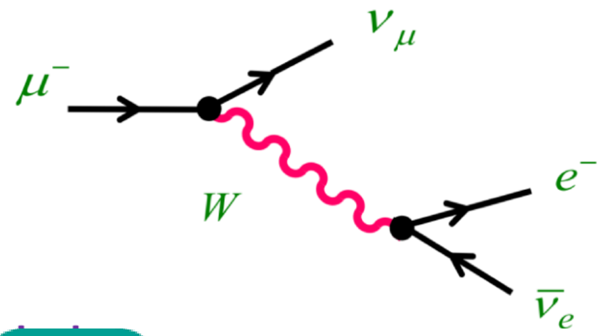
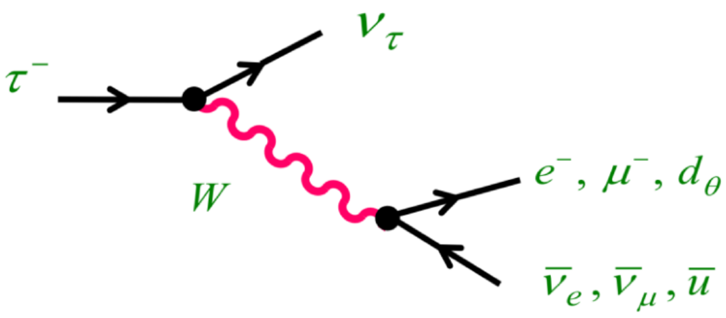
XIVth Latin American Symposium on High Energy Physics

XIV SILAFAE
USFQ, Quito, Ecuador, Nov. 14-18, 2022

New physics tests in lepton decays





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DISCLAIMER

I will not discuss very interesting NP searches that can be performed with charged leptons. Namely:

- **Electron & muon anomalous magnetic moments**
- **Electron & muon electric dipole moments**
- **$\mu\, e \rightarrow \mu\, e$ for $a_\mu^{HVP,LO}$**
- **LU anomalies in semileptonic decays of heavy mesons**
- **LNV**
- **LFV (lepton decays & conversion in nuclei)**
- **Baryogenesis through leptogenesis**
- **…**

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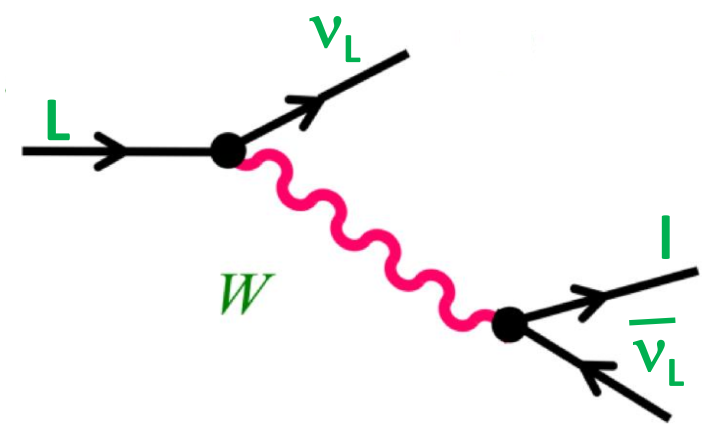
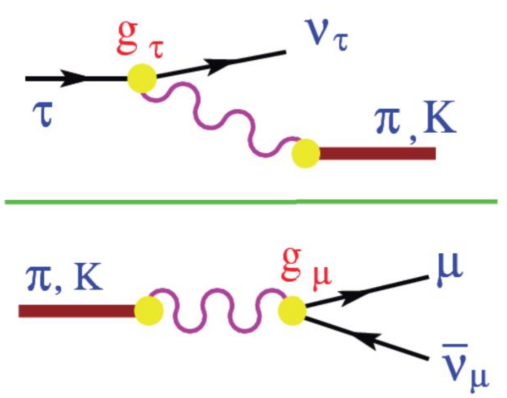


Michel parameters in the presence of massive Dirac and Majorana neutrinos



Improved radiative corrections for

$\tau \to \pi\ (K)\ \nu_\tau\ [\gamma]$ and reliable new physics tests

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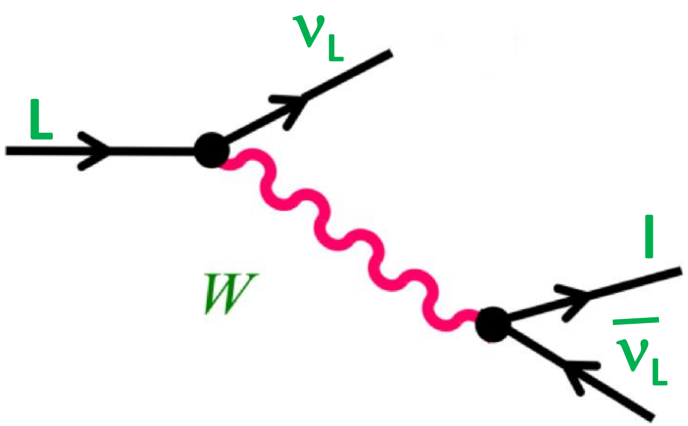

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Michel parameters in the presence of massive Dirac and Majorana neutrinos

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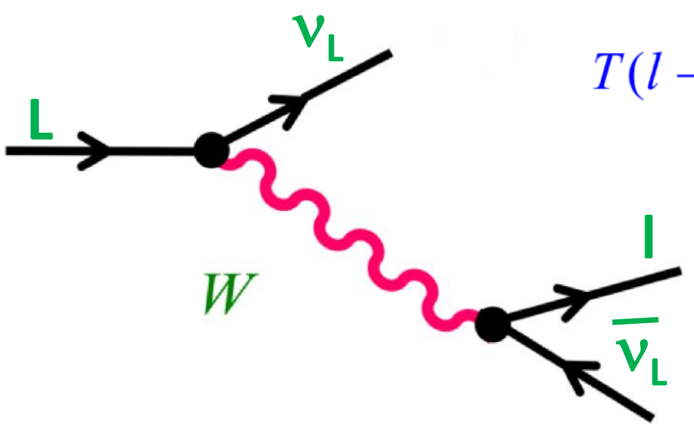
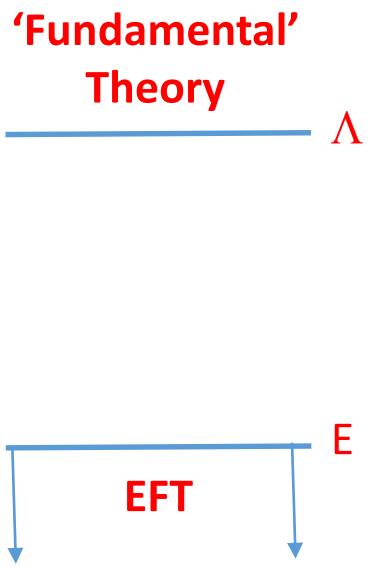


$$T(l \to \nu_l \, l' \bar{\nu}_{l'}) \sim \frac{g^2}{M_W^2 - q^2} \xrightarrow{q^2 << M_W^2} \frac{g^2}{M_W^2} = 4\sqrt{2}\, G_F$$



The most general, derivative-free, four-lepton interaction Hamiltonian, consistent with Lorentz invariance is:

$$\mathcal{H} = 4\frac{G_{ll'}}{\sqrt{2}} \sum_{n,\epsilon,\omega} g^n_{\epsilon\omega} \left[\bar{l'}_\epsilon \Gamma^n (\nu_{l'})_\sigma \right] \left[(\bar{\nu}_l)_\lambda \Gamma_n l_\omega \right] + h.c.$$

Where $\epsilon, \omega, \sigma, \lambda$ label the chiralities (L, R) of fermions, and $n = S, V, T$ the type of interaction: scalar $(\Gamma^S = I)$,vector $(\Gamma^V = \gamma^\mu)$ and tensor $(\Gamma^T = \sigma^{\mu\nu}/\sqrt{2})$.

$\tau_\mu \longrightarrow |g^S_{\epsilon\omega}| \leq 2, |g^V_{\epsilon\omega}| \leq 1$ and $|g^T_{\epsilon\omega}| \leq 1/3$.

For the case of massless neutrinos, the differential decay rate is:

$$\frac{d\Gamma}{dxd\cos\theta} = \frac{m_1}{4\pi^3}\omega^4 G_{ll'}^2\sqrt{x^2 - x_0^2}\left(F(x) - \frac{\xi}{3}\mathcal{P}\sqrt{x^2 - x_0^2}\cos\theta A(x)\right)$$
$$\times\left[1 + \hat{\zeta}\cdot\vec{\mathcal{P}}_{l'}(x,\theta)\right],$$

where $\mathcal{P}$ is the degree of the initial lepton polarization, θ is the angle between the l^- spin and the final charged-lepton momenta, $\omega \equiv (m_1^2 + m_4^2)/2m_1$, $x \equiv E_4/\omega$ is the reduced energy and $x_0 \equiv m_4/\omega$, $\hat{\zeta}$ is an arbitrary direction parallel to the final charged-lepton spin and the polarization vector $\vec{\mathcal{P}}_{l'}$ is:

$$\vec{\mathcal{P}}_{l'} = P_{T_1}\cdot\hat{x} + P_{T_2}\cdot\hat{y} + P_L\cdot\hat{z}.$$

The components of $\vec{\mathcal{P}}_{l'}$ are, respectively:

$$P_{T_1} = \mathcal{P}\sin\theta\cdot F_{T_1}(x)\ /\ \left\{F(x) - \frac{\xi}{3}\mathcal{P}\sqrt{x^2 - x_0^2}\cos\theta A(x)\right\},$$

$$P_{T_2} = \mathcal{P}\sin\theta\cdot F_{T_2}(x)\ /\ \left\{F(x) - \frac{\xi}{3}\mathcal{P}\sqrt{x^2 - x_0^2}\cos\theta A(x)\right\},$$

$$P_L = \frac{-F_{IP}(x) + \mathcal{P}\cos\theta\cdot F_{AP}(x)}{F(x) - \frac{\xi}{3}\mathcal{P}\sqrt{x^2 - x_0^2}\cos\theta A(x)}.$$

These functions are written in terms of the well-known **Michel Parameters** $(\rho, \eta, \delta, \xi, \eta'', \xi', \xi'', \alpha', \beta')$:

$$F(x) = x(1-x) + \frac{2}{9}\rho\left(4x^2 - 3x - x_0^2\right) + \eta x_0(1-x),$$

$$A(x) = 1 - x + \frac{2}{3}\delta\left(4x - 4 + \sqrt{1-x_0^2}\right),$$

$$F_{T_1}(x) = \frac{1}{12}\left[-2\left(\xi'' + 12\left(\rho - \frac{3}{4}\right)\right)(1-x)x_0 - 3\eta(x^2 - x_0^2) + \eta''(-3x^2 + 4x - x_0^2)\right],$$

$$F_{T_2}(x) = \frac{1}{3}\sqrt{x^2 - x_0^2}\left[3\frac{\alpha'}{\mathcal{A}}(1-x) + 2\frac{\beta'}{\mathcal{A}}\sqrt{1-x_0^2}\right],$$

$$F_{IP}(x) = \frac{1}{54}\sqrt{x^2 - x_0^2}\left[9\xi'\left(-2x + 2 + \sqrt{1-x_0^2}\right) + 4\xi\left(\delta - \frac{3}{4}\right)\left(4x - 4 + \sqrt{1-x_0^2}\right)\right],$$

$$F_{AP}(x) = \frac{1}{6}\left[\xi''(2x^2 - x - x_0^2) + 4\left(\rho - \frac{3}{4}\right)(4x^2 - 3x - x_0^2) + 2\eta''(1-x)x_0\right].$$

As an example:

$$\eta = \frac{1}{2}\mathrm{Re}[g^V_{LL}g^{S*}_{RR} + g^V_{RR}g^{S*}_{LL} + g^V_{LR}(g^{S*}_{RL} + 6g^{T*}_{RL}) + g^V_{RL}(g^{S*}_{LR} + 6g^{T*}_{LR})].$$

In the SM, $\rho = \delta = 3/4$, $\eta = \eta^{''} = \alpha^{'} = \beta^{'} = 0$ and $\xi = \xi^{'} = \xi^{''} = 1$.

	$\mu^- \to e^- \nu_\mu \bar{\nu}_e$	$\tau^- \to e^- \nu_\tau \bar{\nu}_e$	$\tau^- \to \mu^- \nu_\tau \bar{\nu}_\mu$
ρ	0.74979 ± 0.00026	0.747 ± 0.010	0.763 ± 0.020
η	0.057 ± 0.034	—	0.094 ± 0.073
		$\eta = 0.016 \pm 0.013$	
ξ	$1.0009^{+0.0016}_{-0.0007}$	0.994 ± 0.040	1.030 ± 0.059
$\xi\delta$	$0.7511^{+0.0012}_{-0.0006}$	0.734 ± 0.028	0.778 ± 0.037
$\xi^{'}$	1.00 ± 0.04	—	—
$\xi^{''}$	0.65 ± 0.36	—	—

The total decay rate is:

$$\Gamma_{l \to l'} = \frac{\hat{G}_{ll'}^2 m_1^5}{192\pi^3} f(m_4^2/m_1^2) \left(1 + \delta_{RC}^{ll'}\right),$$

where

$$\hat{G}_{ll'} \equiv G_{ll'} \sqrt{1 + 4\eta \frac{m_4}{m_1} \frac{g(m_4^2/m_1^2)}{f(m_4^2/m_1^2)}}$$

$f(x) = 1 - 8x - 12x^2 \log(x) + 8x^3 - x^4, g(x) = 1 + 9x - 9x^2 - x^3 + 6x(1+x)log(x)$ and the SM radiative correction $\delta_{RC}^{ll'}$ has been included.

$$\delta_{RC}^{ll'} = \frac{\alpha}{2\pi} \left[\frac{25}{4} - \pi^2 + \mathcal{O}\left(\frac{m_4^2}{m_1^2} \right) \right] + \dots$$

$$G_{ll'}^2 = \left[\frac{g^2}{4\sqrt{2} M_W^2} (1 + \Delta r) \right]^2 \left[1 + \frac{3}{5} \frac{m_1^2}{M_W^2} + \frac{9}{5} \frac{m_4^2}{M_W^2} + \mathcal{O}\left(\frac{m_4^4}{m_1^2 M_W^2} \right) \right]$$

The current neutrino ($\nu_{L,R}$) is assumed to be the superposition of the mass-eigenstate neutrinos (N_j) with the mass m_j, that is,

$$\nu_{lL} = \sum_j U_{lj} N_{jL}, \quad \nu_{lR} = \sum_j V_{lj} N_{jR},$$

where $j = \{1, 2, ..., n\}$ with n the number of mass-eigenstate neutrinos. Thus, we can write the effective Hamiltonian in the mass basis, for the process $l^- \longrightarrow l'^- \overline{N}_j N_k$.

$$\mathcal{H} = 4\frac{G_{ll'}}{\sqrt{2}} \sum_{j,k} \Big\{ g^S_{LL} \left[\overline{l'_L} V_{l'j} N_{jR}\right] \left[\overline{N}_{kR} V^*_{lk} l_L\right] + g^V_{LL} \left[\overline{l'_L} \gamma^\mu U_{l'j} N_{jL}\right] \left[\overline{N}_{kL} U^*_{lk} \gamma_\mu l_L\right]$$

$$+ g^S_{RR} \left[\overline{l'_R} U_{l'j} N_{jL}\right] \left[\overline{N}_{kL} U^*_{lk} l_R\right] + g^V_{RR} \left[\overline{l'_R} \gamma^\mu V_{l'j} N_{jR}\right] \left[\overline{N}_{kR} V^*_{lk} \gamma_\mu l_R\right]$$

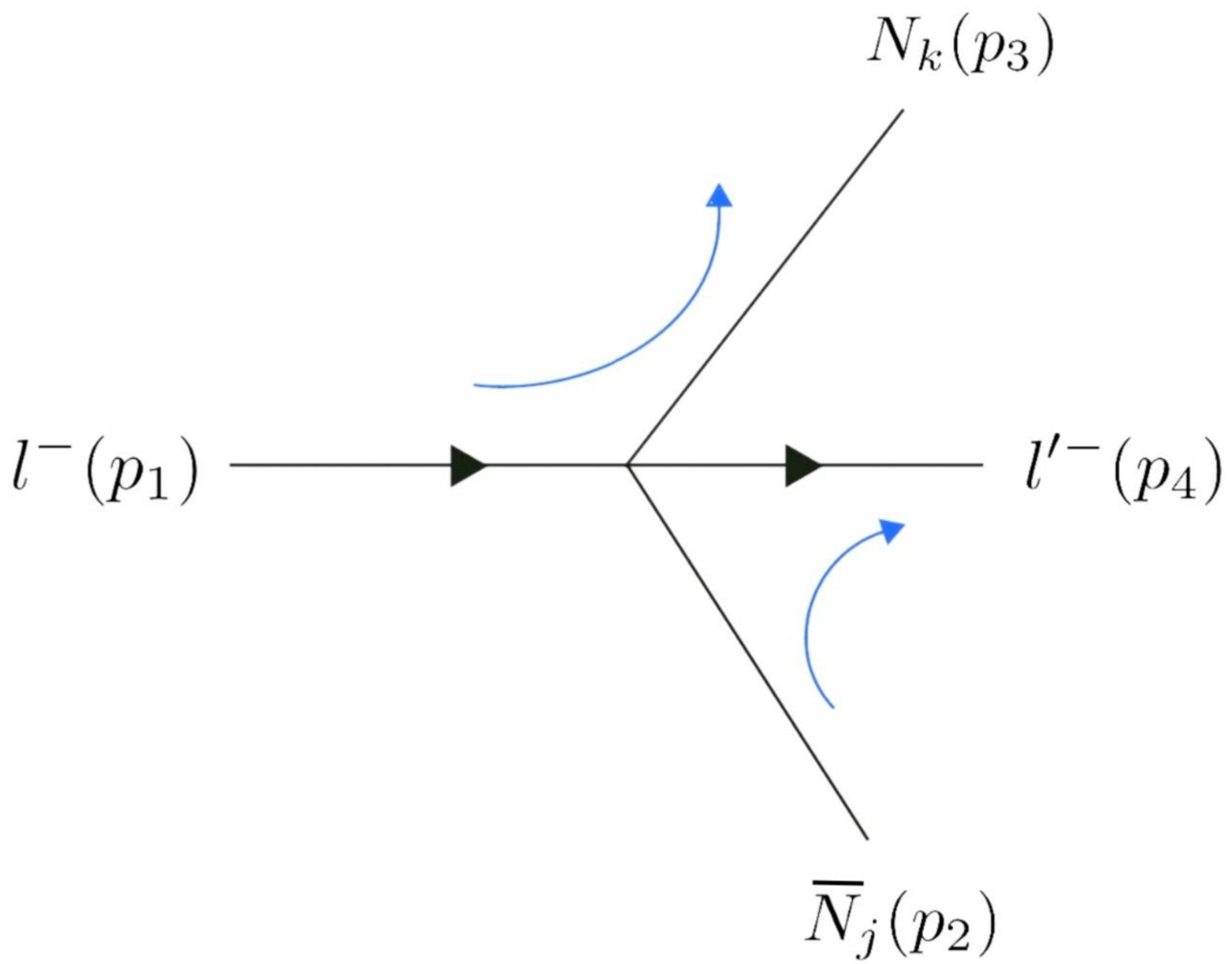
$$+ g^S_{LR} \left[\overline{l'_L} V_{l'j} N_{jR}\right] \left[\overline{N}_{kL} U^*_{lk} l_R\right] + g^V_{LR} \left[\overline{l'_L} \gamma^\mu U_{l'j} N_{jL}\right] \left[\overline{N}_{kR} V^*_{lk} \gamma_\mu l_R\right]$$

$$+ g^T_{LR} \left[\overline{l'_L} \frac{\sigma^{\mu\nu}}{\sqrt{2}} V_{l'j} N_{jR}\right] \left[\overline{N}_{kL} U^*_{lk} \frac{\sigma_{\mu\nu}}{\sqrt{2}} l_R\right] + g^S_{RL} \left[\overline{l'_R} U_{l'j} N_{jL}\right] \left[\overline{N}_{kR} V^*_{lk} l_L\right]$$

$$+ g^V_{RL} \left[\overline{l'_R} \gamma^\mu V_{l'j} N_{jR}\right] \left[\overline{N}_{kL} U^*_{lk} \gamma_\mu l_L\right] + g^T_{RL} \left[\overline{l'_R} \frac{\sigma^{\mu\nu}}{\sqrt{2}} U_{l'j} N_{jL}\right] \left[\overline{N}_{kR} V^*_{lk} \frac{\sigma_{\mu\nu}}{\sqrt{2}} l_L\right] \Big\}.$$

Note that $\overline{N}$ represents an antineutrino for the Dirac neutrino case, but should be identified with N for the Majorana neutrino case ($N = N^c = C\overline{N}^T$).

Dirac Neutrinos



- Neutrino $\neq$ Antineutrino.
- One possible first-order Feynman diagram.
- Well defined fermionic flux.

The Hamiltonian for the case of Majorana neutrinos is

$$\mathcal{H} = 4\frac{G_{ll'}}{\sqrt{2}} \sum_{j,k} \Big\{ g^S_{LL} \left[\bar{l}'_L V_{l'j} N_{jR} \right] \left[N_{kR} V^*_{lk} l_L \right] + g^V_{LL} \left[\bar{l}'_L \gamma^\mu U_{l'j} N_{jL} \right] \left[N_{kL} U^*_{lk} \gamma_\mu l_L \right]$$

$$+ g^S_{RR} \left[\bar{l}'_R U_{l'j} N_{jL} \right] \left[N_{kL} U^*_{lk} l_R \right] + g^V_{RR} \left[\bar{l}'_R \gamma^\mu V_{l'j} N_{jR} \right] \left[N_{kR} V^*_{lk} \gamma_\mu l_R \right]$$

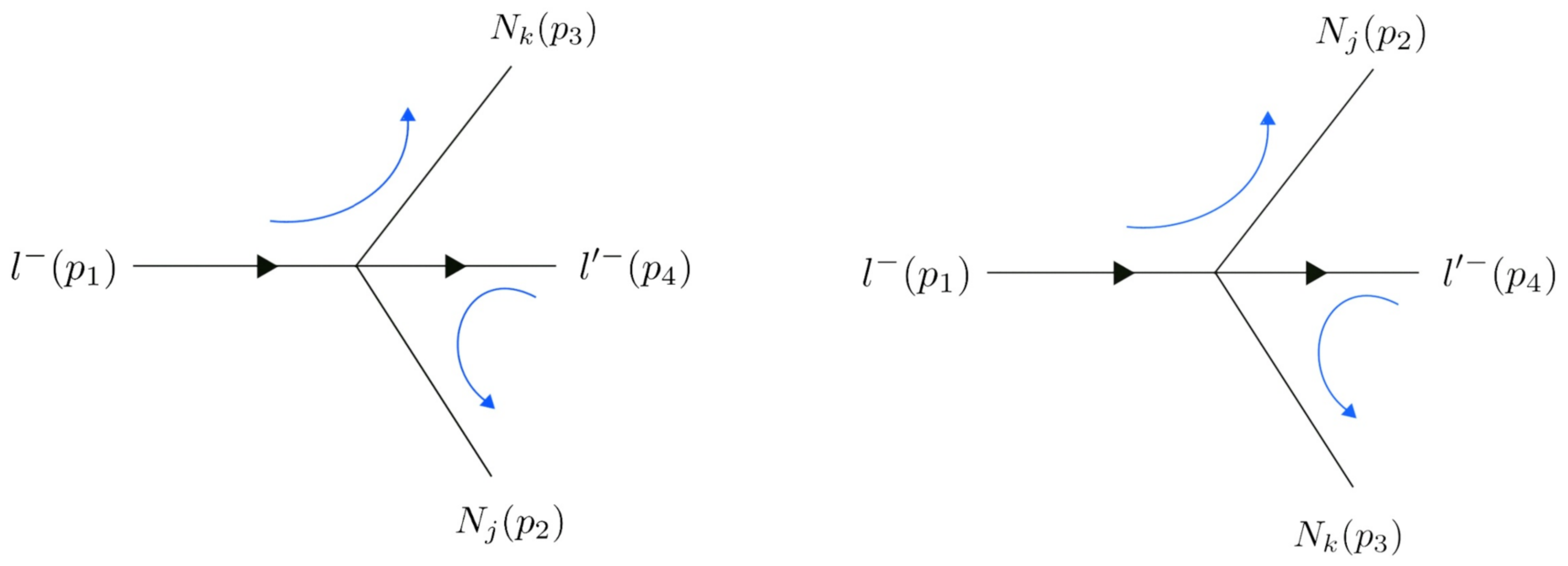
$$+ g^S_{LR} \left[\bar{l}'_L V_{l'j} N_{jR} \right] \left[N_{kL} U^*_{lk} l_R \right] + g^V_{LR} \left[\bar{l}'_L \gamma^\mu U_{l'j} N_{jL} \right] \left[N_{kR} V^*_{lk} \gamma_\mu l_R \right]$$

$$+ g^T_{LR} \left[\bar{l}'_L \frac{\sigma^{\mu\nu}}{\sqrt{2}} V_{l'j} N_{jR} \right] \left[N_{kL} U^*_{lk} \frac{\sigma_{\mu\nu}}{\sqrt{2}} l_R \right] + g^S_{RL} \left[\bar{l}'_R U_{l'j} N_{jL} \right] \left[N_{kR} V^*_{lk} l_L \right]$$

$$+ g^V_{RL} \left[\bar{l}'_R \gamma^\mu V_{l'j} N_{jR} \right] \left[N_{kL} U^*_{lk} \gamma_\mu l_L \right] + g^T_{RL} \left[\bar{l}'_R \frac{\sigma^{\mu\nu}}{\sqrt{2}} U_{l'j} N_{jL} \right] \left[N_{kR} V^*_{lk} \frac{\sigma_{\mu\nu}}{\sqrt{2}} l_L \right] \Big\}.$$

Majorana Neutrinos

The possible first order Feynman diagrams for the $l^- \longrightarrow l'^- N_j N_k$ decay are:



The first diagram leads to the same matrix element as the Dirac case, while the second diagram is only possible in the Majorana neutrino case and we already defined the orientation for each fermion chain.

Majorana Neutrinos

Then, after integrating over the neutrinos momenta, the decay rate will have the following dependence on the amplitude:

$$d\Gamma \propto \frac{1}{2}\sum_{j,k} |\mathcal{M}^{D}_{jk} - \mathcal{M}^{M}_{jk}|^2$$

$$= \frac{1}{2}\sum_{j,k} \left\{|\mathcal{M}^{D}_{jk}|^2 + |\mathcal{M}^{M}_{jk}|^2 - 2\,\mathrm{Re}(\mathcal{M}^{D}_{jk}\mathcal{M}^{M*}_{jk})\right\}$$

$$= \sum_{j,k} |\mathcal{M}^{D}_{jk}|^2 - \sum_{j,k} \mathrm{Re}(\mathcal{M}^{D}_{jk}\mathcal{M}^{M*}_{jk}).$$

The interference term distinguishes between Dirac and Majorana cases, which is sometimes called the Majorana term.

The differential decay rate taking into account finite Dirac or Majorana neutrino masses is:

$$\frac{d\Gamma}{dx d\cos\theta} = \sum_{j,k} \frac{m_1}{4\pi^3} \omega^4 G_{ll'}^2 \sqrt{x^2 - x_0^2}$$
$$\times \Big(\big(F_{IS}(x) + F'_{IS}(x) + F''_{IS}(x)\big) - \mathcal{P}\cos\theta \big(F_{AS}(x) + F'_{AS}(x) + F''_{AS}(x)\big)\Big)$$
$$\times \left[1 + \hat{\zeta} \cdot \vec{\mathcal{P}}_{l'}(x,\theta)\right],$$

Linear in ν masses

Quadratic in ν masses

where

$$\vec{\mathcal{P}}_{l'} = P_{T_1} \cdot \hat{x} + P_{T_2} \cdot \hat{y} + P_L \cdot \hat{z}.$$

and the components of $\vec{\mathcal{P}}_{l'}$ are, respectively,

‘ is linear, ´´ is quadratic in ν masses

$$P_{T_1} = \mathcal{P}\sin\theta \cdot \big(F_{T_1}(x) + F'_{T_1}(x) + F''_{T_1}(x)\big) \ / \ N,$$
$$P_{T_2} = \mathcal{P}\sin\theta \cdot \big(F_{T_2}(x) + F'_{T_2}(x) + F''_{T_2}(x)\big) \ / \ N,$$
$$P_L = \Big(- \big(F_{IP}(x) + F'_{IP}(x) + F''_{IP}(x)\big) + \mathcal{P}\cos\theta \cdot \big(F_{AP}(x) + F'_{AP}(x) + F''_{AP}(x)\big)\Big) \ / \ N.$$

with N the normalization factor:
$$N = \big(F_{IS}(x) + F'_{IS}(x) + F''_{IS}(x)\big) - \mathcal{P}\cos\theta\big(F_{AS}(x) + F'_{AS}(x) + F''_{AS}(x)\big) .$$

Total Decay Rate

Finally, integrating over all energy and angular configurations we obtained:

$$\Gamma_{l\to l'} = \sum_{j,k} \frac{\hat{G}^2_{ll'} m_1^5}{192\pi^3} f(m_4^2/m_1^2) \left(1 + \delta_{RC}^{ll'}\right),$$

where

Linear in ν masses

$$\hat{G}_{ll'} \equiv G_{ll'} \left\{ (I)_{jk} + 4(\eta)_{jk} \frac{m_4}{m_1} \frac{g(m_4^2/m_1^2)}{f(m_4^2/m_1^2)} - 2\frac{m_j}{m_1} \left[(\kappa_L^+)_{jk} \frac{f'(m_4^2/m_1^2)}{f(m_4^2/m_1^2)} + (\kappa_R^+)_{kj} \frac{m_4}{m_1} \frac{g'(m_4^2/m_1^2)}{f(m_4^2/m_1^2)} \right] \right.$$
$$\left. - 4\frac{m_j m_k}{m_1^2} \left[(C^+)_{jk} \frac{f''(m_4^2/m_1^2)}{f(m_4^2/m_1^2)} + 3(H^+)_{jk} \frac{m_4}{m_1} \frac{g''(m_4^2/m_1^2)}{f(m_4^2/m_1^2)} \right] \right\}^{1/2},$$

Quadratic in ν masses

with the functions defined as:

$$f'(x) = -1 + 6x - 2x^3 + 3x^2 \left(4 \operatorname{arctanh}\left(\frac{x-1}{x+1}\right) - 1\right),$$

$$f''(x) = 1 - 3x + 3x^2 - x^3,$$

$$g'(x) = 2 - 6x^2 + x^3 + 3x \left(4 \operatorname{arctanh}\left(\frac{x-1}{x+1}\right) + 1\right),$$

$$g''(x) = 1 - x^2 + 2x \log(x).$$

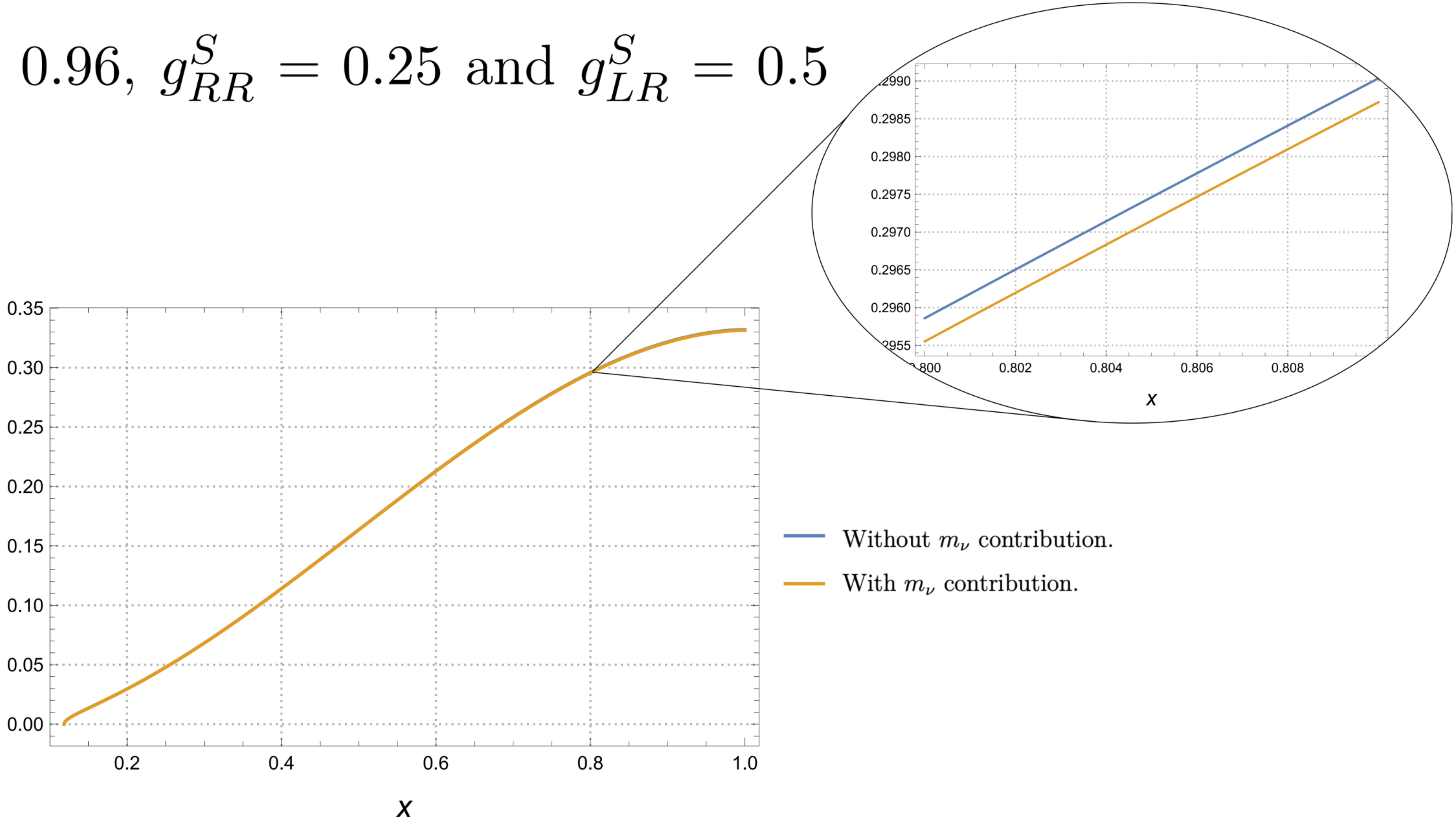
Considering the constraints on an invisible heavy neutrino[1], we can estimate the suppression of the neutrino mass dependent terms compared with the ones without this dependence (standard Michel distribution).

Neutrino	Mass (MeV)	Mixing $\|U_{l4}\|^2$	Process
Heavy ($l = e$)	0.001 - 0.45	10^{-3}	$n \to p + e + \nu_4$
	10 - 55	10^{-8}	$\pi \to e\nu_4$
	135 - 350	10^{-6}	$k \to e\nu_4$
Heavy ($l = \mu$)	10 - 30	10^{-4}	$\pi \to \mu\nu_4$
	70 - 300	10^{-5}	$k \to \mu\nu_4$
	175 - 300	10^{-8}	$k \to \mu\nu_4$
Heavy ($l = \tau$)	100 - 1.2×10^3	$10^{-7} - 10^{-3}$	$\tau \to \nu_4 + 3\pi$
	1×10^3-60×10^3	$10^{-5} - 10^{-3}$	$Z \to \nu\nu_4$

[1] A. de Gouvea and A. Kobach, Phys.Rev.D 93 (2016).

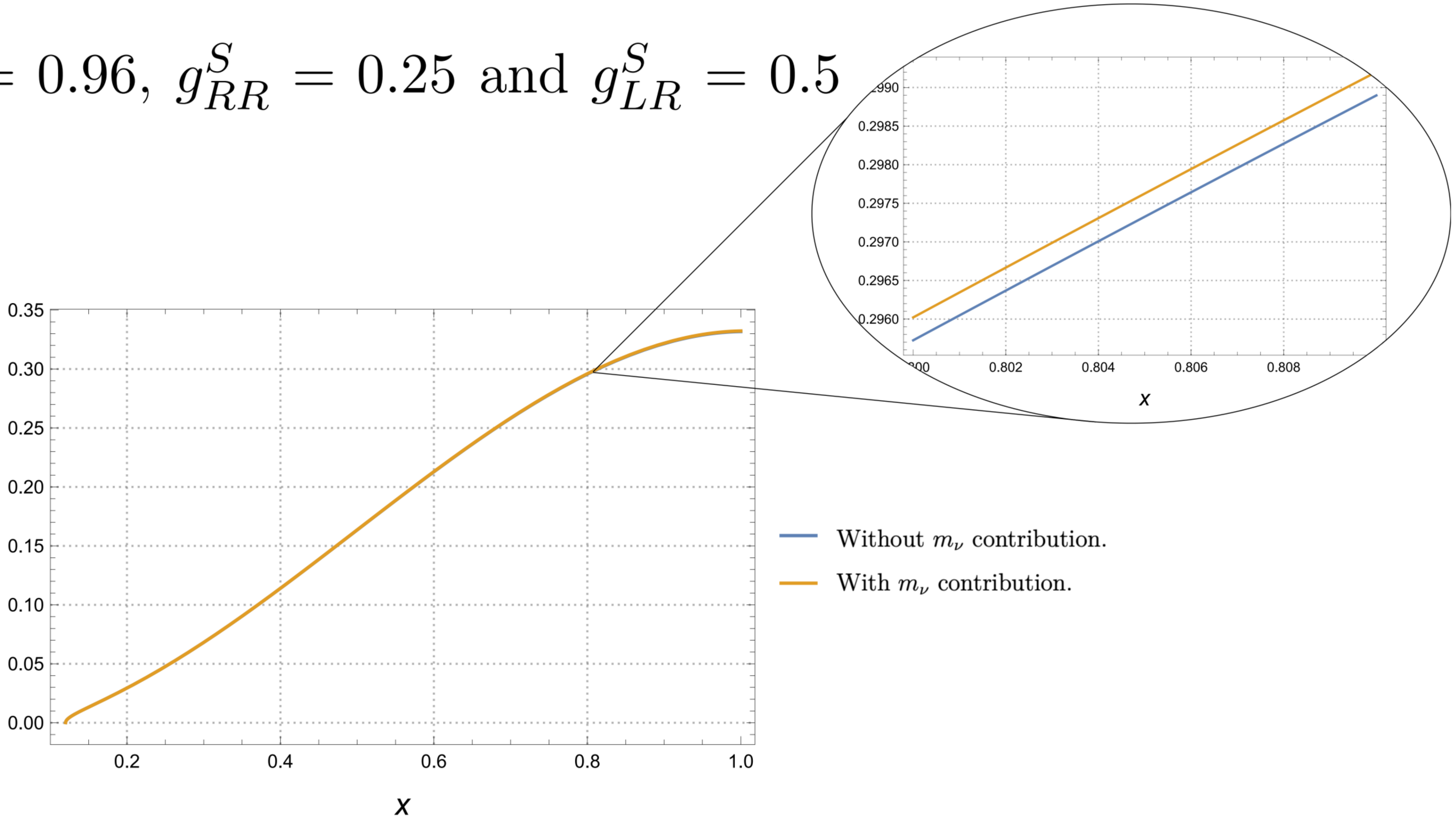
Neutrino	Mass (MeV)	Mixing Suppression	Linear Term Suppression (m_ν)	Quadratic Term Suppression (m_ν^2)
Light (2)	1×10^{-6}	—	10^{-9}	10^{-18}
Heavy (1) ($l = e$)	0.001 - 0.45	10^{-3}	$10^{-9} - 10^{-7}$	$10^{-18} - 10^{-16}$
	10 - 55	10^{-8}	10^{-10}	10^{-19}
	135 - 350	10^{-6}	10^{-7}	10^{-16}
Heavy (1) ($l = \mu$)	10 - 30	10^{-4}	10^{-6}	10^{-15}
	70 - 300	10^{-5}	$10^{-7} - 10^{-6}$	$10^{-16} - 10^{-15}$
	175 - 300	10^{-8}	10^{-9}	10^{-18}
Heavy (1) ($l = \tau$)	100 - 1.2×10^{3}	$10^{-7} - 10^{-3}$	$10^{-8} - 10^{-3}$	$10^{-18} - 10^{-12}$
	$1\times10^{3} - 60\times10^{3}$	$10^{-5} - 10^{-3}$	$10^{-5} - 10^{-3}$	$10^{-14} - 10^{-12}$
Heavy (2) ($\mu \to eNN$)	10 - 30	10^{-12}	10^{-14}	10^{-16}
	175 - 300	$10^{-14} - 10^{-11}$	$10^{-15} - 10^{-12}$	$10^{-16} - 10^{-13}$
Heavy (2) ($\tau \to eNN$)	135 - 350	$10^{-13} - 10^{-9}$	$10^{-14} - 10^{-10}$	$10^{-14} - 10^{-10}$
Heavy (2) ($\tau \to \mu NN$)	100 - 300	$10^{-12} - 10^{-8}$	$10^{-13} - 10^{-9}$	$10^{-14} - 10^{-10}$
	175 - 350	$10^{-15} - 10^{-11}$	$10^{-16} - 10^{-12}$	$10^{-16} - 10^{-12}$

$$g^V_{LL} = 0.96,\ g^S_{RR} = 0.25 \text{ and } g^S_{LR} = 0.5$$



(a) Dirac neutrinos.

$$g^V_{LL} = 0.96,\ g^S_{RR} = 0.25 \text{ and } g^S_{LR} = 0.5$$



(b) Majorana neutrinos.

$g^V_{LL} = 0.96,\ g^S_{RR} = 0.25$ and $g^S_{LR} = 0.5$

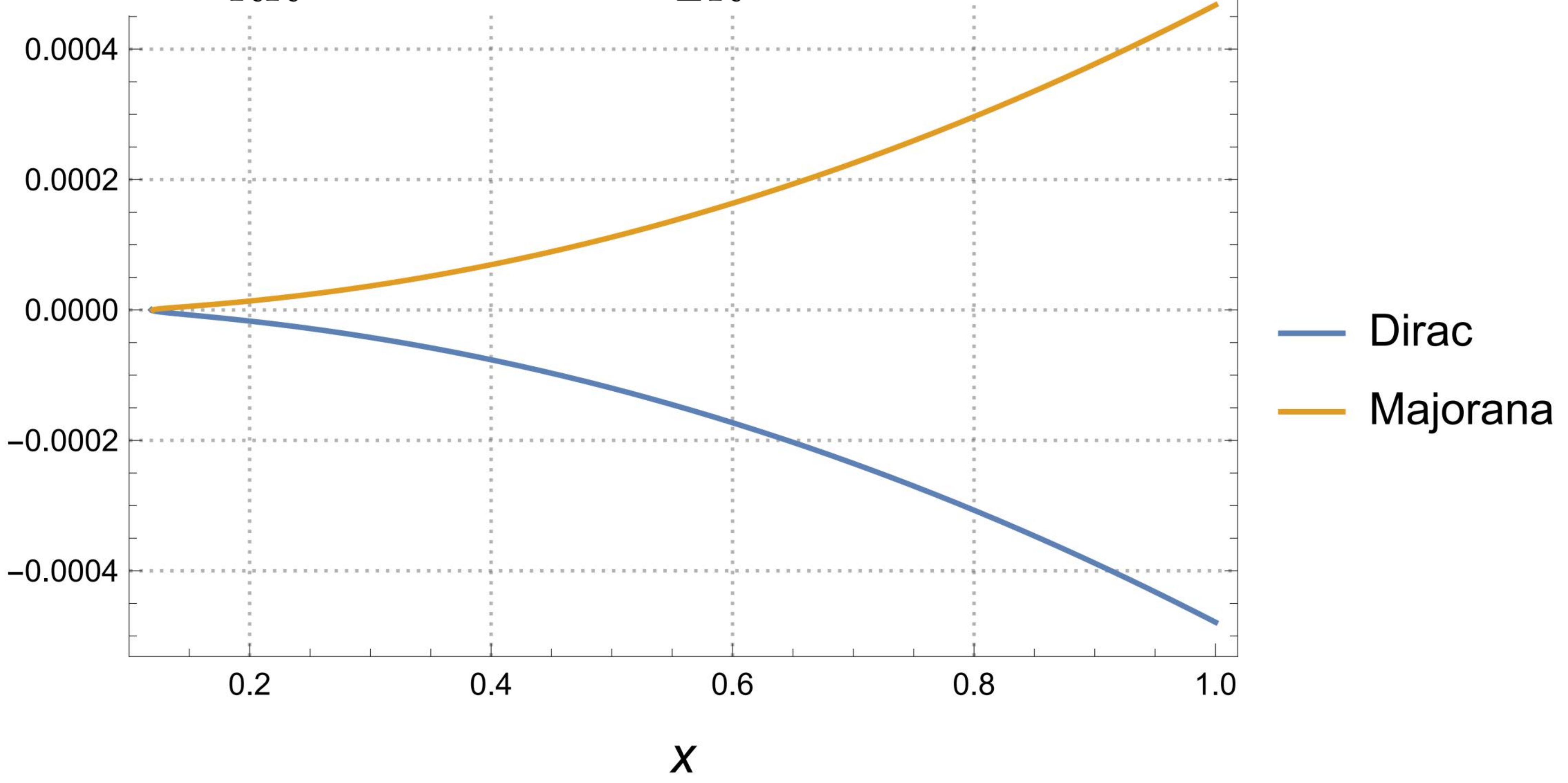


Figure 6: Neutrino mass contribution to Dirac and Majorana distributions.

- In this work we have studied the leptonic decay $l^- \longrightarrow l'^- N_j N_k$, where N_j and N_k are mass-eigenstate neutrinos.
- We have constructed its matrix element by using the most general four-lepton effective interaction Hamiltonian and obtained the specific energy and angular distribution of the final charged lepton, complemented with the decaying and final charged-lepton polarization and the effects of Dirac and Majorana neutrino masses.
- We have introduced generalized Michel parameters, that arise due to considering finite neutrino masses and a specific neutrino nature.
- We discuss their properties and main differences, together with some examples of its application to model-dependent theories.
- Specifically, for the case of τ-decay with one heavy final-state neutrino with a mass around $10^2 - 10^3 MeV$ the linear term suppression could be of order 10^{-3}, low enough to be measured in current and forthcoming experiments.
- Finally, it would also be interesting to analyze other type of leptonic decays, such as radiative muon and tau decay with Dirac and Majorana neutrinos, where new information could be obtained.

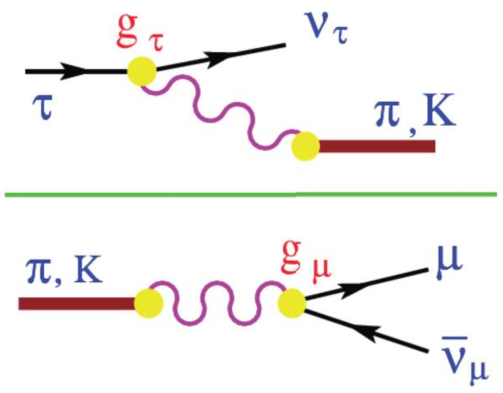
What is new:

- We write our expressions in the PDG parametrization form, in a way that complements all previous results, facilitating their application to model-dependent scenarios.
- We classify the Dirac and Majorana contributions with the help of a flag parameter $\epsilon = 0, 1$, making easier to distinguish between Dirac and Majorana nature of neutrinos.
- We also introduced and discussed the leading W-boson propagator correction to the differential decay rate including the final charged-lepton polarization.

Previous work on Michel parameters: Michel'50, Bouchiat-Michel'57, Shrock'82, Doi-Kotani-Takasugi'85, Mursula-Scheck'85, Fetscher-Gerber-Johnson'86, Langacker-London'89,Fetscher'94, Stahl-Voss'97,Flores Tlalpa-López Castro-Roig'16,Arbuzov-Kopylova'16,... And of course all the essential work on the required RadCors and the precise measurements.

XIVth Latin American Symposium on High Energy Physics

XIV SILAFAE
USFQ, Quito, Ecuador, Nov. 14-18, 2022



Improved radiative corrections for $\tau \to \pi\ (K)\ \nu_\tau\ [\gamma]$ and reliable new physics tests

Pablo Roig
Cinvestav (Mexico)

In collaboration with:
M.A. Arroyo-Ureña (BUAP & Cinvestav, Mexico)
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G. López-Castro (Cinvestav, Mexico)
I. Rosell (UCH-CEU,Valencia,Spain)

Cinvestav

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PRD 104 (2021) 9, L091502 [arXiv:2107.04603]

OUTLINE

1) Motivation

2) $P \to \mu\, \nu_\mu\, [\gamma]$ (P=π,K)

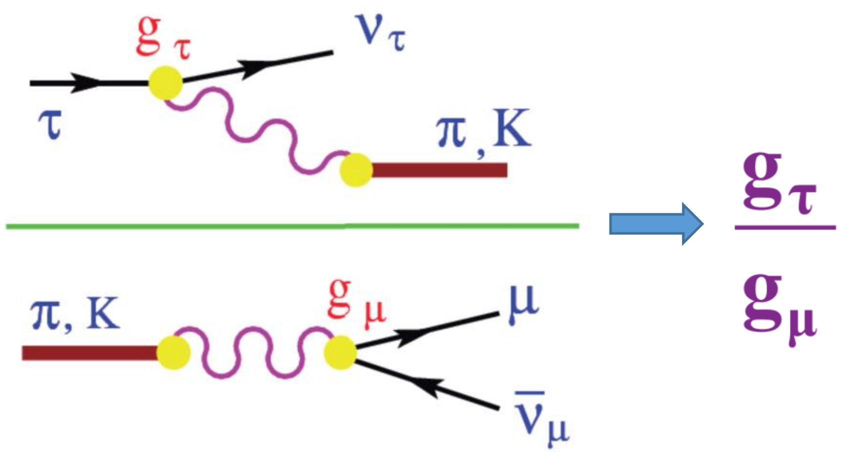
3) $\tau \to P\, \nu_\tau\, [\gamma]$ (P=π,K)

4) Calculation of $R_{\tau/P} \equiv \frac{\Gamma(\tau \to P\nu_\tau[\gamma])}{\Gamma(P \to \mu\nu_\mu[\gamma])}$

5) Results

6) Applications

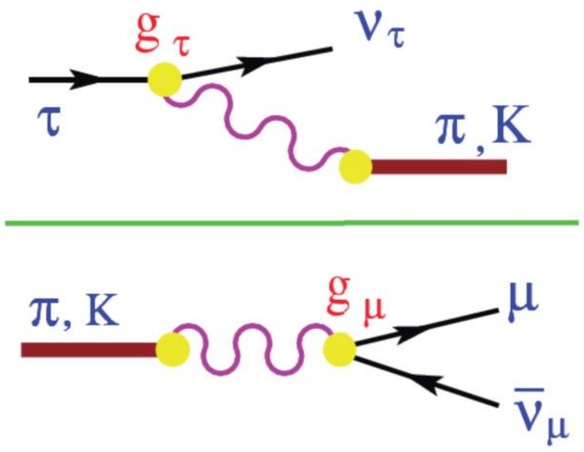
7) Conclusions



1. Motivation

- ✓ Lepton Universality (LU) as a basic tenet of the Standard Model (SM).
  - ✓ A few anomalies observed in semileptonic B meson decays*. (See talks by Irina and E. Rojas)
  - ✓ Lower energy observables currently provide the most precise test of LU**.
- ✓ We aim to test muon-tau lepton universality through the ratio (P = π, K)***:

$$R_{\tau/P} \equiv \frac{\Gamma(\tau \to P\nu_\tau[\gamma])}{\Gamma(P \to \mu\nu_\mu[\gamma])} = \left|\frac{g_\tau}{g_\mu}\right|_P^2 R^{(0)}_{\tau/P}\left(1+\delta R_{\tau/P}\right)$$



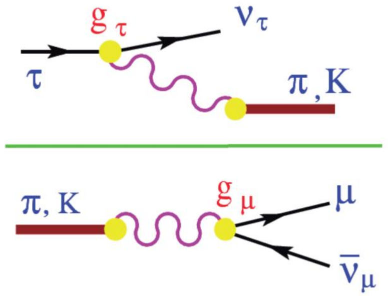
  - ✓ $g_\tau = g_\mu$ according to LU.
  - ✓ $R_{\tau/P}^{(0)}$ is the LO result $$R^{(0)}_{\tau/P} = \frac{1}{2}\frac{M_\tau^3}{m_\mu^2 m_P}\frac{(1-m_P^2/M_\tau^2)^2}{(1-m_\mu^2/m_P^2)^2}\,.$$
  - ✓ $\delta R_{\tau/P}$ encodes the radiative corrections.
- ✓ $\delta R_{\tau/P}$ was calculated by Decker & Finkemeier (DF'95) ^ :
  - ✓ $\delta R_{\tau/\pi} = (0.16 \pm 0.14)\%$ and $\delta R_{\tau/K} = (0.90 \pm 0.22)\%$.
- ✓ Important phenomenological and theoretical reasons to address the analysis again.

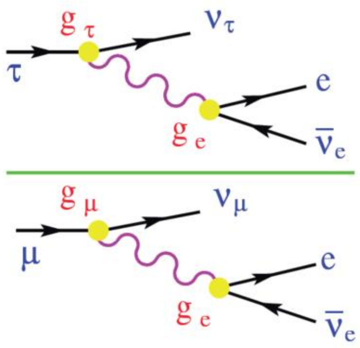
* Albrecht et al.'21
** Bryman et al.'21

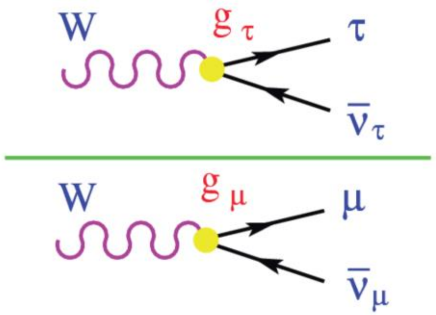
*** Marciano & Sirlin'93
^ Decker & Finkemeier'95

1. Motivation

- Phenomenological disagreement in LU tests:
  - Using $\frac{\Gamma(\tau \to P\nu_\tau[\gamma])}{\Gamma(P \to \mu\nu_\mu[\gamma])}$ and DF'95*, HFLAV** reported:
    - $|g_\tau/g_\mu|_\pi$ = 0.9958 ± 0.0026 (at 1.6σ of LU)
    - $|g_\tau/g_\mu|_K$ = 0.9879 ± 0.0063 (at 1.9σ of LU)
  - Using $\frac{\Gamma(\tau \to e\bar{\nu}_e\nu_\tau[\gamma])}{\Gamma(\mu \to e\bar{\nu}_e\nu_\mu[\gamma])}$, HFLAV** reported:
    - $|g_\tau/g_\mu|$ = 1.0010 ± 0.0014 (at 0.7σ of LU)
  - Using $\frac{\Gamma(W \to \tau\nu_\tau)}{\Gamma(W \to \mu\nu_\mu)}$, CMS and ATLAS*** and reported:
    - $|g_\tau/g_\mu|$ = 0.995 ± 0.006 (at 0.8σ of LU)







* Decker & Finkemeier'95
** HFLAV'21
*** CMS'21, ATLAS'21

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✓ Theoretical issues within DF'95*:

- ✓ Hadronic form factors are different for real- and virtual-photon corrections, do not satisfy the correct QCD short-distance behavior, violate unitarity, analicity and the chiral limit at leading non-trivial orders.
- ✓ A cutoff to regulate the loop integrals (separating long- and short-distance corrections)
- ✓ Unrealistic uncertainties (purely $O(e^2p^2)$ ChPT size).

* Decker & Finkemeier'95
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✓ $|g_\tau/g_\mu| = 0.995 \pm 0.006$ (at 0.8σ of LU)

✓ By-products of the project:

✓ Radiative corrections in $\Gamma(\tau \to P\nu_\tau[\gamma])$.

✓ CKM unitarity test via $\Gamma(\tau \to K\nu_\tau[\gamma])$ or via the ratio $\Gamma(\tau \to K\nu_\tau[\gamma]) / \Gamma(\tau \to \pi\nu_\tau[\gamma])$.

✓ Constraints on possible non-standard interactions in $\Gamma(\tau \to P\nu_\tau[\gamma])$^.

✓ Theoretical issues within DF'95*:

✓ Hadronic form factors are different for real- and virtual-photon corrections, do not satisfy the correct QCD short-distance behavior, violate unitarity, analicity and the chiral limit at leading non-trivial orders.

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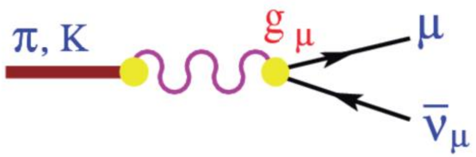
* Decker & Finkemeier'95
** HFLAV'21
*** CMS'21, ATLAS'21

^ Cirigliano et al.'10 '19, '21
^ González-Alonso & Martín-Camalich '16
^ Gonzàlez-Solís et al. '20

2. P → μ ν_μ [γ]   (P=π,K)

✓ Calculated unambigously within the Standard Model (Chiral Perturbation Theory, ChPT*).

✓ Notation by Marciano & Sirlin** and numbers by Cirigliano & Rosell*** (D=d,s for π,K and $F_\pi \approx 92.2$ MeV):



LO result

short-distance EW correction ≈ 1.0232**

structure independent (SI) contributions (point-like approximation)^

$$\Gamma(P \to \mu\nu_\mu[\gamma]) = \frac{G_F^2 |V_{uD}|^2 F_P^2}{4\pi} m_P m_\mu^2 \left(1 - \frac{m_\mu^2}{m_P^2}\right)^2 S_{\mathrm{EW}} \left\{1 + \frac{\alpha}{\pi} F(m_\mu^2/m_P^2)\right\} \times$$
$$\left\{1 - \frac{\alpha}{\pi}\left[\frac{3}{2}\log\frac{m_\rho}{m_P} + c_1^{(P)} + \frac{m_\mu^2}{m_\rho^2}\left(c_2^{(P)} \log\frac{m_\rho^2}{m_\mu^2} + c_3^{(P)} + c_4^{(P)}(m_\mu/m_P)\right) - \frac{m_P^2}{m_\rho^2}\tilde{c}_2^{(P)} \log\frac{m_\rho^2}{m_\mu^2}\right]\right\}$$

structure-dependent (SD) contributions
[coefficents reported in Cirigliano & IR'07]

✓ The only model-dependence is the determination of the counterterms in $c_1^{(P)}$ and $c_3^{(P)}$:

- ✓ Large-N_C expansion of QCD: ChPT is enlarged by including the lightest multiplets of spin-one resonances such that the relevant Green functions are well-behaved at high energies†.

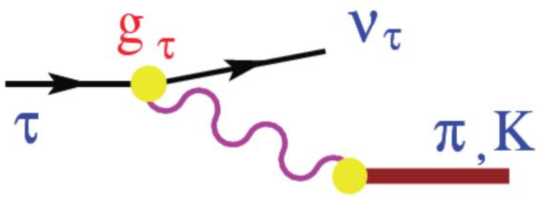
* Weinberg'79
* Gasser & Leutwyler'84 '85
** Marciano & Sirlin'93

*** Cirigliano & IR'07
^ Kinoshita'59

† Ecker et al.'89
† Cirigliano et al.'06

3. $\tau \to P\,\nu_\tau\,[\gamma]$ (P=π,K)

- ✓ Calculated within an effective approach encoding the hadronization:
  - ✓ Large-N_C expansion of QCD: ChPT is enlarged by including the lightest multiplets of spin-one resonances such that the relevant Green functions are well-behaved at high energies*.
- ✓ We follow a similar notation to $P\to\mu\nu_\mu[\gamma]$ (D=d,s for π,K and $F_\pi \approx$ 92.2 MeV):



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$$\Gamma(\tau\to P\nu_\tau[\gamma]) = \frac{G_F^2|V_{uD}|^2F_P^2}{8\pi}M_\tau^3\left(1-\frac{m_P^2}{M_\tau^2}\right)^2 S_{\rm EW}\left\{1+\frac{\alpha}{\pi}G(m_P^2/M_\tau^2)\right\}\times$$
$$\left\{1-\frac{3\alpha}{2\pi}\log\frac{m_\rho}{M_\tau}+\delta_{\tau P}\big|_{\rm rSD}+\delta_{\tau P}\big|_{\rm vSD}\right\}$$

real-photon structure-dependent (rSD) contributions

virtual-photon structure-dependent (vSD) contributions

- ✓ Real-photon structure-dependent (rSD) contributions from Guo & Roig'10^.
- ✓ Virtual-photon structure-dependent (vSD) contributions not calculated in the literature.

* Ecker et al.'89
* Cirigliano et al.'06
** Erler'02

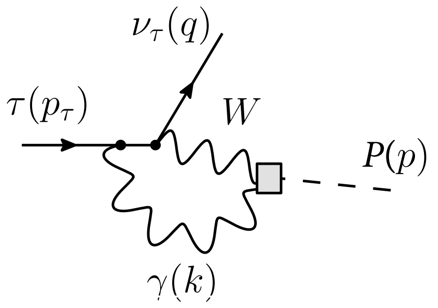
*** Kinoshita'59
^ Guo & Roig'10

3. $\tau \to P\,\nu_\tau\,[\gamma]$ (P=π,K)

✓ Virtual-photon structure-dependent contribution (vSD):

$$i\mathcal{M}[\tau\to P\nu_\tau]|_{\rm SD}=G_F V_{uD} e^2\int\frac{d^dk}{(2\pi)^d}\frac{\ell^{\mu\nu}}{k^2[(p_\tau+k)^2-M_\tau^2]}\left[i\epsilon_{\mu\nu\lambda\rho}k^\lambda p^\rho F_V^P(W^2,k^2)+F_A^P(W^2,k^2)\lambda_{1\mu\nu}+2B(k^2)\lambda_{2\mu\nu}\right]$$

$$\begin{aligned}\ell^{\mu\nu} &= \bar{u}(q)\gamma^\mu(1-\gamma_5)[(\not{p}_\tau+\not{k})+M_\tau]\gamma^\nu u(p_\tau)\\ \lambda_{1\mu\nu} &= \left[(p+k)^2+k^2-m_P^2\right]g_{\mu\nu}-2k_\mu p_\nu\\ \lambda_{2\mu\nu} &= k^2 g_{\mu\nu}-\frac{k^2(p+k)_\mu p_\nu}{(p+k)^2-m_P^2}\end{aligned}$$



✓ Form factors from Guo & Roig'10 and Guevara et al.'13,'21*:

$$\begin{aligned}F_V^P(W^2,k^2) &= \frac{-N_C M_V^4}{24\pi^2 F_P(k^2-M_V^2)(W^2-M_V^2)}\\ F_A^P(W^2,k^2) &= \frac{F_P}{2}\frac{M_A^2-2M_V^2-k^2}{(M_V^2-k^2)(M_A^2-W^2)}\\ B(k^2) &= \frac{F_P}{M_V^2-k^2}\end{aligned}$$

✓ Well-behaved two- and three-point Green functions.

✓ Chiral and U(3) limits.

✓ M_V and M_A vector- and axial-vector resonance mass: $M_V=M_\rho$ and $M_A=M_{a1}$ (π case); $M_V=M_{K*}$ and $M_A\approx M_{f1}$ (K case).

* Guo & Roig'10
* Guevara et al.'13,'21

3. $\tau \to P\, \nu_\tau\, [\gamma]$ $(P=\pi,K)$

✓ Virtual-photon structure-dependent contribution (vSD):

$$iM[\tau\to P\nu_\tau]|_{\rm SD}=G_F V_{uD} e^2\int\frac{d^dk}{(2\pi)^d}\frac{\ell^{\mu\nu}}{k^2[(p_\tau+k)^2-M_\tau^2]}\left[i\epsilon_{\mu\nu\lambda\rho}k^\lambda p^\rho F_V^P(W^2,k^2)+F_A^P(W^2,k^2)\lambda_{1\mu\nu}+2B(k^2)\lambda_{2\mu\nu}\right]$$

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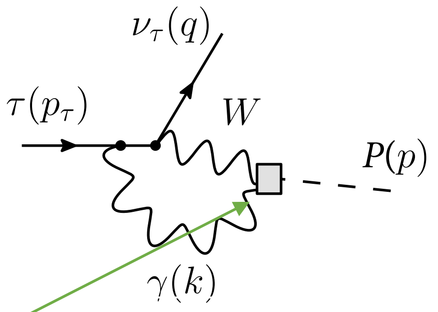

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* Guo & Roig'10
* Guevara et al.'13,'21

4. Calculation of $R_{\tau/P} = R_{\tau/P}^{(0)}\,(1 + \delta R_{\tau/P}) = R_{\tau/P}^{(0)}\,(1 + \delta_{\tau P} - \delta_{P\mu})$

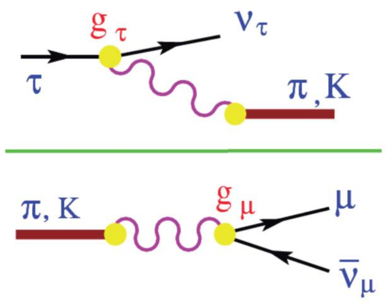
1. Structure-independent contribution (point-like approximation): SI.

✓ We confirm the results by DF'95*.

$$\delta R_{\tau/P}\big|_{\rm SI} = \frac{\alpha}{2\pi}\left\{\frac{3}{2}\log\frac{M_\tau^2 m_P^2}{m_\mu^4} + \frac{3}{2} + g\left(\frac{m_P^2}{M_\tau^2}\right) - f\left(\frac{m_\mu^2}{m_P^2}\right)\right\}$$

$$f(x) = 2\left(\frac{1+x}{1-x}\log x - 2\right)\log(1-x) - \frac{x(8-5x)}{2(1-x)^2}\log x + 4\frac{1+x}{1-x}{\rm Li}_2(x) - \frac{x}{1-x}\left(\frac{3}{2} + \frac{4}{3}\pi^2\right)$$

$$g(x) = 2\left(\frac{1+x}{1-x}\log x - 2\right)\log(1-x) - \frac{x(2-5x)}{2(1-x)^2}\log x + 4\frac{1+x}{1-x}{\rm Li}_2(x) + \frac{x}{1-x}\left(\frac{3}{2} - \frac{4}{3}\pi^2\right)$$



$\delta R_{\tau/\pi}|_{SI} = 1.05\%$ and $\delta R_{\tau/K}|_{SI} = 1.67\%$

Real-photon structure-dependent contribution: rSD.

✓ $\delta_{P\mu}|_{rSD}$ from Cirigliano & IR'07**: $\delta_{\pi\mu}|_{rSD} = -1.3\cdot10^{-8}$ and $\delta_{K\mu}|_{rSD} = -1.7\cdot10^{-5}$.

✓ $\delta_{\tau P}|_{rSD}$ from Guo & Roig'10***: $\delta_{\tau\pi}|_{rSD} = 0.15\%$ and $\delta_{\tau K}|_{rSD} = (0.18 \pm 0.05)\%$.

$\delta R_{\tau/\pi}|_{rSD} = 0.15\%$ and $\delta R_{\tau/K}|_{rSD} = (0.18 \pm 0.15)\%$

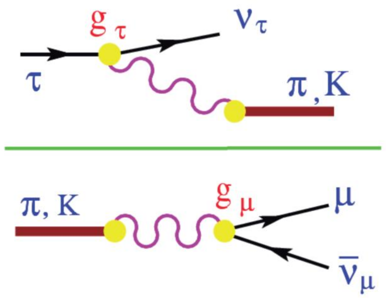
* Decker & Finkemeier'95
** Cirigliano & Rosell'07
*** Guo & Roig'10

4. Calculation of $R_{\tau/P} = R_{\tau/P}^{(0)} (1 + \delta R_{\tau/P}) = R_{\tau/P}^{(0)} (1 + \delta_{\tau P} - \delta_{P\mu})$

3. Virtual-photon structure-dependent contribution: vSD.

- ✓ $\delta_{P\mu}|_{vSD}$ from Cirigliano & IR'07*: $\delta_{\pi\mu}|_{vSD} = (0.54 \pm 0.12)\%$ and $\delta_{K\mu}|_{vSD} = (0.43 \pm 0.12)\%$.
- ✓ $\delta_{\tau P}|_{vSD}$, **new calculation**: $\delta_{\tau\pi}|_{vSD} = (-0.48 \pm 0.56)\%$ and $\delta_{\tau K}|_{vSD} = (-0.45 \pm 0.57)\%$.

$$\delta R_{\tau/\pi}|_{vSD} = (-1.02 \pm 0.57)\% \text{ and } \delta R_{\tau/K}|_{vSD} = (-0.88 \pm 0.58)\%$$



* Cirigliano & IR'07

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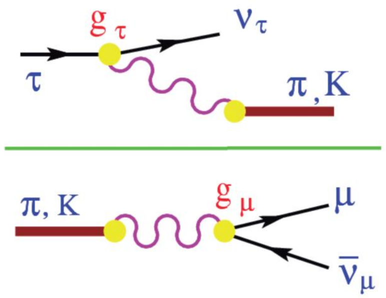
$$\delta R_{\tau/\pi}|_{vSD} = (-1.02 \pm 0.57)\% \text{ and } \delta R_{\tau/K}|_{vSD} = (-0.88 \pm 0.58)\%$$



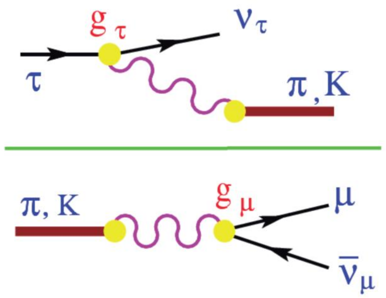
- ✓ Uncertainties dominated by $\delta_{\tau P}|_{vSD}$:
  - ✓ P decays within ChPT [counterterms can be determined by matching ChPT with the resonance effective approach at higher energies], whereas τ decays within resonance effective approach [no matching to determine the counterterms].
  - ✓ Estimation of the model-dependence by comparing our results with a less general scenario where only well-behaved two-point Green functions and a reduced resonance Lagrangian is used: ±0.22% and ±0.24% for the pion and the kaon case.
  - ✓ Estimation of the counterterms by considering the running between 0.5 and 1.0 GeV: ±0.52% (similar procedure in Marciano & Sirlin'93). **Conservative estimate**, since vSD counterterms affecting in P decays imply similar corrections to our estimation of the vSD counterterms in τ decays.

* Cirigliano & Rosell'07

5. Results

Contribution	$\delta R_{\tau/\pi}$	$\delta R_{\tau/K}$	Ref.
SI	$+1.05\%$	$+1.67\%$	*
rSD	$+0.15\%$	$+(0.18 \pm 0.05)\%$	**
vSD	$-(1.02 \pm 0.57)\%$	$-(0.88 \pm 0.58)\%$	new
Total	$+(0.18 \pm 0.57)\%$	$+(0.97 \pm 0.58)\%$	new

Errors are not reported if they are lower than 0.01%.



Central values agree remarkably with DF'95, merely a coincidence: $\delta R_{\tau/\pi} = (0.16 \pm 0.14)\%$ and $\delta R_{\tau/K} = (0.90 \pm 0.22)\%$, **but** in that work:

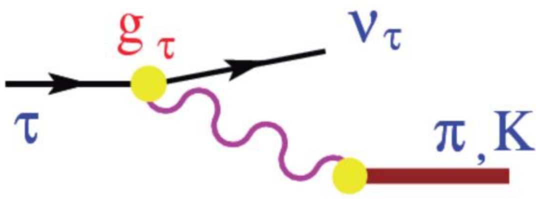
✓ problematic hadronization: form factors are different for real- and virtual-photon corrections, do not satisfy the correct QCD short-distance behavior, violate unitarity, analicity and the chiral limit at leading non-trivial orders.

✓ a cutoff to regulate the loop integrals, splitting unphysically long- and short-distance regimes.

✓ unrealistic uncertainties (purely $O(e^2p^2)$ ChPT size).

* Decker & Finkemeier'95
** Cirigliano & Rosell'07
** Guo & Roig'10

6. Application I: Radiative corrections in $\Gamma(\tau \to P\nu_\tau[\gamma])$



short-distance EW correction ≈ 1.0201*

$$\Gamma(\tau \to P\nu_\tau[\gamma]) = \frac{G_F^2 |V_{uD}|^2 F_P^2}{8\pi} M_\tau^3 \left(1 - \frac{m_P^2}{M_\tau^2}\right)^2 S_{\rm EW} (1 + \delta_{\tau P})$$

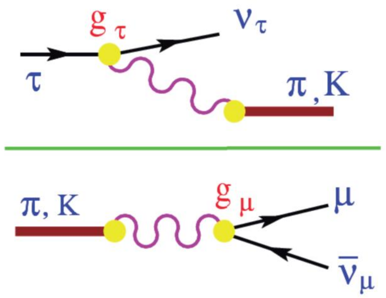
✓ $\delta_{\tau P}$ includes SI and SD radiative corrections.

$$\delta_{\tau P} = \frac{\alpha}{2\pi}\left(g\left(\frac{m_P^2}{M_\tau^2}\right) + \frac{19}{4} - \frac{2\pi^2}{3} - 3\log\frac{m_\rho}{M_\tau}\right) + \delta_{\tau P}\big|_{\rm rSD} + \delta_{\tau P}\big|_{\rm vSD} = \begin{cases} \delta_{\tau\pi} = (-0.24 \pm 0.56)\% \\ \delta_{\tau K} = (-0.15 \pm 0.57)\% \end{cases}$$

* Erler'02

6. Application II: lepton universality test

$$R_{\tau/P} \equiv \frac{\Gamma(\tau \to P\nu_\tau[\gamma])}{\Gamma(P \to \mu\nu_\mu[\gamma])} = \left|\frac{g_\tau}{g_\mu}\right|_P^2 \frac{1}{2}\frac{M_\tau^3}{m_\mu^2 m_P}\frac{(1-m_P^2/M_\tau^2)^2}{(1-m_\mu^2/m_P^2)^2}\left(1+\delta R_{\tau/P}\right)$$



$$\left|\frac{g_\tau}{g_\mu}\right|_\pi = 0.9964 \pm 0.0028_{\rm th} \pm 0.0025_{\rm exp} = 0.9964 \pm 0.0038$$

$$\left|\frac{g_\tau}{g_\mu}\right|_K = 0.9857 \pm 0.0028_{\rm th} \pm 0.0072_{\rm exp} = 0.9857 \pm 0.0078$$

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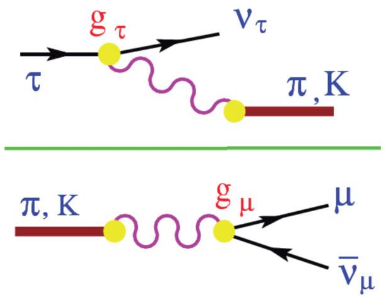
PDG

$\delta R_{\tau/\pi} = (0.18 \pm 0.57)\%$
$\delta R_{\tau/K} = (0.97 \pm 0.58)\%$



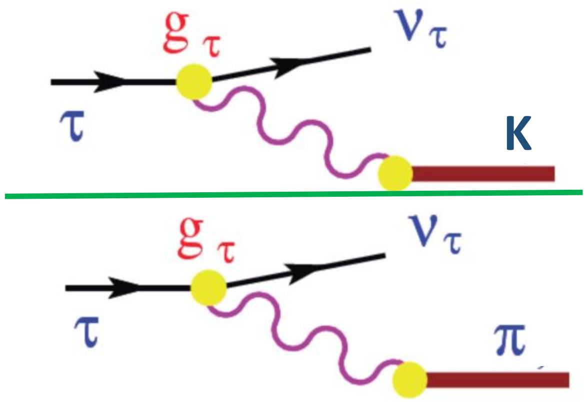
$$\left|\frac{g_\tau}{g_\mu}\right|_\pi = 0.9964 \pm 0.0028_{\rm th} \pm 0.0025_{\rm exp} = 0.9964 \pm 0.0038$$

$$\left|\frac{g_\tau}{g_\mu}\right|_K = 0.9857 \pm 0.0028_{\rm th} \pm 0.0072_{\rm exp} = 0.9857 \pm 0.0078$$

- ✓ π case: at 0.9σ of LU vs. 1.6σ of LU in HFLAV'21* using DF'95**
- ✓ K case: at 1.8σ of LU vs. 1.9σ of LU in HFLAV'21* using DF'95**

* HFLAV'21
** Decker & Finkemeier'95

6. Application III: CKM unitarity test in the ratio $\Gamma(\tau \to K\nu_\tau[\gamma]) / \Gamma(\tau \to \pi\nu_\tau[\gamma])$



$$\frac{\Gamma(\tau \to K\nu_\tau[\gamma])}{\Gamma(\tau \to \pi\nu_\tau[\gamma])} = \frac{|V_{us}|^2}{|V_{ud}|^2}\frac{F_K^2}{F_\pi^2}\frac{(1-m_K^2/M_\tau^2)^2}{(1-m_\pi^2/M_\tau^2)^2}(1+\delta)$$

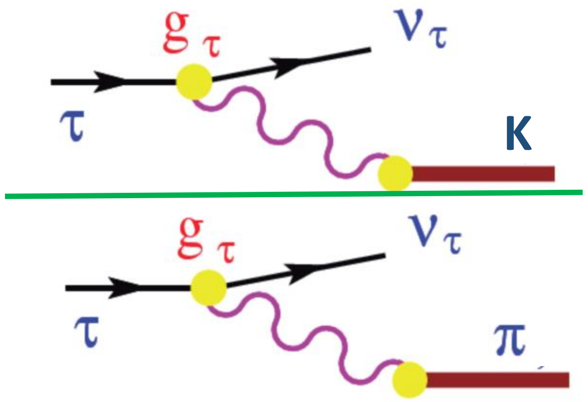
$$\left|\frac{V_{us}}{V_{ud}}\right| = 0.2288 \pm 0.0010_{\rm th} \pm 0.0017_{\rm exp} = 0.2288 \pm 0.0020$$

6. Application III: CKM unitarity test in the ratio $\Gamma(\tau \to K\nu_\tau[\gamma]) / \Gamma(\tau \to \pi\nu_\tau[\gamma])$

FLAG'20*:
$F_K/F_\pi = 1.1932 \pm 0.0019$

$$\delta = \delta_{\tau K} - \delta_{\tau\pi} = +(0.10 \pm 0.80)\%$$

PDG



$$\frac{\Gamma(\tau \to K\nu_\tau[\gamma])}{\Gamma(\tau \to \pi\nu_\tau[\gamma])} = \frac{|V_{us}|^2}{|V_{ud}|^2}\frac{F_K^2}{F_\pi^2}\frac{(1-m_K^2/M_\tau^2)^2}{(1-m_\pi^2/M_\tau^2)^2}(1+\delta)$$

$$\left|\frac{V_{us}}{V_{ud}}\right| = 0.2288 \pm 0.0010_{\rm th} \pm 0.0017_{\rm exp} = 0.2288 \pm 0.0020$$

- ✓ 2.1σ away from CKM unitarity, considering $|V_{ud}|=0.97373\pm0.00031$**.
- ✓ To be compared with $|V_{us}/V_{ud}|=0.2291\pm0.0009$***, obtained with kaon semileptonic decays. Our error does not reach this level due to lack of statistics in τ decays.

* FLAG'20
** Hardy & Towner'20
*** Seng et al.'21

6. Application III: CKM unitarity test in the ratio $\Gamma(\tau \to K\nu_\tau[\gamma]) / \Gamma(\tau \to \pi\nu_\tau[\gamma])$

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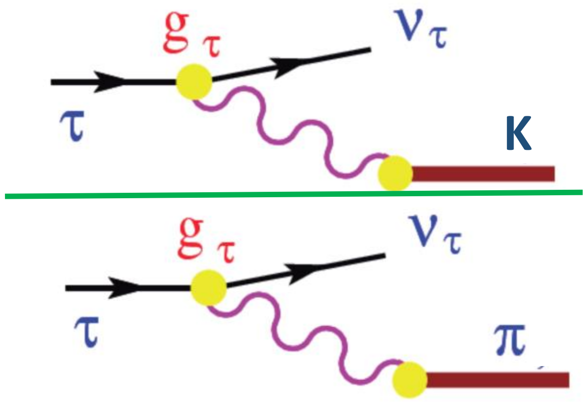

$$\frac{\Gamma(\tau \to K\nu_\tau[\gamma])}{\Gamma(\tau \to \pi\nu_\tau[\gamma])} = \frac{|V_{us}|^2}{|V_{ud}|^2}\frac{F_K^2}{F_\pi^2}\frac{(1-m_K^2/M_\tau^2)^2}{(1-m_\pi^2/M_\tau^2)^2}(1+\delta)$$

Conservative estimation of the errors in δ, since we have directly propagated the uncertainties of $\delta_{\tau K}$ and $\delta_{\tau\pi}$.

$$\left|\frac{V_{us}}{V_{ud}}\right| = 0.2288 \pm 0.0010_{\rm th} \pm 0.0017_{\rm exp} = 0.2288 \pm 0.0020$$

- ✓ 2.1σ away from CKM unitarity, considering $|V_{ud}|=0.97373\pm0.00031$**.
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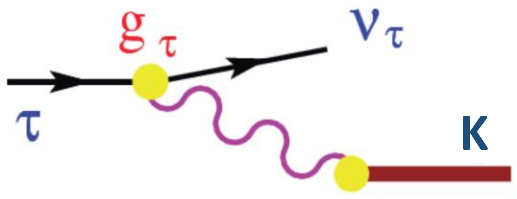
* FLAG'20
** Hardy & Towner'20
*** Seng et al.'21

6. Application IV: CKM unitarity test in $\Gamma(\tau \to K\nu_\tau[\gamma])$



$$\Gamma(\tau \to K\nu_\tau[\gamma]) = \frac{G_F^2 F_K^2}{8\pi}|V_{us}|^2 M_\tau^3 \left(1 - \frac{m_K^2}{M_\tau^2}\right)^2 S_{\rm EW}\ (1 + \delta_{\tau K})$$

$$|V_{us}| = 0.2220 \pm 0.0008_{\rm th} \pm 0.0016_{\rm exp} = 0.2220 \pm 0.0018$$

6. Application IV: CKM unitarity test in $\Gamma(\tau \to K\nu_\tau[\gamma])$

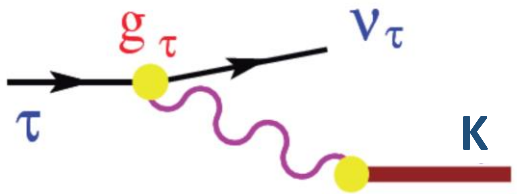
PDG

FLAG'20*: $\sqrt{2}F_K = (155.7 \pm 0.3)$ MeV

short-distance EW correction ≈ 1.0201**

$\delta_{\tau K} = (-0.15 \pm 0.57)\%$

$$\Gamma(\tau \to K\nu_\tau[\gamma]) = \frac{G_F^2 F_K^2}{8\pi}|V_{us}|^2 M_\tau^3 \left(1 - \frac{m_K^2}{M_\tau^2}\right)^2 S_{\rm EW}(1+\delta_{\tau K})$$

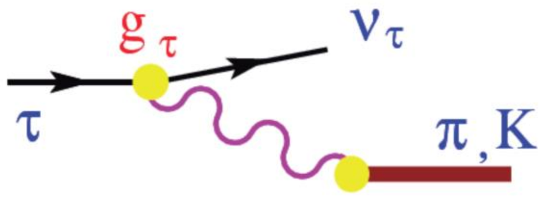


$$|V_{us}| = 0.2220 \pm 0.0008_{\rm th} \pm 0.0016_{\rm exp} = 0.2220 \pm 0.0018$$

- ✓ 2.6σ away from CKM unitarity, considering $|V_{ud}|=0.97373\pm0.00031$***.
- ✓ To be compared with $|V_{us}|=0.2234\pm0.0015$^ or $|V_{us}|=0.2231\pm0.0006$†, obtained this last one with kaon semileptonic decays. Our error does not reach this level due to lack of statistics in τ decays.

* FLAG'20
** Erler'02
*** Hardy & Towner'20
^ HFLAV'21
† Seng et al.'21

6. Application V: constraining non-standard interactions in $\Gamma(\tau \to P\nu_\tau[\gamma])$



$$\Gamma(\tau \to P\nu_\tau[\gamma]) = \frac{G_F^2 |\widetilde{V}_{uD}|^2 F_P^2}{8\pi} M_\tau^3 \left(1 - \frac{m_P^2}{M_\tau^2}\right)^2 S_{\rm EW} \left(1 + \delta_{\tau P} + 2\Delta^{\tau P}\right)$$

Values of $\Delta^{\tau P}$ reported in the $\overline{\rm MS}$-scheme and at a scale of μ=2 GeV.

$$\Delta^{\tau P} = \epsilon_L^\tau - \epsilon_L^e - \epsilon_R^\tau - \epsilon_R^e - \frac{m_P^2}{M_\tau(m_u + m_D)} \epsilon_P^\tau = \begin{cases} \Delta^{\tau\pi} = -(0.15 \pm 0.72) \cdot 10^{-2} \\ \Delta^{\tau K} = -(0.36 \pm 1.18) \cdot 10^{-2} \end{cases}$$

6. Application V: constraining non-standard interactions in $\Gamma(\tau \to P\nu_\tau[\gamma])$

$|V_{ud}| = 0.97373 \pm 0.00031$*
$|V_{us}/V_{ud}| = 0.2288 \pm 0.0020$

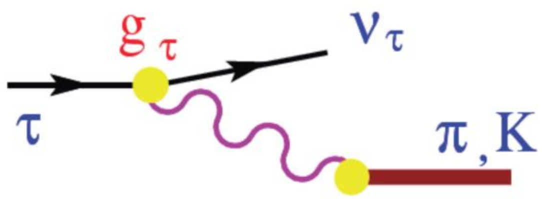
FLAG'20*:
$\sqrt{2}F_\pi = (130.2 \pm 0.8)$ MeV
$\sqrt{2}F_K = (155.7 \pm 0.3)$ MeV

short-distance EW correction ≈ 1.0201**

$\delta_{\tau\pi} = (-0.24 \pm 0.56)\%$
$\delta_{\tau K} = (-0.15 \pm 0.57)\%$

PDG

$$\Gamma(\tau \to P\nu_\tau[\gamma]) = \frac{G_F^2 |\widetilde{V}_{uD}|^2 F_P^2}{8\pi} M_\tau^3 \left(1 - \frac{m_P^2}{M_\tau^2}\right)^2 S_{\rm EW} \left(1 + \delta_{\tau P} + 2\Delta^{\tau P}\right)$$



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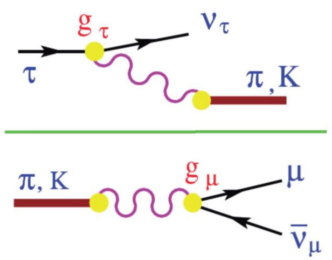
- ✓ To be compared with $\Delta^{\tau\pi} = -(0.15 \pm 0.67)\cdot 10^{-2}$ of Cirigliano et al.'19^.
- ✓ To be compared with $\Delta^{\tau\pi} = -(0.12 \pm 0.68)\cdot 10^{-2}$ and $\Delta^{\tau K} = (-0.41 \pm 0.93)\cdot 10^{-2}$ of González-Solís et al.'20†.

* Hardy & Towner'20
** FLAG'20
*** Erler'02

^ Cirigliano et al.'19
† Gonzàlez-Solís et al. '20

7. Conclusions



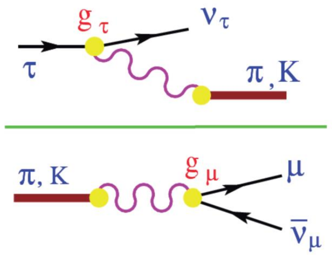
- ✓ The observable and **our result**:

$$R_{\tau/P} \equiv \frac{\Gamma(\tau \to P\nu_\tau[\gamma])}{\Gamma(P \to \mu\nu_\mu[\gamma])} = \left|\frac{g_\tau}{g_\mu}\right|_P^2 R^{(0)}_{\tau/P}\left(1+\delta R_{\tau/P}\right) \quad \longrightarrow \quad \begin{cases} \delta R_{\tau/\pi} = (0.18 \pm 0.57)\% \\ \delta R_{\tau/K} = (0.97 \pm 0.58)\% \end{cases}$$

- ✓ Framework: ChPT for π decays and a resonance extension of ChPT for τ decays.
- ✓ Consistent with DF'95*, but with more robust assumptions and yielding a reliable uncertainty.
- ✓ Applications:
  - ✓ Theoretical determination of radiative corrections in $\Gamma(\tau \to P\nu_\tau[\gamma])$.
  - ✓ $|g_\tau/g_\mu|_P$ at 0.9σ (π) and 1.8σ (K) of LU, reducing HFLAV'21** disagreement with LU.
  - ✓ CKM unitarity in $\Gamma(\tau\to K\nu_\tau[\gamma])/\Gamma(\tau\to\pi\nu_\tau[\gamma])$: $|V_{us}/V_{ud}| = 0.2288 \pm 0.0020$, at 2.1$\sigma$ from unitarity.
  - ✓ CKM unitarity in $\Gamma(\tau\to K\nu_\tau[\gamma])$: $|V_{us}| = 0.2220 \pm 0.0018$, at 2.6$\sigma$ from unitarity.
  - ✓ Constraining non-standard interactions in $\Gamma(\tau \to P\nu_\tau[\gamma])$: update of $\Delta^{\tau P}$.
- ✓ Our results have been incorporated in the very recent HFLAV'22.

* Decker & Finkemeier'95
** HFLAV'21

Reliable NP tests for present & future exps.



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RadCors for $\pi\pi$ τ decays evaluated in Miranda&Roig'20. For other 2-meson modes, Escribano-Miranda-Roig, to appear very soon

* Decker & Finkemeier'95
** HFLAV'21

Comparison with Decker & Finkemeier’95 (DF’95) in the π case

Contribution	$\delta R_{\tau\pi}$ by DF’95 [μ_{cut} =1.5 GeV]	our $\delta R_{\tau\pi}$
SI	$+0.84\%^*$	$+1.05\%$
rSD	$+0.05\%$	$+0.15\%$
vSD	$-0.49\%^*$	$-(1.02 \pm 0.57)\%$
short-distance	$-0.25\%^*$	0
Total	$+(0.16 \pm 0.14)\%^*$	$+(0.18 \pm 0.57)\%$

- ✓ Virtual corrections by DF’95 are μ_{cut}-dependent, since long- and short-distance photonic contributions were separated unphysically: figures with an asterisk are cutoff-dependent.
- ✓ The quoted error in the radiative correction of DF’95 arises from uncertainties in hadronic parameters of SD contributions and from variations in the cutoff parameter, μ_{cut}.
- ✓ For the SI contribution in DF’95 we have added to the result obtained in the point-like approximation (1.05%) the term coming from cutting off the loops at μ_{cut} (−0.21%).
- ✓ Different contributions of $\delta R_{\tau/K}$ are not provided in DF’95, which prevents a comparison.
- ✓ Although central values for the sum of all the corrections agree remarkably, this is a coincidence, since central values for the SD corrections are largely different within both approaches.