TMD distributions at the next-to-leading power.

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Outline

What and why?

Tools: background field approach, position space computation and multipole expansion

Results and discussion:
NLO matching for quark TMDPDFs and LO matching for gluon TMDPDFs
What is the matching?
Selection of first few terms in the light-cone OPE for the TMD operator

Small-b matching schematically

\[ F(x, b) = C(x, \ln(\mu b)) \otimes f(x, \mu) + \mathcal{O}(b^2) \]

Contains quark-gluon mixing!

Why is it necessary?
It greatly increase agreement between theory and experiment
Reduces parametric freedom in model building
Going to $O(a_s)$ allows evolution and inclusion of finite contributions
Quark TMD distributions

\[ \Phi^{[\Gamma]}(x, b) = \frac{1}{2} \int \frac{dz}{2\pi} e^{-ixzp^+} \langle p, S|\bar{q}(zn + b)[zn + b, -\infty n + b]\Gamma[-\infty, 0]q(0)|p, S\rangle \]

\[ \Gamma \in \{\gamma^+, \gamma^+\gamma_5, i\sigma^\alpha\gamma_5\} \]

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TMD twist

Quark pol.

Proton pol.

Collinear twist of the matching distribution

Vladimirov, et al., JHEP 01 (2022) 110
Collinear distributions

Twist-2

**Quark case**

\[ \Phi[\Gamma](x) = \frac{1}{2} \int \frac{dz}{2\pi} e^{-ixzp^+} \langle p, S | q(zn)|zn, 0 \rangle \Gamma q(0) | p, S \rangle \]

**Gluon case**

\[ \int \frac{dz}{2\pi} e^{-izxp^+} \langle p, S | F_{\mu^+}(zn)|zn, 0 \rangle F_{\nu^+}(0) | p, S \rangle \]

\[ = \frac{x p^+}{2} \left( - \frac{g_T^{\mu\nu} f_g(x)}{1 - \epsilon} - \frac{ic_T^{\mu\nu} \Delta f_g(x)}{(1 - \epsilon)(1 - 2\epsilon)} \right) \]

\( \epsilon \)-factors to ensure same result in any dimension

Twist-3

\[ \langle p, S | g f^{ABC} F^\mu_{A^+}(zn) F^\nu_{B^+}(zn) F^\rho_{C^+}(zn) | p, S \rangle \]

\[ = (p^+)^3 M \int [dx] e^{-ip^+(x_1z_1 + x_2z_2 + x_3z_3)} \sum_i t_{i\mu\nu} F_i^{+}(x_1, x_2, x_3) \]

\[ \langle p, S | g d^{ABC} F^\mu_{A^+}(zn) F^\nu_{B^+}(zn) F^\rho_{C^+}(zn) | p, S \rangle \]

\[ = (p^+)^3 M \int [dx] e^{-ip^+(x_1z_1 + x_2z_2 + x_3z_3)} \sum_i t_{i\mu\nu} F_i^{-}(x_1, x_2, x_3) \]

evanescent structures!
Small-b expansion, tree level

No special technique needed

Expand the operator and use equation of motion to remove `bad' components

$$
\Phi^{[\Gamma]}(x, b) = \phi^{[\Gamma]}(x) + b^\mu \phi^{[\Gamma]}_\mu(x) + b^\mu b^\nu \phi^{[\Gamma]}_{\mu\nu}(x) + \ldots
$$

$$
\phi^{[\Gamma]}(x) = \frac{1}{2} \int \frac{dz}{2\pi} e^{-ixzp^+} \langle p, S|\bar{q}(z, n)[zn, 0]\Gamma q(0)|p, S\rangle
$$

$$
\phi^{[\Gamma]}_\mu(x) = \frac{1}{2} \int \frac{dz}{2\pi} e^{-ixzp^+} \langle p, S|\bar{q}(z, n)[zn, -\infty n] \slashed{D}_\mu[-\infty n, 0]\Gamma q(0)|p, S\rangle
$$

Reduction to standard collinear twist-3 distributions is tricky. One possible approach:

It keeps track of T-evenness/oddness of the `parent' TMD distribution

Moos, et al., JHEP 12 (2020) 145
Small-b expansion, tree level

Twist-2 is trivial

\[ f_1(x, b) \sim f_1(x) + \mathcal{O}(b^2) \]
\[ g_1(x, b) \sim g_1(x) + \mathcal{O}(b^2) \]
\[ h_1(x, b) \sim h_1(x) + \mathcal{O}(b^2) \]

Twist-2 T-odd TMDs have `simple' matching to pure twist-3 collinear distributions

\[ f_{1T}^\perp(x, b) = \pm \pi T(-x, 0, x) \quad \text{Upper (lower) sign for Drell-Yan (SIDIS)-like} \]
\[ h_{1}^\perp(x, b) = \mp \pi E(-x, 0, x) \quad \text{Gauge link structure} \]

Twist-2 T-even TMDs have a more complex matching with both a twist-2 and twist-3 collinear contributions

\[ h_{1L}^{\perp, \text{tw2}}(x, b) = -x^2 \int_x^1 \frac{dy}{y} h_1(y) \]
\[ g_{1T}^{\text{tw2}}(x, b) = x \int_x^1 \frac{dy}{y} g_1(y) \]
\[ g_{1T}^{\text{tw3}}(x, b) = 2x \int [dy] \int_0^1 d\alpha \delta(x - \alpha y_3) \left( \frac{\Delta T(y_{1,2,3})}{y_2^2} + \frac{T(y_{1,2,3}) - \Delta T(y_{1,2,3})}{2y_2y_3} \right) \]
\[ h_{1L}^{\perp, \text{tw3}}(x, b) = -2x \int_0^1 d\alpha \int [dy] \alpha \delta(x - \alpha y_3) H(y_1, y_2, y_3) \frac{y_3 - y_2}{y_2^2 y_3} \]
One loop computation

Background field approach

Why? It allows to, up to a certain degree, ignore the specific external state and focus only on the operator. It is also extremely handy for higher-twist operators.

Basic idea: mode separation

\[ \langle S_1 | u | S_2 \rangle = \int D\Phi \: \Psi^*_{S_1}[\Phi] u[\Phi] \Psi_{S_2}[\Phi] e^{iS[\Phi]} \]

\[ \Phi = \phi(\mu) + \psi(\mu) \]

\[ \text{mom} < \mu \quad \text{Slow modes aka background modes} \]

\[ \text{mom} > \mu \quad \text{Fast modes aka dynamical modes} \]

Suppose that hadron contains only slow modes, i.e.

\[ \Psi_{S_1}[\phi + \psi] \sim \Psi_{S_1}[\psi] \]

\[ \langle S_1 | u | S_2 \rangle \sim \int D\psi \: \Psi^*_{S_1}[\psi] \Psi_{S_2}[\psi] e^{iS[\psi]} \left[ \int D\phi u[\phi + \psi] e^{iS[\psi + \phi] - iS[\psi]} \right] \]

Perturbative computable
How do we separate the fast and slow modes in TMD physics?

\[ \{(n \cdot \partial), (\bar{n} \cdot \partial), \partial_T \}\psi \lesssim p^+ \{1, \lambda^2, \lambda\} \psi \quad \lambda \ll 1 \]

The hadron (external state) defines with its momentum a direction
The fields scale depending on their momentum w.r.t. the external hadron

Background field approach preserves gauge invariance at each step

One can define two different gauges for dynamical and background fields
Common choice: Feynman-like gauges for dynamical sector and light-cone gauge for background

One can also derive scaling for the “good” and “bad” components of the fields
Flow of the computation

The matrix element for a TMD is presented in a functional-integral form. QCD fields are split into the dynamical and background, with corresponding momentum counting.

Expand in the coupling and in the number of fields
Integration of the dynamical modes to obtain the effective operator

Reduce the effective operator to combination of definite-twist operators via EOM

\[
\begin{align*}
\Phi_{\text{renor.}}(\mu, \zeta) &= \left( Z_{\text{UV}}^{-1}(\mu, \zeta) R^{-1}(\zeta) C_{\text{bare}} \otimes Z_\phi(\mu_{\text{OPE}}) \right) \otimes \Phi_{\text{renor.}}(\mu_{\text{OPE}}) \\
&= C_{\text{renor.}}(\mu, \zeta, \mu_{\text{OPE}})
\end{align*}
\]

\[
C_{\text{renorm}}^{\text{NLO}} = \mu^{2\epsilon} e^{\epsilon \gamma_E} C_{\text{bare}}^{\text{NLO}} + \left[ \mu^{2\epsilon} e^{\epsilon \gamma_E} 2 \left( \frac{-b^2}{4} \right)^\epsilon \right] C_F \Gamma(-\epsilon) \left( L_b - L_\zeta + 2 \ln \left( \frac{\delta^+}{p^+} \right) - \psi(-\epsilon) - \gamma_E \right) - C_F \left( \frac{2}{\epsilon^2} + \frac{3 + 2\zeta}{\epsilon} \right) - \frac{a_s}{\epsilon} \mathcal{H} \otimes C_{\text{LO}}
\]
NLO diagrams

Disclaimer: we work with massless quark fields
For all the distributions, the final result has the following form:

\[
F(x, b; \mu, \zeta) = F^{(0)}(x) + a_s \left\{ C_F \left(-L_b^2 + 2L_b l_\zeta + 3L_b - \frac{\pi^2}{6}\right) F^{(0)}(x) - 2L_b \mathbb{H} \otimes F^{(0)}(x) + F^{(1)}(x) \right\} + \mathcal{O}(a_s^2, b^2)
\]

- Tree-level
- Evolution kernel
- Finite part of the coefficient function

\[
\mu^2 \frac{d F^{(0)}(x)}{d\mu^2} = 2a_s \mathbb{H} \otimes F^{(0)}(x)
\]

\[
L_b = \ln \left( \frac{(-b^2)\mu^2}{4e^{-2\gamma_E}} \right)
\]
\`Bonus’ of the computation

In the gluon sector it was needed the tree-level matching of the gluon TMDs at twist-3

\[
\mathbb{O}^{\mu\alpha\nu}(z) = F^{\mu+}(zn + b)[zn, \pm\infty n] \overleftarrow{D}^{\alpha}[\pm\infty n, 0] F^{\nu+}(0)
\]

\[
\langle p, S | \mathbb{O}^{\mu\alpha\nu}(z) \rangle_{\text{tw2}} | p, S \rangle = \frac{\varepsilon_{\mu\nu} s_T^\alpha M}{2(1 - \varepsilon)(1 - 2\varepsilon)} \int_0^1 d\alpha \int_{-\infty}^{\infty} dy e^{iy\alpha p^+ z}\left(\alpha p^+ y\right)^2 \Delta f_g(y),
\]

\[
\langle p, S | \mathbb{O}^{\mu\alpha\nu}(z) \rangle_{\text{tw3}} | p, S \rangle = t_2^{\mu\alpha\nu} M FDF_2^{\text{tw3}}(z) + t_4^{\mu\alpha\nu} M FDF_4^{\text{tw3}}(z) + t_6^{\mu\alpha\nu} M FDF_6^{\text{tw3}}(z)
\]

\[
FDF_2^{\text{tw3}}(z) = \mp ip_+^2 \pi \int_{-1}^{1} dy F_2^+(y, 0, y) e^{iy p^+ z}
\]

\[
FDF_4^{\text{tw3}}(z) = \mp ip_+^2 \pi \int_{-1}^{1} dy F_4^+(y, 0, y) e^{iy p^+ z}
\]

\[
FDF_6^{\text{tw3}}(z) = p_+^2 \int [dx] \left(2F_2^+ + F_4^+ + F_6^+\right) \int_0^1 du \left(\frac{3x_1 + 2x_3}{x_2^2} u^2 e^{-ix_1 p^+ z} + \frac{x_3}{x_2^2} u^2 e^{ix_3 p^+ z}\right)
\]

\[
+ p_+^2 \sum_q \int [dx] 2T_q(x_1, x_2, x_3) \int_0^1 du u^2 e^{-ip_+ zux_2}
\]

Upper sign for SIDIS
Lower sign for DY
Conclusions

Complete NLO small-b matching for all quark TMDPDFs up to collinear twist-3 accuracy

Complete LO small-b matching for all gluon TMDPDFs up to collinear twist-3 accuracy

Background field approach: versatile and powerful tool to disentangle the operator loop structure from the external states

For the future

Complete the NLO for the gluon distributions

Explore NLO for TMDFFs

Explore matching to collinear twist-4 (pretzelosity)

Small-b matching for TMDs of TMD twist-3