Five texture zeros in the lepton sector and neutrino oscillations at DUNE

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1. Introduction

In this work, we have assumed special structures for the charged and neutral mass matrices in the lepton sector, assuming only Dirac neutrinos. The structure of the model is SM + three singlets right handed neutrinos.

2. Forms with five textures zeros

The Dirac Lagrangian mass term for the lepton sector is given by

$$-\mathcal{L} = \bar{\nu}_L M_n \nu_R + \bar{l}_L M_l l_R + h.c, \qquad (1)$$

This implies: $U_{PMNS} = U_l^\dagger U_\nu$ is the mixing matrix.

A WBT: $M_n \to M_n^R = U M_n U^\dagger, M_l \to M_l^R = U M_l U^\dagger$. Both representations $M_{\nu,l}$ and $M_{\nu,l}^R$ are equivalent, because

$$U_{PMNS} = U_l^\dagger U_\nu = U_l^\dagger U U^\dagger U_\nu = U_l^{R\dagger} U_\nu^R = U_{PMNS}^R \qquad (2)$$

The textures considered in this work are the ones are shown in the next table,

Form	M_n	M_l
RRR$_4$	$\begin{pmatrix} 0 & 0 & \lvert b_n\rvert\, e^{i\alpha_1} \\ 0 & c_n & \lvert d_n\rvert\, e^{i\alpha_2} \\ \lvert b_n\rvert\, e^{-i\alpha_1} & \lvert d_n\rvert\, e^{-i\alpha_2} & a_n \end{pmatrix}$	$\begin{pmatrix} 0 & \lvert b_l\rvert\, e^{i\beta_1} & 0 \\ \lvert b_l\rvert\, e^{-i\beta_1} & c_l & 0 \\ 0 & 0 & a_l \end{pmatrix}$
T$_1$	$\begin{pmatrix} 0 & 0 & \lvert b_n\rvert\, e^{i\alpha_1} \\ 0 & c_n & \lvert d_n\rvert\, e^{i\alpha_2} \\ \lvert b_n\rvert\, e^{-i\alpha_1} & \lvert d_n\rvert\, e^{-i\alpha_2} & a_n \end{pmatrix}$	$\begin{pmatrix} 0 & \lvert c_l\rvert\, e^{i\beta_1} & 0 \\ \lvert c_l\rvert\, e^{-i\beta_1} & 0 & \lvert b_l\rvert\, e^{i\beta_2} \\ 0 & \lvert b_l\rvert\, e^{-i\beta_2} & a_l \end{pmatrix}$

Factoring the phases: $\Phi M' \Phi^*$, where, Φ = Diag$(1, e^{i\phi_1}, e^{i\phi_2})$. Such that the M' matrix now is real.

The real rotation matrices, R_l and R_n that diagonalize each sector can be found using the invariants, Det$\{M_{(n,l)}^{\text{diag}}\}$ = Det$\{M_{(n,l)}\}$Tr$\{M_{(n,l)}^{\text{diag}}\}$ = Tr$\{M_{(n,l)}\}$, Tr$\{[M_{(n,l)}^{\text{diag}}]^2\}$ = Tr$\{[M_{(n,l)}]^2\}$, where $M_{(n,l)}^{\text{diag}}$ = Diag$\{m_{(1,e)}, -m_{(2,\mu)}, m_{(3,\tau)}\}$, and the lepton mixing matrix can be written as: $K = R_l \Phi R_n^T$. From this matrix we find the three mixing angles in the form: $\tan\theta_{12} = |K_{e,2}|/|K_{e,1}|$, $\sin\theta_{13} = |K_{e,3}|$, $\tan\theta_{23} = |K_{\mu,3}|/|K_{\tau,3}|$, and, The Jarlskog invariant $J_{CP} = \mathcal{I}\{K_{e1}^* K_{\mu3}^* K_{e3} K_{\mu1}\}$.

3. Analysis of the forms

m_e (MeV)	m_μ (MeV)	m_τ (MeV)
0.511	105.658	1776.860

$\sin^2\theta_{12} \pm \sigma(\sin^2\theta_{12})$	$\sin^2\theta_{13} \pm \sigma(\sin^2\theta_{13})$	$\sin^2\theta_{23} \pm \sigma(\sin^2\theta_{23})$
0.320 ± 0.016	0.0220 ± 0.0007	0.574 ± 0.014

Δm_{21}^2 (eV^2)	Δm_{31}^2 (eV^2)
7.50×10^{-5}	2.55×10^{-3}

Neutrino masses m_i can be written in the form: $m_1 = m_0\,, m_2 = \sqrt{m_0^2 + \Delta m_{21}^2}\,, m_3 = \sqrt{m_0^2 + \Delta m_{31}^2}$ where Δm_{21}^2 (Δm_{31}^2) is the solar (atmospheric) mass-squared difference.

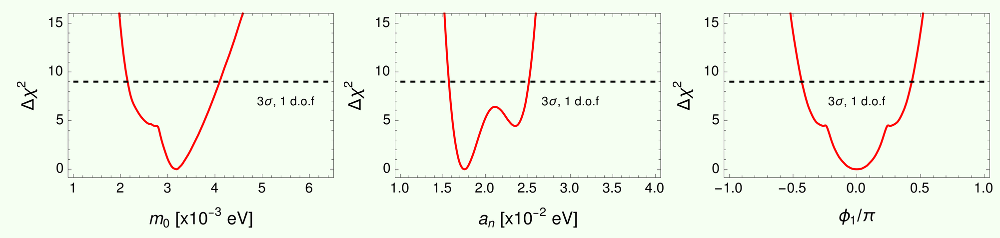
3.1 Analysis of the RRR_4-form

For this particular form we find the relations:

$$\begin{aligned} c_n &= -a_n + m_1 - m_2 + m_3, \\ |b_n| &= \sqrt{m_1 m_2 m_3/(-a_n + m_1 - m_2 + m_3)}, \\ |d_n| &= \sqrt{\frac{(a_n - m_1 + m_2)(a_n - m_1 - m_3)(a_n + m_2 - m_3)}{-a_n + m_1 - m_2 + m_3}}, \\ a_l &= m_e - m_\mu, \\ |b_l| &= m_\tau, \\ |c_l| &= \sqrt{m_e m_\mu}\,. \end{aligned}$$

It is possible to diagonalize the mass matrices at function of m_1, m_2, m_3, a_n and m_e, m_μ, m_τ, respectively.

In order to constrain the model parameters $\vec{\lambda}$ in each form, the following statistical test was implemented: $\chi^2(\vec{\lambda}) = \sum_{i<j}\left(\frac{\sin^2\theta_{ij} - \sin^2\tilde{\theta}_{ij}}{\sigma(\sin^2\theta_{ij})}\right)^2$, with $i, j = 1, 2, 3$.

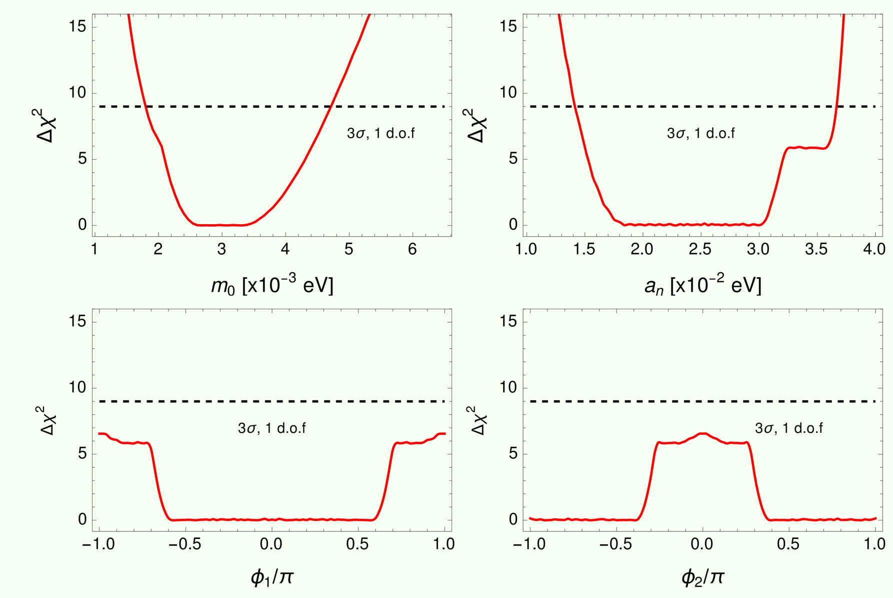


Parameter	Best fit	3σ range
m_0 ($\times 10^{-3}$ eV)	3.2	$[2.2, 4.1]$
a_n ($\times 10^{-2}$ eV)	1.8	$[1.6, 2.5]$
ϕ_1/π	0	$[-0.4, 0.4]$

The $\chi^2_{\text{min.}} = 1.4$.

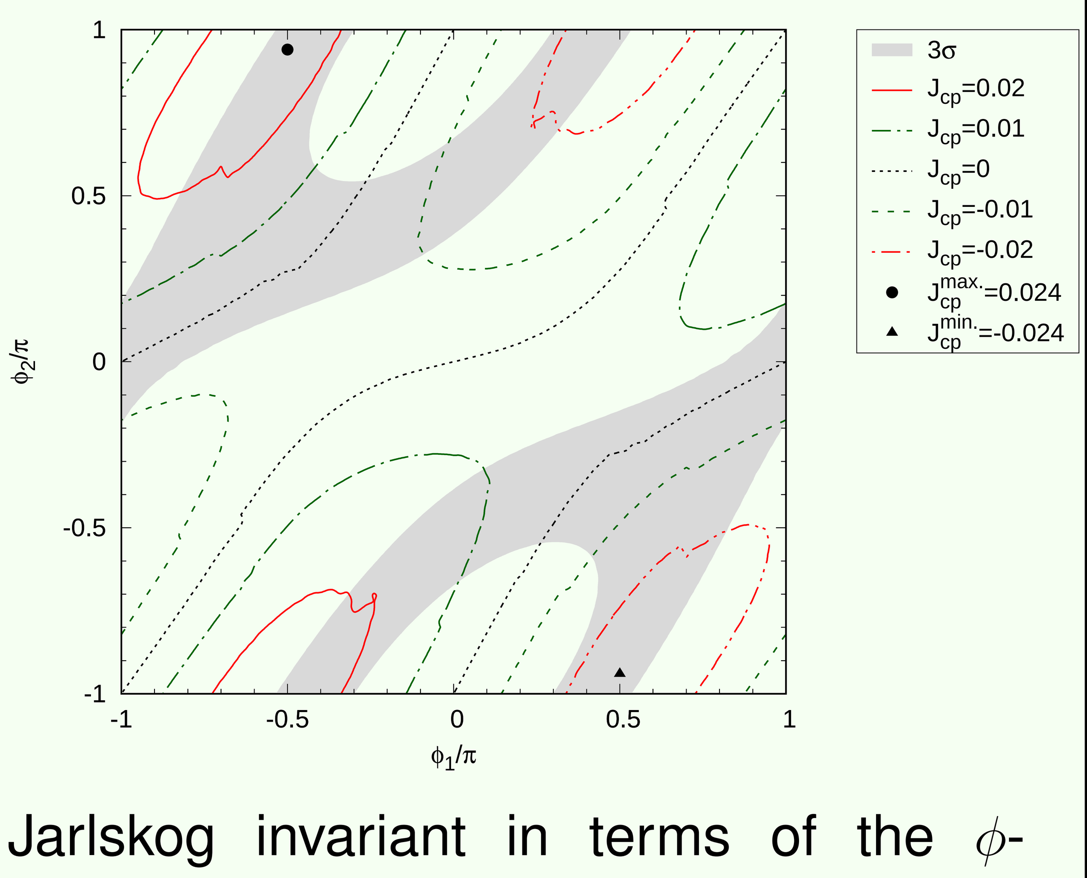
3.2 Analysis of the T$_1$-form

In this case the four parameters $\vec{\lambda} = \{m_0, a_n, \phi_1, \phi_2\}$ contribute to the lepton mixing. The results are:



Parameter	Best fit	3σ range
m_0 ($\times 10^{-3}$eV)	3.3	$[1.8, 4.7]$
a_n ($\times 10^{-2}$eV)	2.3	$[1.4, 3.7]$
ϕ_1/π	0.4	Unconstrained
ϕ_2/π	0.9	Unconstrained

The χ^2 value at the minimum is $\chi^2_{\text{min.}} = 0$



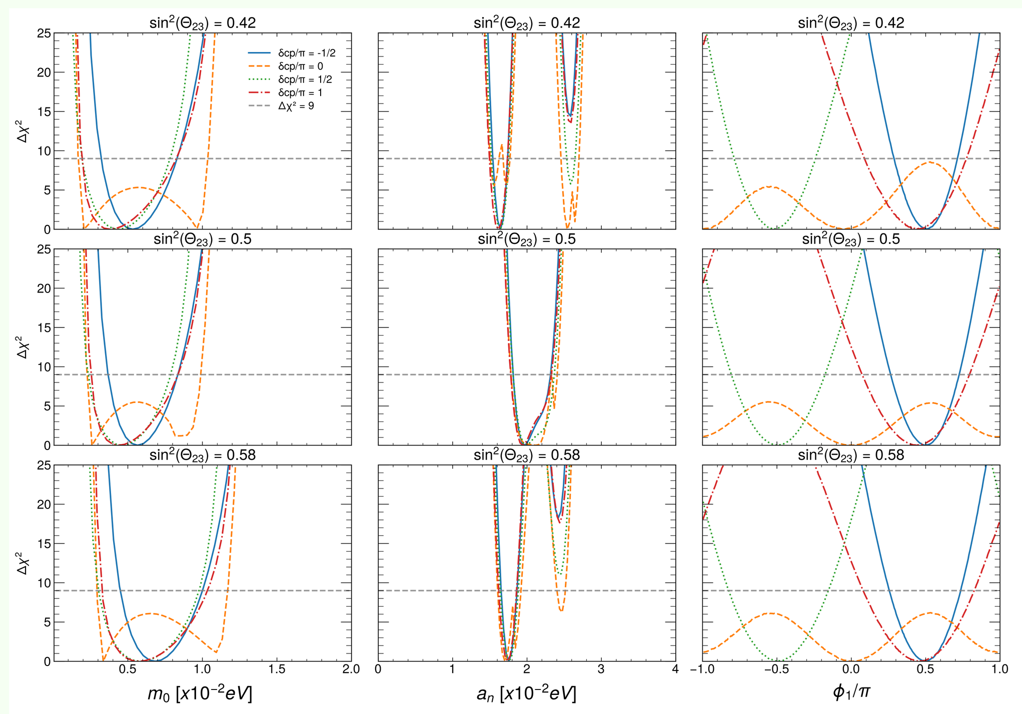
Jarlskog invariant in terms of the ϕ-phases.

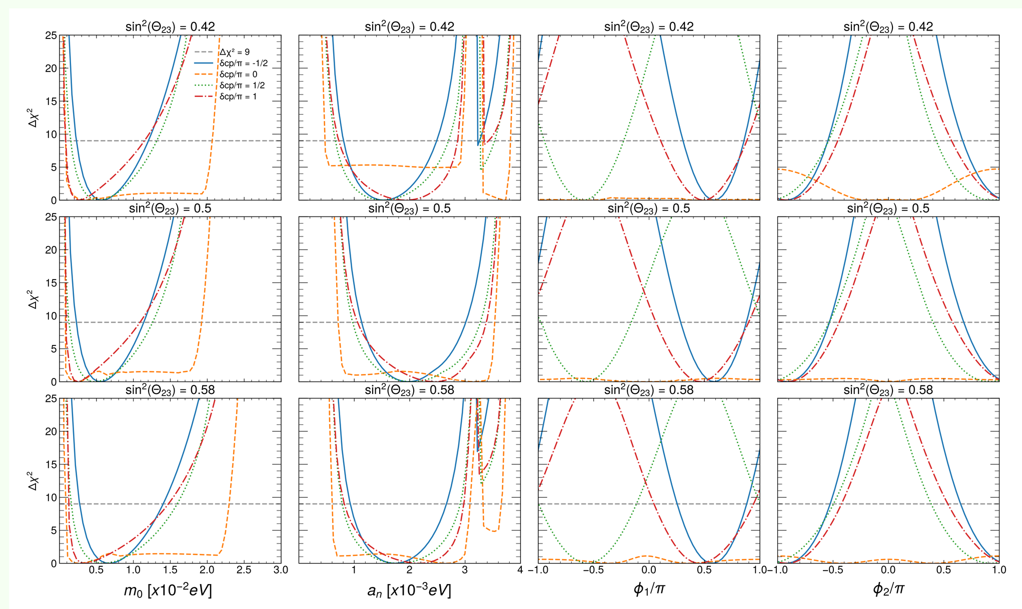
4. DUNE sensitivity to the mixing parameters

The DUNE experiment is a long-baseline multi-purpose neutrino oscillation experiment that is under construction. We calculated the lepton mixing matrix for each form showed in the first table and their were implemented in the GLoBES C-library probability engine. We simulated the disappearance and appearance process from $(\nu_\mu, \bar{\nu}_\mu)$.

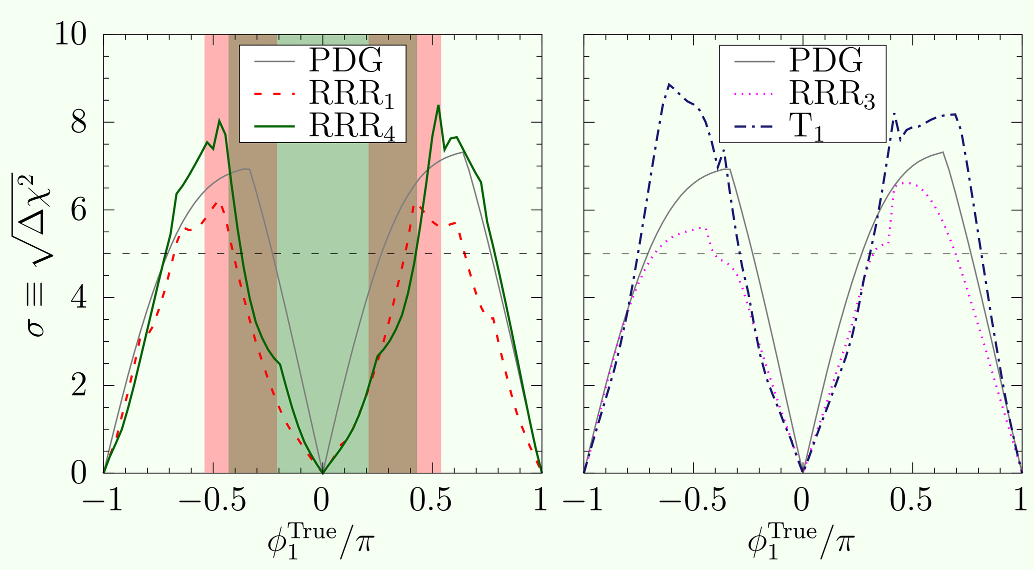
4.1 Results for the RRR$_4$ and T_1-form

$\Delta\chi^2$-profiles for each one of the three mixing parameters. ($\Delta\chi^2$-profiles for each one of the four mixing parameters $\vec{\lambda} = \{m_0, a_n, \phi_1, \phi_2\}$ respectively)





4.2 DUNE sensitivity to the CP-violating phases



In the left (right) panel the sensitivity obtained for the RRR$_4$ (T$_1$) forms.

5. Conclusions

Texture zeros diminish the mathematical parameters in the models, they produce an alternative to the PDG parametrization, that can explain all observables of neutrino physics.

6. References

This work is in a review process in a journal (PRD), for more details see:: https://arxiv.org/abs/2207.04072.