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A fresh look at the classical sphaleron rate of hot gauge theory

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Work in progress

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Contents

- Review of sphaleron transitions in pure gauge theory
- Revisiting the classical sphaleron rate with a new measurement
- Some preliminary results

SU(N) topological charge and sphalerons

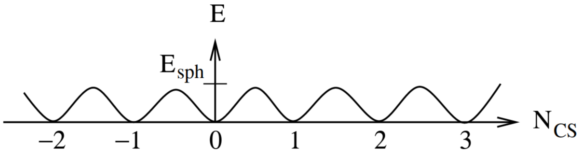
Consider the charge density

$$\chi(x) = \frac{g^2}{16\pi^2} \,\mathrm{Tr}\, F_{\mu\nu}\tilde{F}^{\mu\nu}$$

- Topological: $Q(t) \equiv \int dt' \int d^3\mathbf{x}\, \chi(t', \mathbf{x}) = N_{\mathrm{CS}}(t) - N_{\mathrm{CS}}(0)$, with N_{CS} the Chern-Simons number

- Thermal fluctuations induce transitions between vacua at different N_{CS}. The sphaleron rate (diffusion rate of N_{CS}) is

$$\Gamma_{\mathrm{sph}} = \lim_{t,V\to\infty} \frac{\langle Q(t)^2 \rangle}{Vt} \propto \alpha^5 T^4$$



Why we care about sphalerons

- Sphalerons mediate baryon number violation in $\mathrm{SU}(2)_L$: $\partial_\mu J^\mu_B \propto \chi(x)$. For electroweak theory, weak-coupling methods are reasonably accurate and the Γ_{sph} is known arXiv:1404.3565

- In QCD: Chirality violation in heavy ion collisions. arXiv:0711.0950 However, the $\mathrm{SU}(3)$ rate at physically relevant coupling is less well understood

- Axion dynamics in thermal medium: Consider $\mathcal{L} \supset -\phi\chi/f_a$. Then real-time fluctuations of the topological charge induce a friction term to axion EOM. For slowly varying ϕ, the friction coefficient is given by Γ_{sph}. McLerran, Mottola, Shaposhnikov, Phys.Rev.D 43 (1991)

- etc...

Classical simulations and the UV problem

- Computing a nonperturbative Minkowskian quantity like $\Gamma_{\rm sph}$ is hard! But momentum modes with $\ll \pi T$ (including sphalerons) are Bose enhanced, and we can use classical (Hamiltonian) simulations.

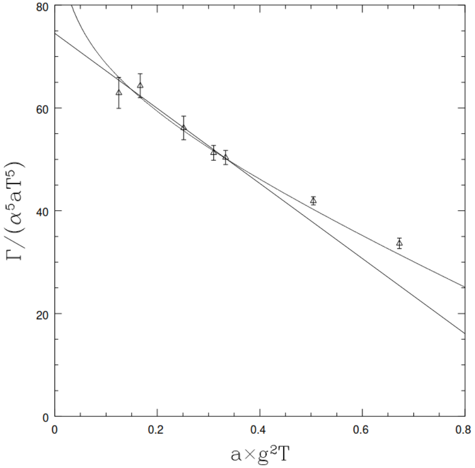
- Classical thermodynamics has Rayleigh-Jeans UV divergences. In simulations, the cutoff $\omega \sim 1/a$ is explicit and the classical theory can be used as an effective description, but the $a \to 0$ limit cannot be taken!

- Time scale of sphaleron transitions, $t_{\rm sph} \sim \alpha^2 T$, is sensitive to damping from hard thermal modes (scales $\gtrsim T$) arXiv:hep-ph/9609481

- On lattice these are cut at $1/a$ and one instead has $t_{\rm sph} \sim \alpha^2 T^2 a$ $\implies$ rate itself is $\Gamma_{\rm sph} \sim \alpha^5 T^5 a$

Scaling of classical Γ_{sph}

Figure: arXiv:hep-ph/9906259



There is also a logarithmic correction, $\Gamma \sim \alpha^5 T^5 a \log(ag^2 T)$

hep-ph/9801430

Final comments on classical methods

- For extreme weak coupling $1/(\log(1/g)) \ll 1$ an effective Langevin method is possible with finite $a \to 0$ limit (Bödeker EFT)

- The EFT is applicable for the electroweak sphaleron rate, but for QCD $\Gamma_{\rm sph}$ its validity is questionable arXiv:1011.1167
$\Longrightarrow$ Classical Hamiltonian approach is still motivated at least for qualitative studies

- **This work:** Revisit the Hamiltonian approach in $\mathrm{SU}(2)$, $\mathrm{SU}(3)$ with a slight twist

Contents

Time correlations of the charge density

- The diffusion rate Γ is related (by a Green-Kubo relation) to a time-symmetric 2-point function: arXiv:hep-th/0205051

$$C_s(t) = \int d^3\mathbf{x} \left\langle \frac{1}{2} \{\chi(t,\mathbf{x}), \chi(0,\mathbf{0})\} \right\rangle$$

$$C_s(\omega) = \int_{-\infty}^{\infty} dt\, e^{i\omega t} C_s(t), \qquad \Gamma_{\text{sph}} = \lim_{\omega\to 0} C_s(\omega).$$

- Sometimes also the $\omega \neq 0$ transform is interesting, *e.g.* in warm axion inflation the friction really depends on $C_s(\omega)$ Laine, Procacci, JCAP 06 (2021)

- Ultimately, our goal is to study nonperturbative features of the correlator at non-vanishing $\omega \ll T$. Focus on $\mathrm{SU}(2)$ and $\mathrm{SU}(3)$

Choosing the observable

- To measure $\chi(t, \mathbf{x})$ we need a lattice counterpart of $\mathrm{Tr}\, F_{\mu\nu}\tilde{F}^{\mu\nu}$. We use

$$igF_{jk} = \left[Q_{jk}(x) - Q_{kj}(x)\right]/8a$$

  where Q_{jk} is a 2×2 "clover"

- This definition has correct IR properties, but is not topological due to field discontinuities at the lattice scale

- In earlier studies of Γ_{sph} the gauge configurations are smoothed with gradient flow and coarse-grained before calculating the charge (or N_{CS}) hep-ph/9805264

- A smoothed correlator is not the same object that affects *e.g.* axion evolution, so we will not use this approach $\implies$ our observables are *different*

Lattice setup

Partition function in $A_0 = 0$ gauge:

$$Z^{(\mathrm{cl})} = \int \mathcal{D}U_i \mathcal{D}\mathcal{E}_i \; \delta(G) \exp\Big\{ -\frac{1}{ag^2T} \sum_{\mathbf{x}} \Big[\sum_{i,j} \mathrm{Tr}\,(1 - P_{ij}) + \sum_i \mathrm{Tr}\,\mathcal{E}_i^2 \Big] \Big\},$$

i.e. Kogut-Susskind formulation in the $\hbar \to 0$ limit.

- $\mathcal{E}_i$ is the "electric field", $\mathcal{E}_i \in \mathfrak{su}(N)$

- The $\delta(G)$ ăenforces **Gauss' law**:

$$G(x) = \sum_i \Big[\mathcal{E}_i(\mathbf{x}) - U_i^\dagger(\mathbf{x} - a\mathbf{i})\mathcal{E}_i(\mathbf{x} - a\mathbf{i})U_i(\mathbf{x} - a\mathbf{i}) \Big]$$

- Our algorithm for generating field configurations satisfying $G = 0$ is from G. Moore, Nucl.Phys.B 480 (1996)

Generating the time dependence

- Classical EOM (with leapfrog, temporal spacing $a_t = 0.02a$):

$$\mathcal{E}_i^a(x + a_t\mathbf{0}) = \mathcal{E}(x) + 2\frac{a_t}{a}\sum_{j\neq i} \mathrm{Im}\,\mathrm{Tr}\,T^a \left\{P_{ji}\left(x + \frac{a_t\mathbf{0}}{2}\right) + P_{-ji}\left(x + \frac{a_t\mathbf{0}}{2}\right)\right\}$$

$$U_i\left(x + \frac{1}{2}a_t\mathbf{0}\right) = \exp\left[i\frac{a_t}{a}\mathcal{E}_i\left(x\right)\right] U_i\left(x - \frac{1}{2}a_t\mathbf{0}\right)$$

- The simulation in practice: generate a configuration from the thermal ensemble, evolve it with the EOM to measure $\chi(t)$, repeat…

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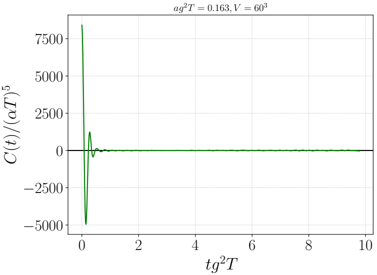
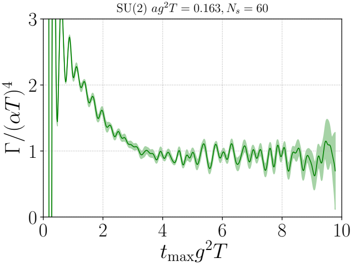
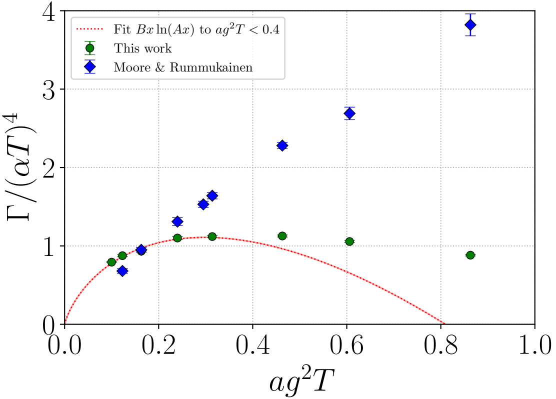
Integrating the $C_s(t)$ correlator





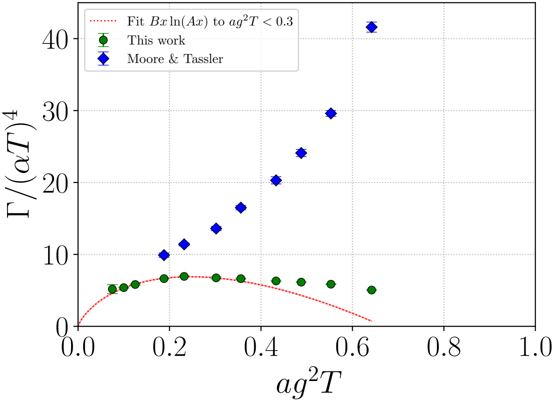
- Turns out that our construction of $\Gamma_{\text{sph}} = \int_{-\infty}^{\infty} dt\, C_s(t)$ has delicate UV/IR cancellations

- In practice we can integrate only up to some $t = t_{\max}$
  $\Longrightarrow$ ăneed to choose large enough $t_{\max}$ that the cancellations stabilize



At large ag^2T, the results differ because the UV physics is treated differently.
If the IR is done properly, we expect $\Gamma_{\mathrm{sph}} \sim \alpha^5 T^5 a$ăor $\sim \alpha^5 T^5 a \log(ag^2T)$ scaling at weak coupling. We observe much more log-dominated scaling than the old results

SU(3) comparison



Compare with G. Moore, M. Tassler, JHEP 02 (2011). Qualitatively similar to the SU(2) case.

Conclusions

- We have studied the statistical real-time correlator of topological charge density using classical simulations of $\mathrm{SU}(2)$ăand $\mathrm{SU}(3)$
- The sphaleron rate can be directly extracted from the correlation function without the need to smooth out UV fluctuations.
- Our results for Γ_{sph} at small ag^2T are in rough qualitative agreement with old results that use UV smoothed fields. We expected more linear scaling, so questions remain:
  - Are we still at too large ag^2T?
  - Are there other IR effects apart from sphalerons that contribute to $C(t)$?
  - …

Finite size effects (backup)