Construction of a traversable wormhole from a suitable embedding function

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Introduction

Since the seminal work by Morris and Thorne [1], the study of traversable wormholes has remain as an attractive research line for many years. However, the detection of gravitational waves by LIGO/Virgo from the merger of binary black holes [2-4] has driven the attention on the possibility of considering wormholes as black holes mimickers [5-7]. To be more precise, as the ringdown phase of black hole mergers is dominated by the quasinormal modes of the final object, it has been claimed that wormholes can mimic black holes based on the similitude of their quasinormal modes spectrum.

In this work, it is our main goal to construct an asymptotically flat traversable wormhole supported by a finite amount of exotic matter by assuming a general embedding function. Besides, we explore its stability thorough its response to scalar perturbation. The response of a wormhole to perturbations is dominated by damped oscillations called quasi--normal modes. In this work, we shall use the recently developed WKB approximation which has brought the attention of the community [8].

A wormhole model

Let us consider the spherically symmetric line element

$$ds^2 = -e^{2\phi}dt^2 + dr^2/(1-b/r) + r^2(d\theta^2 + \sin^2\theta d\phi^2),$$

With a null redshift function. Now consider the embedding function

$$z(r) = \sqrt{\log\left(a + \left(\frac{cr}{r_0} + d\right)^2\right)}$$

Where a,c and d are free parameters.

Imposing the existence of a throat and the flare out condition

$$a = 1 - c^2 - 2cd - d^2$$

The shape function is

$$b(r) = \frac{c^2\zeta^2 r}{c^2\zeta^2 + r_0^2\left(a+\zeta^2\right)^2\log\left(a+\zeta^2\right)}$$

Where

$$\zeta = \left(\frac{cr}{r_0} + d\right).$$

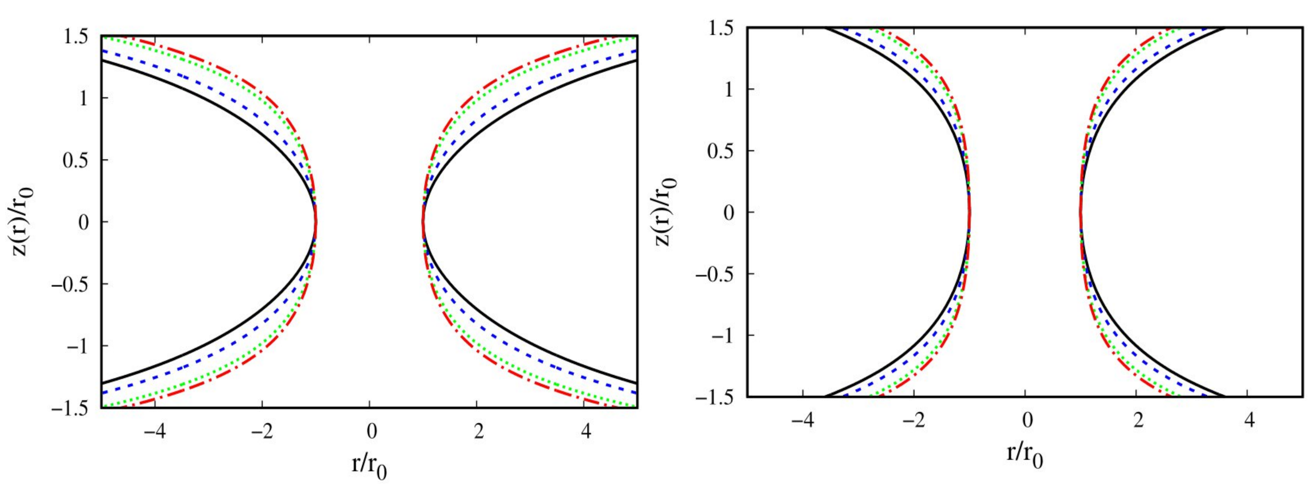

Fig. 1. Embedding diagram for c = 0.4 and c = 0.8 , d = 0.6 (blue), d = 1.4 (green), d = 1.8 (red)

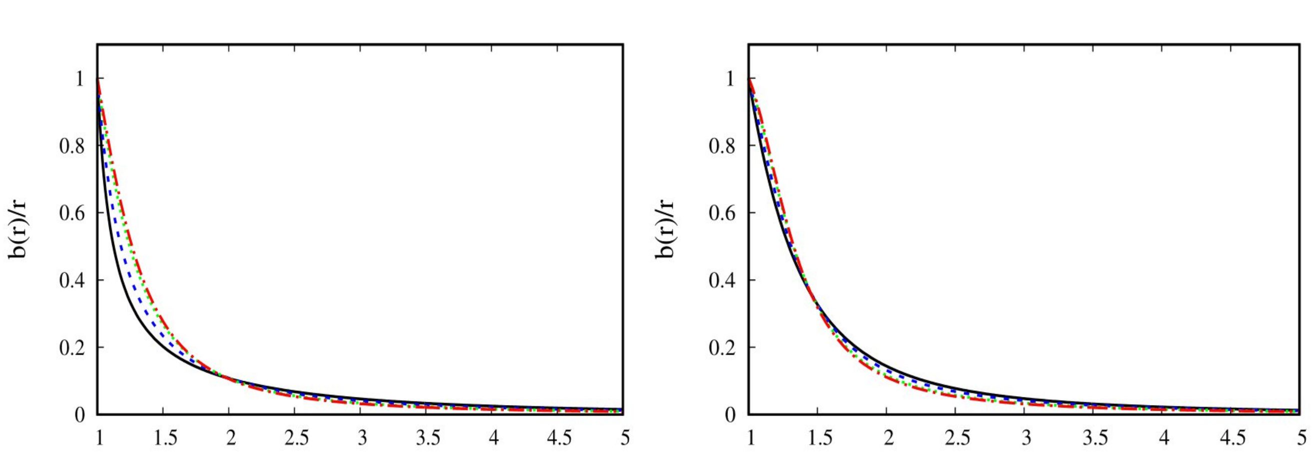

Fig. 2. Plots of   as a function of for c = 0.4 and c = 0.8, d = 0.6 (blue), d = 1.4 (green), d = 1.8 (red)

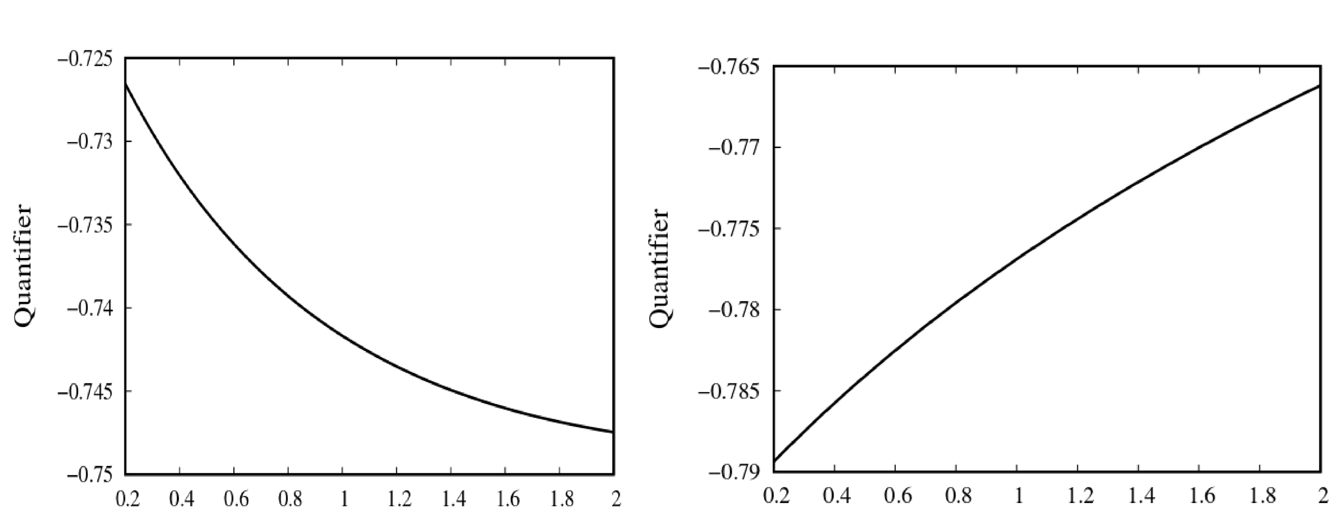

Fig. 3. Quantifier as a function of d for c = 0.4 and c = 0.8

Quasinormal modes

For a scalar field perturbation the equation governing the evolution of the perturbation can be reduced to a like–Schrödinger equation given by

$$\left(\frac{d^2}{dx^2} + \omega^2 - V(x)\right)\chi(x) = 0, \quad x(r) = \int_{r_0}^{r} \frac{1}{\sqrt{1-b(r')/r'}}dr', \quad V_L(r) = e^{2\phi}\left(\frac{L(L+1)}{r^2} - \frac{rb'-b}{2r^3} + \frac{\phi'}{r}\left(1-\frac{b}{r}\right)\right)$$

Where x is the tortoise radial cordinate and V(r) is an effective potential. The solution is

$$\chi(x) \sim C_{\pm}\exp(\mp i)$$

Corresponding to purely out-going waves at infinity, are the QNM with frequency $\omega = Re(\omega) + iIm(\omega)$.

We shall implement the WKB approach taking advantage of the Schrödinger-like equation with a potential barrier. For the 6th order aproximation the equation reads

$$i\frac{\omega^2 - V_0}{\sqrt{-2V_0''}} - \sum_{j=2}^{6}\Lambda_j = n + \frac{1}{2},$$

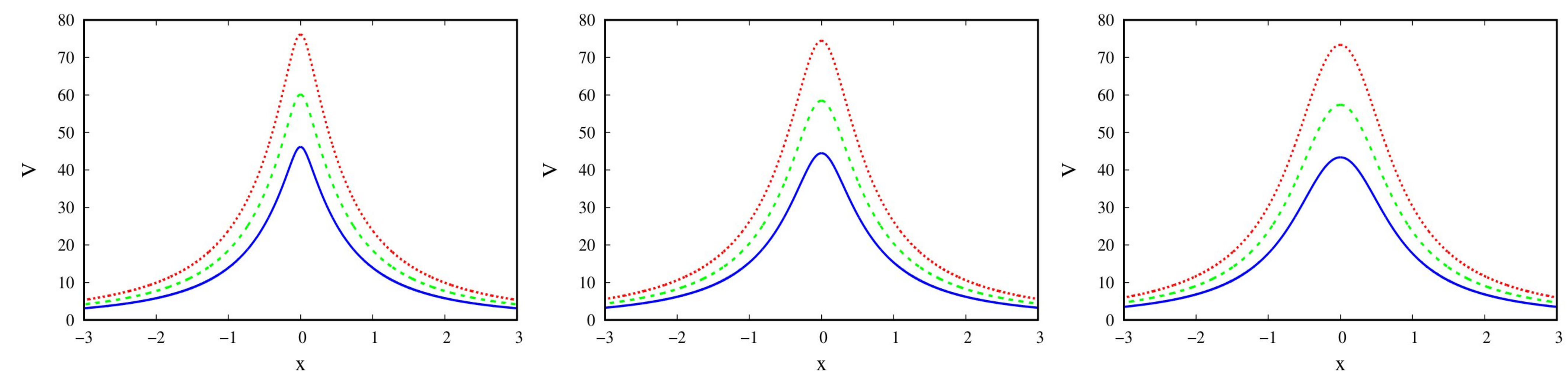

Fig. 4. The potential of perturbation as a function of x (the tortoise coordinate) for l = 6 (blue), l = 7 (green), l = 8 (red). We have set c = 0.4 , d = 0.2 (left panel), d = 0.6 (center panel) and d = 1.4 (right panel).

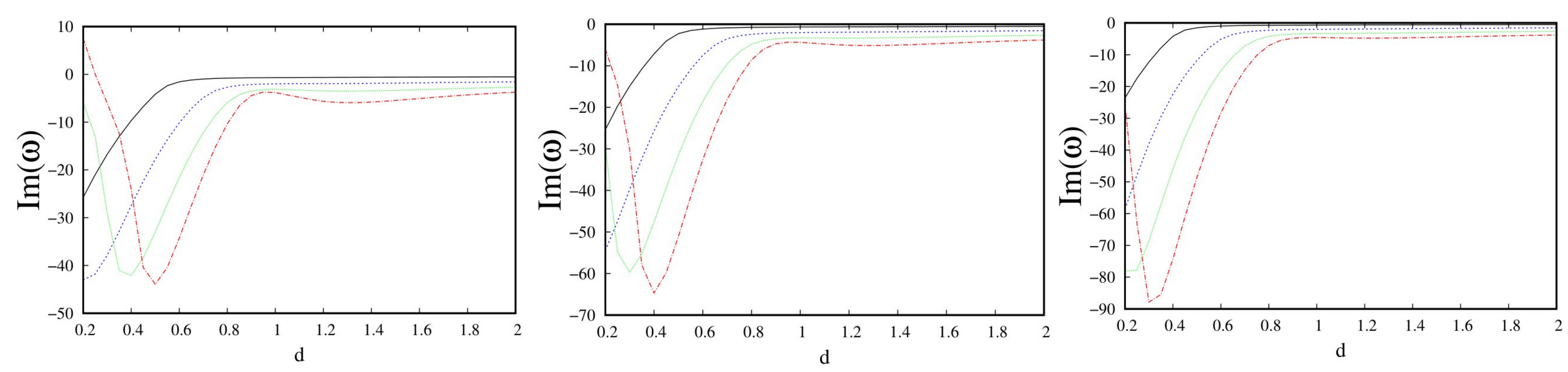

Fig. 5. Imaginary part of the frequency as a function of d for n = 0 (black), n = 1 (blue), n = 2 (green), n = 3 (red). We have set c = 0.4, l = 6(left panel), l = 7 (center panel) and l = 8 (right panel).

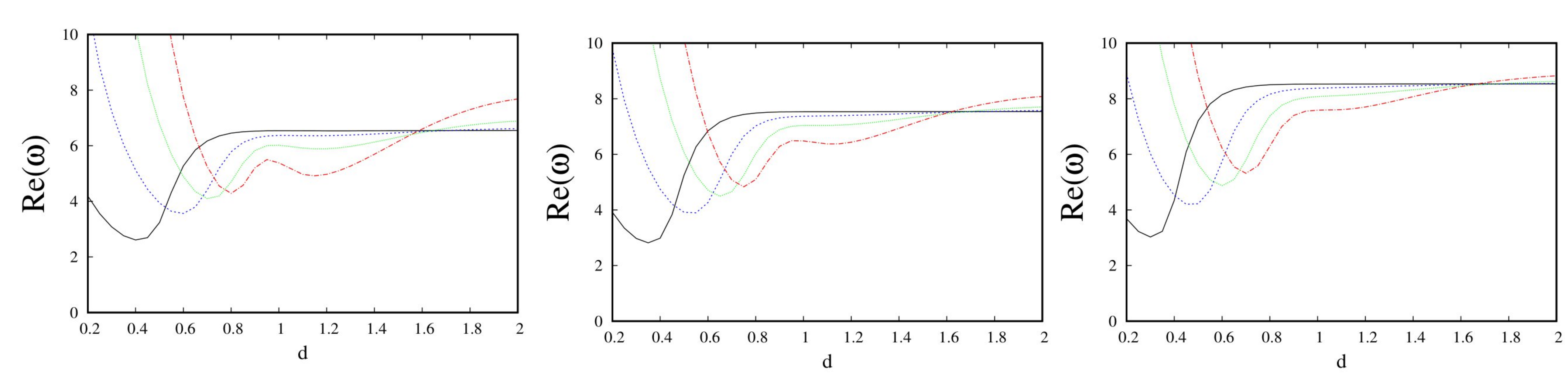

Fig. 6. Real part of the frequency as a function of d for n = 0 (black), n = 1 (blue), n = 2 (green), n = 3 (red). We have set c = 0.4 (first row), l = 6 (left panel), l = 7 (center panel) and l = 8 (right panel).

Conclusion

In this work we obtained a traversable wormhole with vanishing radial tidal force by proposing a general embedding function with some free parameters.We demanded the existence of a minimum radius and the flaring out condition. In order to explore how the geometry behaves in terms of the remaining parameter we analyzed both the quantifier of the exotic matter and the quasi normal modes of the solution.We observed that the solution requires a finite amount of exotic matter that decreases for certain values of the parameters involved. Besides,the imaginary part of the quasinormal frequencies remains negative which leads to a suitable damping factor for the signal.

References

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