Advances in fixed-order computations

Chiara Signorile-Signorile

QCD@LHC
01/12/2022
Motivation

LHC continues to confirm the Standard Model

BUT

still several tensions with SM are present:

- flavour anomalies
- g-2
- dark matter

see Marzia Bordone, Eluned Smith’s talk!

High precision theoretical predictions
Is percent precision a reality?

- Frontiers of experimental precision

- Determination of the interaction luminosity

<table>
<thead>
<tr>
<th>Source</th>
<th>2015 (%)</th>
<th>2016 (%)</th>
</tr>
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<tbody>
<tr>
<td>Total normalization uncertainty</td>
<td>1.3</td>
<td>1.0</td>
</tr>
<tr>
<td>Total integration uncertainty</td>
<td>1.0</td>
<td>0.7</td>
</tr>
<tr>
<td>Total uncertainty</td>
<td>1.6</td>
<td>1.2</td>
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Data sample

<table>
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<tr>
<th></th>
<th>2015+16</th>
<th>2017</th>
<th>2018</th>
<th>Comb.</th>
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<tbody>
<tr>
<td>Integrated luminosity (fb⁻¹)</td>
<td>36.2</td>
<td>44.3</td>
<td>58.5</td>
<td>139.0</td>
</tr>
<tr>
<td>Total uncertainty (fb⁻¹)</td>
<td>0.8</td>
<td>1.0</td>
<td>1.2</td>
<td>2.4</td>
</tr>
</tbody>
</table>

Resolution on observed energy of particles and hadronic jets

The final uncertainties on the jet energy scale are below 3% across the phase space considered by most analyses ($p_T > 30$ GeV and $|\eta| < 5.0$). In the barrel region we reach an uncertainty below 1% for $p_T > 30$ GeV, when excluding the jet-flavor uncertainties, provided separately for different jet-flavor mixtures. At its lowest, the core uncertainty (excluding optional time-dependent and flavor systematics) is 0.32% for jets with $p_T$ between 165 and 330 GeV, and $|\eta| < 0.8$. These results set a new benchmark for jet energy scale determination at hadron colliders.

Statistical limitations are expected to be overcome by HL-LHC

(HL-LHC). The HL-LHC will collide protons against protons at 14 TeV centre-of-mass energy with an instantaneous luminosity a factor of five greater than the LHC and will accumulate ten times more data, resulting in an integrated luminosity of 3 ab⁻¹.
Is percent precision a reality?

The success of a percent level phenomenology program relies on our ability to interpret and predict the outcome LHC measurement.

[Collins, Soper, Sterman 0409313]

Hard collisions at the LHC are described in terms of quark and gluon cross sections

\[ d\sigma = \sum_{ij} \int dx_1 dx_2 f_{i\bar{p}}(x_1) f_{\bar{p}j}(x_2) d\hat{\sigma}_{ij}(x_1 x_2 s) \left( 1 + \mathcal{O}\left( \frac{\Lambda_{QCD}}{Q^n} \right) \right), \quad n \geq 1 \]

Parton distribution functions

Hard scattering (perturbative quantum field theory)

Non perturbative effects (fragmentation, hadronisation)

aim for few % level!
Motivations

NNLO HADRON-COLLIDER CALCULATIONS VS. TIME

Explosion of calculations starting in 2014

[Cieri@WG]
Motivations

NNLO HADRON-COLLIDER CALCULATIONS VS. TIME

Explosion of calculations starting in 2014

N^3LO HADRON-COLLIDER CALCULATIONS VS. TIME

First calculations

QCD fixed-order as of 2022

- most procs. known (some w. public code)
- some procs. known / no public code
- some inputs known (no full calc)

pp → Wbb see Rene Poncelet's talk!
A lot of homework...

<table>
<thead>
<tr>
<th>process</th>
<th>known</th>
<th>desired</th>
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<tbody>
<tr>
<td>( pp \rightarrow H )</td>
<td>( N^3LO_{HTL} )</td>
<td>( N^3LO_{HTL} ) (incl.)</td>
</tr>
<tr>
<td>( pp \rightarrow H + j )</td>
<td>( NNLO_{QCD} )</td>
<td>( NNLO_{QCD} )</td>
</tr>
<tr>
<td>( pp \rightarrow H + 2j )</td>
<td>( NNLO_{QCD} )</td>
<td>( NNLO_{QCD} )</td>
</tr>
<tr>
<td>( pp \rightarrow H + 3j )</td>
<td>( NNLO_{QCD} )</td>
<td>( NNLO_{QCD} )</td>
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<tr>
<td>( pp \rightarrow VH )</td>
<td>( NLO_{(A)} )</td>
<td>( NLO_{(A)} )</td>
</tr>
<tr>
<td>( pp \rightarrow VH + j )</td>
<td>( NNLO_{QCD} )</td>
<td>( NNLO_{QCD} )</td>
</tr>
<tr>
<td>( pp \rightarrow HH )</td>
<td>( NLO_{QCD} )</td>
<td>( NLO_{QCD} )</td>
</tr>
<tr>
<td>( pp \rightarrow H + t )</td>
<td>( NNLO_{QCD} )</td>
<td>( NNLO_{QCD} )</td>
</tr>
<tr>
<td>( pp \rightarrow H + t/t )</td>
<td>( NNLO_{QCD} )</td>
<td>( NNLO_{QCD} )</td>
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</table>

Table 1: Precision wish list: Higgs boson final states. \( N^3LO_{QCD}^{(BFP)} \) means a calculation using the structure function approximation. \( V = W, Z \).
The Drell-Yan process: mixed corrections

\[ \frac{d\hat{\sigma}}{dM_{\ell\ell}} = d\hat{\sigma},_{\text{LO}} (1 + \alpha_s \Delta_{ij, \text{NLO}}^{QCD} + \alpha_{ew} \Delta_{ij, \text{NLO}}^{EW} + \alpha_s^2 \Delta_{ij, \text{NNLO}}^{QCD} + \alpha_s \alpha_{ew} \Delta_{ij, \text{NLO}}^{QCD\otimes EW} + \alpha_s^3 \Delta_{ij, \text{NNLO}}^{QCD} + \cdots) \]

\[ \text{NLO EW } \sim \mathcal{O}(\alpha_{ew}) \sim 1\% \]

\[ \text{QCD}\times\text{EW } \sim \mathcal{O}(\alpha_s \alpha_{ew}) \sim 0.1\% \]

**Couplings:** \( \alpha_s \sim 0.1, \alpha_{ew} \sim 0.01 \)

**Precise EW parameters determination:**

- \( \sin^2 \theta_{ew} \) see Rhys Taus’s talk!

**Ultimate goals of ATLAS, CMS, LHCb \sim 10 \text{ MeV} each, with different experimental conditions and methods.**

- \( m(W)_{\text{ATLAS}} = 80370 \pm 19 \text{ MeV} \)
- \( m(W)_{\text{LHCb}} = 80354 \pm 32 \text{ MeV} \)

> would mean \( \mathcal{O}(0.01\%) \) uncertainty

[Aperio Bella, QCD@LHC22]
Motivation: why Drell-Yan in the high invariant mass region?

- Hunting for New Physics (NP)
  - Search for shape distortions in kinematic distributions
  - Constrain heavy NP in a model-independent way using EFT
    [Barbieri, Pomarol, Rattazzi, Strumia ’04] [Alioli, Farina, Pappadopulo, Ruderman ’17] [Farina et al.’17]

EW contributions enhanced by Sudakov logarithms
[Kuhn, Penin, Smirnov ’00][Ciafaloni, Ciafaloni, Comelli ’01][Denner, Pozzorini ’01]

\[
\alpha_{\text{ew}} \frac{\log^2 \left( \frac{s}{m_Z^2} \right)}{4\pi \sin^2 \theta_W} \sim 10\% , \quad \alpha_{\text{ew}} \frac{\log \left( \frac{s}{m_Z^2} \right)}{4\pi \sin^2 \theta_W} \sim 1.6\% , \quad \sqrt{s} = 2\text{TeV}
\]
Ingredients for off-shell calculation at NNLO

- Fully differential description of mixed QCD-EW effects requires different ingredients

- Different approaches to tackle the problem → results qualitatively in agreement [Bonciani, Buonocore, Grazzini, Kallweit, Rana, Tramontano, Vicini 2106.11953] [Buccioni, Caola, Chawdhry, Devoto, Heller, A. von Manteuffel, Melnikov, Röntsch, S-S 2203.11237]

**Mixed QCD-EW** amount to about -1% of the LO cross-section → larger than expected from coupling magnitude
Associated Higgs production

- $pp \rightarrow t\bar{t}H$ allows for direct measurement of the top-quark Yukawa coupling

- ATLAS and CMS accuracy $\mathcal{O}(20\%)$, expected $\mathcal{O}(2\%)$ at HL-LHC. Current theoretical uncertainties are $\mathcal{O}(10\%)$ [LHC Higgs WG 1610.07922].

- $2 \rightarrow 3$ process with massive coloured particles at NNLO [Catani, Devoto, Grazzini, Kallweit, Mazzitelli, Savoini 2210.07846]

See Chiara Savoini’s talk!
Associated Higgs production

• 2 → 3 process with massive coloured particles at NNLO [Catani, Devoto, Grazzini, Kallweit, Mazzitelli, Savoini 2210.07846]

• Soft Higgs approximation \( p_H \rightarrow 0 \) (exact 2loop amplitudes not known): factorisation of soft limit of the scalar form factor of the heavy quark, lower multiplicity amplitude, eikonal factor for top-Higgs interaction

\[
\mathcal{M}(\{p_i\}, p_H) \sim F(\alpha_s(\mu_R); m_t/\mu_R) \frac{m_t}{\nu} \sum_{i=3,4} \frac{m_t}{p_i \cdot p_H} \mathcal{M}(\{p_i\})
\]

Complete calculation except for the finite part of the 2loop virtual amplitude, computed using soft Higgs approximation.

Double-virtual contribution found to be small + moderate uncertainty inferred from NLO matching → NNLO uncertainties due to soft approx. much smaller than perturbative uncertainties

NNLO ranges from about +4 % at low \( \sqrt{s} \) to about +2 % at 100 TeV

Perturbative uncertainty reduced from ±9 % at NLO to ±(2 − 3) %

*double real* tackled with the \( q_\perp \) subtraction [Catani, Grazzini 0703012]

\[
\begin{array}{|c|c|c|}
\hline
\sigma [\text{pb}] & \sqrt{s} = 13 \text{ TeV} & \sqrt{s} = 100 \text{ TeV} \\
\hline
\sigma_{\text{LO}} & 0.3910^{+31.3\%}_{-22.2\%} & 25.38^{+21.1\%}_{-16.0\%} \\
\sigma_{\text{NLO}} & 0.4875^{+5.6\%}_{-9.1\%} & 36.43^{+9.4\%}_{-8.7\%} \\
\sigma_{\text{NNLO}} & 0.5070 (31)^{+0.9\%}_{-5.0\%} & 37.20 (25)^{+0.1\%}_{-2.2\%} \\
\hline
\end{array}
\]
Multijet processes

- Theory-data comparison of differential multi-jet rates provides information about perturbative QCD and modelling jet production

- Ratio of three-to-two jet rates sensitive to parton splittings and then to $\alpha_s$ (in the ratio some systematic uncertainties cancel, as from PDFs) [CMS 1304.7498][ATLAS 1805.04691]

$$R_{3/2}(X, \mu_R, \mu_F) = \frac{d\sigma_3(\mu_R, \mu_F)/dX}{d\sigma_2(\mu_R, \mu_F)/dX}$$

$\alpha_s$ in ATLAS e CMS, see Tanmay Sarkar’s talk!
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- Involved calculation: 5 coloured partons at the Born level, 7 coloured partons for the double real, 2-loop-5-point amplitudes

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\[ H_T = \sum_{i\in\text{jets}} p_\perp(j_i) \quad \hat{H}_T = \sum_{i\in\text{partons}} p_\perp(j_i) \]

Scale dependence main theoretical uncertainty at NLO, reduced at NNLO

\[ R_{3/2} \] stabilises once NNLO QCD corrections are included

*double virtual matrix element in the leading color approximation [Chicherin, Sotnikov 2009.07803][Abreu et al. 2102.13609] (10% of the total cross section).

*double real tackled with the STRIPPER subtraction [Czakon 1005.0274][Czakon, Heymes 1408.2500]
Multijet processes

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A central goal is to demonstrate the feasibility of three-jet computations with NNLO precision... the enormous computational cost of the calculation ( $\sim 10^6$CPUh) makes it clear that further refinements in the handling of real radiation contributions to NNLO calculations are desirable. [Czakon, Mitov, Poncelet 2106.05331]
Two main topologies contribute to the $t$-channel, single-top production:

- **Factorisable contributions**
  - Structure function approximation
  - Small effects on inclusive cross-section and distributions
  - Non-factorisable contributions: new!
    - Vanish at NLO, colour suppressed by $N_c^2 - 1 = 8$ at NNLO

- Production rate of the same order of magnitude as $t\bar{t}$: $\sigma_t \sim 1/4 \sigma_{t\bar{t}}$
- $t$-channel accounts for $\sim 70\% \sigma_t$
- Electroweak mediated
Single-top production: t-channel

Main bottleneck: evaluate loop amplitudes that remain stable in the degenerate kinematics.

**VV: two-loop four-point amplitude** [Brønnum-Hansen, Melnikov, Quarroz, Wang ’21]
- 4 kinematic scales: $s$, $t$, $m_t^2$, $m_W^2$
- IBP performed analytically with KIRA2.0 and FireFly
- 428 master integrals evaluated numerically (auxiliary mass flow method)
  20 digits in $\sim$ 30 min on a single core

| $\langle A^{(0)} | A^{(2)}_{\text{nf}} \rangle$ | $\epsilon^{-2}$ | $\epsilon^{-1}$ |
|-----------------------------------------------|----------------|----------------|
| IR poles                                    | $-229.0940408654660 - 8.97816333241640i$ | $-301.1802988944764 - 264.1773596529505i$ |
|                                              | $-229.0940408654665 - 8.97816333241973i$ | $-301.1802988944791 - 264.1773596529535i$ |

$$
\sigma_{pp \rightarrow X+t} / 1 \text{ pb} = 117.96 + 0.26 \left( \frac{\alpha_s(\mu_R)}{0.108} \right)^2
$$

1. Non-factorisable corrections are $0.22^{+0.04}_{-0.04} \%$ LO for $\mu_R = m_t$.
2. **Unclear optimal scale choice**: NF corrections appear for the first time at NNLO
3. $\mu_R = 40 \text{ GeV}$ (typical **transfer momentum scale** of top) NF corrections $0.35 \%$ LO.
4. In comparison, **NNLO factorisable** corrections are around $0.7 \%$ NLO.

Chiara Signorile-Signorile

Advances in fixed-order predictions
Selected examples... general comments

Fixed-order calculation of higher-order corrections involves two main technical challenges:

**Amplitudes:**
- Multi-loop integrals involving various masses/many legs

**IR singularities**
- Extraction of soft and collinear singularities
  → fully differential predictions
Loops amplitudes: rapid growth in the complexity as the loop order increases and as more massive particle are considered. Ultimately it is necessary to find a representation for the amplitudes that can be evaluated efficiently and yields numerically reliable results.

\[ g_s^4 \text{Tr}\{T^{a_1} T^{a_2}\} \int \frac{d^d k}{(2\pi)^d} \frac{\text{tr} \left[ (k) \epsilon_1 (k + p_1) \epsilon_2 (k + p_{12}) \epsilon_3 (k + p_{123}) \epsilon_4 \right]}{(k)^2 (k + p_1)^2 (k + p_{12})^2 (k + p_{123})^2} \]

Selected examples... general comments

[Bargiela@ICHEP22]
Virtual corrections

One-loop QCD helicity amplitudes for $pp \to t\bar{t}j$ to $O(\alpha_s^2)$

April 2022

Simon Badger, Matteo Becchetti, Ekta Chaubey, Robin Marzucca, Francesco Sarandrea

Two-loop leading colour helicity amplitudes for $W^\pm\gamma + j$ production at the LHC

Jan. 2022

Simon Badger, Heribertus Bayu Hartanto, Jakub Kryšťof, Simone Zois

Leading-Color Two-Loop Amplitudes for Four Partons and a $W$ Boson in QCD

April 2022

S. Abreu, F. Febres Cordero, I. H. Ita, M. Klinkert, B. Page, and V. Sotnikov

NNLO QCD corrections to $Wb\bar{b}$ production at the LHC

May 2022

Heribertus Bayu Hartanto, Rene Poncelet, Andrei Popescu, and Simone Zois

Two-loop mixed QCD-EW corrections to $q\bar{q} \to Hg$, $qg \to Hq$, and $\bar{q}g \to H\bar{q}$

June 2022

Marco Bonetti, Erik Panzer, and Lorenzo Tancredi

Three-loop helicity amplitudes for quark-gluon scattering in QCD

July 2022

Fabrizio Caola, Amlan Chakraborty, Giulio Gambuti, Andreas von Manteuffel, and Lorenzo Tancredi

Two-loop QCD corrections to the $V \to q\bar{q}g$ helicity amplitudes with axial-vector couplings

Nov. 2022

Thomas Gehrmann, Tiziano Peraro, and Lorenzo Tancredi

See Rene Poncelet, Fabian Lange, Vasily Sotnikov, Piotr Bargiela, Andrew Lifson, Daniel Maitre’s talks!
Loops amplitudes: rapid growth in the complexity as the loop order increases and as more massive particle are considered. Ultimately it is necessary to find a representation for the amplitudes that can be evaluated efficiently and yields numerically reliable results.

Three families of solutions to the problem:

- **Analytic**
  - Fast, precise evaluation (e.g. \( L_n, p F_q, MPL, \ldots \))
  - Wider applications (e.g. changing parameters)

First 3-loop \( 2 \rightarrow 2 \), QCD results (light quarks)

- \( q\bar{q} \rightarrow \gamma\gamma \) [Caola, von Manteuffel, Tancredi 2011.13946]
- \( q\bar{q} \rightarrow q\bar{q} \) [Caola, Chakraborty, Gambuti, von Manteuffel, Tancredi 2108.00055]
- \( gg \rightarrow \gamma\gamma \) [Bargiela, Caola, von Manteuffel, Tancredi 2111.13595]
- \( gg \rightarrow gg \) [Caola, Chakraborty, Gambuti, von Manteuffel, Tancredi 2112.11097]
- \( q\bar{q} \rightarrow gg \) [Caola, Chakraborty, Gambuti, von Manteuffel, Tancredi 2207.03503]

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<th>2L</th>
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<td>20935</td>
</tr>
<tr>
<td># MILs</td>
<td>6</td>
<td>39</td>
</tr>
</tbody>
</table>

**Qgraf** results [kB]:
- Amplitudes before IBPs [kB] | 276 | 54364 | 1973464 |
- Amplitudes after IBPs [kB]  | 12  | 562  | 304499 |
- Expanded amplitudes [kB]    | 136 | 380  | 1195  |
Selected examples... general comments

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- $q\bar{q} \to q\bar{q}$ [Caola, Chakraborty, Gambuti, von Manteuffel, Tancredi 2108.00055]
- $gg \to \gamma\gamma$ [Bargiela, Caola, von Manteuffel, Tancredi 2111.13595]
- $gg \to gg$ [Caola, Chakraborty, Gambuti, von Manteuffel, Tancredi 2112.11097]
- $q\bar{q} \to gg$ [Caola, Chakraborty, Gambuti, von Manteuffel, Tancredi 2207.03503]

$gg \to \gamma\gamma$ @2loop in QCD [Bern, De Freitas, Dixon 0109078]

Master integrals for 3loop 4particle scattering [Henn, Mstlberger, Smirnov, Wasser 2002.09492]

2 years to move from MI to the amplitude
Selected examples... general comments

- **Numerical**
  - Flexible
  - The challenge is to make them fast and reliable

**Sector decomposition**
- $gg \to ZZ$ @N2LO [Agarwal, Jones, von Manteuffel 2011.15113]
- $gg \to ZH$ @N2LO [Chen, Heinrich et al. 2011.12325]

**Differential equations**
- $gg \to ZZ$ @N2LO [Brønnum-Hansen, Wang 2101.12095]
- $\gamma \to q\bar{q}$ @N3LO [Chen, Czakon, Niggetiedt 2109.01917]
- $gg \to H$ @N3LO [Czakon, Niggetiedt 2109.01917]
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  - $gg \to ZZ \text{ @N2LO} \ [Agarwal, Jones, von Manteuffel 2011.15113]$
  - $gg \to ZH \text{ @N2LO} \ [Chen, Heinrich et al. 2011.12325]$

- **Approximate**
  - Not universal
  - Non-trivial to find a good small parameter

  $gg \to HH \text{ @NLO}:$ expansion in the small transverse momentum of the Higgs boson \[Bociani, Degrassi, Giardino, Gröber 1806.11564\]

  High-energy limit together with Padé approximation \[Davies, Heinrich et al. 1907.06408\]

- **Differential equations**
  - $gg \to ZZ \text{ @N2LO} \ [Brønnum-Hansen, Wang 2101.12095]$
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\begin{align*}
\text{(a)} \quad \text{(b)} \quad \text{(c)}
\end{align*}
Selected examples... general comments

Real radiation, the problem:

**extract IR singularities without integrating** over the resolved phase space → obtain **fully differential predictions**

Extraction of real-emission singularities was the main bottleneck for NNLO predictions.

**Example**: di-jet two-loop amplitudes ~ 21 years ago [Anastasiou, Glover, Oleari, Tejeda-Yeomans ‘01],

di-jet production at NNLO ~ 5 ago [Currie, De Ridder, Gehrmann, Glover, Huss, Pires ‘17]

---

**Two-loop QCD corrections to massless identical quark scattering**

2001

C. Anastasiou\textsuperscript{a}, E. W. N. Glover\textsuperscript{a}, C. Oleari\textsuperscript{b} and M. E. Tejeda-Yeomans\textsuperscript{a}

We therefore expect that the problem of the analytic cancellation of the infrared divergences will soon be addressed thereby enabling the construction of numerical programs to provide next-to-next-to-leading order QCD estimates of jet production in hadron collisions.

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**Precise predictions for dijet production at the LHC**

J. Currie\textsuperscript{a}, A. Gehrmann-De Ridder\textsuperscript{b,c}, T. Gehrmann\textsuperscript{c}, E.W.N. Glover\textsuperscript{a}, A. Huss\textsuperscript{b}, J. Pires\textsuperscript{d}

\textsuperscript{a} Institute for Particle Physics Phenomenology, University of Durham, Durham DH1 3LE, UK

\textsuperscript{b} Institute for Theoretical Physics, ETH, CH-8093 Zürich, Switzerland

\textsuperscript{c} Department of Physics, Universität Zürich, Winterthurerstrasse 190, CH-8057 Zürich, Switzerland

\textsuperscript{d} Max-Planck-Institut für Physik, Föhringer Ring 6, D-80805 Munich, Germany
Selected examples... general comments

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**extract IR singularities without integrating** over the resolved phase space → obtain **fully differential predictions**

\[
\sigma(X) = \int_0^{\tau_{\text{cut}}} d\tau \frac{d\sigma(X)}{d\tau} + \int_{\tau_{\text{cut}}}^\tau d\tau \frac{d\sigma(X)}{d\tau}
\]

**Slicing**

- Intuitive core idea
- Non-local in the phase space
- Dependence on the slicing parameter

\[
\int |M|^2 F_J \, d\phi_d = \int (|M|^2 F_J - K) \, d\phi_d + K \, d\phi_d
\]

**Subtraction**

- Conceptually less intuitive
- Local in the phase space
- Numerically stable

Despite the number of different working slicing and subtraction schemes, only few of them can provide (selected) results at N3LO.

Newly developed methods are maturing…

Elegance can be probably sacrificed for the call for phenomenological results.

See Zeno Capatti’s talk!
**Selected examples... general comments**

Real radiation, the problem:

**extract IR singularities without integrating** over the resolved phase space → obtain **fully differential predictions**

\[
\int |\mathcal{M}|^2 F_{J} \, d\phi_{d} = \int (|\mathcal{M}|^2 F_{J} - K) \, d\phi_{4} + K \, d\phi_{d}
\]

**Subtraction**

**Differential N3LO accuracy:**

**Projection to Born**

- Jet production in DIS: P2B+antenna
  [Currie, Gehrmann et al. 180309973]
- Higgs decay to $b\bar{b}$: P2B+Njettiness
  [Mondini, Schiavi, Williams 1904.08960]
- Higgs production via ggF: P2B+antenna
  [Chen, Gehrmann et al. 2102.07607]
- VBF: P2B
  [Chen, Gehrmann et al. 1904.08960]

Also progresses with **Antenna subtraction**

[Jakubčík, Marcoli, Stagnitto 2211.08446]

See Giovanni Stagnitto’s talk!

**Common starting point, common problems:**

- Clear understanding of which **singular configurations** do actually contribute
- Get to the point where the **problem is well defined**
- Solve the **phase space integrals** of the relevant limits

See Gloria Bertolotti’s talk!

Chiara Signorile-Signorile

**Advances in fixed-order predictions**
Selected examples... general comments

Real radiation, the problem:

**extract IR singularities without integrating** over the resolved phase space → obtain **fully differential predictions**

\[
\sigma(X) = \int_0^{\tau_{\text{cut}}} d\tau \frac{d\sigma(X)}{d\tau} + \int_{\tau_{\text{cut}}}^{\infty} d\tau \frac{d\sigma(X)}{d\tau}
\]

- Sensitivity to the slicing parameter induces **large cancellations between IR and hard contributions**
  → **high CPU cost** to take numerics under control → abandoned at NLO
- Recently, increased computing power has made **slicing methods appealing again**, as they can be (in principle) straightforwardly generalised to N3LO

\[
\sigma(X) = \int_0^{\tau_{\text{cut}}} d\tau \frac{d\sigma^{\text{sing}}(X)}{d\tau} + \int_{\tau_{\text{cut}}}^{\infty} d\tau \frac{d\sigma(X)}{d\tau} + \Delta\sigma(X, \tau_{\text{cut}})
\]

\[
\Delta\sigma(X, \tau_{\text{cut}}) = \sum_{i>0} \int_0^{\tau_{\text{cut}}} d\tau \frac{d\sigma^i(X)}{d\tau}
\]

Described by **factorisation theorem** using EFT (SCET)

\[
\lim_{\tau \to 0} d\sigma \sim B_a \otimes B_b \otimes S \otimes J \otimes d\sigma^{\text{LO}}
\]

N-Jettiness at N3LO ongoing [Brüser, Liu, Stahlhofen 1804.09722] [Banerjee, Dhani, Ravindran 1805.02637] [Ebert, Mistlberger, Vita 2006.03055] [Baranowski, Behring et al. 2211.05722] [Chen 1902.10387] [Baranowski et al. 2111.13594]

Δσ(X, τcut) τcut→0 0

Recent progresses in obtaining control of Δ

[Moult et al. 1612.00450] [Boughezal, Liu, Petriello 1612.02911] [Buonocore et al. 1911.10166, 2111.13661] [Camarda, Cieri, Ferrera 2111.14509] [Chen et al. 2203.01565]

See Paolo Torrielli’s talk!
N3LO: just a brief overview


[2203.06730]
Inclusive results at N3LO for $2 \rightarrow 1$

- $pp \rightarrow \gamma^*$ [Duhr, Dulat, Mistlberger 2001.07717]
- $pp \rightarrow W \rightarrow \ell \nu_\ell$ [Duhr, Dulat, Mistlberger 2007.13313]
- $pp \rightarrow h + X$ $ggF$ [Anastasiou et al. 1503.06056][Chen et al. 2102.07607]

- Scale dependence is usually reduced considerably as the perturbative order is increased
- Nice convergence of the perturbative series $\rightarrow$ scale variation band at N3LO often contained within the NNLO band

<table>
<thead>
<tr>
<th>Process</th>
<th>$Q$ [GeV]</th>
<th>K-factor</th>
<th>$\delta$(scale) (%)</th>
<th>$\delta$(PDF + $\alpha_S$)</th>
<th>$\delta$(PDF-TH)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$gg \rightarrow$ Higgs</td>
<td>$m_H$</td>
<td>1.04</td>
<td>$+0.21%$</td>
<td>$+3.2%$</td>
<td>$+1.2%$</td>
</tr>
<tr>
<td>$bb \rightarrow$ Higgs</td>
<td>$m_H$</td>
<td>0.978</td>
<td>$-0.8%$</td>
<td>$+8.4%$</td>
<td>$+2.5%$</td>
</tr>
<tr>
<td>NCDY</td>
<td>30</td>
<td>0.952</td>
<td>$+1.5%$</td>
<td>$-2.8%$</td>
<td>$+2.8%$</td>
</tr>
<tr>
<td></td>
<td>100</td>
<td>0.979</td>
<td>$-0.6%$</td>
<td>$-3.8%$</td>
<td>$+2.5%$</td>
</tr>
<tr>
<td>CCYD(W$^+$)</td>
<td>30</td>
<td>0.953</td>
<td>$+2.3%$</td>
<td>$+3.95%$</td>
<td>$+3.2%$</td>
</tr>
<tr>
<td></td>
<td>150</td>
<td>0.985</td>
<td>$-0.9%$</td>
<td>$+1.9%$</td>
<td>$+2.1%$</td>
</tr>
<tr>
<td>CCYD(W$^-$)</td>
<td>30</td>
<td>0.950</td>
<td>$+1.6%$</td>
<td>$+3.7%$</td>
<td>$+3.2%$</td>
</tr>
<tr>
<td></td>
<td>150</td>
<td>0.984</td>
<td>$+0.6%$</td>
<td>$+2%$</td>
<td>$+2.13%$</td>
</tr>
</tbody>
</table>

Chiara Signorile-Signorile

Advances in fixed-order predictions

[1802.00827]
Progresses at N3LO: take home message

❖ The calculation of fiducial cross section for different process at N3LO brings to some conclusions:

Development of theoretical methods that can be used to describe hard scattering has accelerated in recent years.

Drell-Yan

\[ \sigma_{\text{n}3\text{lo}}^{\text{QCD}} \]

\[ \sigma_{\text{nlo}}^{\text{QCD}} \] [Altarelli, Ellis, Martinelli ’79]

\[ \frac{d\sigma_{\text{QCD}}^{\text{nnlo}}}{d^3p} \text{ (fully diffr. + leptonic decay)} \]

[Anastasiou, Dixon, Melnikov, Petriello ’03, ’04]

[Melnikov, Petriello ’06]

[Catani, Cieri, Ferrera, de Florian, Grazzini ’09]

[Catani, Ferrera, Grazzini ’10]

\[ \frac{d\sigma_{\text{QCD} \times \text{EW}}^{\text{nnlo}}}{d^3p} \]

[Bonciani et al.’21][Buccioni et al.’22]

Complete EW corr. to Z/W prod.

\[ \sigma_{\text{n}3\text{lo}}^{\text{QCD}} \]

[Hamberg, van Neerven, Matsuura ’91]

[Harlander, Kilgore ’02]

\[ \sigma_{\text{n}3\text{lo}}^{\text{QCD}} \]

[Arbuzov et al. ’06][Arbuzov et al. ’08][Dittmaier, Huber ’09]

[Dittmaier, Krämer ’02][Bauer, Wackerste ’04]

[Zykunov ’06][Bauer et al. ’02][Zykunov ’07][Carloni et al. ’07]

[Carloni et al. ’07]

[Duhr, Dulat, Mistlberger ’20]

\[ \sigma_{\text{n}3\text{lo}}^{\text{QCD}} \]

[Camarda, Cieri, Ferrera ’21]

[Chen, Gehrmann, Glover, Huss, Monni, Re, Rottoli, Torrielli ’22]
Progresses at N3LO: take home message

- The calculation of fiducial cross section for different process at N3LO brings to some conclusions:
  - **Development** of theoretical methods that can be used to describe hard scattering **has accelerated in recent years**
  - Beyond being **interesting per se**, these calculations offer the possibility to **better understand physics**

Use **transverse momentum spectrum of the Z boson** to obtain the **strong coupling constant**

[Camarda, Ferrera, Schott 2203.05394]

\[ \alpha_s(m_Z) = 0.1185^{+0.0014}_{-0.0015} \]
The calculation of fiducial cross section for different processes at N3LO brings to some conclusions:

- **Development** of theoretical methods that can be used to describe hard scattering **has accelerated in recent years**
- Beyond being **interesting per se**, these calculations offer the possibility to **better understand physics**
- Lack of **PDFs at N3LO** becomes crucial if percent accuracy is desired. Moreover, PDFs evolution with $\mu_F$ is governed by DGLAP equation. **DGLAP anomalous dimensions needed at 4loop order.** This is an ongoing effort.

#### Approximate N3LO Parton Distribution Functions with Theoretical Uncertainties:

MSHT20aN3LO PDFs

J. McGowen*, T. Cridge†, L. A. Harland-Lang‡, and R.S. Thorne*  
* Department of Physics and Astronomy, University College London, London, WC1E 6BT, UK  
† Rudolf Peierls Centre, Beecroft Building, Parks Road, Oxford, OX1 3PU

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**Gluon Fusion: $gg \rightarrow H (\mu=m_H)$**

- Includes approximations and data-driven fits to parts of N3LO currently known
- 7.6% decrease in Higgs cross section (wrt $\sigma^{n3lo} +$N2LO PDFs)
- PDF part of uncertainty increases

<table>
<thead>
<tr>
<th>$\sigma$ order</th>
<th>PDF order</th>
<th>$\sigma + \Delta\sigma_+ - \Delta\sigma_-$ (pb)</th>
<th>$\sigma$ (pb) + $\Delta\sigma_+ - \Delta\sigma_-$ (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>N3LO</td>
<td>aN3LO (no theory unc.)</td>
<td>45.296 + 0.723 − 0.545</td>
<td>45.296 + 1.60% − 1.22%</td>
</tr>
<tr>
<td></td>
<td>aN3LO ($H_{tq} + K_{tq}$)</td>
<td>45.296 + 0.832 − 0.755</td>
<td>45.296 + 1.84% − 1.67%</td>
</tr>
<tr>
<td></td>
<td>aN3LO ($H_{tq}$)</td>
<td>45.296 + 0.821 − 0.761</td>
<td>45.296 + 1.81% − 1.68%</td>
</tr>
<tr>
<td></td>
<td>NNLO (NNLO)</td>
<td>47.817 + 0.558 − 0.581</td>
<td>47.817 + 1.17% − 1.22%</td>
</tr>
<tr>
<td>NNLO</td>
<td>NNLO</td>
<td>46.206 + 0.541 − 0.564</td>
<td>46.206 + 1.17% − 1.22%</td>
</tr>
</tbody>
</table>
The calculation of fiducial cross section for different process at N3LO brings to some conclusions:

- Development of theoretical methods that can be used to describe hard scattering has accelerated in recent years.

- Beyond being interesting per se, these calculations offer the possibility to better understand physics.

- Lack of PDFs at N3LO becomes crucial if percent accuracy is desired. Moreover, PDFs evolution with $\mu_F$ is governed by DGLAP equation. **DGLAP anomalous dimensions needed at 4-loop order.** This is an ongoing effort.

- Achieving percent-level predictions at N3LO reveals the importance of other effects that were neglected in the past. Classical examples are the mixed QCD-electroweak corrections ($\alpha_s^3 \sim \alpha_{ew} \alpha_s$) and the non-vanishing quark mass contributions.

### Exact top-quark mass dependence in hadronic Higgs production: full NNLO inclusive calculation completed (complicated 3-loop diagrams) [Czakon, Harlander, Klappert, Nieggetiedt 2105.04436]

Large cancellation between different channels gives an overall effect of $-0.26\%$ @ 13 TeV.

<table>
<thead>
<tr>
<th>Channel</th>
<th>$\sigma_{\text{NNLO}}^{\text{HEFT}}$ [pb]</th>
<th>$\sigma_{\text{exact}}^{\text{NNLO}} - \sigma_{\text{NNLO}}^{\text{HEFT}}$ [pb]</th>
<th>$\sigma_{\text{exact}}^{\text{NNLO}} / \sigma_{\text{HEFT}}^{\text{NNLO}} - 1$ [%]</th>
</tr>
</thead>
<tbody>
<tr>
<td>$gg$</td>
<td>7.39 + 8.58 + 3.88</td>
<td>+0.0353</td>
<td>+0.0879 ± 0.0005</td>
</tr>
<tr>
<td>$qg$</td>
<td>0.55 + 0.26</td>
<td>-0.1397</td>
<td>-0.0021 ± 0.0005</td>
</tr>
<tr>
<td>$qq$</td>
<td>0.01 + 0.04</td>
<td>+0.0171</td>
<td>-0.0191 ± 0.0002</td>
</tr>
<tr>
<td>Total</td>
<td>7.39 + 9.15 + 4.18</td>
<td>-0.0873</td>
<td>+0.0667 ± 0.0007</td>
</tr>
</tbody>
</table>

$\sqrt{s} = 8$ TeV

<table>
<thead>
<tr>
<th>Channel</th>
<th>$\sigma_{\text{NNLO}}^{\text{HEFT}}$ [pb]</th>
<th>$\sigma_{\text{exact}}^{\text{NNLO}} - \sigma_{\text{NNLO}}^{\text{HEFT}}$ [pb]</th>
<th>$\sigma_{\text{exact}}^{\text{NNLO}} / \sigma_{\text{HEFT}}^{\text{NNLO}} - 1$ [%]</th>
</tr>
</thead>
<tbody>
<tr>
<td>$gg$</td>
<td>16.30 + 19.64 + 8.76</td>
<td>+0.0345</td>
<td>+0.2431 ± 0.0020</td>
</tr>
<tr>
<td>$qg$</td>
<td>1.49 + 0.84</td>
<td>-0.3696</td>
<td>-0.0115 ± 0.0010</td>
</tr>
<tr>
<td>$qq$</td>
<td>0.02 + 0.10</td>
<td>-0.0322</td>
<td>-0.0501 ± 0.0006</td>
</tr>
<tr>
<td>Total</td>
<td>16.30 + 21.15 + 9.79</td>
<td>-0.3029</td>
<td>+0.1815 ± 0.0023</td>
</tr>
</tbody>
</table>
The calculation of fiducial cross section for different process at N3LO brings to some conclusions:

➡ Development of theoretical methods that can be used to describe hard scattering has accelerated in recent years.

➡ Beyond being interesting *per se*, these calculations offer the possibility to better understand physics.

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➡ Achieving percent-level predictions at N3LO reveals the importance of other effects that were neglected in the past. Classical examples are the mixed QCD-electroweak corrections ($\alpha_s^3 \sim \alpha_s \alpha_{\text{ew}}$) and the non-vanishing quark mass contributions.

➡ Non-perturbative effects may also play a crucial role.

\[ d\sigma = d\sigma_{\text{LO}} + \left( \frac{\alpha_s}{\pi} \right) d\sigma_{\text{NLO}} + \left( \frac{\alpha_s}{\pi} \right)^2 d\sigma_{\text{NNLO}} + \left( \frac{\alpha_s}{\pi} \right)^3 d\sigma_{\text{N3LO}} + \ldots \]

\[ + \left( \frac{\Lambda_{\text{QCD}}}{Q} \right) d\sigma_{\text{linear}} + \ldots \]

with $\Lambda_{\text{QCD}} \sim 300$ MeV, 
$Q \sim 30 - 100$ GeV.
The calculation of fiducial cross section for different processes at N3LO brings to some conclusions:

- **Development** of theoretical methods that can be used to describe hard scattering has accelerated in recent years.

- Beyond being interesting per se, these calculations offer the possibility to better understand physics.

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- Achieving percent-level predictions at N3LO reveals the importance of other effects that were neglected in the past. Classical examples are the mixed QCD-electroweak corrections ($\alpha_s^3 \sim \alpha_{ew} \alpha_s$) and the non-vanishing quark mass contributions.

- Non-perturbative effects may also play a crucial role.

- It is also crucial to create frameworks that make predictions easily accessible (both experimentalist and theorists would benefit from this).
Conclusions

- Fixed-order calculations are one of the pillars of the precision era of the LHC.

- Two main ingredients are required: multi-loop amplitudes and efficient methods to treat real radiation.

- Many advances have been made in both directions

  - Slicing and subtraction schemes have been recently applied to N3LO calculations for simple processes, efficiency remains a problem

  - Many developments in analytic and numerical methods, and in various approximations to describe scattering amplitudes

- Recent progress makes clear that the precision revolution is possible, but poses several conceptual problems that we could, so far, ignore
Conclusions

The path is tough,
The path is tough,

But the view is amazing

Conclusions
Going differential at N3LO

• Charged current production \([Chen, Gehrmann, Glover, Huss, Yang, Zhu '22]\)

\[
\text{SCET+NLOJET} \quad pp \rightarrow W^+ (\rightarrow e^+ \nu) + X \quad \sqrt{s} = 13 \text{ TeV}
\]

• H boson production \([Cieri, Chen, Gehrmann, Glover, Huss '19]\)

\[
\text{SCET+NLOJET} \quad pp \rightarrow W^- (\rightarrow e^- \bar{\nu}) + X \quad \sqrt{s} = 13 \text{ TeV}
\]

• H boson production \([Dulat, Mistlberger, Pelloni '18]\)
Selected examples... general comments

Real radiation, the problem:

**extract IR singularities without integrating** over the resolved phase space → obtain **fully differential predictions**

\[
\sigma(X) = \int_0^{\tau_{\text{cut}}} d\tau \frac{d\sigma(X)}{d\tau} + \int_{\tau_{\text{cut}}}^{\tau_{\text{cut}}} d\tau \frac{d\sigma(X)}{d\tau}
\]

\[
\lim_{\tau \to 0} d\sigma \sim B_a \otimes B_b \otimes S \otimes J \otimes d\sigma^{\text{LO}}
\]

Moder slicing parameters:

- **Transverse momentum** of a colourless or massive coloured system, \( q_\perp \) \[\text{Catani, Grazzini 0703012}\]
- **Jettiness** \[Boughezal, Petriello et al.\] \[Gaunt, Stahlhofen, Tackmann, Walsh 1505.04794\]

\[
\tau_N = \sum_k \min_i \left\{\frac{2}{Q_i} p_i \cdot q_k\right\}
\]

\( q_\perp \) subtraction successfully applied to color singlet production

\( N \)-jettiness can describe coloured final states, many ingredients for N3LO predictions are known

- Quark and gluon jet function @3loop \[Brüser, Liu, Stahlhofen 1804.09722\] \[Banerjee, Dhani, Ravindran 1805.02637\]
- Beam function @3loop \[Ebert, Mistlberger, Vita 2006.03055\] \[Baranowski, Behring et al. 2211.05722\]
- Soft function @3loop still missing, but progresses ongoing… \[Chen 1902.10387\] \[Baranowski et al. 2111.13594\]
Ingredients for higher-order calculations

Under IR singular limits, the RR and RV factorise into: \((\text{universal kernel}) \times \text{(lower multiplicity matrix elements)}\)

**Double soft limit** [Catani, Grazzini 9903516,9810389]

\[
\lim_{k_i, k_j \to 0} \, RR_{n+2}\left(\{k\}_n, k_i, k_j\right) \sim \text{Eik}\left(\{k\}_n, k_i, k_j\right) \otimes B_n\left(\{k\}_n\right)
\]

**Triple collinear limit** [Catani, Grazzini 9903516,9810389]

\[
\lim_{k_i, k_j, k_k \to 0} \, RR_{n+2}\left(\{k\}_{n-1}, k_i, k_j, k_k\right) \sim \frac{1}{s_{ikj}^2} \, P\left(k_i, k_j, k_k\right) \otimes B_n\left(\{k\}_{n-1}, k_{ijk}\right)
\]

**One loop single soft limit** [Catani, Grazzini 0007142]

\[
\lim_{k_i \to 0} \, RV_{n+1}\left(\{k\}_n, k_i\right) \sim \text{Eik}\left(\{k\}_n, k_i\right) \otimes V_n\left(\{k\}_n\right) + \text{Eik}\left(\{k\}_n, k_i\right) \otimes B_n\left(\{k\}_n\right)
\]

**One loop single collinear limit** [Kosower 9901201, Bern, Del Duca, Kilgore, Schmidt 9903516]

\[
\lim_{k_i \parallel k_j \parallel k_k \to 0} \, RV_{n+1}\left(\{k\}_n, k_i\right) \sim \frac{1}{s_{ij}} \left[ P\left(k_i, k_j\right) \otimes V_n\left(\{k\}_n\right) + \overline{P}\left(k_i, k_j\right) \otimes B_n\left(\{k\}_n\right) \right]
\]
Virtual corrections

Loops amplitudes: rapid growth in the complexity as the loop order increases and as more massive particle are considered. Ultimately it is necessary to find representation of the amplitudes that can be evaluated efficiently and yield numerically reliable results.

Three families of solutions to the problem:

- Analytic
  - Fast, precise evaluation (e.g. $L_i$, $p F_q$, $M PL$, …)
  - Wider applications (e.g. changing parameters)

Some examples….
- $2 \rightarrow 2$, 2loop mixed QCD-EW corrections to Drell-Yan [Heller, von Manteuffel, Schabinger, Spiesberger ’21]
- $2 \rightarrow 2$, 2loop mixed QCD-EW corrections to Higgs+j production [Bonetti, Panzer, Tancredi ’22]
- $2 \rightarrow 3$, 2loop QCD corrections to $\gamma\gamma + j$ production [Agarwal, Buccioni, von Manteuffel, Tancredi ’21]
- $2 \rightarrow 1$, 3loop mixed QCD-EW corrections (light quarks) to Higgs production in gluon fusion [Bonetti, Melnikov, Tancredi ’18]

3loop amplitude milestone
- $1 \rightarrow 1$, QCD [Tarasov et al. PRLB 1980]
- $2 \rightarrow 1$, QCD [Moch et al. 0508055]
- $2 \rightarrow 2$, SYM [Henn, Mistberger 1608.00850]
Power corrections in qT subtraction

\[ \Delta \sigma^{N^k \text{LO}} = \sigma^{N^k \text{LO}} - \sigma^{N^{k-1} \text{LO}} \]

- Excellent fixed order prediction from NNLOJET \([\text{Gehrmann et al., 1507.02850}]\), down to \(p_T^{\text{cut}} < 1 \text{ GeV}\).
- Plateau at small \(p_T^{\text{cut}}\) reached only by including recoil effects at fixed order.
- Slicing error estimated varying \(p_T^{\text{cut}} \in [0.45, 1.5] \text{ GeV}\).

\[ \text{ATLAS fiducial cuts (symmetric): } p_T^{\pm} > 27 \text{ GeV}, \ |\eta_{\ell^{\pm}}| < 2.5. \]

See Paolo Torrielli’s talk!

\[ \Delta \sigma^{N^k \text{LO}} = \sigma^{N^k \text{LO}} - \sigma^{N^{k-1} \text{LO}} \]

- Excellent fixed order prediction from NNLOJET \([\text{Gehrmann et al., 1507.02850}]\), down to \(p_T^{\text{cut}} < 1 \text{ GeV}\).
- Product cuts insensitive to linear power corrections: no visible effect of transverse recoil at fixed order.

\[ \text{Product cuts } [\text{Salam, Slade, 2106.08329}]: \]

\[ |\eta_{\ell^{\pm}}| < 2.5. \]

\[ \sqrt{p_T^{\ell^{+}}p_T^{\ell^{-}}} > 27 \text{ GeV}, \ \min(p_T^{\ell^{+}}, p_T^{\ell^{-}}) > 20 \text{ GeV}, \]
Backup on t-channel single-top corrections
Single-top production: different modes

- At LHC top quarks are mainly produced in pairs via strong interactions

\[ q\bar{q} \rightarrow t\bar{t}, \ g\bar{g} \rightarrow t\bar{t} \]

- Large production rate

- Advanced theoretical predictions:

  NLO QCD
  \cite{NasonDawsonEllis88,Beenakkeretal89,Denneretal11,Bevilaquaetal11,Casciolietal14,DennerPellen18}

  and NLO EW corrections
  \cite{Beenakkeretal94,BernreutherFueckerSi06,Kuhnscharfuwer06,HollikKollar08,DennerPellen16}

  total and fully differential NNLO QCD corrections
  \cite{BarreutherCzakonMitov12,CzakonFiedlerMitov13,Czakonetal16,CzakonHeymesMitovPaganiTsinikosZaro17,Catanietal19,BehringCzakonMitovetal19,CzakonMitovPoncelet21,GaoPapanastasiou17,CzakonMitovPoncelet12}

  matching with parton-shower
  \cite{FrixioneNasonWebber03,FrixioneNasonOleari07,Campbelletal15,Mazzitellietal21}

  and soft-gluon resummation
  \cite{FrixioneNasonWebber03,Kidonakis10,Benekeetal12,Kidonakis17}

\textbf{Not comprehensive list of results!}
Single-top production: LHC status

- Measurements compared to theoretical calculations based on: NLO QCD, NLO QCD complemented with NNLL resummation and NNLO QCD (t-channel only)

- **s-channel**: very small and affected by large background

- Theoretical uncertainties smaller than experimental uncertainties for **s-channel** and **t-channel** but comparable for tW.

- **t-channel**: large exp. uncertainties at 13TeV if compared with 8TeV → improvements are expected from HL-LHC.
Single-top production: different modes

- **Single-top production** also relevant

  - Production rate of the same order of magnitude as $t\bar{t}$: $\sigma_i \sim 1/4 \sigma_{t\bar{t}}$

  \[
  W^* \rightarrow q\bar{q} \rightarrow t\bar{b}
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  - Electroweak mediated

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  t-channel: \quad q_b \rightarrow q't
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  s-channel: \quad q\bar{q} \rightarrow W^* \rightarrow t\bar{b}
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Single-top production: theory status, s-channel and $tW$

- **s-channel:**
  - NNLO QCD corrections in production and decay [Liu, Gao ’18]
  - Inclusive corrections are $\mathcal{O}(5\%)$ wrt NLO

<table>
<thead>
<tr>
<th></th>
<th>inclusive</th>
<th>LO</th>
<th>NLO</th>
<th>NNLO</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\sigma(t)$ [pb]</td>
<td>4.775$^{+2.69%}_{-3.50%}$</td>
<td>6.447$^{+1.30%}_{-0.91%}$</td>
<td>6.778$^{+0.76%}_{-0.53%}$</td>
<td></td>
</tr>
<tr>
<td>$\sigma(t)$ [pb]</td>
<td>2.998$^{+2.69%}_{-3.55%}$</td>
<td>4.043$^{+1.33%}_{-0.94%}$</td>
<td>4.249$^{+0.69%}_{-0.48%}$</td>
<td></td>
</tr>
<tr>
<td>$\sigma(t+t)$ [pb]</td>
<td>7.722$^{+2.69%}_{-3.52%}$</td>
<td>10.49$^{+1.36%}_{-0.92%}$</td>
<td>11.03$^{+0.74%}_{-0.51%}$</td>
<td></td>
</tr>
<tr>
<td>$\sigma(t)/\sigma(t)$</td>
<td>1.593$^{+0.05%}_{-0.01%}$</td>
<td>1.598$^{+0.06%}_{-0.03%}$</td>
<td>1.595$^{+0.07%}_{-0.05%}$</td>
<td></td>
</tr>
</tbody>
</table>

- In low $p_{\perp, t}$ region, NNLO corrections can reach $\mathcal{O}(10\%)$ wrt LO.
- No overlap between NLO and NNLO bands in most region: NNLO corrections underestimated by scale variation at NLO.

- **$tW$-channel:**
  - NNLO QCD corrections not known yet (analytic 2-loop amplitudes, leading colour [Chen, Dong, Li, Li, Wang ’22]).
  - Approximate approaches used to infer higher-order corrections [Kidonakis, Yamanaka ’21]
  - Soft-gluon corrections to approximate N3LO for stable top
Single-top production: theory status, $t$-channel (I)

- Two main topologies contribute to the $t$-channel, single-top production:
  - Factorisable contributions

  **NLO QCD** [Bordes, van Eijk ’95] [Campbell, Ellis, Tramontano ’04] [Cao, Yuan ’05] [Cao, Schwienhorst, Benitez, Brock, Yuan ’05] [Harris, Laenen, Phaf, Sullivan, Weinzierl ’02] [Schwienhorst, Yuan, Mueller, Cao ’11]

  **NNLO QCD**

  - First calculated for a stable top-quark using nested soft-collinear subtraction [Brucherseifer, Caola, Melnikov ’14]

  - Structure function approximation → crosstalk neglected invoking colour suppression

  - Small effects on inclusive cross-section and on cross section with $p_T^t$ cuts

  \[
  \begin{align*}
  \text{MSTW}2008, \text{lo, nlo, nnlo PDF, } & \mu_R = \mu_F = m_t = 173.2 \text{ GeV, } \sqrt{s} = 8 \text{ TeV.} \\
  \hline
  p_T & \sigma_{\text{LO}, \text{ pb}} & \sigma_{\text{NLO}, \text{ pb}} & \delta_{\text{NLO}} & \sigma_{\text{NNLO}, \text{ pb}} & \delta_{\text{NNLO}} \\
  \hline
  0 \text{ GeV} & 53.8^{+3.0}_{-4.3} & 55.1^{+1.6}_{-0.9} & +2.4\% & 54.2^{+0.5}_{-0.2} & -1.6\% \\
  20 \text{ GeV} & 46.6^{+1.2}_{-0.5} & 48.9^{+1.2}_{-0.5} & +4.9\% & 48.3^{+0.3}_{-0.02} & -1.2\% \\
  40 \text{ GeV} & 33.4^{+1.7}_{-2.5} & 36.5^{+0.6}_{-0.03} & +9.3\% & 36.5^{+0.1}_{-0.1} & -0.1\% \\
  60 \text{ GeV} & 22.0^{+1.0}_{-1.5} & 25.0^{+0.2}_{-0.3} & +13.6\% & 25.4^{+0.1}_{-0.2} & +1.6\% \\
  \end{align*}
  \]
Single-top production: theory status, $t$-channel (II)

NNLO QCD
- Extension to top-quark decay in the NW approximation, including also NNLO in decay (computed using SCET/jettiness + projection to Born) [Berger, Gao, Yuan, Zhu '16, '17]
- Large corrections for some distributions

- Disagreement with earlier calculation of inclusive cross-section: $\mathcal{O}(1)$ difference in NNLO coefficient
- Independent calculation based on SCET approach [Campbell, Neumann, Sullivan '21]

CT14, lo, nlo, nnlo PDF, $\mu_R = \mu_F = m_t = 172.5$ GeV.
@14TeV: $\Delta \sigma^{NNLO} \sim -0.7\% \sigma^{NLO}$

<table>
<thead>
<tr>
<th>7 TeV pp</th>
<th>14 TeV pp</th>
</tr>
</thead>
<tbody>
<tr>
<td>top</td>
<td>anti-top</td>
</tr>
<tr>
<td>$\sigma_{LO}^{\mu=m_t}$</td>
<td>37.1$^{+7.1}_{-9.5}$</td>
</tr>
<tr>
<td>$\sigma_{DIS}^{LO}$</td>
<td>39.5$^{+6.4}_{-8.6}$</td>
</tr>
<tr>
<td>$\sigma_{NLO}^{\mu=m_t}$</td>
<td>41.4$^{+3.0}_{-2.0}$</td>
</tr>
<tr>
<td>$\sigma_{DIS}^{NLO}$</td>
<td>41.8$^{+3.3}_{-2.0}$</td>
</tr>
<tr>
<td>PDF</td>
<td>$+1.7_{-1.4}$</td>
</tr>
</tbody>
</table>

Chiara Signorile-Signorile

Advances in fixed-order predictions
Non-factorisable corrections: why?

Non-factorisable contributions vanish at NLO due to their colour structure, and are suppressed by a factor $N_c^2 - 1 = 8$ at NNLO.

Factorisable contributions

$$\propto \text{Tr}(t^a t^a) = \frac{N_c^2 - 1}{2}$$

$$\propto \text{Tr}(t^a) = 0$$

Non-factorisable contributions

$$\propto \text{Tr}(t^a t^a) \text{Tr}(t^b t^b) = \frac{(N_c^2 - 1)^2}{4}$$

$$\propto \text{Tr}(t^a t^b) \text{Tr}(t^a t^b) = \frac{N_c^2 - 1}{4}$$
Non-factorisable corrections: why?

**Non-factorisable contributions** vanish at NLO due to their colour structure, and are suppressed by a factor $N_c^2 - 1 = 8$ at NNLO.

However:

- Non-factorisable corrections could be enhanced by a factor $\pi^2 \sim 10$ due to the Glauber phase
  
  → proven for Higgs production in weak boson fusion in the eikonal approximation [Liu, Melnikov et al. '19]

- The actual size of NNLO non-factorisable corrections **cannot be inferred from NLO contributions**, since they vanish

- Recent calculation of **double-virtual contributions** indicate a comparable size of non-factorisable and factorisable corrections [Brønnum-Hansen, Melnikov, Quarroz, Wang '21]

\[
\frac{\sigma_{pp\to dt}^{ub}}{1 \text{ pb}} = 90.3 + 0.3 \left( \frac{\alpha_s(\mu_{nf})}{0.108} \right)^2
\]

Even though **non-factorisable contributions** are suppressed by colour it is not guaranteed that they are actually negligible.

- Thanks to recent progress [Brønnum-Hansen et al. '21] tackling non-factorisable corrections is actually feasible:
  
  → 2-loop virtual amplitudes computed analytically with full dependence on $m_t$
  
  → integrals computed numerically with sufficient precision to be exploited in phenomenological studies.
Results

Differential cross section:

pp collision: $\sqrt{s} = 13\,\text{TeV}$, PDFs: CT14_lo@LO, CT14_nnlo@NNLO, $m_W = 80.379\,\text{GeV}$, $m_t = 173.0\,\text{GeV}$, $\alpha_s(m_t) = 0.108$, $\mu_F = \mu_R = \mu$

1. Non-factorisable corrections are $p_T^\perp$-dependent.

2. Non-factorisable corrections are small and negative at low values of $p_T^\perp$. They vanish at $p_T^\perp \sim 50\,\text{GeV}$ [in agreement with results for virtual corrections]

3. Factorisable corrections vanish around $p_T^\perp \sim 30\,\text{GeV}$.

4. Factorisable and non-factorisable corrections are comparable in the region around the maximum of the $p_T^\perp$ distribution.

[Brønnum-Hansen, Melnikov, Quarroz, Wang ‘21]
Results

Differential cross section:

\( \text{pp collision: } \sqrt{s} = 13 \text{ TeV, PDFs: CT14\_lo@LO, CT14\_nnlo@NNLO, } m_W = 80.379 \text{ GeV, } m_t = 173.0 \text{ GeV, } \alpha_s(m_t) = 0.108, \mu_F = \mu_R = \mu \)

1. Relative non-factorisable correction to top-quark rapidity fairly flat for \( |y_t| < 2.5, \mathcal{O}(0.25\%) \).

2. Sign change around \( |y_t| \sim 3 \)

3. Factorisable corrections change sign around \( |y_t| \sim 1.2 \)

4. For some top-quark rapidity values, factorisable and non-factorisable correction become quite comparable.

5. \( k_t\text{-algorithm} \) to define jets \( p_{\perp}^{\text{jet}} > 30 \text{ GeV, } R=0.4 \).

6. Non-factorisable corrections reach 1.2% at \( p_{\perp}^{\text{jet}} \sim 140 \text{ GeV} \).
Results

Differential cross section:

\[
s = 100 \text{TeV}, \quad \text{PDFs: CT14}_\text{lo@LO, CT14}_\text{nnlo@NNLO}, \quad m_W = 80.379 \text{GeV}, \quad m_t = 173.0 \text{GeV}, \quad \alpha_s(m_t) = 0.108, \quad \mu_F = m_t.
\]

\[
\frac{\sigma_{pp \rightarrow X + t}}{1 \text{ pb}} = 2367.0 + 3.8 \left( \frac{\alpha_s(\mu_R)}{0.108} \right)^2
\]

1. Non-factorisable corrections are 0.16% LO for \( \mu_R = m_t \).

2. For \( \mu_R = 40 \text{GeV} \) non-factorisable corrections are 0.25% LO.

3. \( p_{\perp}^t \) peaked around 40GeV, changes sign around 70GeV.

4. Non-factorisable corrections increase by 0.04 − 0.07% by increasing \( p_{\perp}^{t,\text{cut}} \).

<table>
<thead>
<tr>
<th>( p_{\perp}^{t,\text{cut}} )</th>
<th>( \sigma_{\text{LO}} ) (pb)</th>
<th>( \alpha_{\text{NNLO}}^{\text{anf}} ) (pb)</th>
<th>( \delta_{\text{NNLO}} ) [%]</th>
<th>( \sigma_{\text{NNLO}}^{\text{anf}} ) (pb)</th>
<th>( \delta_{\text{NNLO}} ) [%]</th>
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</thead>
<tbody>
<tr>
<td>0 GeV</td>
<td>2367.02</td>
<td>3.79−0.63 _0.84</td>
<td>0.16−0.03 0.04</td>
<td>5.95</td>
<td>0.25</td>
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<tr>
<td>20 GeV</td>
<td>2317.03</td>
<td>3.89−0.64 _0.86</td>
<td>0.17−0.03 0.04</td>
<td>6.11</td>
<td>0.26</td>
</tr>
<tr>
<td>40 GeV</td>
<td>2216.61</td>
<td>4.14−0.69 _0.92</td>
<td>0.19−0.03 0.04</td>
<td>6.50</td>
<td>0.29</td>
</tr>
<tr>
<td>60 GeV</td>
<td>2121.88</td>
<td>4.28−0.71 _0.95</td>
<td>0.20−0.03 0.04</td>
<td>6.71</td>
<td>0.32</td>
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</tbody>
</table>

Chiara Signorile-Signorile

Advances in fixed-order predictions
IR singularities

Higher-order corrections are affected by **infrared singularities arising from unresolved radiation.**

- **Virtual corrections:**
  - Explicit IR singularities from loop integrations $\rightarrow$ poles in $1/\epsilon$

- **Real corrections:**
  - Singularities after integration over full phase space of radiated parton
    
    $$\int \frac{d^{d-1}k}{(2\pi)^{d-1}2E_k} |M(\{p\}, k)|^2 \sim \int \frac{dE_k}{E_k^{1+2\epsilon}} \frac{d\theta}{\theta^{1+2\epsilon}} \times |M(\{p\})|^2 \sim \frac{1}{4\epsilon^2}.$$

- **Integrating implies losing kinematic information** (needed for distributions, kinematic cuts, …)
- For **non-factorisable** corrections only **soft** limits are relevant $\rightarrow$ only $1/\epsilon$ poles

**Subtraction scheme:** extract singularities without integrating over full phase space of radiated partons

$$\int d\Phi_g = \int \left[ \quad \right] d\Phi_g + \int \quad d\Phi_g$$

Finite in $d=4$, integrable numerically

exposes the same $1/\epsilon$ poles as the virtual correction
Amplitude evaluation

Diagrams generated with QGRAPH and processed with FORM.

$W$ boson forces light quark to be left-handed and we decompose the massive momentum into 2 massless momenta

$$p_4 = p_4^b + \frac{m_t^2}{2n \cdot p_4} n$$

$$\bar{u}_L(p_4) = \langle 4^b | + \frac{m_t}{[n4^b]} | n \rangle, \quad \bar{u}_R(p_4) = [4^b | + \frac{m_t}{\langle n4^b \rangle} \langle n \rangle$$

RV: one-loop five-point amplitude

- 24 diagrams: 8 pentagons and 16 boxes
- 7 kinematic scales

VV: two-loop four-point amplitude [Brønnum-Hansen, Melnikov, Quarroz, Wang '21]

- 18 diagrams: all topologies maximal
- 4 kinematic scales: $s, t, m_t^2, m_W^2$
- 428 master integrals evaluated numerically using the auxiliary mass flow method to 20 digits in $\sim 30$ min on a single core
- 10 sets of $10^4$ points extracted from a grid prepared on the Born squared amplitude
Double-virtual contribution

The pole structure of the two-loop amplitude is well studied, and can be easily cross-checked against literature

\(\text{[Catani '98][Aybat, Dixon, Sterman '06][Becher, Neubert '09][Czakon, Mitov, Sterman '09][Mitov, Sterman, Sung '09, '10][Ferroglia, Neubert, Pecjak, Yang '09]}\)

\[\mathcal{A}_{\gamma} = \mathcal{A}_{\alpha} \Gamma, \quad \frac{d}{d\mu} Z = - \Gamma Z\]

\[\Gamma(p_i, m_i, \mu) = \sum_{(i,j)} \frac{T_i \cdot T_j}{2} \gamma_{\text{cusp}}(\alpha_i) \log \left( \frac{\mu^2}{-s_{ij}} \right) + \sum_{(i,j)} T_i \cdot T_j \gamma_{\text{cusp}}(\alpha_i) \log \left( \frac{m_i \mu}{-s_{ij}} \right)\]

\[+ \sum_{(i,j)} \frac{T_i \cdot T_j}{2} \gamma_{\text{cusp}}(v_{ij}, \alpha_i) + \sum_i \gamma^i(\alpha_i) + \sum_l \gamma^l(\alpha_l)\]

\[+ \sum_{(i,j,k)} i^{abc} T_i^a T_j^b T_k^c F_1(v_{ij}, v_{jk}, v_{ki}) + \sum_i \frac{i^{abc} T_i^a T_i^b T_i^c f_5(v_{ij}, \log \left( \frac{-\sigma_{jk} v_{jk} \cdot P_k}{-\sigma_{k} v_{ij} \cdot P_k} \right)) + \mathcal{O}(\alpha_i^3)}{\mathcal{O}}\]

Several simplifications occur when only non-factorisable corrections are considered \([\text{Brønnum-Hansen, Melnikov, Quarroz, Wang '21]}\)

\[\Gamma_{\text{nf}} = \left( \frac{\alpha_s}{4\pi} \right) \Gamma_{0,\text{nf}} = \left( \frac{\alpha_s}{4\pi} \right) 4 \left[ T_1 \cdot T_2 \log \left( \frac{\mu^2}{-s - ie} \right) + T_2 \cdot T_3 \log \left( \frac{\mu^2}{-u - ie} \right) + T_1 \cdot T_4 \log \left( \frac{\mu m_i}{m_i^2 - u - ie} \right) + T_3 \cdot T_4 \log \left( \frac{\mu m_i}{m_i^2 - s - ie} \right) \right]\]

\[\langle \mathcal{A}^{(0)}_{\text{nf}} | \mathcal{A}^{(2)}_{\text{nf}} \rangle = \frac{-1}{8\epsilon^2} \langle \mathcal{A}^{(0)}_{\text{nf}} | \Gamma_{0,\text{nf}}^2 | \mathcal{A}^{(0)}_{\text{nf}} \rangle + \frac{1}{2\epsilon} \langle \mathcal{A}^{(0)}_{\text{nf}} | \Gamma_{0,\text{nf}} | \mathcal{A}^{(1)}_{\text{nf}} \rangle + \langle \mathcal{A}^{(0)}_{\text{nf}} | \mathcal{F}_{\text{nf}}^{(2)} \rangle\]

\[\langle \mathcal{A}^{(1)}_{\text{nf}} | \mathcal{A}^{(1)}_{\text{nf}} \rangle = \frac{-1}{4\epsilon^2} \langle \mathcal{A}^{(0)}_{\text{nf}} | \Gamma_{0,\text{nf}} | \mathcal{A}^{(0)}_{\text{nf}} \rangle + \frac{1}{2\epsilon} \langle \mathcal{A}^{(1)}_{\text{nf}} | \Gamma_{0,\text{nf}} | \mathcal{A}^{(0)}_{\text{nf}} \rangle + \frac{1}{2\epsilon} \langle \mathcal{A}^{(0)}_{\text{nf}} | \Gamma_{0,\text{nf}}^{(1)} | \mathcal{A}^{(1)}_{\text{nf}} \rangle + \langle \mathcal{F}_{\text{nf}}^{(1)} | \mathcal{F}_{\text{nf}}^{(1)} \rangle\]
Double-virtual contribution: amplitude evaluation [Brønnum-Hansen, Melnikov, Quarroz, Wang ’21]

Process: \( u(p_1) + b(p_2) \rightarrow d(p_3) + t(p_4) \)

Kinematic scales: \( p_i^2 = 0, \ i = 1,2,3 \), \( p_4^2 = m_t^2, s, t, m_W^2 \)

Dimensions: \( d = 4 - 2\epsilon \)

Planar and non-planar amplitudes appear at 2-loop order. However, only a particular combination of them do actually contribute

\[
\mathcal{A}^{(2)}_{nf} = \frac{1}{4} \left\{ T^a, T^b \right\}_{c_3c_1} \left\{ T^a, T^b \right\}_{c_3c_1} \left( A^{(2), pl}_{nf} + A^{(2), npl}_{nf} \right) + \ldots
\]

Upon interference with tree-level amplitude the colour distinction between planar and non-planar diagrams disappears

(Abelian nature of non-factorisable corrections)

\[
\sum_{\text{color}} \mathcal{A}^{(0)\ast} \mathcal{A}^{(2)}_{nf} = \frac{1}{4} \left( N_c^2 - 1 \right) A^{(0)\ast} A^{(2)\ast}_{nf}
\]
Double-virtual contribution: amplitude evaluation [Brønnum-Hansen, Melnikov, Quarroz, Wang ’21]

Process: \( u(p_1) + b(p_2) \rightarrow d(p_3) + t(p_4) \)
Kinematic scales: \( p_i^2 = 0 \), \( i = 1, 2, 3 \), \( p_4^2 = m_t^2, s, t, m_W^2 \)
Dimensions: \( d = 4 - 2\epsilon \)

- **18 diagrams**: generated with QGRAPH [Nogueira ’93] and processed with FORM [Vermaseren ’00] [Kuipers et al. ’15] [Ruijl et al. ’17]. All topologies maximal.
Double-virtual contribution: amplitude evaluation [Brønnum-Hansen, Melnikov, Quarroz, Wang ’21]

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- One- and two-loop amplitudes are written in terms of invariant form factors and independent Lorentz structure

- \( \gamma_5 \) enters through charged weak currents (left-handed projectors)

- Use anti-commuting prescription for \( \gamma_5 \) and move left-handed projectors to act on external massless fermions.

- 11 structures \( S_i(\lambda) \) and corresponding form factor (FF)

\[
Q_i = \sum_{\lambda} S_i^\dagger(\lambda) A_{nf}^{(2)}(\lambda) , \quad i = 1,\ldots,11
\]

- FF do not depend on helicities of external particles \( \rightarrow \) vector current part

- Polarisation sum returns independent traces \( \rightarrow \) scalar products of loop and external momenta (no external spinor)
Double-virtual contribution: amplitude evaluation \([\text{Brønnum-Hansen, Melnikov, Quarroz, Wang ’21}]\)

Process: \(u(p_1) + b(p_2) \rightarrow d(p_3) + t(p_4)\)

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Dimensions: \(d = 4 - 2\epsilon\)

- **Reduction** performed **analytically** with KIRA2.0 \([\text{Klappert, Lange et al. ’20}]\) and FireFly \([\text{Klappert, Lange ’20}]\ [\text{Klappert, Klein et al. ’21}]\)

\[
\left\langle \mathcal{A}^{(0)} \right\rvert \mathcal{A}_{nf}^{(2)} \right\rangle = \frac{1}{4} \left( N_c^2 - 1 \right) \sum_{i=1}^{428} c_i(d, s, t, m_t, m_W) I_i
\]

- Most complicated took 4 days on 20 cores.

- **428 master integrals** \(I_i\).

- **Exact dependence on the top-quark mass and the W mass** (very first reduction to master integral performed for the fixed numerical relation \(m_t^2 = 14/3 m_W^2\) \([\text{Assadsolimani et al. ’14}]\))
Double-virtual contribution: amplitude evaluation [Brønnum-Hansen, Melnikov, Quarroz, Wang '21]

Process: $u(p_1) + b(p_2) \rightarrow d(p_3) + t(p_4)$

Kinematic scales: $p_i^2 = 0, \ i = 1,2,3, \ p_4^2 = m_t^2, s,t,m_W$

Dimensions: $d = 4 - 2\varepsilon$

- Compute master integrals using the auxiliary mass flow method [Liu, Ma, Wang '18] [Liu, Ma, Tao, Zhang '21]

$$I \propto \lim_{\eta \rightarrow 0^+} \int \prod_{i=1}^{2} \prod_{a=1}^{9} \frac{1}{[q_a^2 - (m^2_a - i\eta)]^{\nu_a}}$$

- Add imaginary part to the W boson mass $m_W^2 \rightarrow m_W^2 - i\eta$

- Solve system of DE at each phase space point: $\partial_x I = MI, m_W^2 - i\eta = m_W^2(1 + x)$

- Boundary condition $x \rightarrow -i\infty$, physical point $x = 0$.

- Some of the boundary integrals are hard to compute: add imaginary part to the top mass $m_t^2 \rightarrow m_t^2 - i\eta$

- DE in $m_t$. Boundary $\eta \rightarrow \infty$. Physical point $\eta \rightarrow 0$. 

Chiara Signorile-Signorile

Advances in fixed-order predictions
Double-virtual contribution: amplitude evaluation [Brønnum-Hansen, Melnikov, Quarroz, Wang ’21]

Process: \( u(p_1) + b(p_2) \rightarrow d(p_3) + t(p_4) \)
Kinematic scales: \( p_i^2 = 0, \ i = 1,2,3, \quad p_4^2 = m_t^2, s, t, m_W \)
Dimensions: \( d = 4 - 2\epsilon \)

- t’Hooft-Veltman scheme: external momenta in \( d = 4 \) and internal momenta in \( d = 4 - 2\epsilon \)
- To be non-vanishing a matrix events in \( d = 4 - 2\epsilon \) between two \( d = 4 \) spinors requires an even number of matrices with support in \(-2\epsilon\) space
- \( \epsilon \) dependence can be explicitly and unambiguously extracted
Backup on mixed QCD-EW corrections
Phenomenology: fiducial cross section

Definition of the fiducial cross section:

\[ \sqrt{s} = 13.6 \text{ TeV} \]
\[ m_l = 0 \]
\[ m_{ll} > 200 \text{ GeV (dressed leptons)} \]
\[ R_{l_l} = 0.1 \text{ (dressed leptons)} \]
\[ p_{l_l}^\perp > 30 \text{ GeV} \]
\[ \sqrt{p_{l_l}^\perp p_{l_l}^\perp} > 35 \text{ GeV} \]
\[ |y_l| < 2.5 \]
\[ \mu_F = \mu_R = m_{ll}/2 \text{ (dressed leptons)} \]
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\[ \sigma = \sigma^{(0,0)} + \delta\sigma^{(1,0)} + \delta\sigma^{(0,1)} + \delta\sigma^{(2,0)} + \delta\sigma^{(1,1)} + \ldots \]

LO  NLO QCD  NLO EW  NNLO QCD  NNLO QCDxEW
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What do we learn?

\( \checkmark \) NLO QCD \( \sim +20\% \)
\( \checkmark \) NLO EW \( \sim -3\% \)
\( \checkmark \) NNLO QCD \( \sim +0.8\% \)
\( \checkmark \) QCDxEW \( \sim -1\% \)
Phenomenology: fiducial cross section

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\[ \sigma = \sigma^{(0,0)} + \delta\sigma^{(1,0)} + \delta\sigma^{(0,1)} + \delta\sigma^{(2,0)} + \delta\sigma^{(1,1)} + \ldots \]

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What do we learn?

\[ \checkmark \text{ NLO QCD} \sim +20\% \quad \delta\text{QCD} \sim 8C_F\alpha_s/(2\pi) \sim 0.2 \]

\[ \checkmark \text{ NLO EW} \sim -3\% \]

\[ \checkmark \text{ NNLO QCD} \sim +0.8\% \]

\[ \checkmark \text{ QCD}\times\text{EW} \sim -1\% \]
**Phenomenology: fiducial cross section**

Definition of the fiducial cross section:

\[
\sigma = \sigma(0,0) + \delta\sigma(1,0) + \delta\sigma(0,1) + \delta\sigma(2,0) + \delta\sigma(1,1) + \ldots
\]

- LO
- NLO QCD
- NLO EW
- NNLO QCD
- NNLO QCDxEW

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**What do we learn?**

- \( \checkmark \) NLO QCD ~ +20\% \( \rightarrow \) \( \delta^{QCD} \sim 8 C_F \alpha_s/(2\pi) \sim 0.2 \)
- \( \checkmark \) NLO EW ~ -3\% \( \rightarrow \) \( \delta^{ew} \sim \alpha_{ew} / \sin^2\theta_W \sim 0.03 \)
- \( \checkmark \) NNLO QCD ~ +0.8\%

CHIARA SIGNORILE-SIGNORILE

**Advances in fixed-order predictions**
Phenomenology: fiducial cross section

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What do we learn?

- $\sqrt{s} = 13.6$ TeV
- $m_l = 0$
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- $R_{\ell} = 0.1$ (dressed leptons)
- $p_{\perp}^l > 30$ GeV
- $\sqrt{p_{\perp}^t p_{\perp}^t} > 35$ GeV
- $|y_l| < 2.5$
- $\mu_F = \mu_R = m_{ll}/2$ (dressed leptons)

NNPDF31_nnlo_as_0118_luxqed

- ✓ NLO QCD ~ +20% $\rightarrow \delta^{QCD} \sim 8C_F \alpha_s/(2\pi) \sim 0.2$
- ✓ NLO EW ~ -3% $\rightarrow \delta^{ew} \sim \alpha_{ew}/\sin^2\theta_W \sim 0.03$
- ✓ NNLO QCD ~ +0.8% $\rightarrow$ unexpected : $\alpha_s^2 \sim 0.014$
  strong qq and qg cancellation
- ✓ QCDxEW ~ -1%
What do we learn?

\[ \sum f b = 13.6 \text{ TeV} \]
\[ m_l = 0 \]
\[ m_{ll} > 200 \text{ GeV (dressed leptons)} \]
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\[ |y| < 2.5 \]

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&\text{LO} \quad \text{NLO QCD} \quad \text{NLO EW} \quad \text{NNLO QCD} \quad \text{NNLO QCDxEW}
\end{align*}

\[
\begin{array}{|c|c|c|c|c|}
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\end{array}
\]

\begin{itemize}
\item \( \sqrt{\sigma^{(0,0)}} \sim +20\% \)
\item \( \sqrt{\sigma^{(0,1)}} \sim -3\% \)
\item \( \sqrt{\sigma^{(1,0)}} \sim 0.8\% \)
\item \( \sqrt{\sigma^{(1,1)}} \sim -1\% \)
\end{itemize}

\[ \delta^{QCD} \sim 8C_F\alpha_s/(2\pi) \sim 0.2 \]
\[ \delta^{ew} \sim \alpha_{ew}/\sin^2\theta_W \sim 0.03 \]

unexpected: \( \alpha_s^2 \sim 0.014 \)

strong qq and qg cancellation

unexpected: \( \alpha_s\alpha_{ew} \sim 0.00089 \)
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What do we learn?

- \( \checkmark \) NLO QCD \( \sim +20\% \) \[ \delta^{\text{QCD}} \sim 8C_F \alpha_s/(2\pi) \sim 0.2 \]
- \( \checkmark \) NLO EW \( \sim -3\% \) \[ \delta^{\text{ew}} \sim \alpha_{ew}/\sin^2\theta_W \sim 0.03 \]
- \( \checkmark \) NNLO QCD \( \sim +0.8\% \) \[ \text{unexpected: } \alpha_s^2 \sim 0.014 \]
  strong qq and qg cancellation
- \( \checkmark \) QCD\times EW \( \sim -1\% \) \[ \text{unexpected: } \alpha_s \alpha_{ew} \sim 0.00089 \]

\( \rightarrow \) QCD\times EW corrections larger than NNLO QCD ones!
Theoretical uncertainties

Envelope of QCD+EW related uncertainties

\[ d\sigma = d\sigma^{(0,0)} \left( 1 + \frac{\alpha_s}{2\pi} \delta^{(1,0)} + \left( \frac{\alpha_s}{2\pi} \right)^2 \delta^{(2,0)} + \frac{\alpha_{ew}}{2\pi} \delta^{(0,1)} + \frac{\alpha_s}{2\pi} \frac{\alpha_{ew}}{2\pi} \delta^{(1,1)} + \ldots \right) \]

- **QCD**: factor 2 rescaling (up and down) of central scale \( \mu = m_{\ell\ell}/2 \)
- **EW**: variation of input parameters: \( G_\mu \) scheme vs \( \alpha(m_Z) \) scheme

\[ \sigma^{(0,0)} + \delta\sigma^{(1,0)} + \delta\sigma^{(0,1)} + \delta\sigma^{(2,0)} = 1928.3^{+1.8}_{-0.15} \text{ fb.} \]

The mixed QCD-EW corrections \( \sim - 1\% \)

\[ \sigma^{(0,0)} + \delta\sigma^{(1,0)} + \delta\sigma^{(0,1)} + \delta\sigma^{(2,0)} + \delta\sigma^{(1,1)} = 1912.6^{+0.65}_{-0.0}\% \text{ fb.} \]

Theoretical uncertainty below percent after inclusion of mixed corrections

\( \rightarrow \) pure EW scheme uncertainty reduced from \( \sim 1\% \) to about \( \sim 0.5\% \)

Results do not include uncertainties from PDFs: uncertainty on \( q\bar{q} \) luminosity \( \sim 5\% \) for \( m_{ll} \sim 2\text{TeV} \).
At high invariant mass ($m_{ll} > 1.5$ TeV) the factorised approximation captures more than 90% of the exact result.

LO NLO QCD NLO EW NNLO QCD QCDxEW

\[ \sigma = \sigma^{(0,0)} + \delta\sigma^{(1,0)} + \delta\sigma^{(0,1)} + \delta\sigma^{(2,0)} + \delta\sigma^{(1,1)} + \ldots \]

Different invariant mass windows: accuracy of the factorised approximation

\[ \frac{\delta\sigma^{(1,1)}_{\text{fact.}}}{\sigma^{(0,0)}} = \frac{\delta\sigma^{(1,0)}}{\sigma^{(0,0)}} \cdot \frac{\delta\sigma^{(0,1)}}{\sigma^{(0,0)}} \rightarrow (\text{NLO QCD}) \cdot (\text{NLO EW}) \]

<table>
<thead>
<tr>
<th>$\Phi$</th>
<th>$\sigma^{(0,0)}$</th>
<th>$\delta\sigma^{(1,0)}$</th>
<th>$\delta\sigma^{(0,1)}$</th>
<th>$\delta\sigma^{(2,0)}$</th>
<th>$\delta\sigma^{(1,1)}$</th>
<th>$\delta\sigma^{(1,1)}_{\text{fact.}}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Phi^{(1)}$</td>
<td>200 GeV &lt; $m_{ll}$ &lt; 300 GeV,</td>
<td>1169.8</td>
<td>254.3</td>
<td>-30.98</td>
<td>10.18</td>
<td>-10.74</td>
</tr>
<tr>
<td>$\Phi^{(2)}$</td>
<td>300 GeV &lt; $m_{ll}$ &lt; 500 GeV,</td>
<td>368.29</td>
<td>71.91</td>
<td>-11.891</td>
<td>2.85</td>
<td>-4.05</td>
</tr>
<tr>
<td>$\Phi^{(3)}$</td>
<td>500 GeV &lt; $m_{ll}$ &lt; 1.5 TeV,</td>
<td>82.08</td>
<td>14.31</td>
<td>-4.094</td>
<td>0.691</td>
<td>-1.01</td>
</tr>
<tr>
<td>$\Phi^{(4)} \times 10$</td>
<td>9.107</td>
<td>1.577</td>
<td>-1.124</td>
<td>0.146</td>
<td>-0.206</td>
<td>-0.1946</td>
</tr>
</tbody>
</table>

\( \delta\sigma^{(1,1)}_{\text{fact.}} \) flat QCD corr. \( \sim 20\% \) LO

EW corr. grow from \( \sim -3\% \) LO to \( \sim -12\% \) LO
[expected from 1-loop Sudakov logs]

\( \Phi^{(4)} \times 10 \)

Factorised approx. improves as invariant mass grows

\( \Phi^{(4)} \times 10 \)

At high invariant mass ($m_{ll} > 1.5$ TeV) the factorised approx. captures more than 90% of the exact result.

\[ \rightarrow \text{Expected: factorised approx. correctly reproduces the leading Sudakov logs, which dominate at high invariant masses} \]
Kinematic distributions: invariant mass

✓ Impact of NLO EW corr. on NLO QCD:
  - small at low invariant mass: $\sim -2\% @ 200\text{GeV}$
  - reaches $\sim -15\% @ 3\text{TeV}$

✓ Impact of NLO EW+QCD×EW corr. on NLO QCD:
  - similar shape as NLO EW corr.
  - reaches $\sim -18\% @ 3\text{TeV}$

✓ Impact of mixed QCD×EW corr. on NLO (QCD+EW):
  - non entirely flat shape
  - large at low invariant mass: $\sim -0.8\% @ 200\text{GeV}$
  - reaches $\sim -3\% @ 3\text{TeV}$

✓ Factorised approximation:
  - under estimation of mixed corr. in $m_{\ell\ell} \in [200,1000] \text{GeV}$ region
  - better agreement in $m_{\ell\ell} \in [1,3] \text{TeV}$ region
Angular distribution and forward-backward asymmetry

Angular distributions can test quark to lepton interactions

Forward-backward asymmetry has been measured in the high invariant mass region [CMS 2202.12327]

\[ A_{FB} = \frac{\sigma_F - \sigma_B}{\sigma_F + \sigma_B} = 0.1580^{+0.15\%}_{-0.07\%} \]

Mixed QCD-EW corrections changes the value by about 2 permille
→ comparable with the uncertainties

<table>
<thead>
<tr>
<th></th>
<th>( \tilde{A}_{FB} )</th>
<th>( A_{FB} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \Phi^{(1)} )</td>
<td>0.1442^{+0.05%}_{-0.31%}</td>
<td>0.1440^{+0.11%}_{-0.09%}</td>
</tr>
<tr>
<td>( \Phi^{(2)} )</td>
<td>0.1852^{+0.08%}_{-0.40%}</td>
<td>0.1847^{+0.10%}_{-0.19%}</td>
</tr>
<tr>
<td>( \Phi^{(3)} )</td>
<td>0.2401^{+0.13%}_{-0.64%}</td>
<td>0.2388^{+0.06%}_{-0.47%}</td>
</tr>
<tr>
<td>( \Phi^{(4)} )</td>
<td>0.3070^{+0.49%}_{-1.5%}</td>
<td>0.3031^{+0.19%}_{-1.2%}</td>
</tr>
</tbody>
</table>

Mixed QCD-EW corrections affect \( A_{FB} \) at the percent level for \( m_{ll} \gtrsim 1 \text{ TeV} \)
→ this shift should become observable at HL-LHC