

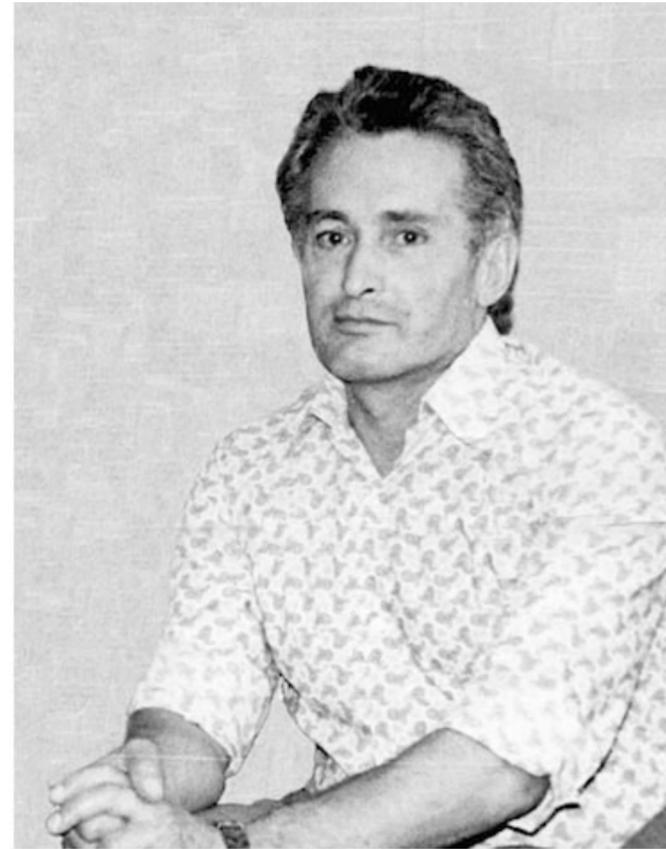
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Fifty Years of Supersymmetry

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(1970). Golfand and Likhtman



Anticommutator

$$\{\bar{Q}_{\dot{\alpha}}, Q_{\alpha}\} = 2P_{\dot{\alpha}\alpha} = 2(\sigma^{\mu})_{\dot{\alpha}\alpha} P_{\mu}$$

Four (Weyl) supercharges

Energy-momentum 4-vector

Super-Poincaré algebra (14= 10 even + 4 odd)

All Jacobi identities are satisfied!

Demise of CM no-go theorem

$$\vec{P} = 0, \quad P_0 = H \longrightarrow \{\bar{Q}_{\dot{\alpha}}, Q_{\alpha}\} = 2\delta_{\dot{\alpha}\alpha} H$$

The ground state energy is exactly ZERO (Likhtman, ...-... Pauli)

Golfand and Likhtman, Nov. 1970;
N=1 superalgebra

Super-QED

Their story

ются в уравнения для определения операторов $H^1(t)$ и $W^1(t)$.

Уравнения (6) сводятся к системе линейных однородных уравнений для постоянных коэффициентов, которые вводятся в качестве неопределенных констант связи в наиболее общий вид гамильтониана взаимодействия. Эту систему уравнений удалось решить в случае, когда $H_1(t)$ является произведением трех полей, два из которых преобразуются по представлению (2) и комплексно-сопряженному (2), а третье — по представлению (3). Система уравнений (6) в этом случае имеет единственное решение, а отличны от нуля лишь операторы $W_1(t)$, $H_1(t)$, $H_2(t)$. Зная точный вид гамильтониана в представлении взаимодействия, можно восстановить по нему лагранжиан в гейзенберговском представлении:

$$\begin{aligned}
 L(x) = & (\partial_\alpha \phi^* - igA_\alpha \phi^*)(\partial_\alpha \phi + igA_\alpha \phi) - m^2 \phi^* \phi + (\partial_\alpha \omega^* - igA_\alpha \omega^*) \times \\
 & \times (\partial_\alpha \omega + igA_\alpha \omega) - m^2 \omega^* \omega + \frac{i}{2} \psi_1 \gamma_\alpha \overleftrightarrow{\partial}_\alpha \psi_1 - m \bar{\psi}_1 \psi_1 - g \psi_1 \gamma_\alpha \psi_1 A_\alpha + \\
 & + \frac{i}{2} \bar{\psi}_2 \gamma_\alpha \overleftrightarrow{\partial}_\alpha \psi_2 - \mu \bar{\psi}_2 \psi_2 - \frac{1}{2} (\partial_\beta A_\alpha)^2 + \frac{\mu^2}{2} A_\alpha A_\alpha + \frac{1}{2} (\partial_\alpha X)^2 - \frac{\mu^2}{2} X^2 + \\
 & + g\mu(\phi^* \phi - \omega \omega^*) X - \frac{g^2}{2} (\phi^* \phi - \omega^* \omega)^2 + \sqrt{2} g (\bar{\psi}_1 \bar{s} \psi_2 \phi + \bar{\psi}_2 \bar{s} \psi_1 \phi^*) - \\
 & - \sqrt{2} g (\psi_1^c \bar{s} \psi_2 \omega^* + \bar{\psi}_2 \bar{s} \psi_1^c \omega) .
 \end{aligned}
 \tag{7}$$

Таким образом, получена модель взаимодействия квантованных полей с несохранением четности, инвариантная относительно алгебры (1).

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Литература

[1] С.Швебер. Введение в релятивистскую квантовую теорию поля . ИИЛ, 1963.

¹⁾ Обоснованию этого постулата, а также сравнению его с обычной формулировкой требования инвариантности теории относительно группы преобразований будет посвящена отдельная работа.

JETP
Letters
1971:

SQED
with γ
mass &
massive
matter

(Couple of
typos)

Поступила в редакцию
10 марта 1971 г.

March 10, 1971

The only reference

The simplest 4D model fith four supercharges, $\mathcal{N} = 1$

$$\langle \text{vac} | \{ \bar{Q}_{\dot{\alpha}}, Q_{\alpha} \} | \text{vac} \rangle = 2\delta_{\dot{\alpha}\alpha} \langle \text{vac} | H | \text{vac} \rangle = 2\delta_{\dot{\alpha}\alpha} E_{\text{vac}}$$

$=0$

$=0$

Likhtman

$$\left. \begin{aligned} Q_{\alpha} | \text{boson} \rangle &= | \text{fermion} \rangle, \\ Q_{\alpha} | \text{fermion} \rangle &= | \text{boson} \rangle \end{aligned} \right\}$$

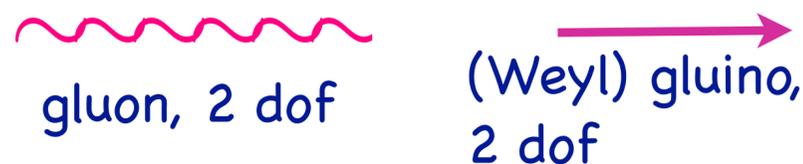
Degenerate masses, equal numbers of d.o.f.

Pauli, 1947

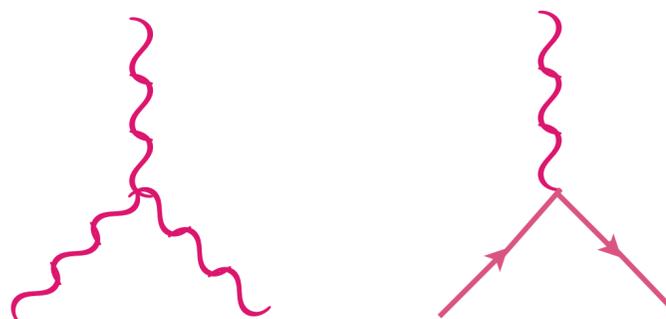
Non-Abelian brother of theo on page 3



SUSY Yang-Mills

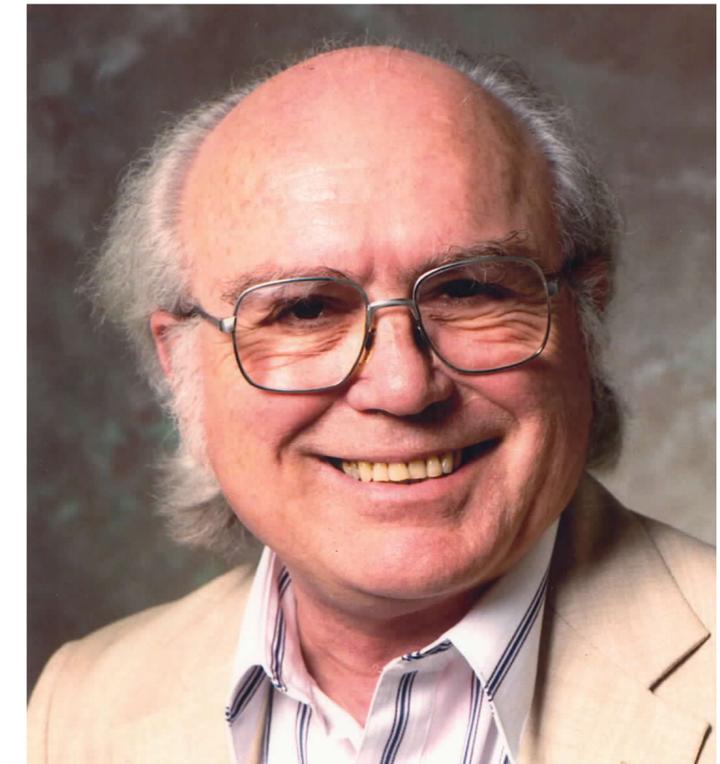
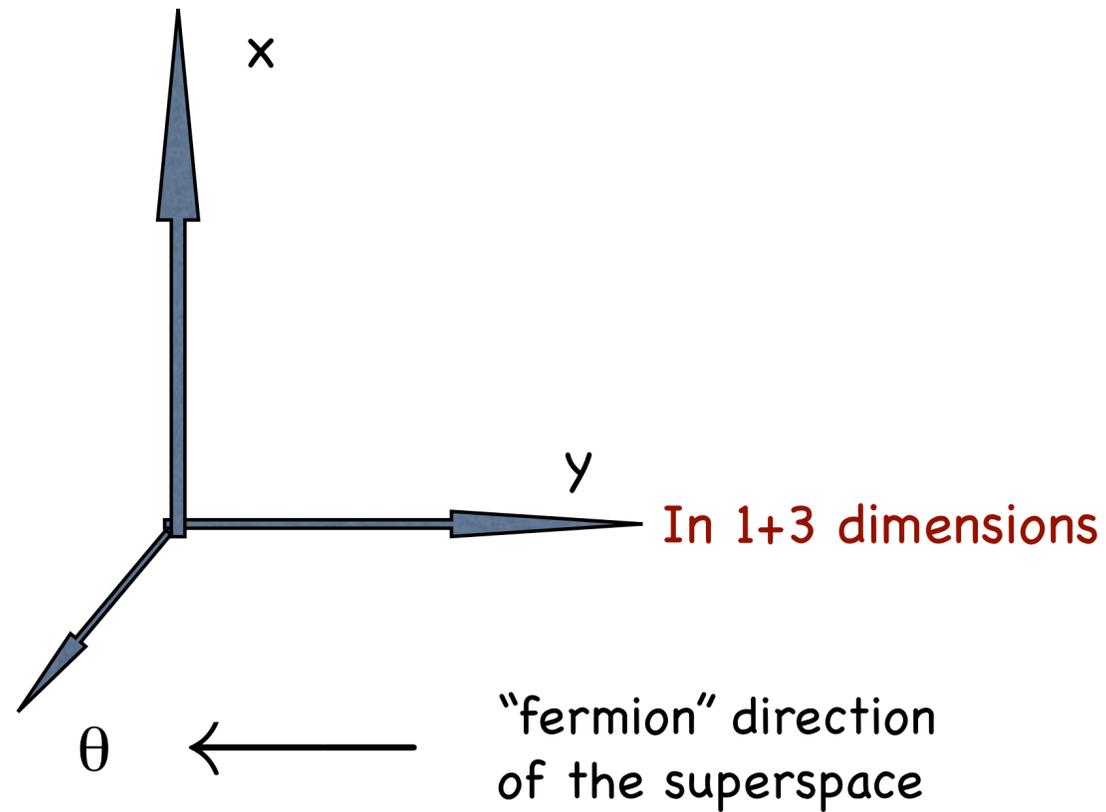


$$\mathcal{L} = -\frac{1}{4g^2} G_{\mu\nu}^a G^{\mu\nu a} + \frac{i}{2} \bar{\lambda} \not{D} \lambda$$



1973 and later,

Julius Wess & Bruno Zumino, ...



Superspace! $\{t, x, y, z\} \longrightarrow \{t, x, y, z; \theta_\alpha^i\}$

$$\theta^2 = 0$$

Salam and
Strathdee, 1975

In **4D** we can have $\theta_\alpha, \bar{\theta}_{\dot{\alpha}} \leftrightarrow \mathcal{N} = 1$

Extended SUSY $\left\{ \begin{array}{l} \theta_\alpha^i, \bar{\theta}_{\dot{\alpha}}^i \quad (i = 1, 2) \quad \leftrightarrow \mathcal{N} = 2 \\ \theta_\alpha^i, \bar{\theta}_{\dot{\alpha}}^i \quad (i = 1, 2, 3, 4) \quad \leftrightarrow \mathcal{N} = 4 \end{array} \right.$

With larger \mathcal{N} power of SUSY is stronger!

In the late 1970 people turned to SUSY-based phenomenology.

👑 Dynamical SUSY breaking, Witten, N.P. B185, 513 (1981);

👑 MSSM, 1981, Dimopoulos, Georgi (naturalness, DM), N.P. B193, 150 (1981)

Then → avalanche

Roughly, 3000 papers/year, 10/day

Why?

Natural Solution of Hierarchy problem; LSP = DM

↑
*physics
esthetics*

+ depth and beauty

$$(\mathcal{E}_{\text{vac}})^{1/4} \sim M_{\text{Pl}} = \sqrt{\frac{\hbar c}{G_N}}$$
$$M_{\text{Higgs}} \approx 125.10 \text{ GeV}$$

Also, gauge coupling unification is better in MSSM

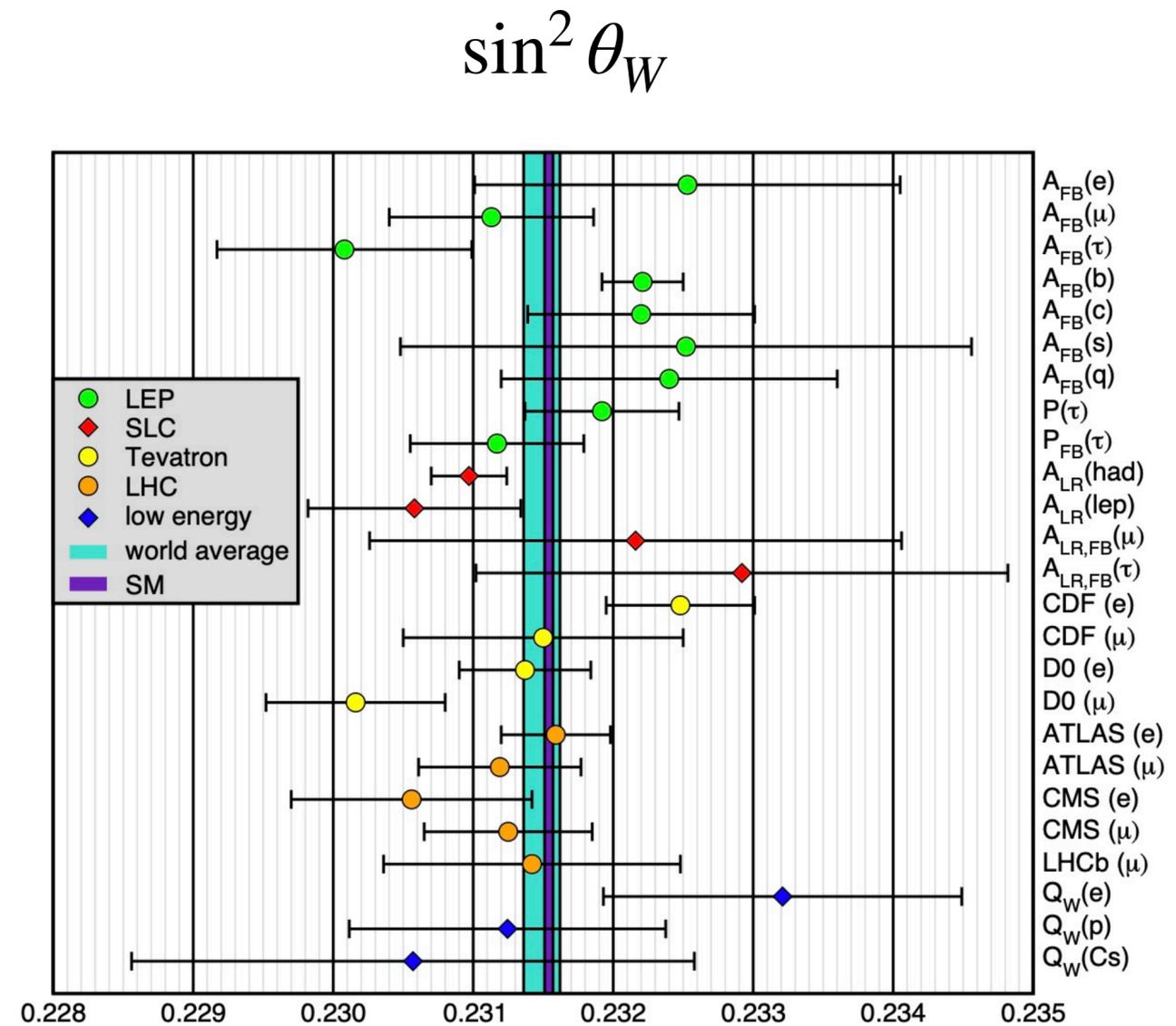
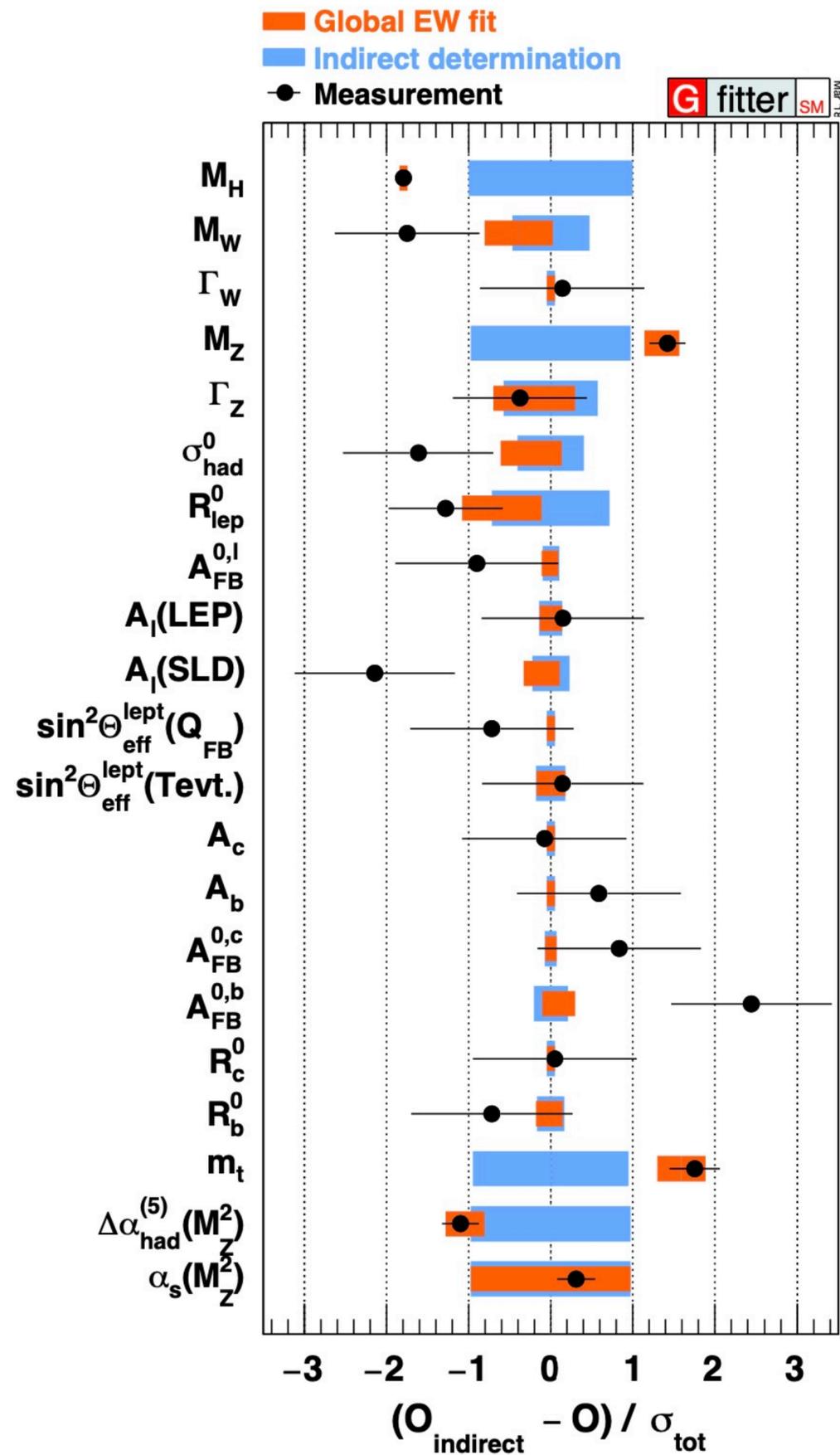
Supersymmetry protects bosons!

Loop graphs for M_{Higgs} diverge quadratically.

Hope was that

$$M_{\text{Pl}}^2 \rightarrow M_{\text{SUSY}}^2 \text{ with } M_{\text{SUSY}} \sim \text{few hundred GeV}$$

Global SM Fit (forget blue)



As data from the LHC has continued to accumulate, the originally favored models of supersymmetry (e.g. $MSSM_1$) have been largely ruled out. For example, gluinos have been excluded up to a mass of 2TeV – an order of magnitude larger than expected.

With masses of superpartners at least in the TeV range or higher there is no hope to appropriately suppress loop corrections as needed to keep

$$M_{\text{Higgs}} \approx 125.10 \text{ GeV}$$

$$MSSM_1 \rightarrow N^k MSSM_{\text{baroque}}$$

After 50 years the principle of naturalness seems to be gone
(Panic???)

Since Galileo and Newton physics operated differently:

Experiment (observation) \rightarrow Theory \rightarrow Prediction
 \rightarrow Falsification (or triumph) ...

👑 Naturalness was a belief.

It is not something imposed by existing data/observations;

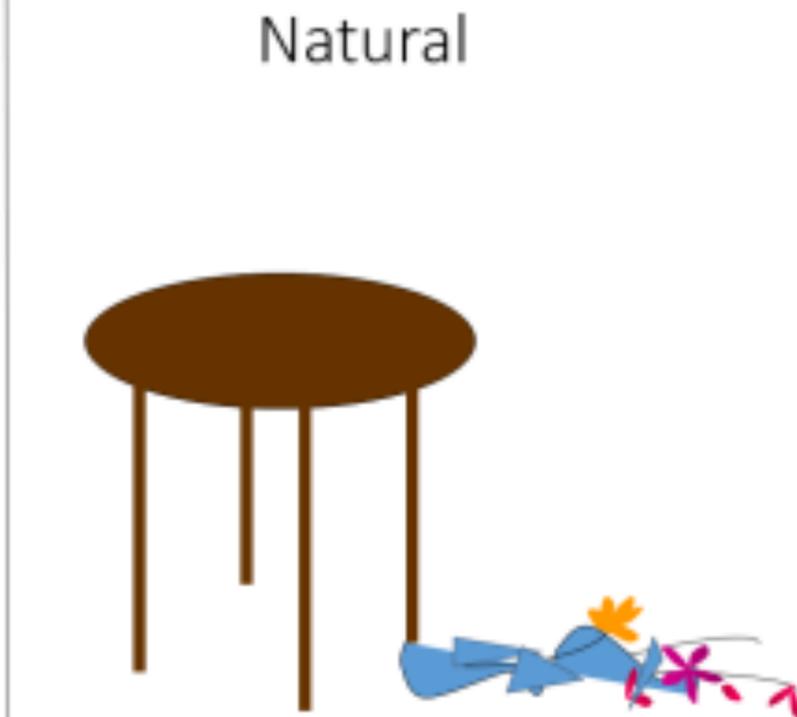
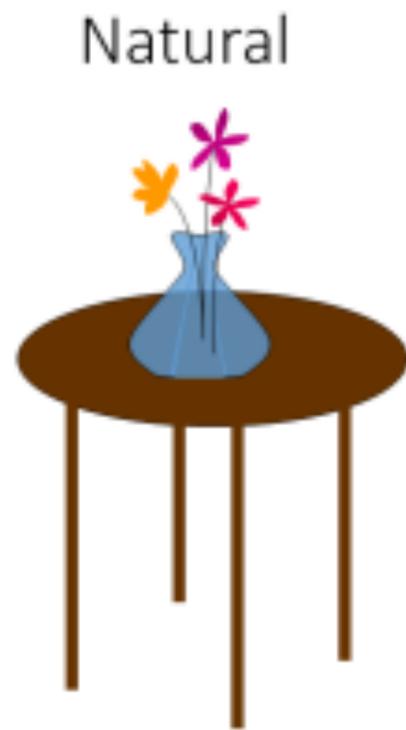
👑 So was the belief that string theory is the ultimate theory of everything. 😞😞😞

Both beliefs faded away (postponed indefinitely???)

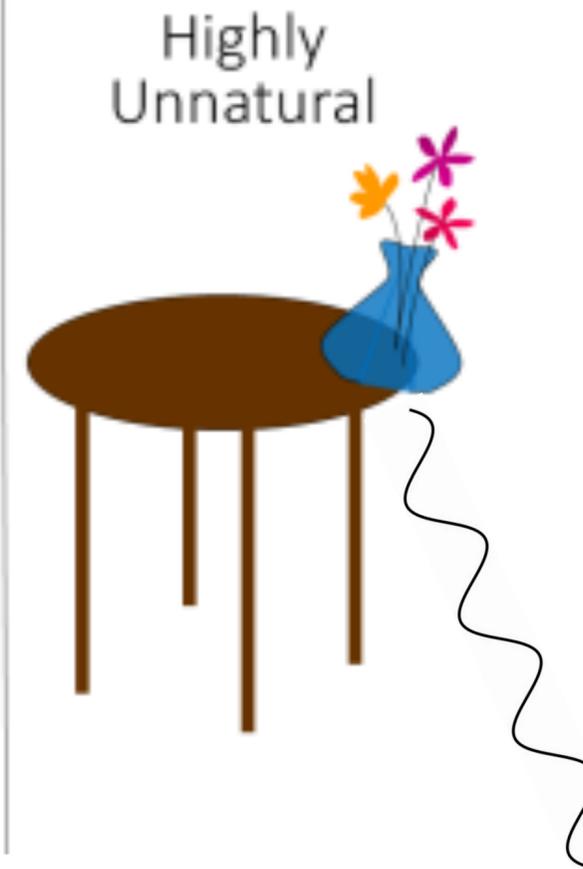
Attempts to base physics on philosophy rather than the other way around is punished by nature

We have to live without naturalness and the ultimate theory of everything and continue to do physics

OR?



M. Strassler 2013



Why this glue is “natural” ?

Multiverse;
Intelligent design/ Anthropic principle } Philosophy/Religion → Fantasy science

Does it mean that SUSY or string theory should not have been thoroughly studied?

Absolutely not! Remember the history of Yang-Mills theory...

But... ambitions should be scaled down, from super-global, "ultimate theory" to less global!

Goals to be redefined and adjusted in correspondence with the Galileo-Newton paradigm.

★ 📌 This is my personal belief, **NOT** a physics statement, and may or may not be followed, as you wish.

The Power of Supersymmetry:

For the first time ever in non-trivial QFT in four dimensions at **strong coupling** one can obtain exact predictions/solutions:

- 👉 $E_{\text{vac}} = 0$ (1971)
- 👉 Number of vacua (1981)
- 👉 Exact β functions (1983)
- 👉 Exact instanton-induced superpotentials (1984)
- 👉 Gluino condensate (1988)
- 👉 Seiberg dualities $SU(N)_{N_f} \leftrightarrow SU(N_f - N)_{N_f}$ (1994)
- 👉 Crescendo: Seiberg-Witten sol'n of $\mathcal{N} = 2$ SYM at low energies (1994)
- 👉 $\mathcal{N} = 4$ Yang – Mills is superconformal !
- 👉 BPS protected sectors: masses, domain wall tensions, vortex flux tensions (1995-2015)
- 👉 2D-4D correspondence ...

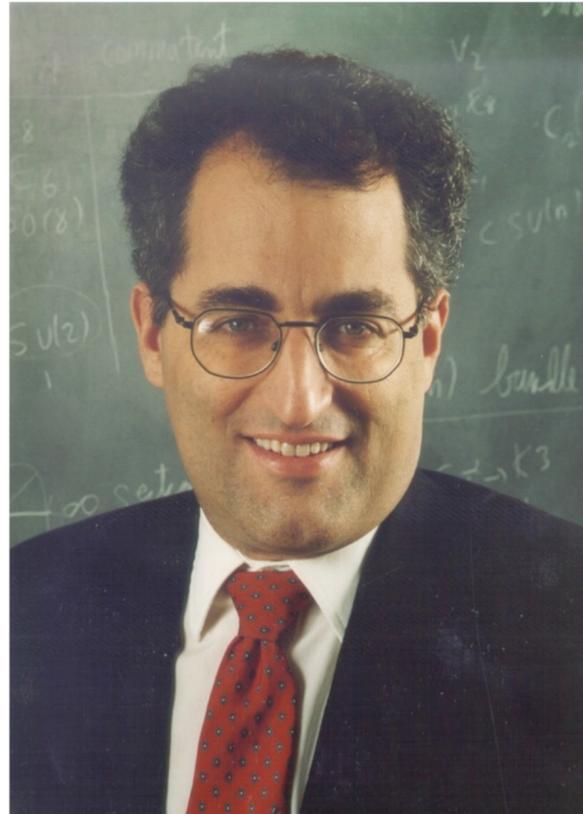
SUSY has revealed its power in physics! 🙌

Supersymmetry at the service of strong coupling QFT

- 👑 The first 50 years of QFT → weak coupling perturbation theory
+symmetry based considerations: *Triumph of QED, success of SM,
asymptotic freedom of QCD*
- 👑 Since 1970s, QFT = realm of strong coupling (primarily), both in HEP
Condensed Matter and even gravity at distances $L \sim (M_{\text{Pl}})^{-1}$ or
strong fields (quantum regime), 4D, 3D, 2D...
*Quark (color) confinement, non-Abelian strings, ...
SYK, Fractional Quantum Hall, ...*

Crescendo. Seiberg-Witten Culmination (1994)

☺ First demonstration of the dual Meissner effect ☺



$\mathcal{N} = 2$ SYM, 8 supercharges

- gluons+complex scalar superpartner
- two gluinos
- Georgi-Glashow model built in

Any point on the complex plane of $\text{Tr}\varphi^2$ can serve as vacuum

$SU(2) \rightarrow U(1)$, Higgsing in the same way as in Georgi-Glashow \longrightarrow monopoles exist; at $|\text{Tr}\varphi^2| \gg \Lambda^2$ seen/calculable quasi classically

Monopoles become light if $|\varphi^3| \lesssim \Lambda \rightarrow$ At two points, massless!

Let us turn to the (second) cousin of pp 4 & 5 and apply the power of SUSY to penetrate to strong coupling!!!

👉 In one of two vacua at $\text{Tr } \varphi^2 = \pm \Lambda^2$ we have massless monopoles; (Condense upon small deformation)

👉 At energies $\ll \Lambda$ the theory reduces to $\mathcal{N} = 2$ SQED;

👉 Small breaking of $\mathcal{N} = 2 \rightarrow \mathcal{N} = 1$ is introduced (analyze to leading order in breaking);

👉 This theory can be dualized! 😊. $\vec{E} \leftrightarrow \vec{B}$

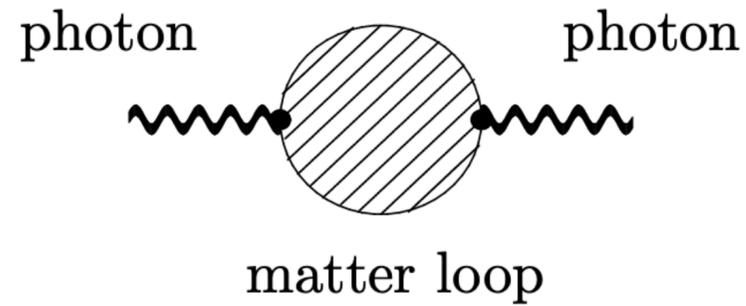
👉 Dual matter (i.e. monopoles) condense, U(1) is spontaneously broken, ANO string is formed

GOLDEN DREAM COMES TRUE ! Well... Sort of ...

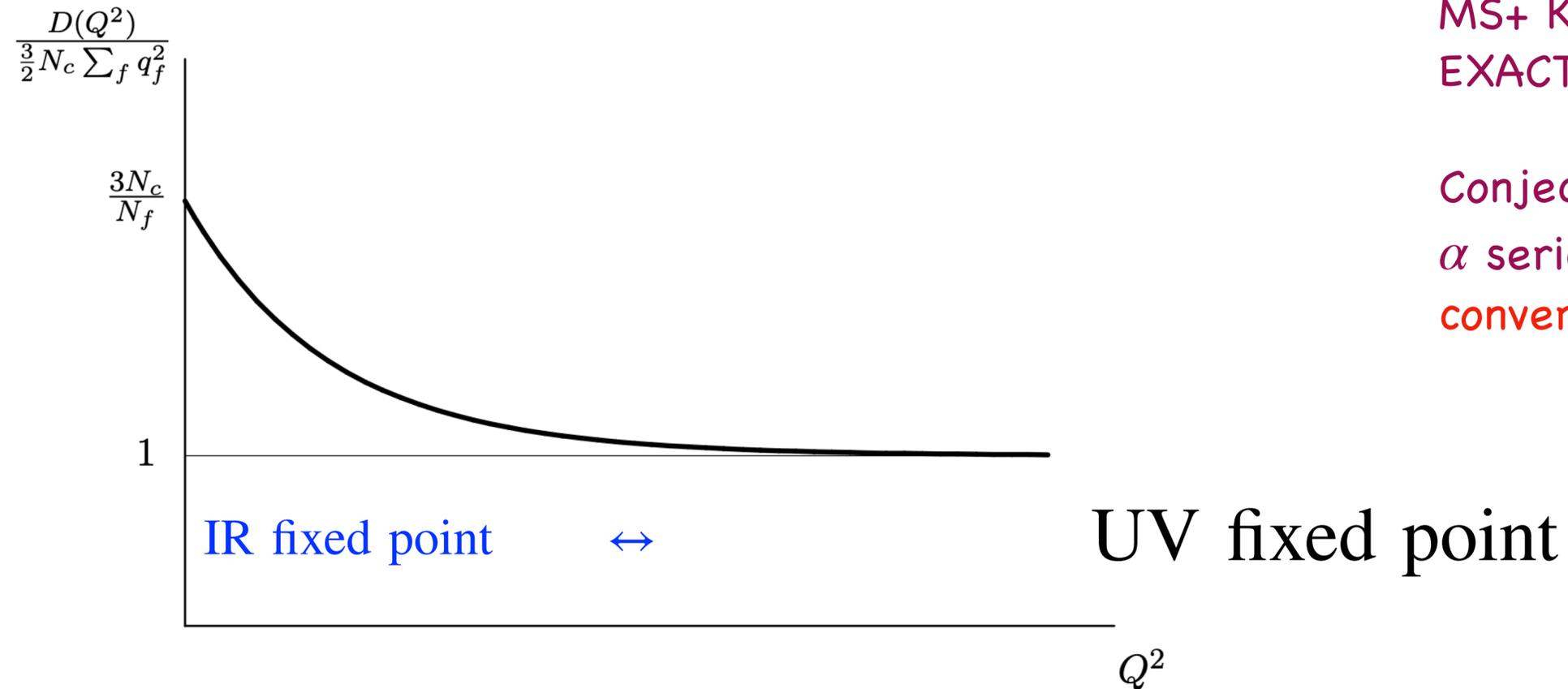
GOLDEN DREAM is to descend down to $\mathcal{N} = 0$!

Supersymmetric analog of R ($e^+e^- \rightarrow$ hadrons)

Recently



$$D(Q^2) = \frac{3}{2} N \sum_f q_f^2 [1 - \gamma(\alpha_s(Q^2))]$$



MS+ K. V. Stepanyantz,
EXACT

Conjectured (not yet proven)
 α series is well-defined and
convergent

$R = D(Q^2) \cdot \left(\frac{3}{2} N_c \sum_f q_f^2\right)^{-1}$ versus Q^2 . The horizontal lines corresponds to $N_f = 3N_c$, i.e. the right edge of the conformal window.

Fifty Years is the age of maturity

Compare: "old" quantum mechanics ~1900-1950;

"new" quantum mechanics ~1925-1975

Feynman's path integral
1948; Aharonov-Bohm
effect 1950s

Fundamental aspects fully worked out, basic principle established, Cornucopia of applications!



QM-50

SUSY-50 Report card: (1) nothing on the pheno side;

(2) many powerful results in strong coupling QFT, with high conceptual value but -- alas -- so far only qualitative value as far as **our** world is concerned.

Future???

