d\nu \int d\nu' \int_{\delta-i\infty}^{\delta+i\infty} \frac{d\omega}{2\pi i} \left(\frac{s}{s_0}\right)^{\omega}$$

$$\times \langle \frac{d\Phi_{AA}}{\vec{q}_1^{\,2}} |n,\nu\rangle \langle n,\nu|\hat{G}_\omega|n',\nu'\rangle \langle n'\nu'| \frac{d\Phi_{BB}}{\vec{q}_2^{\,2}}\rangle$$

- **Projection** onto the eigenfunction of the BFKL kernel

$$\langle \frac{d\Phi_{AA}}{\vec{q}^{\,2}} |n,\nu\rangle = \int \frac{d^{2-2\epsilon}q}{\pi\sqrt{2}} (\vec{q}^{\,2})^{i\nu-\frac{3}{2}} e^{in\phi} d\Phi_{AA}^{(0)}(\vec{q}\,) \equiv d\Phi_{AA}^{(0)}(n,\nu)$$

- LO projected impact factor

$$\frac{d\Phi_{PP}^{\{H\}(0)}(x_H, \vec{p}_H, n, \nu)}{dx_H d^2\vec{p}_H} = \frac{g_H^2}{8(1-\epsilon)\sqrt{N^2-1}} \frac{(\vec{p}_H^{\,2})^{i\nu-\frac{1}{2}} e^{in\phi_H}}{\pi\sqrt{2}} f_g(x_H)$$

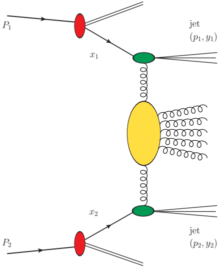
- Scheme for *cancellation of divergences*
  - *i.* Rapidity divergences → removed by the BFKL counterterm
  - *ii.* UV divergences → QCD coupling renormalization
  - *iii.* Soft divergences → cancelled in the real plus virtual combination
  - *iiii.* Surviving initial-state IR divergences → gPDF renormalization

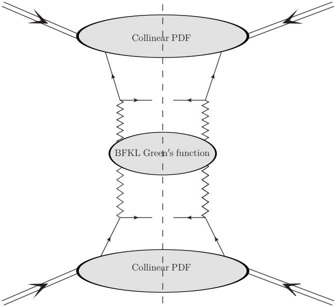
Hybrid collinear/high-energy factorization

Mueller-Navelet jets

- Inclusive production of two rapidity-separated jets in proton-proton collision
- Large energy logarithms → BFKL resummed partonic cross section
- Moderate values of parton x → collinear PDFs

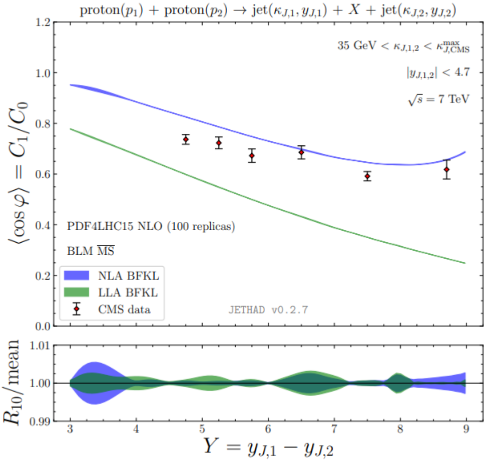
[A.H. Mueller, H. Navelet (1987)]





- Hybrid formalism: can be extended to several type of semi-hard reactions

Mueller-Navelet: Theory vs Experiment



[C. Marquet, C. Royon (2009)]

[B. Ducloué, L. Szymanowski, S. Wallon (2013,2014)]

[F. Caporale, D.Yu. Ivanov, B. Murdaca, A. Papa (2014,2015)]

In this slide: [F.G. Celiberto (2021)]

Mueller-Navelet: Theory vs Experiment

- CMS **@**7Tev with symmetric p_T-ranges, only!

  [CMS collaboration (2016)]

- LHC kinematic **domain** in between the sectors described by BFKL and DGLAP approaches
- Clearer manifestation of high-energy signatures expected at increasing energies

Mueller-Navelet: Theory vs Experiment

- CMS **@**7Tev with symmetric p_T-ranges, only!

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- LHC kinematic **domain** in between the sectors described by BFKL and DGLAP approaches
- Clearer manifestation of high-energy signatures expected at increasing energies

- Strong manifestation of higher-order **instabilities** via scale variation

NLA BFKL corrections to cross section with opposite sign with respect to the leading order (LO) result and large in absolute value...

◇ ...call for some optimization procedure...
◇ ...choose scales to mimic the most relevant subleading terms

- BLM [S.J. Brodsky, G.P. Lepage, P.B. Mackenzie (1983)]
  - ✓ preserve the conformal invariance of an observable...
  - ✓ ...by making vanish its β_0-dependent part

* "Exact" BLM:

suppress NLO IFs + NLO Kernel β_0-dependent factors

Impact factors for partially inclusive processes

NLO impact factors

- Jet impact factor and Mueller Navelet jets

  [J. Bartels, D. Colferai, G.P. Vacca (2002, 2003)]

  [F. Caporale, D.Yu. Ivanov, B. Murdaca, A. Papa, A. Perri (2011)]

- Light hadron IF

  [D.Yu. Ivanov, A. Papa (2012)]

- Heavy hadrons and Quarkonium IFs in VFNS (high-p_T of the hadron)

  [F.G. Celiberto, M.F, D.Yu. Ivanov, A. Papa (2021)]

  [F.G. Celiberto, M.F (2022)]

- Forward Higgs IF* ($m_t \to \infty$)

  [M. Nefedov (2019)], [M. Hentschinski, K. Kutak, A. van Hameren (2021)]

  [F.G. Celiberto, M.F, D.Yu. Ivanov, M.M.A. Mohammed, A. Papa (2022)]

LO impact factors

- Drell-Yan di-lepton IF

  [L. Motyka, M. Sadzikowski, T. Stebela (2015)]

- J/ψ hadroproduction IF in a massive scheme

  [R. Boussarie, B. Ducloué, L. Szymanowski, S. Wallon (2018)]

- $Q\bar{Q}$-pair photo/hadroproduction IF in a massive scheme

  [I.F. Ginzburg, S.L. Panfil and V.G. Serbo (1987)]

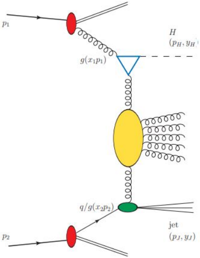
  [A. Bolognino, F.G. Celiberto, M.F, D.Yu. Ivanov, A. Papa (2019)]

Higgs plus jet

- Inclusive **Higgs plus jet** production in proton-proton collision

  i. Full NLL Green function + Partial NLO impact factors (full m_t-dep.)
  [F.G. Celiberto, D. Yu. Ivanov, M.M.A. Mohammed, A. Papa (2021)]

  ii. LL BFKL in HEJ framework + LO impact factors (full m_t,m_b-dep.)
  [J. R. Andersen et al. (2022)]



$$\frac{d\sigma_{\rm pp}}{dy_H\, dy_J\, d|\vec{p}_H|\, d|\vec{p}_J|\, d\phi_1\, d\phi_2} = \frac{1}{(2\pi)^2}$$

$$\times \int \frac{d^2\vec{q}_1}{\vec{q}_1^{\,2}} (\mathcal{V}_H^{(g)}(\vec{q}_1, s_0, x_1, \vec{p}_H) \otimes f_g(x_1))$$

$$\times \int_{\delta-i\infty}^{\delta+i\infty} \frac{d\omega}{2\pi i} \left(\frac{x_1 x_2 s}{s_0}\right)^{\omega} G_\omega(\vec{q}_1, \vec{q}_2)$$

$$\times \int \frac{d^2\vec{q}_2}{\vec{q}_2^{\,2}} \left(\sum_r \mathcal{V}_J^{(p)}(\vec{q}_2, s_0, x_2, \vec{p}_J) \otimes f_r(x_2)\right)$$

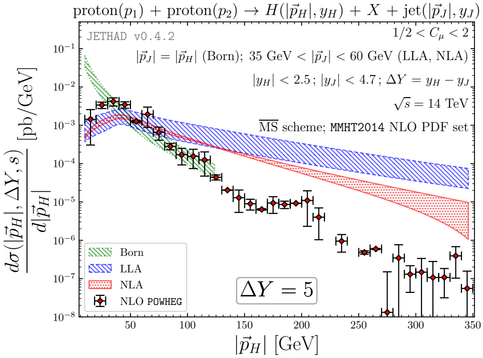
- Hadronic cross section expanded in **azimuthal coefficients**

$$\frac{d\sigma_{\rm pp}}{dy_H\, dy_J\, d|\vec{p}_H|\, d|\vec{p}_J|\, d\phi_1\, d\phi_2} = \frac{1}{(2\pi)^2}\left[\mathcal{C}_0 + 2\sum_{n=1}^{\infty} \cos(n\phi)\mathcal{C}_n\right]$$

Higgs p_T-distribution

$$\frac{d\sigma\left(|\vec{p}_H|, \Delta Y, s\right)}{d|\vec{p}_H| d\Delta Y} = \int_{p_J^{min}}^{p_J^{max}} d|\vec{p}_J| \int_{y_H^{min}}^{y_H^{max}} dy_H \int_{y_J^{min}}^{y_J^{max}} dy_J \delta\left(y_H - y_J - \Delta Y\right) C_0$$

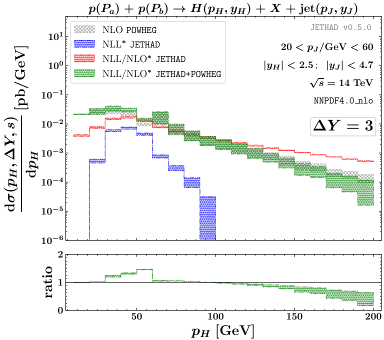
- JETHAD vs POWHEG

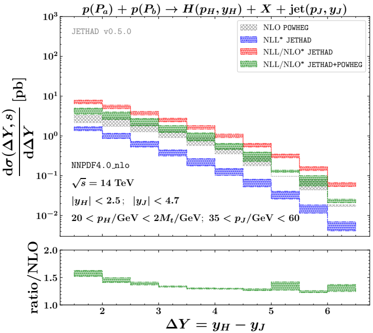


[F.G. Celiberto, M.M.A. Mohammed, D.Yu. Ivanov, A. Papa (2021)]

- **Additive matching procedure**

$$d\sigma^{\mathrm{NLL/NLO}}(\Delta Y, s) = \underbrace{d\sigma^{\mathrm{NLO}}(\Delta Y, s)}_{\text{POWHEG}} + \underbrace{d\sigma^{\mathrm{NLL}}(\Delta Y, s)}_{\text{JETHAD}} - \underbrace{\Delta d\sigma^{\mathrm{NLL/NLO}}(\Delta Y, s)}_{\text{NLO double counting}}$$

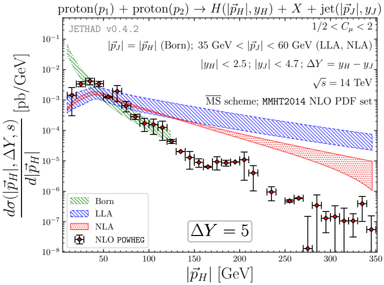


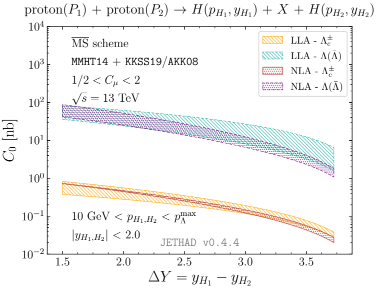


[**Preliminary results presented by F.G. Celiberto at Higgs 2022**]

Stabilization effects

- Stabilization effects in Higgs and heavy flavor production
- Λ-baryon FFs
  - heavy species $\longrightarrow \Lambda_c$
    KKSS19 [B.A. Kniehl, G. Kramer, I. Schienbein, H. Spiesberger (2020)]
  - light species $\longrightarrow \Lambda$
    AKK08 [S.Albino, B.A. Kniehl, and G. Kramer (2008)]





[F.G. Celiberto, D.Yu. Ivanov, M.M.A. Mohammed, A. Papa (2020)]

[F.G. Celiberto, D.Yu. Ivanov, M. F., A. Papa (2021)]