Beyond threshold resummation to Drell-Yan and Higgs productions at the LHC

V. Ravindran
The Institute of Mathematical Sciences,
Chennai, India

with
A.H.Ajjath, Pooja Mukherjee
Aparna Sankar and Surabhi Tiwari

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Plan of my talk

- Inclusive reactions
- Soft and Collinear partons
- Framework to study Next to Soft logarithms
- Framework to Resum Next to Soft terms N space
- Conclusions

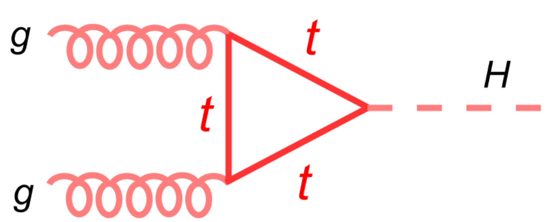
Inclusive Higgs production

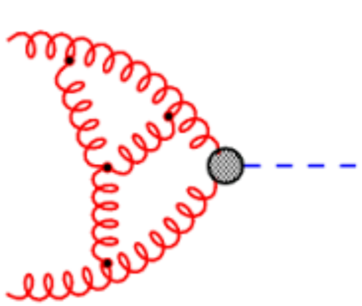
Higgs Effective theory

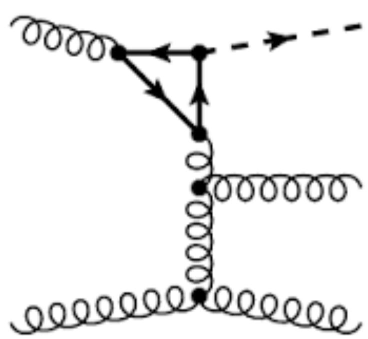
Anastasiou, Duhr, Dulat and Mistleberger (`19)

$$\hat{\sigma}(z) = \hat{\sigma}^{LO}(z) + \alpha_S \hat{\sigma}^{NLO}(z) + \alpha_S^2 \hat{\sigma}^{NNLO}(z) + \alpha_S^3 \hat{\sigma}^{N3LO}(z) + \mathcal{O}(\alpha_S^4)$$

✓ ✓ ✓ ✓

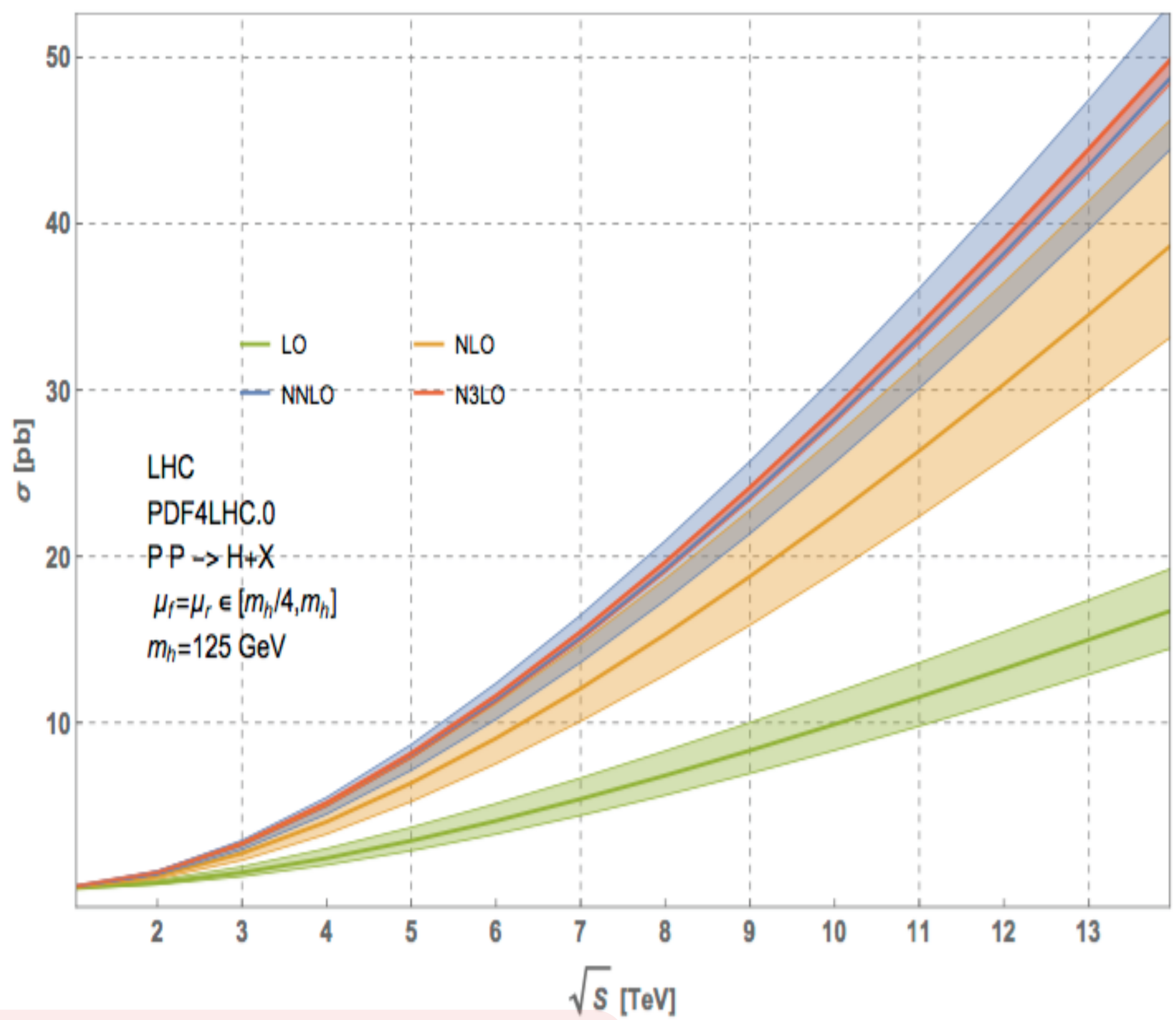






LO	$15.05 \pm 14.8\%$
NLO	$38.2 \pm 16.6\%$
NNLO	$45.1 \pm 8.8\%$
N3LO	$45.2 \pm 1.9\%$

pb



State of the art prediction

Inclusive Drell-Yan production

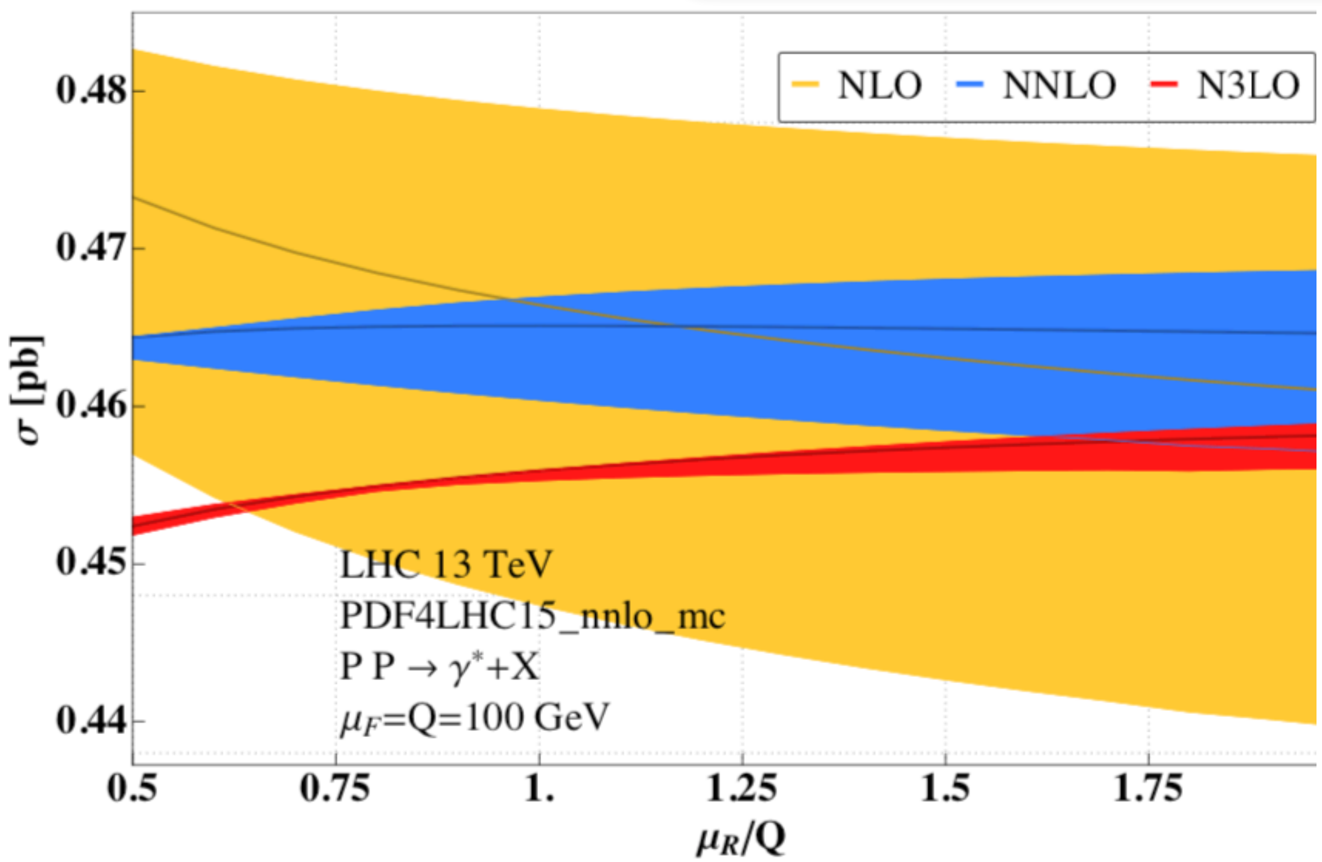
Fixed order prediction at N3LO

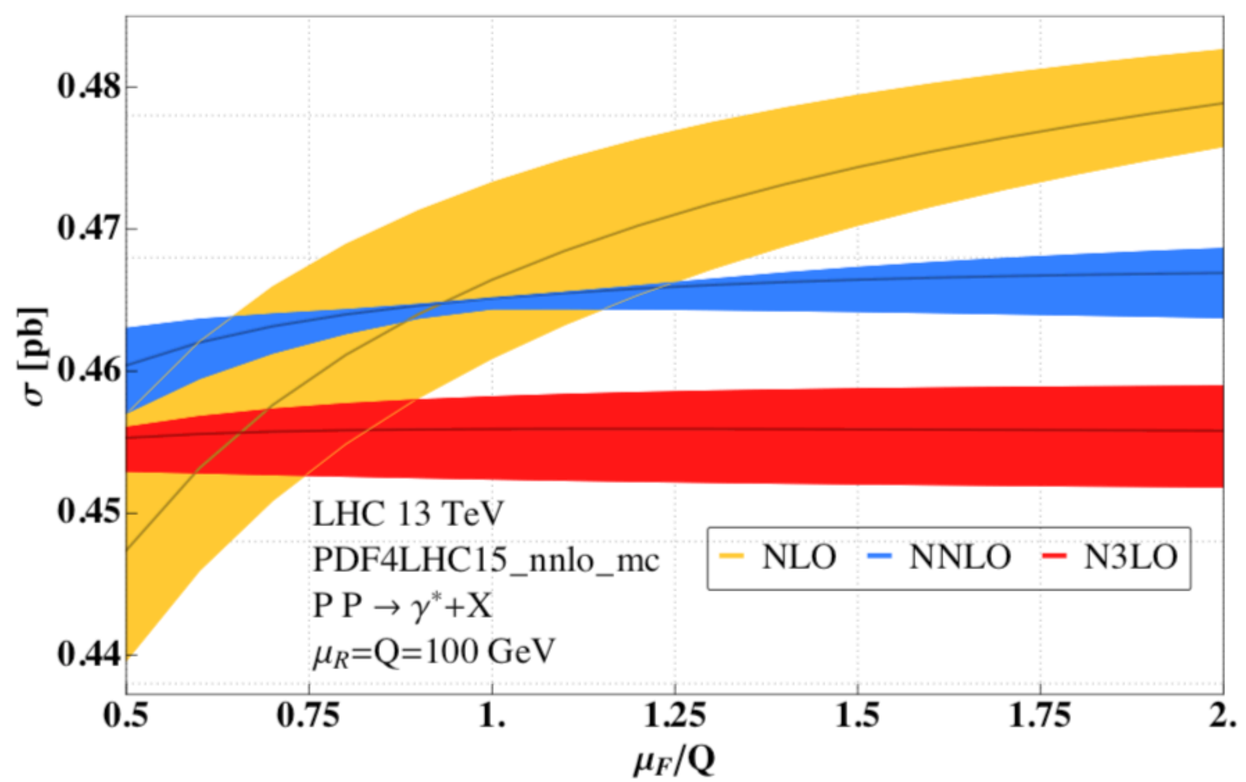
Duhr, Dulat and Mistleberger (`20)

$$\hat{\sigma}(z) = \hat{\sigma}^{LO}(z) + \alpha_S \hat{\sigma}^{NLO}(z) + \alpha_S^2 \hat{\sigma}^{NNLO}(z) + \alpha_S^3 \hat{\sigma}^{N3LO}(z) + \mathcal{O}(\alpha_S^4)$$

✓ ✓ ✓ ✓

State of the art prediction





What is next?

Soft + Virtual + Hard

DY/Higgs production cross section:

Partonic Flux

$$\sigma(q^2,\tau) = \sigma_0(\mu_R^2) \int \frac{dz}{z} \Phi_{ab}\left(\frac{\tau}{z}, \mu_F^2\right) \Delta_{ab}(q^2, \mu_R^2, \mu_F^2, z)$$

Partonic cross section:

$$\Delta_{ab}(q^2,\mu_i^2,z) = \Delta_{ab}^{SV}(q^2,\mu_i^2,z) + \Delta_{ab}^{H}(q^2,\mu_i^2,z)$$

Soft + Virtual

Hard

$$\delta(1-z_i)$$

$$\left(\frac{\ln(1-z_i)}{(1-z_i)}\right)_+$$

Polylogs or HPLS $Li_n(g(z)), H(\vec{n}, h(z))$

$$\log^k(1-z_i), \quad k = 0, \cdots \infty$$

$$(1-z_i)^k, \quad k = 1, \cdots \infty$$

Infrared Structure

Soft plus Virtual

$$\Delta_{ab}^{SV}(z) = \delta_{ab}\Delta_{a,\delta}\delta(1-z) + \delta_{ab}\sum_{j=0}^{\infty}\Delta_{a,\mathcal{D}_j}\left(\frac{\log^j(1-z)}{1-z}\right)_+$$

Perturbatively Calculable

$$\Delta_{a,\delta} = \sum_k a_s^k \Delta_{a,\delta}^{(k)}$$

$$\Delta_{a,\mathcal{D}_j} = \sum_k a_s^k \Delta_{a,\mathcal{D}_j}^{(k)}$$

Sensitive to Soft and Collinear parsons

$$\frac{1}{(p+k)^2} = \frac{1}{2p^0k^0\,(1-\cos\theta)}$$

$k^0 \to 0$ Soft divergence

$\cos\theta \to 0$ Collinear divergence

- Soft and Collinear regions are Universal
- Divergences are controlled by Soft and Collinear Anomalous dimensions - ex: Cusp A, B, f etc
- RGE in IR sector allows for All Order Prediction

SV + Next to Soft + all that

Near threshold: $z = 1$

Hard part

$$\Delta_{ab}^{H}(z) = \Delta_{ab}^{NSV}(z) + \Delta_{ab}^{N^n SV}(z)$$

- Next to SV (NSV)

$$\Delta_{ab}^{NSV}(z) = \sum_{k=0}^{\infty} C_i^{NSV} \log^k(1-z)$$

- Next to next to....to soft (NnSV)

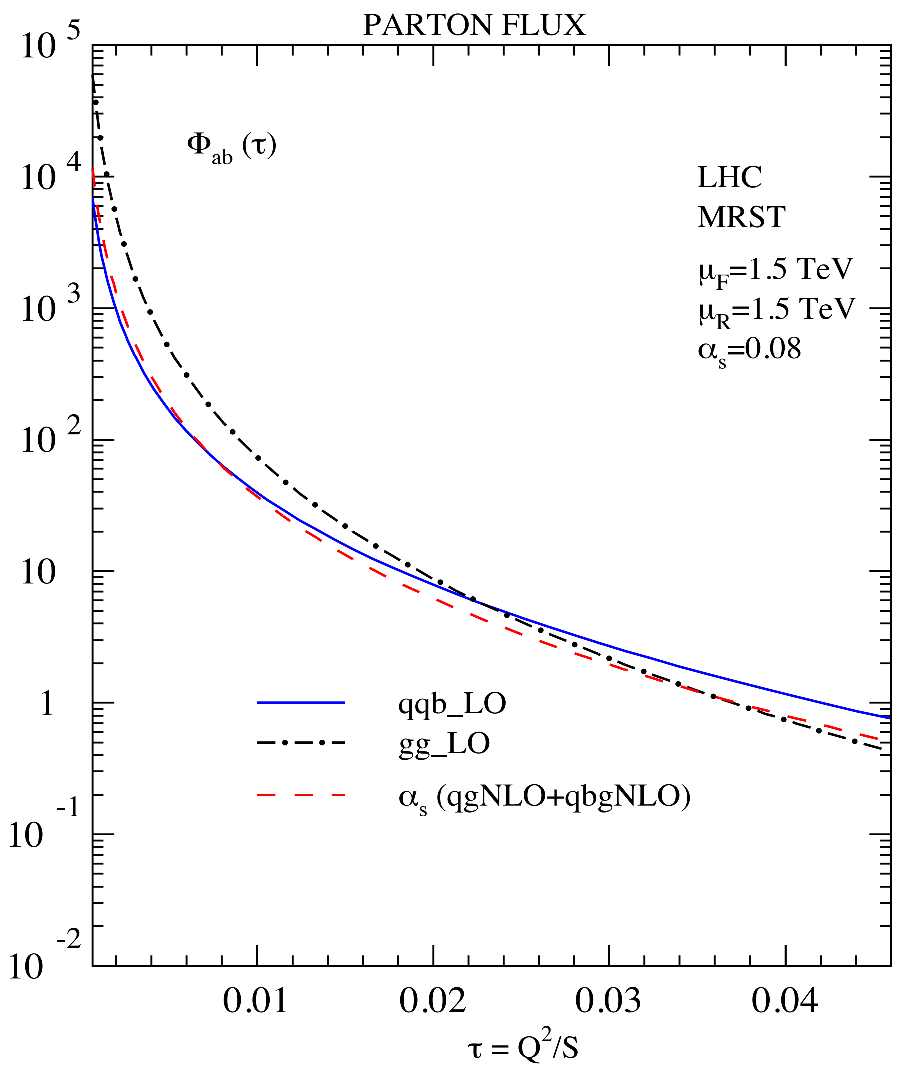
$$\Delta_{ab}^{N^n SV}(z) = \sum_{k=1}^{\infty} d_k (1-z)^k$$

Why Threshold corrections?

partonic scaling variable $z = \frac{\tau}{x}$

Catani et al, Harlander, Kilgore

$$2S\, d\sigma^{P_1 P_2}(\tau, m_h) = \sum_{ab} \int_\tau^1 \frac{dx}{x} \Phi_{ab}(x)\; 2\hat{s}\, d\hat{\sigma}^{ab}\left(\frac{\tau}{x}, m_h\right) \qquad \tau = \frac{m_h^2}{S}$$



Gluon flux is largest at LHC

- Parton flux $\Phi_{ab}(x)$ becomes large when $x \to x_{min} = \tau$
- Dominant contribution to Higgs production comes from the region when $x \to \tau$
- It is sufficient if we know the partonic cross section when $x \to \tau$
- $x \to \tau$ is called *soft limit*.
- Expand the partonic cross section around $x = \tau$.

$z \to 1$

- Dominantly come from virtual and soft gluon emission processes (SV)

Sterman-Catani-Trentedue

Sterman (`87),Catani,Trentedue `89

SUMMATION OF LARGE CORRECTIONS TO SHORT-DISTANCE HADRONIC CROSS SECTIONS

George STERMAN*

Institute for Advanced Study, Princeton, NJ 08540, USA

RESUMMATION OF THE QCD PERTURBATIVE SERIES FOR HARD PROCESSES

S. CATANI

Istituto Nazionale di Fisica Nucleare, Sezione di Firenze, Dipartimento di Fisica, Università di Firenze, I-50125 Florence, Italy

L. TRENTADUE

Dipartimento di Fisica, Università di Parma, INFN, Gruppo Collegato di Parma, I-43100 Parma, Italy

$$\Delta_N(Q^2) \underset{N\to\infty}{=} \exp\left(2\int_0^1 \mathrm{d}x\, \frac{x^{N-1}-1}{1-x} \int_{Q^2}^{Q^2(1-x)} \frac{\mathrm{d}k^2}{k^2} A\left(\alpha_s\left((1-x)Q^2\right)\right) + \frac{3}{2}\frac{C_F}{\pi}\int_0^1 \mathrm{d}x\, \frac{x^{N-1}-1}{1-x}\,\alpha_s\left((1-x)Q^2\right)\right) + \mathrm{O}\left(\alpha_s(\alpha_s \ln N)^n\right). \qquad (3.25)$$

Exponentiation of S+V and

Predictions for Threshold logs to all orders

Importance of beyond NSV

Higgs production in the gluon fusion: N3LO

Anastasiou et al JHEP 03 (2015)

$$
\begin{aligned}
\eta^{(3)}_{gg}(z)\Big|_{(1-z)^0} &= -256\log^5(1-z) && (\to \quad 115.33\%)\\
&+ 959\log^4(1-z) && (\to \quad 101.07\%)\\
&+ 1254.029198\ldots\log^3(1-z) && (\to \quad -32.15\%)\\
&- 11089.328274\ldots\log^2(1-z) && (\to \quad -89.41\%)\\
&+ 15738.441212\ldots\log(1-z) && (\to \quad -55.50\%)\\
&- 5872.588877\ldots && (\to \quad -14.31\%)
\end{aligned}
$$

- The total SV contribution in z-space -> -2.25 % of the Born
- The total NSV contribution in z-space -> 25 % of the Born !

Importance of beyond NSV

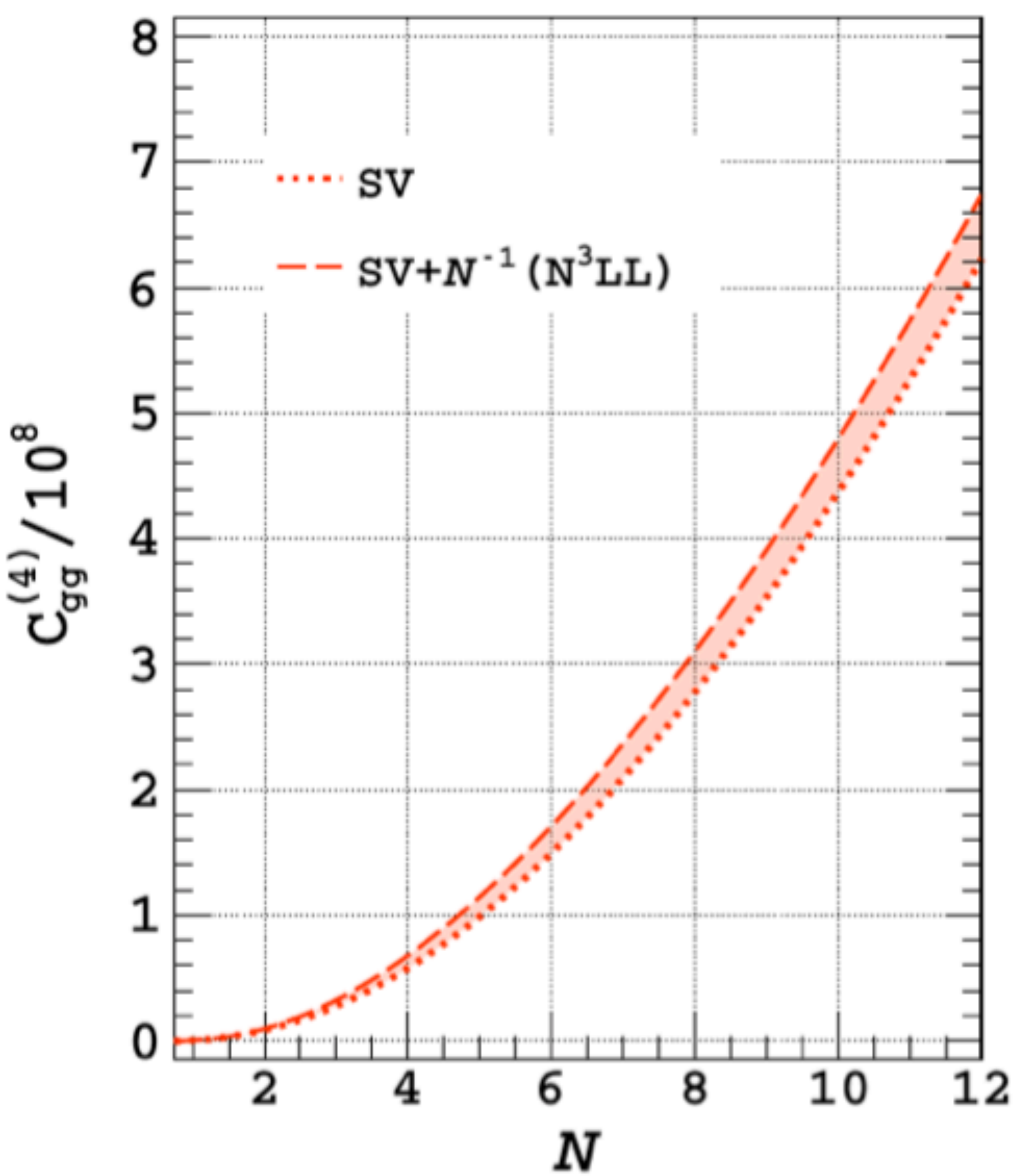
- The total SV contribution in Mellin N-space (conjugate space) -> 18 % of the Born
- The total NSV contribution in N-space -> 11 % of the Born !

$$
\begin{aligned}
M\left[\eta_{gg}^{(3)}\right](N) \simeq\, & 36\log^6 N && (\to 0.0013\%) \\
& + 170.679\ldots\log^5 N && (\to 0.0226\%) \\
& + 744.849\ldots\log^4 N && (\to 0.2570\%) \\
& + 1405.185\ldots\log^3 N && (\to 1.0707\%) \\
& + 2676.129\ldots\log^2 N && (\to 4.0200\%) \\
& + 1897.141\ldots\log N && (\to 5.1293\%) \\
& + 1783.692\ldots && (\to 8.0336\%) \\
& + 108\frac{\log^5 N}{N} && (\to 0.0105\%) \\
& + 615.696\ldots\frac{\log^4 N}{N} && (\to 0.1418\%) \\
& + 2036.407\ldots\frac{\log^3 N}{N} && (\to 0.9718\%) \\
& + 3305.246\ldots\frac{\log^2 N}{N} && (\to 2.9487\%) \\
& + 3459.105\ldots\frac{\log N}{N} && (\to 5.2933\%) \\
& + 703.037\ldots\frac{1}{N} && (\to 1.7137\%).
\end{aligned}
$$

Approximate four-loop QCD correction to Higgs boson production

Phys.LettB.807 124446

G. Das*[a], S. Moch†[b] and A. Vogt‡[c]



Sizeable contribution from terms beyond SV (the NSV terms) due to the large coefficients

Understanding the NSV sector is important because:

- Numerically sizeable
- Provide check of higher-order corrections
- Help to reduce the scale uncertainites
- Stabilize fixed-order calculations

Previous Works:

The earliest evidence that IR effects can be studied at NLP

[Low, Burnett, Kroll]

Early attempts :

[Kraemer, Laenen, Spira (98)] [Akhoury, Sotiropoulos & Sterman (98)]

Important Results & Predictions using Physical Kernel Approach & explicit computation:

[Moch , Vogt et al. (09-20)] [Anastasiou, Duhr, Dulat et al.(14)]

Universality of NLP effects and LL Resummation:

[Laenen, Magnea, et al. (08-19)] [Grunberg, Grunberg & Ravindran (09)]
[Ball, Bonvini, Forte, Marzani, Ridolfi (13)] [Del Duca et al. (17)]

Subleading Factorisation and LL Resummation at NLP using SCET:

[Larkoski, Nelli , Stewart et al. (14)] [Kolodrubetz, Moult, Neill ,Stewart et al. (17)] [Beneke et al. (19-20)]

And many other works...

Some of the recent works:

- Factorisation and RG invariance approach to study NSV resummation effects
  [Ajjath, Pooja Mukherjeei, Ravindran , Phys. Rev. D 105, 094035 , Phys. Rev. D 105, L091503, (2020)]

- On next to soft threshold corrections to DIS and SIA processe [Ajjath, Pooja Mukherjee, Ravindran, A.Sankar, S.Tiwari, JHEP 04 (2021) 131]

- Next-to SV resummed Drell-Yan cross section beyond Leading-logarithm
  [Ajjath, Pooja Mukherjee, Ravindran, A.S,ankar S.Tiwari, Eur. Phys. J. C 82, 234, (2021)]

- Resummed Higgs boson cross section at next-to SV to NNLO + NNLL [Ajjath, Pooja Mukherjee, Ravindran, A.Sankar, S.Tiwari, Eur. Phys. J. C 82, 774, (2021)]

- Next-to-soft corrections for Drell-Yan and Higgs boson rapidity distributions beyond N3LO [Ajjath, Pooja Mukherjee, Ravindran, A.Sankar S.Tiwari, Phys.Rev.D 103 (2021) L111502, (2021)]

- Next-to SV resummed rapidity distribution for Drell-Yan to NNLO + NNLL
  [Ajjath, Pooja Mukherjee, Ravindran, A.Sankar, S.Tiwari, Phys. Rev. D 106, 034005, (2022)]

- Resummed next-to-soft corrections to rapidity distribution [Ravindran, A.S,ankar, S.Tiwari]

Revisiting parton evolution and the large-x limit

Yu. L. Dokshitzer[1*], G. Marchesini[2,1] and G. P. Salam[1]
[1]LPTHE, Universities of Paris-VI and VII and CNRS, Paris, France
[2]University of Milano–Bicocca and INFN Sezione di Milano, Milan, Italy

Dokshitzer-Marchesini-Salam (DMS) equation

$$\mu^2 \frac{\partial}{\partial \mu^2} \psi(x, \mu^2) = \int_x^1 \frac{dz}{z} \psi\left(\frac{x}{z}, z^\sigma \mu^2\right) \mathcal{P}\left(z, \alpha_s\left(\frac{\mu^2}{z}\right)\right) .$$

Threshold Expansion (near x=1)

$$P(x) = \frac{A\, x}{(1-x)_+} + B\,\delta(1-x) + C \ln(1-x) + D + \mathcal{O}((1-x)\log^p(1-x)) . \qquad (13)$$

Predictions for Ci and Di

$$C_1 = 0\,, \quad C_2 = -\sigma A_1^2\,, \quad C_3 = -2\sigma A_1 A_2\,, \quad C_4 = -\sigma(A_2^2 + 2A_1 A_3)\,, \quad \text{etc.} \quad (11b)$$

$$D_1 = 0, \quad D_2 = -A_1(\sigma B_1 + \beta_0)\,, \quad D_3 = -A_1(\sigma B_2 + \beta_1) - A_2(\sigma B_1 + \mathbf{2} \cdot \beta_0)\,, \quad \text{etc.} \quad (16)$$

Exact For DIS structure function:!

Not right!

On next-to-eikonal corrections to threshold resummation for the Drell-Yan and DIS cross sections

Eric Laenen[1] Lorenzo Magnea[2] Gerben Stavenga[3]

Inspired by DMS equation

- For DIS structure function:

$$\ln\left[\widehat{F}_2(N)\right] = \mathcal{F}_{\rm DIS}\left(\alpha_s(Q^2)\right) + \int_0^1 dz\, z^{N-1}\left\{\frac{1}{1-z}B\left[\alpha_s\left(\frac{(1-z)Q^2}{z}\right)\right]\right.$$
$$\left. + \int_{Q^2}^{(1-z)Q^2/z}\frac{dq^2}{q^2}P_s\left[z,\alpha_s(q^2)\right] + \int_{(1-z)^2Q^2/z}^{(1-z)Q^2/z}\frac{dq^2}{q^2}\,\delta P\left[z,\alpha_s(q^2)\right]\right\}_+ . \qquad (37)$$

- For Drell-Yan process

$$\ln\left[\widehat{\omega}(N)\right] = \mathcal{F}_{\rm DY}\left(\alpha_s(Q^2)\right) + \int_0^1 dz\, z^{N-1}\left\{\frac{1}{1-z}D\left[\alpha_s\left(\frac{(1-z)^2Q^2}{z}\right)\right]\right.$$
$$\left. + 2\int_{Q^2}^{(1-z)^2Q^2/z}\frac{dq^2}{q^2}P_s\left[z,\alpha_s(q^2)\right]\right\}_+ , \qquad (31)$$

Two loop predictions are somewhat closer to exact results!

On Higgs-exchange DIS, physical evolution kernels and fourth-order splitting functions at large x

G. Soar[a], S. Moch[b], J.A.M. Vermaseren[c] and A. Vogt[a]

Physical Evolution Kernel

$$\frac{d}{d\ln m_H^2}\mathcal{F}_{gg} = K_{gg}\otimes\mathcal{F}_{gg} \equiv \sum_{\ell=0}^{\infty} a_s^{\ell+1} K_{gg}^{(\ell)}\otimes\mathcal{F}_{gg}$$

From Exact 2nd and 3rd results:

$$K_{gg}^{(1)}\Big|_{N^{-1}} = -\left(8\beta_0 C_A + 32 C_A^2\right)\ln N + \mathcal{O}(1),$$

$$K_{gg}^{(2)}\Big|_{N^{-1}} = -\left(16\beta_0^2 C_A + 112\beta_0 C_A^2\right)\ln^2 N + \mathcal{O}(\ln N),$$

$$K_{gg}^{(3)}\Big|_{N^{-1}} = -\left(32\beta_0^3 C_A + \xi_H^{(3)}\beta_0^2 C_A^2\right)\ln^3 N + \mathcal{O}(\ln^2 N),$$

Successful predictions for certain ``Next to SV , log(1-z) terms at 4th order in QCD for Splitting fn. and CFs

Infrared Structure

Expanding Hard Pard around z = 1

Next to SV

$$\Delta_{ab}^{NSV}(z) = \sum_{k=0}^{\infty} C_i^{NSV} \log^k(1-z)$$

- Is Next to Soft universal like SV ?
- Are the log(1-z) terms controlled by Certain IR anomalous dimensions?
- Does IR Renormalisation Group Eqn. exist?
- Can one exponentiate, resum and predict higher order logs?

Factorisation: Diagonal

Begin with Mass factorised cross section

$$\frac{1}{z}\hat{\sigma}_{ab}(\varepsilon) = \sigma_0 \sum_{a'b'} \Gamma^T_{aa'}(\mu_F^2,\varepsilon) \otimes \left(\frac{1}{z}\Delta_{a'b'}(\mu_F^2,\varepsilon)\right) \otimes \Gamma_{b'b}(\mu_F^2,\varepsilon)$$

For Drell-Yan process:

Diagonal Channel:

$$\frac{\hat{\sigma}_{q\bar{q}}}{z\sigma_0} = \Gamma^T_{qq} \otimes \frac{\Delta_{qq}}{z} \otimes \Gamma_{q\bar{q}} + \Gamma^T_{qq} \otimes \frac{\Delta_{qg}}{z} \otimes \Gamma_{g\bar{q}} + \cdots$$

In the threshold limit z -> 1 , keeping only $\left(\frac{\ln(1-z_i)}{(1-z_i)}\right)_+$ $\delta(1-z_i)$ SV

$\log^k(1-z_i), \quad k = 0, \cdots \infty$ next to SV

$$\frac{\hat{\sigma}^{\rm sv+nsv}_{q\bar{q}}}{z\sigma_0} = \Gamma^T_{qq} \otimes \Delta^{\rm sv+nsv}_{q\bar{q}} \otimes \Gamma_{\bar{q}\bar{q}}\,.$$

dropping

$(1-z_i)^k, \quad k = 1, \cdots \infty$

Remarkably Simple form !

Factorisation: Off-diagonal

Begin with Mass factorised cross section

$$\frac{1}{z}\hat{\sigma}_{ab}(\varepsilon) = \sigma_0 \sum_{a'b'} \Gamma^T_{aa'}(\mu_F^2, \varepsilon) \otimes \left(\frac{1}{z}\Delta_{a'b'}(\mu_F^2, \varepsilon)\right) \otimes \Gamma_{b'b}(\mu_F^2, \varepsilon)$$

For Drell-Yan process:

Off-diagonal Channel:

$$\frac{\hat{\sigma}_{qg}}{z\sigma_0} = \Gamma^T_{qq} \otimes \Delta_{qq} \otimes \Gamma_{qg} + \Gamma^T_{qq} \otimes \Delta_{qg} \otimes \Gamma_{gg} + \cdots$$

In the threshold limit z -> 1 , keeping only $\log^k(1-z_i), \quad k = 0, \cdots \infty$ next to SV

$$\frac{\hat{\sigma}^{\mathrm{sv+nsv}}_{qg}}{z\sigma_0} = \Gamma^T_{qq} \otimes \Delta^{\mathrm{sv+nsv}}_{q\bar{q}} \otimes \Gamma_{\bar{q}g} + \Gamma^T_{qq} \otimes \Delta^{\mathrm{nsv}}_{qg} \otimes \Gamma_{gg}\,.$$

dropping $(1-z_i)^k, \quad k = 1, \cdots \infty$

Getting complicated due to Mixing of channels

Our Results:

SV+ NSV Coefficient function for Diagonal Channels c = q, Qb, b,Bb,g

$$\Delta_c(z) = \Delta^{SV}_{c\bar{c}}(q^2,\mu_R^2,\mu_F^2,z) + \Delta^{NSV}_{c\bar{c}}(q^2,\mu_R^2,\mu_F^2,z)$$

Main Results of our work are

- Exponentiation of SV+NSV terms to all orders z-space

$$\Delta_c(z) = \mathcal{C}\exp\Big(\Psi^c(q^2,\mu_R^2,\mu_F^2,z,\varepsilon)\Big)\Big|_{\varepsilon=0},$$

- Resummation of SV+NSV terms to all orders N-space

$$\Delta^c_N(q^2) = C^c_0(q^2)\ e^{\Psi^c_{sv,N}(q^2,\omega)+\Psi^c_{nsv,N}(q^2,\omega)}$$

We use

- Factorisation of universal IR configuration
- Renormalisation Group Invariance

Mellin Convolution in z-space:

$$\mathcal{C}e^{f(z)} = \delta(1-z) + \frac{1}{1!}f(z) + \frac{1}{2!}f(z)\otimes f(z) + \cdots .$$

Factorisation

Master Formula

$$\Psi^c(q^2, \mu_R^2, \mu_F^2, z, \varepsilon) = \mathcal{C} \log\left(\Delta_c(q^2, \mu_R^2, \mu_F^2, z)\right)$$

$$\Psi^c(q^2, \mu_R^2, \mu_F^2, z, \varepsilon) = \left(\ln\left(Z_{UV,c}(\hat{a}_s, \mu^2, \mu_R^2, \varepsilon)\right)^2 + \ln\left|\hat{F}_c(\hat{a}_s, \mu^2, Q^2, \varepsilon)\right|^2\right)\delta(1-z)$$
$$+2\Phi^c(\hat{a}_s, \mu^2, q^2, z, \varepsilon) - 2\mathcal{C}\ln\Gamma_{cc}(\hat{a}_s, \mu^2, \mu_F^2, z, \varepsilon)$$

$$\mathcal{S}_c = \mathcal{C}\exp\left(2\Phi^c\right)$$

- $\hat{F}_c(Q^2)$ Form factor
- $Z_{UV,c}(\mu_R^2)$ UV Renormalisation const
- $\Phi^c(q^2, z)$ Soft+next to soft distribution fn.
- $\Gamma_{cc}(\mu_F^2, z)$ Altarelli-Parisi kernel

Renormalisation Group Equations

Renormalisation costant:

$$\log[Z_{c,UV}]^2 \qquad \mu_R^2 \frac{d}{d\mu_R^2} \log Z_{c,UV}(\mu_R^2) = \sum_{i=1}^{\infty} a_s^i(\mu_R^2) \quad \gamma_{i-1}^c \qquad \alpha_s^3$$

Form factor

$$\log|\hat{F}^c|^2 \qquad Q^2 \frac{d}{dQ^2} \log \hat{F}^c = \frac{1}{2}\left[K^c\left(\hat{a}_s, \frac{\mu_R^2}{\mu^2}, \varepsilon\right) + G^c\left(\hat{a}_s, \frac{Q^2}{\mu_R^2}, \frac{\mu_R^2}{\mu^2}, \varepsilon\right)\right] \qquad \alpha_s^3$$

DGLAP Kernel

$$\log\Gamma_{cc} \qquad \mu_F^2 \frac{d}{d\mu_F^2} \Gamma(z, \mu_F^2, \varepsilon) = \frac{1}{2} P\left(z, \mu_F^2\right) \otimes \Gamma\left(z, \mu_F^2, \varepsilon\right) \qquad \alpha_s^3$$

Soft Function

$$\Phi^c \qquad q^2 \frac{d}{dq^2} \Phi^c = \frac{1}{2}\left[\overline{K}^c\left(\hat{a}_s, \frac{\mu_R^2}{\mu^2}, \varepsilon, z\right) + \overline{G}^c\left(\hat{a}_s, \frac{q^2}{\mu_R^2}, \frac{\mu_R^2}{\mu^2}, \varepsilon, z\right)\right] \qquad \alpha_s^3$$

Integral Representation

Integral representation:

$$\Delta_c(q^2, z) = C_0^c(q^2) \ \ \mathcal{C} \exp\left(2\Psi_{\mathcal{D}}^c(q^2, z)\right),$$

Exponent:

$$\Psi_{\mathcal{D}}^c(q^2, z) = \frac{1}{2}\int_{\mu_F^2}^{q^2(1-z)^2} \frac{d\lambda^2}{\lambda^2} P'_{cc}(a_s(\lambda^2), z) + \mathcal{Q}^c(a_s(q^2(1-z)^2), z)$$

C_0^c Process dependent constant

$P'_{cc}(z)$ Process independent (Universal):
SV+NSV part of Alatarell-Parisi splitting function

$\mathcal{Q}^c(z)$ Process dependent function

Perturbative predictions

$$\Delta_c(z) = \mathcal{C} \exp\left(\Psi^c\left(q^2, \mu_R^2, \mu_F^2, z, \varepsilon\right)\right)\Big|_{\varepsilon=0}$$

$$= \sum_{k=0}^{\infty} \Delta^c_{\mathcal{D},k} \left(\frac{\log^k(1-z)}{1-z}\right)_+ + \Delta^c_\delta\, \delta(1-z)$$ SV

$$+ \sum_{k=0}^{\infty} \Delta^c_{L,k}\, \log^k(1-z)$$ NSV

Perturbative expansion: $$\Delta^c_{J,k} = \sum_{i=0}^{\infty} a_s^i \Delta^{c,(i)}_{J,k}$$

Predictive Power:

Lower order results decide ``Certain `` higher order SV and NSV terms to all orders

All order perturbative predictions

All order exponentiation can predict to all orders from lower orders:

$$\Delta_c(z) = \mathcal{C} \exp\left(\Psi^c\left(q^2, \mu_R^2, \mu_F^2, z, \varepsilon\right)\right)\Big|_{\varepsilon=0}$$

$$= \sum_{i=0}^{\infty} a_s^i \, \Delta_c^{(i)}(z)$$

$$\mathcal{D}_k = \left(\frac{\log^k(1-z)}{1-z}\right)_+$$

$$L_z = \log(1-z)$$

GIVEN				PREDICTIONS		
$\Psi_c^{(1)}$	$\Psi_c^{(2)}$	$\Psi_c^{(3)}$	$\Psi_c^{(n)}$	$\Delta_c^{(2)}$	$\Delta_c^{(3)}$	$\Delta_c^{(i)}$
$\mathcal{D}_0, \mathcal{D}_1, \delta$ L_z^1, L_z^0				$\mathcal{D}_3, \mathcal{D}_2$ L_z^3	$\mathcal{D}_5, \mathcal{D}_4$ L_z^5	$\mathcal{D}_{(2i-1)}, \mathcal{D}_{(2i-2)}$ $L_z^{(2i-1)}$
	$\mathcal{D}_0, \mathcal{D}_1, \delta$ L_z^2, L_z^1, L_z^0				$\mathcal{D}_3, \mathcal{D}_2$ L_z^4	$\mathcal{D}_{(2i-3)}, \mathcal{D}_{(2i-4)}$ $L_z^{(2i-2)}$
		$\mathcal{D}_0, \mathcal{D}_1, \delta$ $L_z^3, \cdots, L_z^0$				$\mathcal{D}_{(2i-5)}, \mathcal{D}_{(2i-6)}$ $L_z^{(2i-3)}$
			$\mathcal{D}_0, \mathcal{D}_1, \delta$ $L_z^n, \cdots, L_z^0$			$\mathcal{D}_{(2i-(2n-1))}, \mathcal{D}_{(2i-2n)}$ $L_z^{(2i-n)}$

Third order predictions for Drell-Yan production

Drell-Yan production

Duhr et al

$$\Delta_q(z) = \delta(1-z) + a_s \Delta_q^{(1)}(z) + a_s^2 \Delta_q^{(2)}(z) + a_s^3 \Delta_q^{(3)}(z) + \cdots$$

3rd order prediction:

Using known results upto 2nd order

$$\Delta_q^{(3)} = \Delta_q^{SV,(3)}(z) + \sum_{k=0}^{5} C_k \, \log^k(1-z) + \cdots$$

N3LO predictions

	$\log^5(1-z)$		$\log^4(1-z)$		$\log^3(1-z)$	
C_F^3	-512	-512	1728	1728	$2272 + 3072\ \zeta 2$	$2272 + 3072\ \zeta_2$
$C_F^2 n_f$	0	0	$-\frac{1280}{9}$	$-\frac{1280}{9}$	$\frac{19456}{27}$	$\frac{6464}{9} + \chi_1$
$C_A C_F^2$	0	0	$\frac{7040}{9}$	$\frac{7040}{9}$	$-\frac{111904}{27} + 512\ \zeta_2$	$-\frac{37184}{9} + 512\zeta_2 + \chi_2$
$C_F n_f^2$	0	0	0	0	$-\frac{256}{27}$	$-\frac{256}{27}$
$C_A C_F n_f$	0	0	0	0	$\frac{2816}{27}$	$\frac{2816}{27}$
$C_A^2 C_F$	0	0	0	0	$-\frac{7744}{27}$	$-\frac{7744}{27}$

Third order predictions for Higgs production

Anastasiou et al

Higgs boson production g+g -> Higgs

$$\Delta_g(z) = \delta(1-z) + a_s\Delta_g^{(1)}(z) + a_s^2\Delta_g^{(2)}(z) + a_s^3\Delta_g^{(3)}(z) + \cdots$$

3rd order prediction: Using known results upto 2nd order

$$\Delta_g^{(3)} = \Delta_g^{SV,(3)}(z) + \sum_{k=0}^{5} C_k \log^k(1-z) + \cdots$$

N3LO predictions

	$\log^5(1-z)$		$\log^4(1-z)$		$\log^3(1-z)$	
C_a^3	-512	-512	$\frac{22592}{9}$	$\frac{22592}{9}$	$-\frac{111008}{27} + 3584\ \zeta_2$	$-\frac{111008}{27} + 3584\ \zeta_2$
$C_a^2 n_f$	0	0	$-\frac{1280}{9}$	$-\frac{1280}{9}$	$\frac{6560}{9}$	$\frac{19616}{27} + \chi_1$
$C_a n_f^2$	0	0	0	0	$-\frac{256}{27}$	$-\frac{256}{27}$

Third order predictions for Higgs production

Duhr et al

Higgs boson production in b+Bbar -> g

$$\Delta_b(z) = \delta(1-z) + a_s \Delta_b^{(1)}(z) + a_s^2 \Delta_b^{(2)}(z) + a_s^3 \Delta_b^{(3)}(z) + \cdots$$

3rd order prediction: Using known results upto 2nd order

$$\Delta_b^{(3)} = \Delta_b^{SV,(3)}(z) + \sum_{k=0}^{5} C_k \ \log^k(1-z) + \cdots$$

N3LO predictions

	$\log^5(1-z)$		$\log^4(1-z)$		$\log^3(1-z)$	
C_F^3	-512	-512	1728	1728	$2272 + 3072\ \zeta 2$	$2272 + 3072\ \zeta_2$
$C_F^2 n_f$	0	0	$-\frac{1280}{9}$	$-\frac{1280}{9}$	$\frac{19456}{27}$	$\frac{6464}{9} + \chi_1$
$C_A C_F^2$	0	0	$\frac{7040}{9}$	$\frac{7040}{9}$	$-\frac{111904}{27} + 512\ \zeta_2$	$-\frac{37184}{9} + 512\zeta_2 + \chi_2$
$C_F n_f^2$	0	0	0	0	$-\frac{256}{27}$	$-\frac{256}{27}$
$C_A C_F n_f$	0	0	0	0	$\frac{2816}{27}$	$\frac{2816}{27}$
$C_A^2 C_F$	0	0	0	0	$-\frac{7744}{27}$	$-\frac{7744}{27}$

Fourth order prediction

4th order QCD prediction for Drell-Yan production

Vogt,Moch et al,
DeFlorian et al, Das et al

$$\Delta_q^{(4)} = \left(-\frac{4096}{3}C_F^4\right)\log^7(1-x) + \left(\frac{39424}{9}C_F^3C_A + \frac{19712}{3}C_F^4 - \frac{7168}{9}n_fC_F^3\right)$$
$$\times\log^6(1-x) + \left(-\frac{123904}{27}C_F^2C_A^2 - \frac{805376}{27}C_F^3C_A + 9088C_F^4 + \frac{45056}{27}n_fC_F^2C_A\right.$$
$$\left. + \frac{139520}{27}n_fC_F^3 - \frac{4096}{27}n_f^2C_F^2 + 3072\zeta_2C_F^3C_A + 20480\zeta_2C_F^4\right)\log^5(1-x)$$

4th order QCD prediction for Higgs production in gluon fusion

$$\Delta_g^{(4)} = \left(-\frac{4096}{3}C_A^4\right)\log^7(1-x) + \left(\frac{98560}{9}C_A^4 - \frac{7168}{9}n_fC_A^3\right)\log^6(1-x)$$
$$+\left(-\frac{298240}{9}C_A^4 + \frac{174208}{27}n_fC_A^3 - \frac{4096}{27}n_f^2C_A^2 + 23552\zeta_2C_A^4\right)\log^5(1-x).$$

Mellin Moments and large N

Mellin Moment:

$$f_N = \int_0^1 dz\ z^{N-1} f(z)$$

Threshold limit $z \to 1$ in z-Space translates to

$N \to \infty$ in N-Space

$N \to \infty$ Taking into account SV and NSV terms

$$\left(\frac{\log(1-z)}{1-z}\right)_+ = \frac{\log^2 N}{N} - \frac{\log N}{2N} + \mathcal{O}\left(\frac{1}{N^2}\right)$$

$$\log^k(1-z) = \frac{\log^k N}{N} + \mathcal{O}\left(\frac{1}{N^2}\right)$$

Structure of NSV logs

Structure of Next to SV terms

$$\Delta_N^c = 1 + a_s\Big[c_1^2 \log^2 N + c_1^1 \log N + c_1^0 + d_1^1 \frac{\log N}{N} + \mathcal{O}(1/N)\Big]$$

$$+a_s^2\Big[c_2^4 \log^4 N + \cdots + c_2^0 + d_2^3 \frac{\log^3 N}{N} + \cdots + \mathcal{O}(1/N)\Big]$$

$$+\cdots$$

$$+a_s^n\Big[c_n^{2n} \log^{2n} N + \cdots + d_n^{2n-1} \frac{\log^{2n-1} N}{N} + \cdots + \mathcal{O}(1/N)\Big]$$

$a_s \log N$ is of order \`one\` when a_s is very small at every order 1/N

Can one resume terms of the form $\frac{\log^k N}{N}$ to all orders

Resummation of NSV logs

$$\Delta_c = C_0^c(q^2) \; e^{\Psi^c_{sv,N}(q^2,\omega)+\Psi^c_{nsv,N}(q^2,\omega)}$$

	LL	NLL		NnLL
	$a_s \frac{1}{N} \log N$	$a_s^2 \frac{1}{N} \log^2 N$		$a_s^n \frac{1}{N} \log^n N$
	$a_s^3 \frac{1}{N} \log^3 N$	$a_s^4 \frac{1}{N} \log^4 N$		
	$a_s^5 \frac{1}{N} \log^5 N$	$a_s^6 \frac{1}{N} \log^6 N$	…	
	⋮	⋮		⋮
Resummed terms:	$a_s^i \frac{1}{N} \log^{2i-1} N$	$a_s^i \frac{1}{N} \log^{2i-2} N$		$a_s^i \frac{1}{N} \log^{2i-n} N$
Exponents	$g_1, g_2,$ $h_{0,k}, h_{1,k}$	$g_3, h_{2,k}$		$g_{n+1}, h_{n,k}$

Resummation of NSV logs

Resummed NSV

$$\Psi^c_{nsv,N} = \frac{1}{N} \sum_{n=0}^{\infty} \sum_{k=0}^{n} h_{nk}(w)\ a_s^n \log^k N$$

Resumed Logarithms

$$\log^k(1-z) \to \frac{1}{N} \log^k N$$

LL → $a_s^i \frac{1}{N} \log^{2i-1} N$

NLL → $a_s^i \frac{1}{N} \log^{2i-2} N$

NNLL → $a_s^i \frac{1}{N} \log^{2i-n} N$

Matched cross section

Mellin Inversion of the resummed result and add it to fixed order results

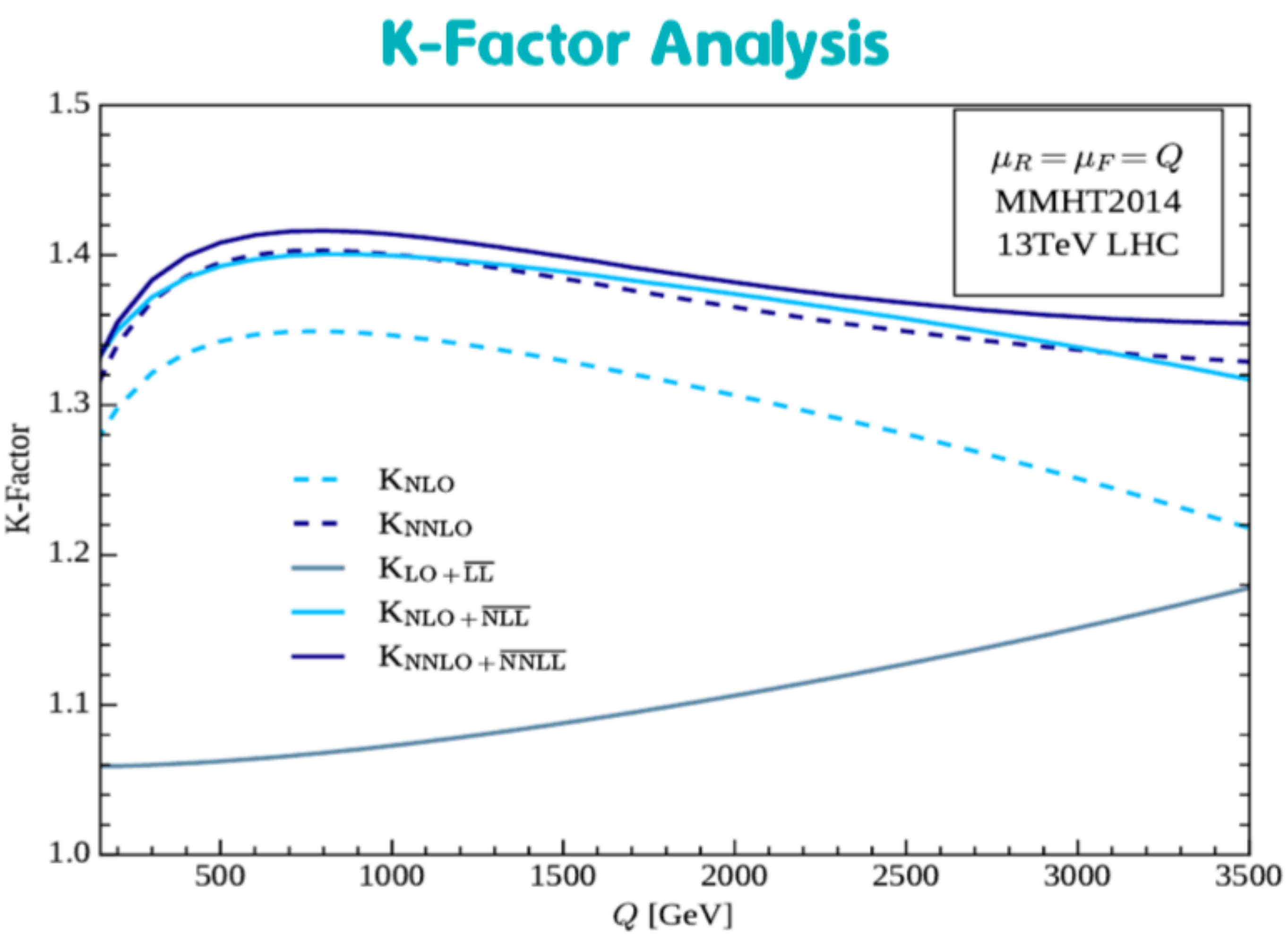
$$\sigma_N^{\mathrm{N^nLO}+\overline{\mathrm{N^nLL}}} = \sigma_N^{\mathrm{N^nLO}} + \sigma^{(0)} \sum_{ab\in\{q,\bar{q}\}} \int_{c-i\infty}^{c+i\infty} \frac{dN}{2\pi i} (\tau)^{-N} \delta_{a\bar{b}} f_{a,N}(\mu_F^2) f_{b,N}(\mu_F^2)$$

$$\times \left(\Delta_{q,N}\Big|_{\overline{\mathrm{N^nLL}}} - \Delta_{q,N}\Big|_{tr\ \mathrm{N^nLO}} \right) .$$

The resummed results are matched to the fixed order result in order to avoid any double counting of threshold logarithms

The contour C in the Mellin inversion is chosen according to Minimal prescription

K-factor for DY



$$K(Q) = \frac{\frac{d\sigma}{dQ}(\mu_R = \mu_F = Q)}{\frac{d\sigma^{LO}}{dQ}(\mu_R = \mu_F = Q)}$$

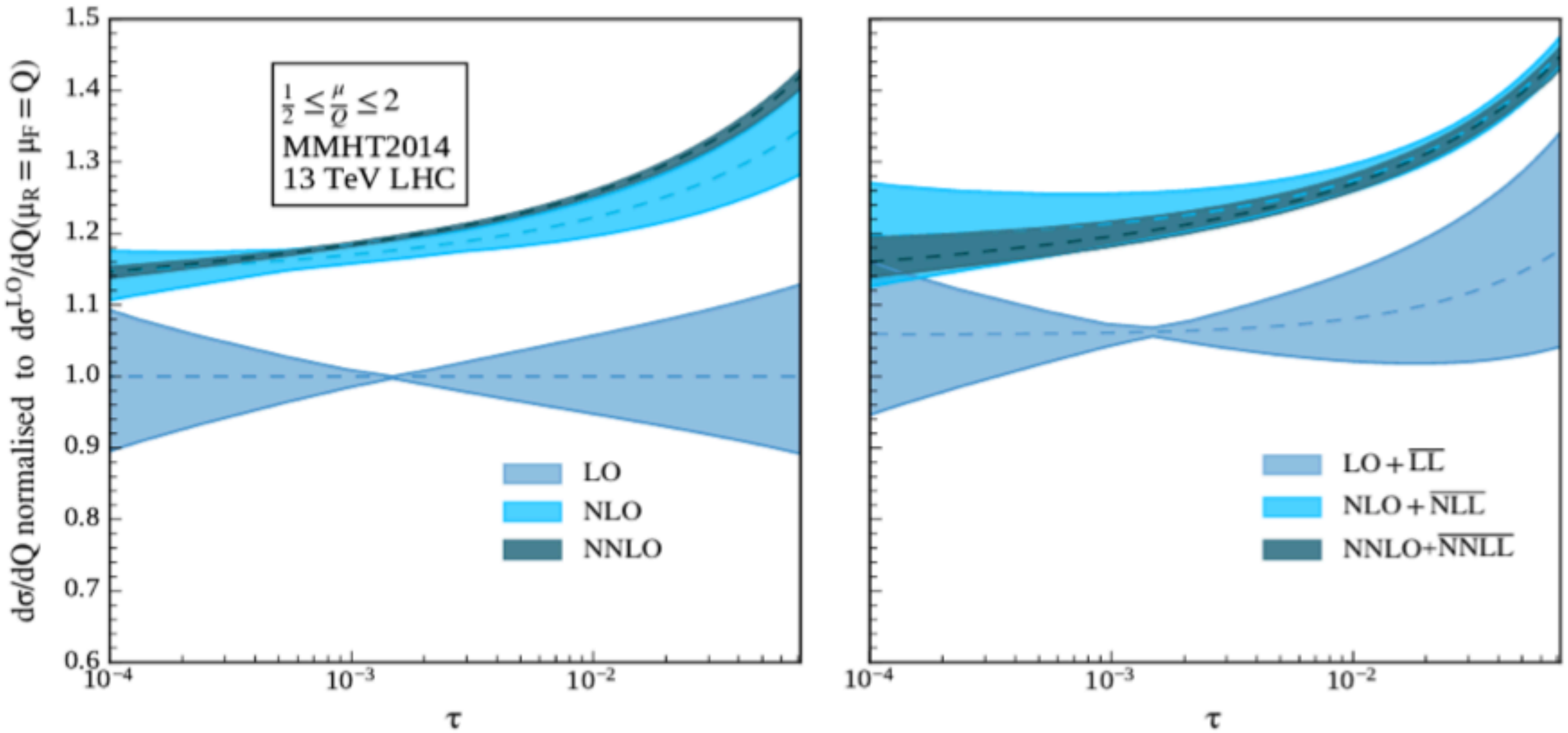
- Enhancement

	Q=500 GeV	2000 GeV
LO- $\rightarrow \mathrm{LO} + \overline{\mathrm{LL}}$	**6.2 %**	**10.6%**
NLO- $\rightarrow \mathrm{NLO} + \overline{\mathrm{NLL}}$	**3.7 % ,**	**5.2%**
NNLO $\rightarrow \mathrm{NNLO} + \overline{\mathrm{NNLL}}$	**0.94 %**	**1.2%**

- Resummed curves are closer
- They decreases as we go for higher order resummed contributions
- Perturbative convergence

$\mu_R = \mu_F = Q(\mathrm{GeV})$	$\mathrm{LO} + \overline{\mathrm{LL}}$	NLO	$\mathrm{NLO} + \overline{\mathrm{NLL}}$	NNLO	$\mathrm{NNLO} + \overline{\mathrm{NNLL}}$
500	1.0624	1.3425	1.3925	1.3950	1.4082
1000	1.0728	1.3464	1.3995	1.4004	1.4138
2000	1.1062	1.3064	1.3739	1.3652	1.3818

Scale uncertainty for DY



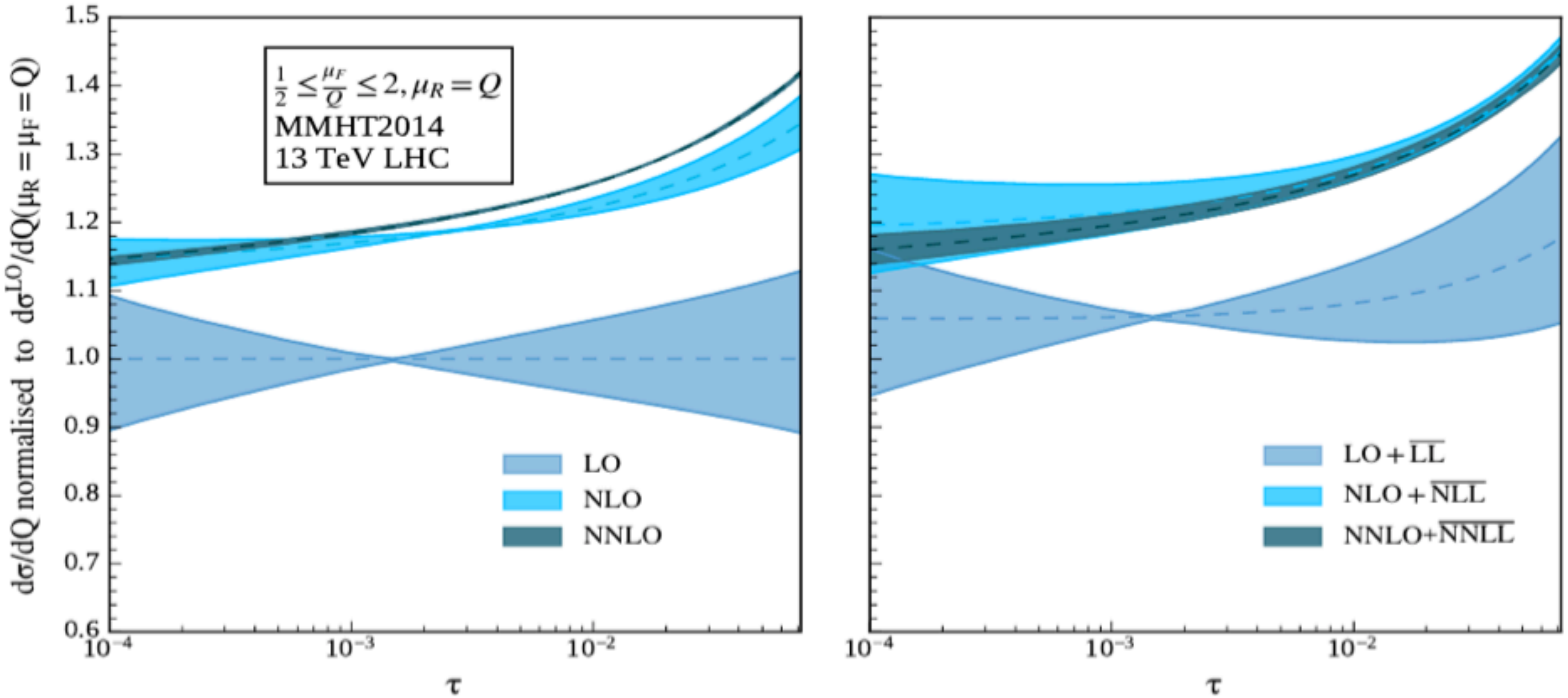
- Systematic reduction of the uncertainties at different logarithmic accuracies
- Improvement at the NLO+NLL than at the NNLO+NNLL in comparison to their respective F.O predictions

7-point var: μ = {μF , μR} is varied in the range [1/2Q, 2Q] keeping the ratio μR /μF not larger than 2 and smaller than 1/2.

Q	LO	LO+$\overline{\text{LL}}$	NLO	NLO+$\overline{\text{NLL}}$	NNLO	NNLO+$\overline{\text{NNLL}}$
1000	$2.3476^{+4.10\%}_{-3.92\%}$	$2.5184^{+4.49\%}_{-4.25\%}$	$3.1609^{+1.79\%}_{-1.69\%}$	$3.2857^{+2.08\%}_{-1.18\%}$	$3.2876^{+0.20\%}_{-0.31\%}$	$3.3191^{+1.13\%}_{-0.86\%}$
2000	$0.0501^{+8.50\%}_{-7.46\%}$	$0.0554^{+9.10\%}_{-7.91\%}$	$0.0654^{+2.83\%}_{-2.98\%}$	$0.0688^{+1.43\%}_{-1.23\%}$	$0.0684^{+0.37\%}_{-0.62\%}$	$0.0692^{+0.89\%}_{-0.78\%}$

Cross section in 10^−5 pb/GeV

Factorisation scale uncertainty for DY



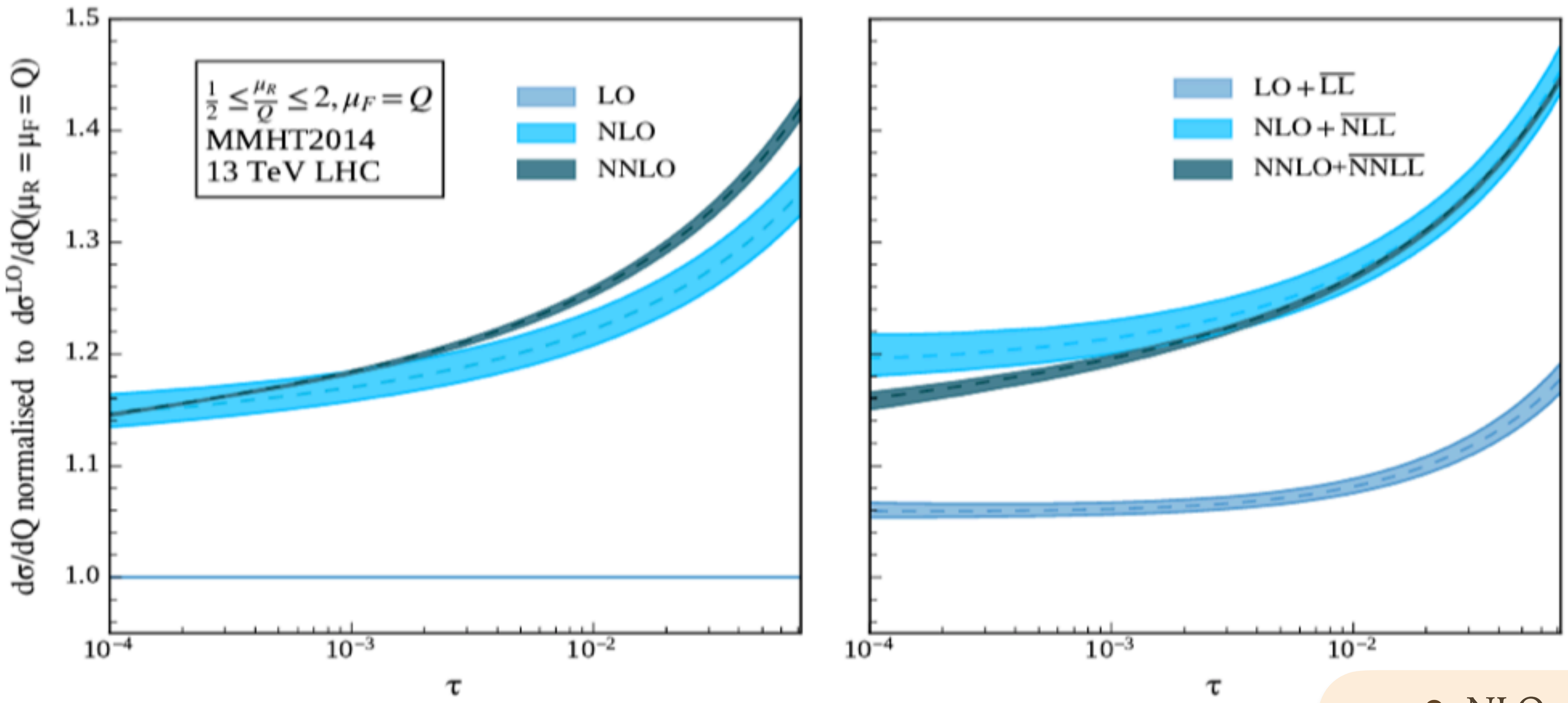
NLO : 22% from $q\bar{q}$
−5% from qg
$NNLO$: 4.9% from $q\bar{q}$
−2.8% from qg

- resummed bands look similar to that of 7-point bands --width of the 7-point bands mainly comes from the μF uncertainties

- NLO band gets improved with the inclusion of NLL, but NNLO band increses with the inclusion of NNLL

- Missing qg contribution increases the scale dependence

Renormalisation scale uncertainty for DY



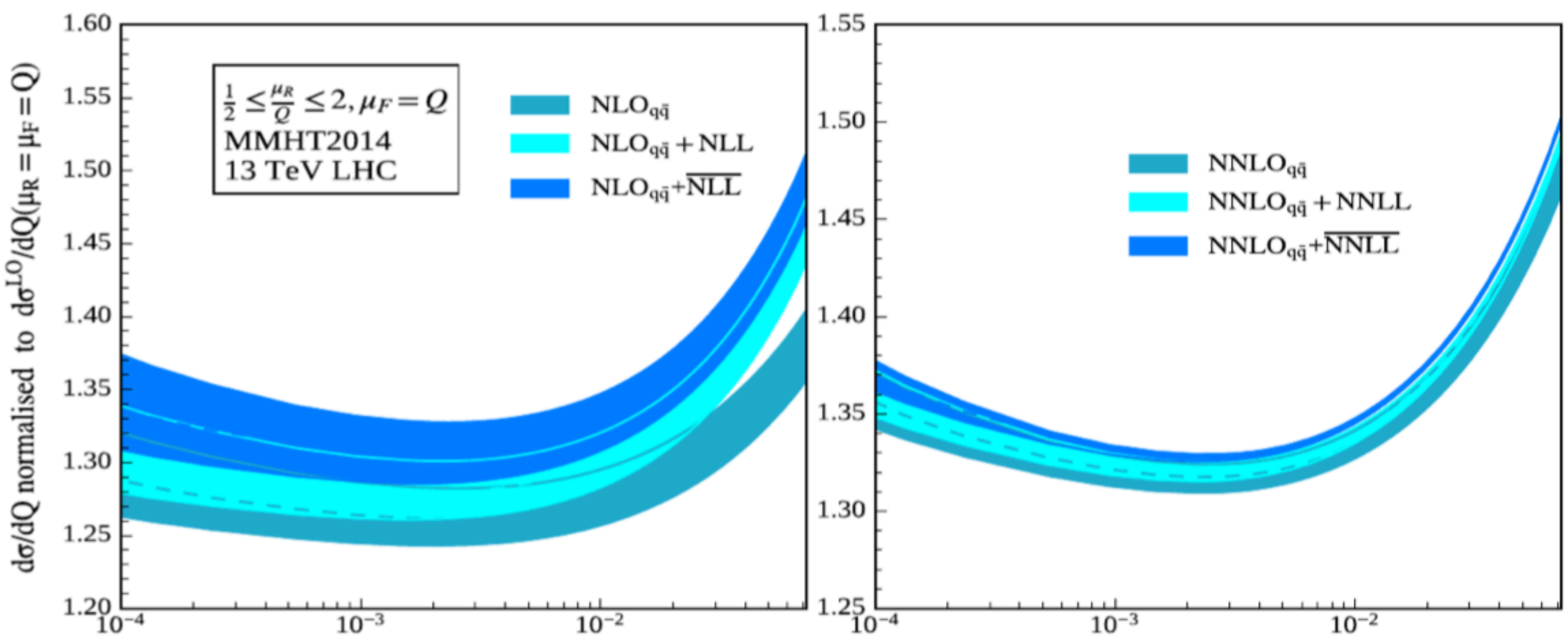
- NNLO+NNLL the error band becomes substantially thinner
- Each partonic channel is invariant under μ R variation and hence inclusion of More corrections within a channel is expected to reduce the uncertainity

- $\text{NLO}: {}^{+1.46\%}_{-1.28\%}$
- $\text{NLO} + \overline{\text{NLL}}: {}^{+1.35\%}_{-1.23\%}$
- $\text{NNLO}: {}^{+0.37\%}_{-0.46\%}$
- $\text{NNLO} + \overline{\text{NNLL}}: {}^{+0.02\%}_{-0.23\%}$

Scale dependence gets reduced significantly due to the inclusion NSV resummation

Renormalisation scale uncertainty for q qb in DY

Uncertainities w.r.t renormalisation scale variation in the quark antiquark channel

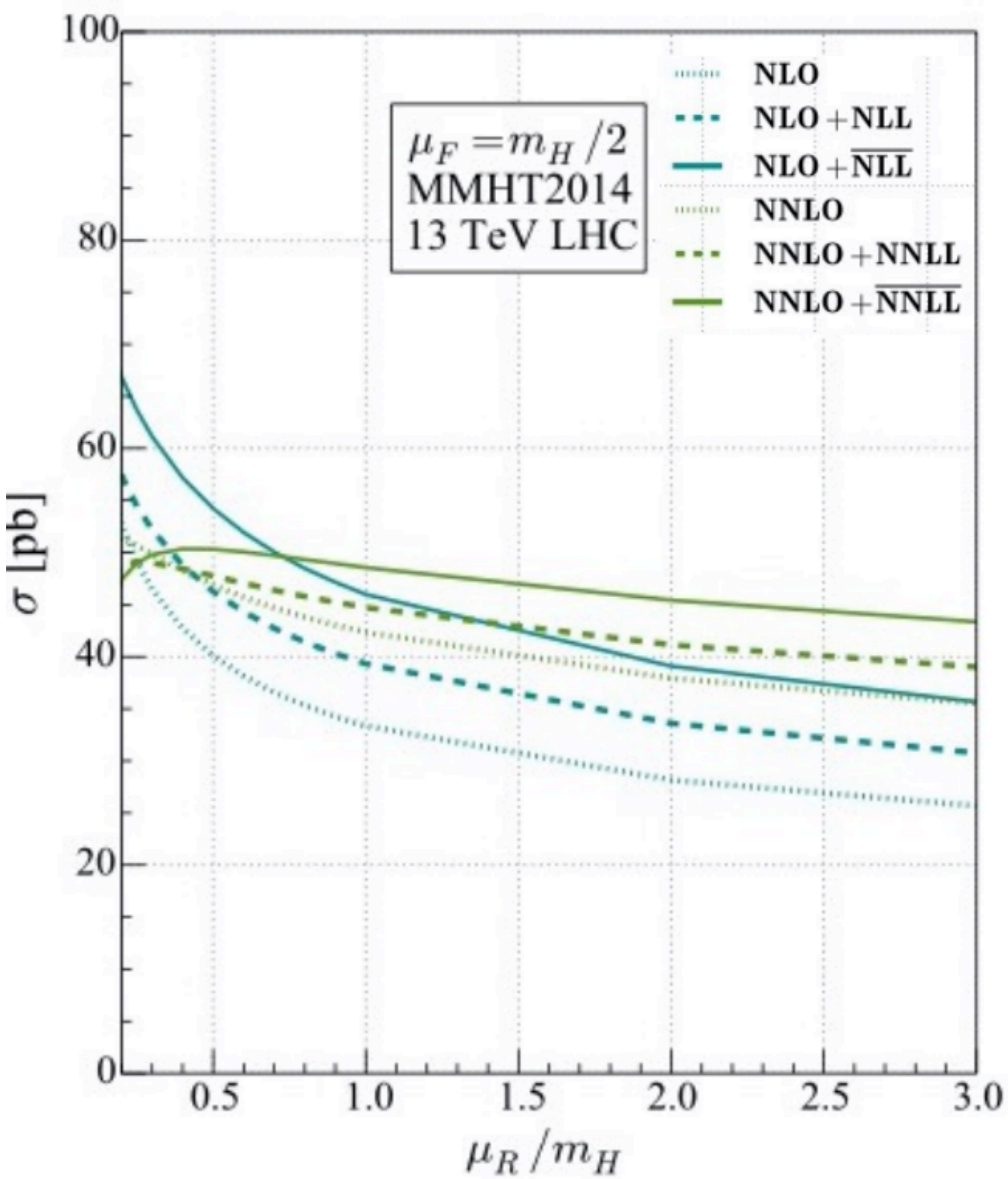


$Q = \mu_R = \mu_F$	$NNLO_{q\bar{q}}$	$NNLO_{q\bar{q}} + NNLL$	$NNLO_{q\bar{q}} + \overline{NNLL}$
1000	$3.5260^{+0.49\%}_{-0.58\%}$	$3.5376^{+0.25\%}_{-0.39\%}$	$3.5576^{+0.006\%}_{-0.20\%}$
2000	$0.0717^{+0.54\%}_{-0.62\%}$	$0.0721^{+0.19\%}_{-0.33\%}$	$0.0725^{+0.0\%}_{-0.15\%}$

Cross section in 10^−5 pb/GeV for $q\bar{q}$ channel

Interestingly, the behaviour of NNLO_qqb + NNLLbar is significantly improved from the corresponding SV results, NNLO_qqb + NNLL, for a wide range of Q.

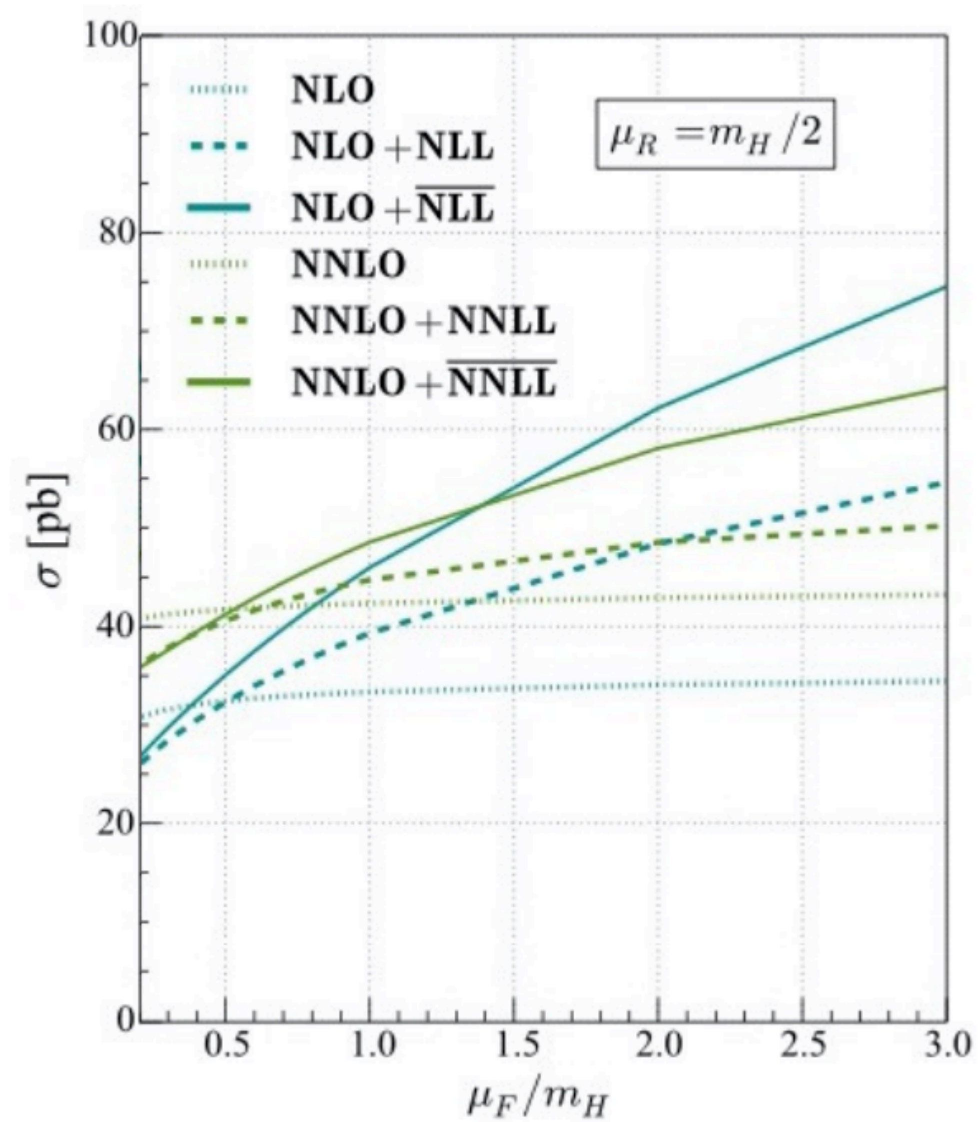
Renormalisation scale uncertainty for Higgs

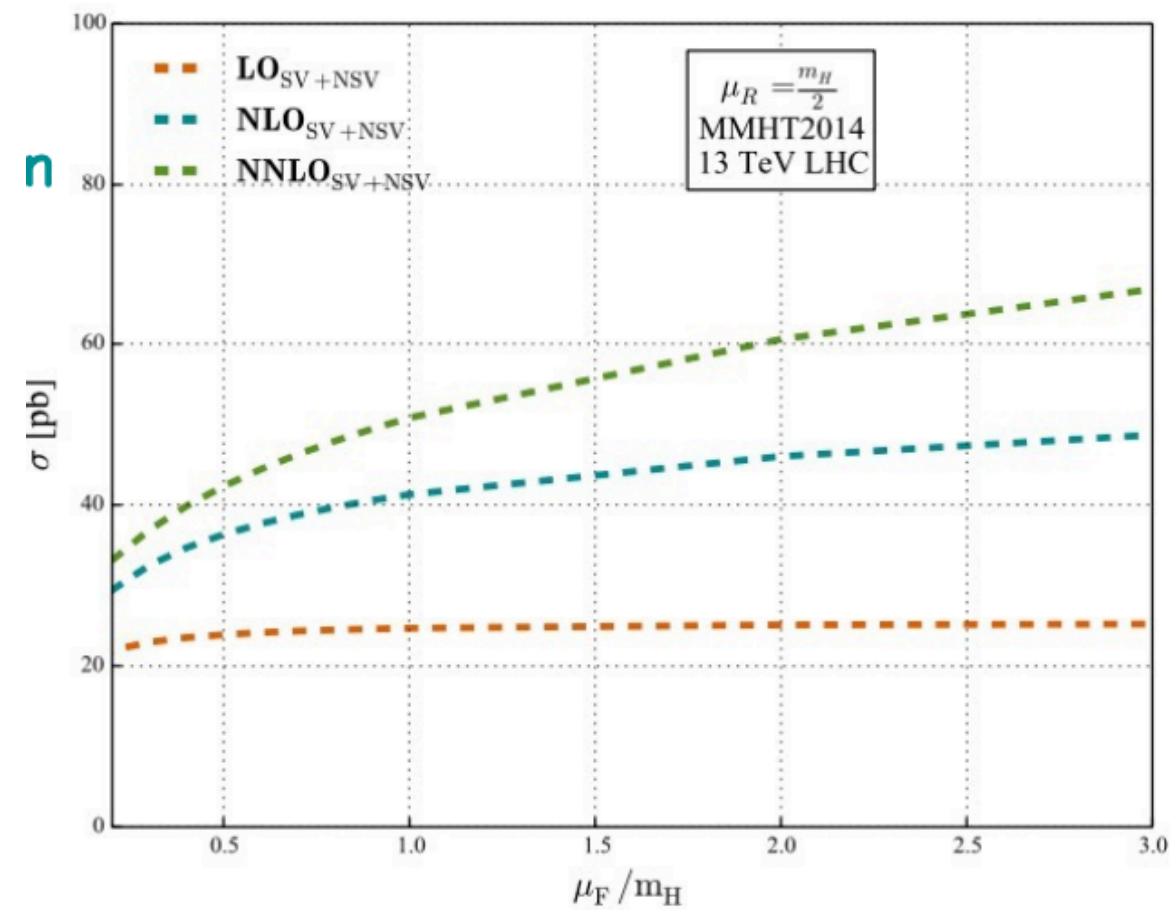


- Renormalisation scale variation is less for resummed results as compared to fixed order
- However, the scale variation is comparable for SV resumed and SV+NSV resumed at NLO accuracy
- No significant improvement by the inclusion of SV but Comprehensible improvement by NSV Res results at NNLO
- SV is dominant at NLO with 73.16% contribution
- NSV is dominant at NNLO with 58.9% contribution while SV is only 15.8%

NLO	NLO+NLL	NLO+$\overline{\text{NLL}}$	NNLO	NNLO+NNLL	NNLO+$\overline{\text{NNLL}}$
$39.1681^{+9.09}_{-6.73}$	$38.0142^{+7.06}_{-5.70}$	$41.0325^{+7.06}_{-5.97}$	$46.4304^{+4.11}_{-4.70}$	$45.0904^{+4.32}_{-4.52}$	$44.9685^{+2.94}_{-3.74}$

Factorisation scale uncertainty for Higgs





- Fixed order truncated at SV+NSV contribution shows significant variations
- Factorization scale dependence due to NSV contribution cancels with beyond NSV terms
- Increase in dependence with the increase in order of accuracy μ
- % contribution of beyond NSV term increases with the order of accuracy

Conclusions

- Studied Next to soft terms in Inclusive reactions
- Set up a Framework in z-space
- Framework to Resum Next to Soft terms N space
- Z-space prediction at third and fourth order
- N-space resummation to all orders for NSV
- Resummation in non-diagonal terms is still open problem