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Axially symmetric systems, rotating black holes, and gravitational decoupling

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Plan of the talk

1. Newman-Janis algorithm (bird's eye)

2. Gravitational Decoupling

3. Gravitational Decoupling for rotating black holes

4. Final comments

Newman-Janis algorithm (bird's eye)

$$G_{\mu\nu} = R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = T_{\mu\nu}$$

"Natural" units $G = c = 8\pi = 1$

Static and spherically symmetric space-time

$$ds^2 = e^{\nu(r)} dt^2 - e^{\lambda(r)} dr^2 - r^2 d\Omega^2$$

$$T^{\mu}{}_{\nu} = diag(\rho, -p_r, -p_t, -p_t)$$

$$\begin{cases} \rho = \dfrac{1}{r^2} - e^{-\lambda}\left(\dfrac{1}{r^2} - \dfrac{\lambda'}{r}\right) \\ p_r = \dfrac{1}{r^2} - e^{-\lambda}\left(\dfrac{1}{r^2} + \dfrac{\nu'}{r}\right) \\ p_t = -\dfrac{e^{-\lambda}}{4}\left(\lambda'\nu' - \dfrac{2(\nu' - \lambda')}{r} - \nu'^2 - 2\nu''\right) \end{cases}$$

Newman-Janis algorithm (bird's eye)

1. Complexification

$$r \to r + ia\cos\theta$$

2. Suitable metric variables

$$\{e^{\nu}, e^{\lambda}\} \to \{A(r,\theta,a), B(r,\theta,a)\}$$

3. Static limit

$$\lim_{a\to 0}\{A, B\} \to \{e^{\nu}, e^{\lambda}\}$$

E.T. Newman and A.I. Janis, J. Math. Phys. 6, 915 (1965).

Newman-Janis algorithm (bird's eye)

Ambiguity!!!!

$$r^2 \to (r + iacos\theta)(r - iacos\theta) = r^2 + a^2cos^2\theta$$

$$\frac{1}{r} \to \frac{1}{2}\left(\frac{1}{r + iacos\theta} + \frac{1}{r - iacos\theta}\right) = \frac{r}{r^2 + a^2cos^2\theta}$$

$$r^2 \to r(r + iacos\theta)^{1/2}(r - iacos\theta)^{1/2}$$

All of them coincide when a is turned off!!!

Alternative:

Newman-Janis without complexification

M. Azreg-Aïnou, Phys. Rev. D 90, 064041 (2014)

Gravitational Decoupling

What if? $T_{\mu\nu} = T_{\mu\nu(0)} + S_{\mu\nu}$

$$\begin{cases} T^{\mu}{}_{\nu(0)} = diag(\rho_0, -p_{r0}, -p_{t0}, -p_{t0}) \\ S^{\mu}{}_{\nu} = diag(\rho_S, -p_{rS}, -p_{tS}, -p_{tS}) \end{cases}$$

$$\rho_0 + \rho_S = \frac{1}{r^2} - e^{-\lambda}\left(\frac{1}{r^2} - \frac{\lambda'}{r}\right)$$

$$p_{r0} + p_{rS} = \frac{1}{r^2} - e^{-\lambda}\left(\frac{1}{r^2} + \frac{\nu'}{r}\right)$$

$$p_{t0} + p_{tS} = -\frac{e^{-\lambda}}{4}\left(\lambda'\nu' - \frac{2(\nu' - \lambda')}{r} - \nu'^2 - 2\nu''\right)$$

J. Ovalle, Phys. Rev. D95, 104019 (2017), 1704.05899.

Gravitational Decoupling

Let us assume $$e^{-\lambda} = 1 - \frac{2m(r)}{r}$$

$$m = m_0 + m_S$$

$$\rho_0 + \rho_S = \frac{2m'}{r^2}$$

$$p_{r0} + p_{rS} = \frac{\nu'}{r^2} - \left(\frac{2\nu'}{r^2} + \frac{2}{r^3}\right) m$$

$$p_{t0} + p_{tS} = \frac{\nu'}{2r} + \frac{1}{4}\nu'^2 + m'\left(-\frac{1}{r^2} - \frac{\nu'}{2r}\right) + \frac{\nu''}{2} + m\left(\frac{1}{r^3} - \frac{\nu'}{2r^2} - \frac{\nu'^2}{2r} - \frac{\nu''}{r}\right)$$

Linear in the mass function!

Gravitational Decoupling

$$e^{-\lambda} = 1 - \frac{2m_0}{r} - \frac{2m_S}{r} = e^{-\mu} + f$$

$$\rho_0 + \rho_S = \frac{1}{r^2} - \mathrm{e}^{-\mu}\left(\frac{1}{r^2} - \frac{\mu'}{r}\right) - \frac{f}{r^2} - \frac{f'}{r}$$

$$p_{r0} + p_{rS} = -\frac{1}{r^2} + \mathrm{e}^{-\mu}\left(\frac{1}{r^2} + \frac{\nu'}{r}\right) + f\left(\frac{1}{r^2} + \frac{\nu'}{r}\right)$$

$$p_{t0} + p_{tS} = \mathrm{e}^{-\mu}\left(-\frac{\mu'}{2r} + \frac{\nu'}{2r} - \frac{1}{4}\mu'\nu' + \frac{1}{4}\nu'^2 + \frac{\nu''}{2}\right) + f'\left(\frac{1}{2r} + \frac{\nu'}{4}\right)$$

$$+f\left(\frac{\nu'}{2r} + \frac{1}{4}\nu'^2 + \frac{\nu''}{2}\right)$$

Gravitational Decoupling

Seed sector $\{\nu, \mu, \rho_0, p_{\mathrm{r}0}, p_{\mathrm{t}0}\}$

$$\rho_0 = \frac{1}{r^2} - \mathrm{e}^{-\mu}\left(\frac{1}{r^2} - \frac{\mu'}{r}\right)$$

$$p_{r0} = -\frac{1}{r^2} + \mathrm{e}^{-\mu}\left(\frac{1}{r^2} + \frac{\nu'}{r}\right)$$

$$p_{t0} = \mathrm{e}^{-\mu}\left(-\frac{\mu'}{2r} + \frac{\nu'}{2r} - \frac{1}{4}\mu'\nu' + \frac{1}{4}\nu'^2 + \frac{\nu''}{2}\right)$$

Decoupling sector $\{f, \rho_S, p_{\mathrm{r}S}, p_{\mathrm{t}S}\}$

$$\rho_S = -\frac{f}{r^2} - \frac{f'}{r}$$

$$p_{rS} = f\left(\frac{1}{r^2} + \frac{\nu'}{r}\right)$$

$$p_{tS} = f'\left(\frac{1}{2r} + \frac{\nu'}{4}\right) + f\left(\frac{\nu'}{2r} + \frac{1}{4}\nu'^2 + \frac{\nu''}{2}\right)$$

Given a seed metric $\{\nu, \mu\}$, the first set is trivial and the second set corresponds to three equations with four unknowns!!

Gravitational Decoupling

What if we asume?

$$e^{\nu} = 1 - \frac{2m}{r}$$

$$e^{-\lambda} = 1 - \frac{2m}{r}$$

$$\rho_0 + \rho_S = \frac{2m'}{r^2}$$

$$p_{r0} + p_{rS} = -\frac{2m'}{r^2}$$

$$p_{t0} + p_{tS} = -\frac{m''}{r}$$

→

$$m = m_0 + m_S$$

→

$$\nu \to \xi + g$$

$$e^{-\lambda} \to e^{-\mu} + f$$

Gravitational Decoupling

Seed sector (trivial)

$$\rho_0 = \frac{2{m_0}'}{r^2}$$

$$p_{r0} = -\frac{2{m_0}'}{r^2}$$

$$p_{t0} = -\frac{{m_0}''}{r}$$

Decoupling sector $\{m_S, \rho_S, p_{\mathrm{t}S}\}$

$$\rho_S = \frac{2{m_S}'}{r^2}$$

$$p_{rS} = -\frac{2{m_S}'}{r^2}$$

$$p_{tS} = -\frac{{m_S}''}{r}$$

Strategy?

1. Provide a “suitable” mass function
2. Equation of state.

Gravitational Decoupling for Rotating black holes

$$ds^2 = \left(1 - \frac{2rm(r)}{\rho^2}\right)dt^2 - \frac{4arm(r)sin^2\theta}{\rho^2}dtd\phi - \frac{\rho^2}{\Delta}dr^2 - \rho^2 d\theta^2 - \frac{\Sigma sin^2\theta}{\rho^2}d\phi^2$$

The simplest generalization of the Kerr metric!!

$$\rho^2 = r^2 + a^2 cos^2\theta$$

$$\Delta = r^2 - 2rm + a^2$$

$$\Sigma = (r^2 + a^2)^2 - a^2 \Delta sin^2\theta$$

$$a = J/M$$

Note that

- reduces to the static case when $a \to 0$
- $m = M$ = constant corresponds to Kerr
- $m(r)$ depends on the radial coordinate only, so is the same when $a = 0$

E.C, J. Ovalle, R. Casadio, Phys. Rev. D 103, 044020 (2021)

Gravitational Decoupling for Rotating black holes

$$\epsilon = -p_r = \frac{2r^2}{\rho^4} m'$$

$$p_\theta = p_\phi = -\frac{r}{\rho^2} m'' + \frac{2(r^2 - \rho^2)}{\rho^4} m'$$

1. Linear in m

2. Non-linear in a

Condition for the decoupling

$$m = m_0 + m_S$$

$$a = a_0 = a_S$$

$$\epsilon_0 + \epsilon_S = -(p_r + p_{rS}) = \frac{2r^2}{\rho^4} {m_0}' + \frac{2r^2}{\rho^4} {m_S}'$$

$$p_{\theta 0} + p_{\theta S} = p_{\phi 0} + p_{\phi S} = -\frac{r}{\rho^2} {m_0}'' + \frac{2(r^2 - \rho^2)}{\rho^4} {m_0}' - \frac{r}{\rho^2} {m_S}'' + \frac{2(r^2 - \rho^2)}{\rho^4} {m_S}'$$

E.C, J. Ovalle, R. Casadio, Phys. Rev. D 103, 044020 (2021)

Gravitational Decoupling for Rotating black holes

Strategy to construct a Kerr-like black holes

1. Set $m = M =$ constant. The problem reduces to solving two equations with three unknowns

2. (a) Consider a suitable mass function or a suitable equation of state

   (b) Obtain the mass function from a well-behaved static seed (as Newmann-Janis)

E.C, J. Ovalle, R. Casadio, Phys. Rev. D 103, 044020 (2021)

Gravitational Decoupling for Rotating black holes

Example

$$e^{\nu} = e^{-\lambda} = 1 - \frac{2M}{r} + \alpha e^{-r(M-\ell_0/2)} \qquad \ell_0 = \alpha\ell$$

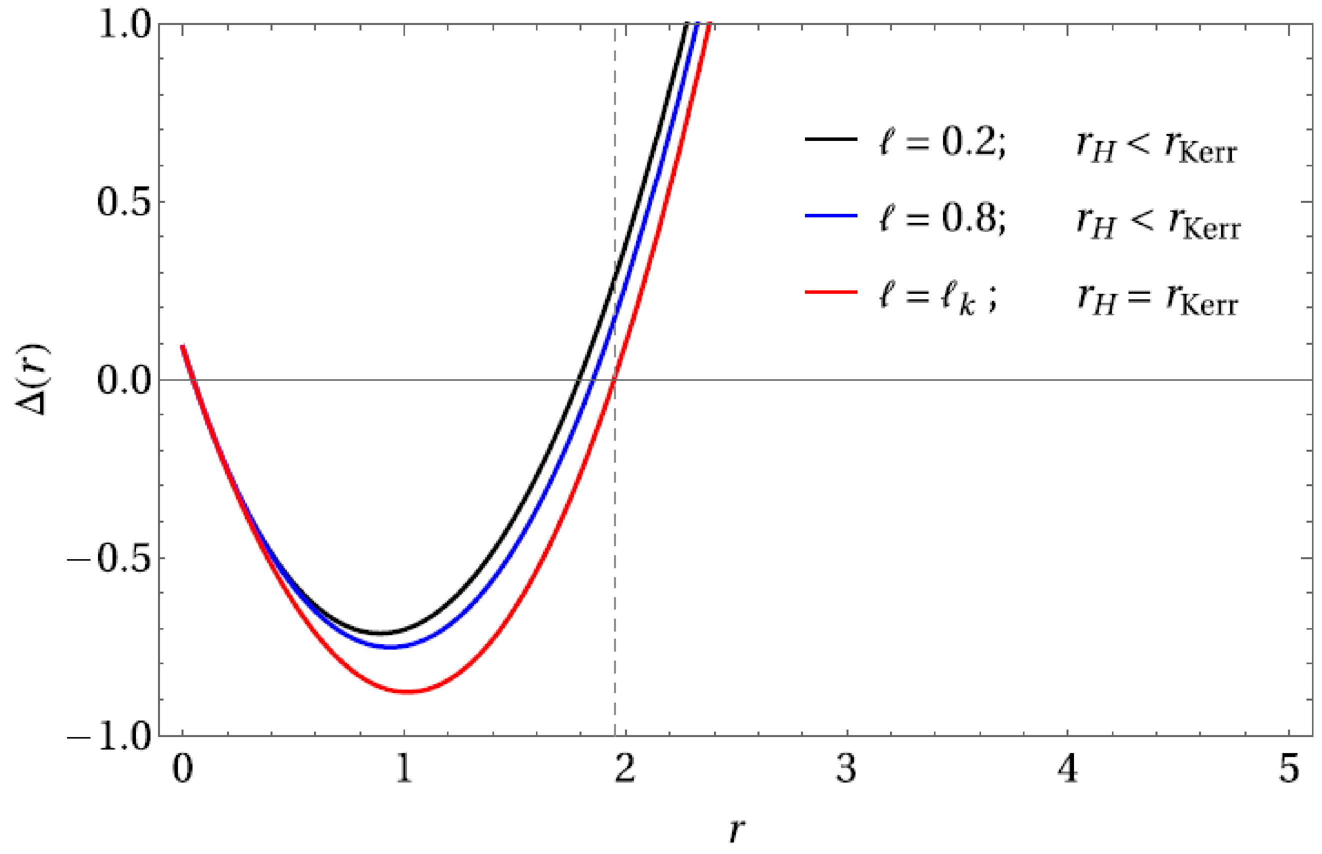
- The solution satisfies the SEC
- Reduces to Schwarzschild when $\alpha \to 0$
- The mas function reads

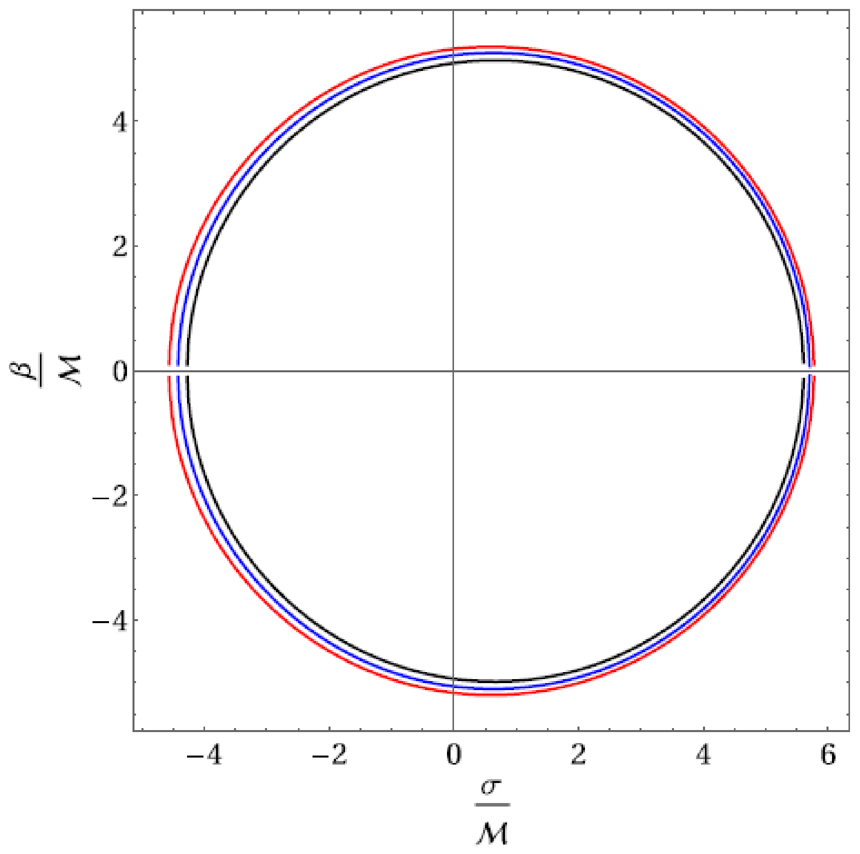
$$m = M - \frac{\alpha r}{2M} e^{-r(M-\ell_0/2)}$$

- The horizon condition is given by

$$r_H{}^2 - 2r_H M + \frac{\alpha r_H{}^2}{M} e^{-r_H(M-\ell_0/2)} + a^2 = 0$$

Gravitational Decoupling for Rotating black holes





E.C, J. Ovalle, R. Casadio, Phys. Rev. D 103, 044020 (2021)

Final comments

- Gravitational decoupling allows the full decoupling of sources for rotating Kerr-like systems

- The construction of rotating black holes can be performed by assuming a suitable mass function or some equation of state.

- We can assume a seed static metric (like the Newman-Janis algorithm)

- What is the “source” of Kerr (hard!)

Thanks!