Hubble tension and matter inhomogeneities: a theoretical perspective

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arXiv:2107.14377 (Working on referee’s replies)

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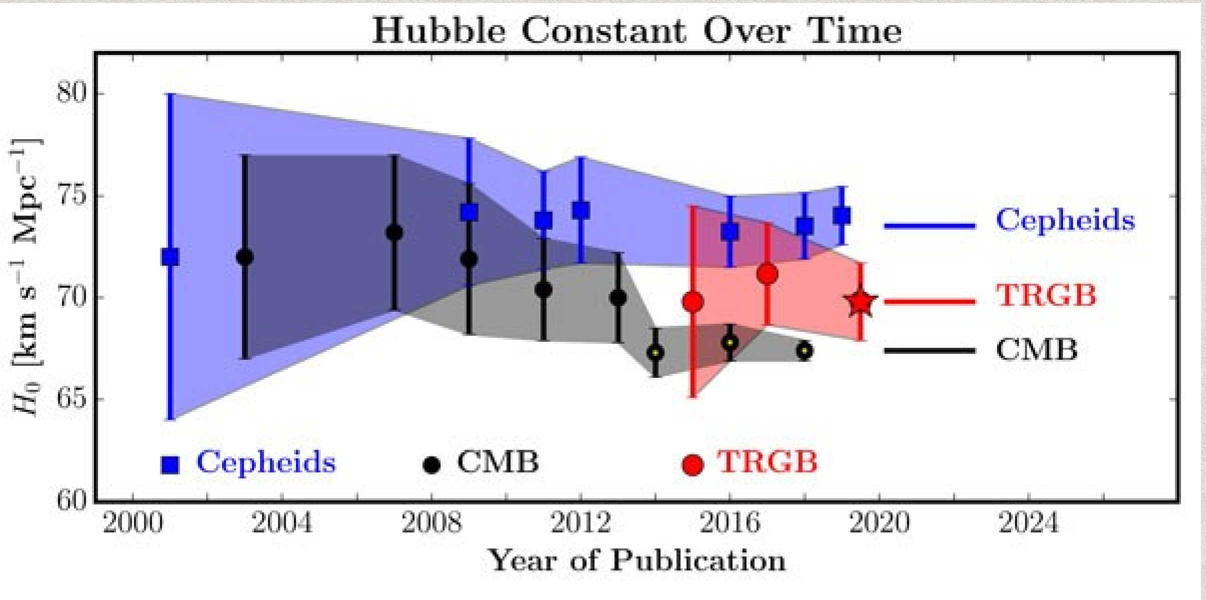
Hubble Tension over time



https://doi.org/10.1146/annurev-nucl-111119-041046

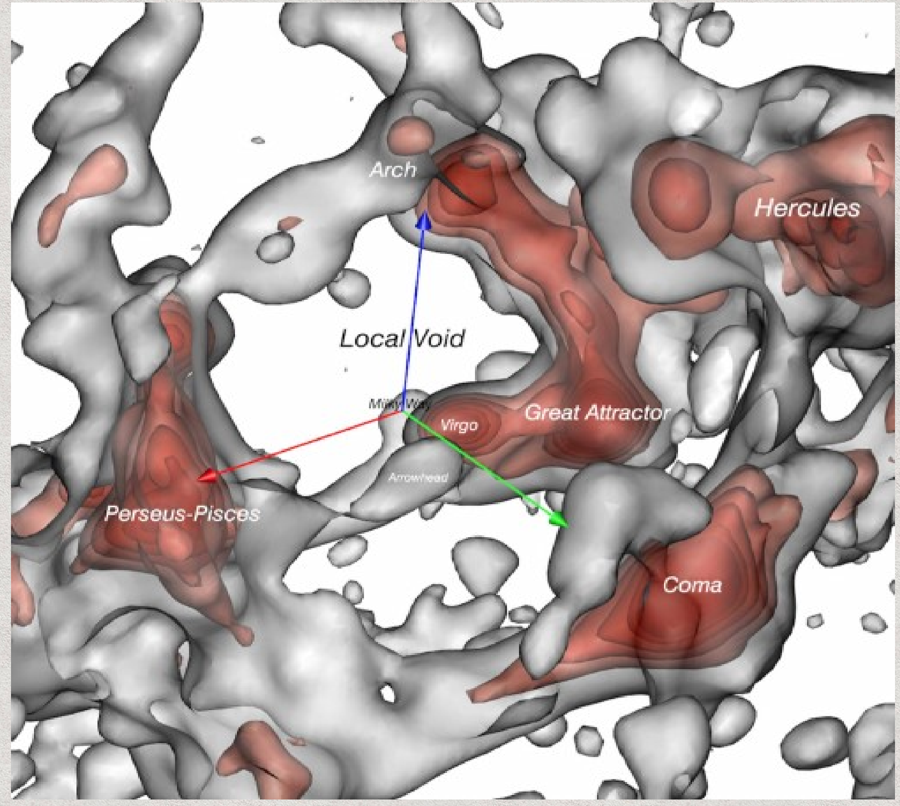
Some solutions...

- Time-varying Dark Energy (DE) models.
- Extended models (For example: J. Alfaro, M. San Martín and C.R. https://doi.org/10.3847/1538-4357/abddc3)
- Modifications in the early-time physics.

Our approach

- We studied the possibility that local Hubble measurements differ from the Planck data because the local density of the universe is different from the global.
- This idea has been studied before by Adam Riess and other authors. In general, they concluded that our local void is not enough (in the sense of gravitational potential) to alleviate the controversy.

The local void: Our neighborhood



- Cosmicflows-3. R. Brent Tully et all. https://doi.org/10.3847/1538-4357/ab2597

Our approach

- We assume a FLRW metric with scalar perturbations in a Newtonian gauge with no curvature to describe and inhomogeneity that is spherically symmetric.
- This approach considers the temporal evolution of the field. This evolution is essential because from redshift 0.15 until now elapse approx. 2 Gyrs. This period allows the galaxies to move through space due to the field. In other words, our description allows a temporal evolution of a local spherically symmetric overdensity/subdensity, which affects the real dependence of the luminosity distance measured by a local observer centered on the origin of this inhomogeneity.

Notation

- Any observable measured parameter, which is determined by assuming a non-pertubed FLRW metric, is denoted by the superscript Lo (from local).
- On the other hand, big scale parameters are defined by superscript Pl (from Planck).

Our approach

- We will use two different series with different meanings

$$a^{\mathrm{Pl}}(\tau) \approx 1 + H_0^{\mathrm{Pl}}\tau - \frac{1}{2}q_0^{\mathrm{Pl}}\left(H\tau\right)^2 + \frac{1}{6}j_0^{\mathrm{Pl}}\left(H\tau\right)^3 + \mathcal{O}\left(\tau\right)^4$$

- and

$$a^{\mathrm{Lo}}(\tau) \approx 1 + H_0^{\mathrm{Lo}}\tau - \frac{1}{2}q_0^{\mathrm{Lo}}\left(H\tau\right)^2 + \frac{1}{6}j_0^{\mathrm{Lo}}\left(H\tau\right)^3 + \mathcal{O}\left(\tau\right)^4 .$$

Redshift dependence

- Consider a perturbed metric assuming the Newtonian gauge with scalar perturbations

$$ds^2 = -c^2 \left(1 + 2\Phi(t,\vec{x})\right) dt^2 + a(t)^2 \left(1 - 2\Psi(t,\vec{x})\right) \left(dx^2 + dy^2 + dz^2\right).$$

Redshift dependence

- If we do not include anisotropies in the Energy-momentum tensor (i.e. $\Phi = \Psi$), the perturbed redshift (at first order) is

$$z + \delta z$$

where

$$z = \frac{a(t_0)}{a(t)} - 1$$

and

$$\delta z = \frac{a(t_0)}{a(t)} \Big[-\Phi(r\hat{n}, t) + \Phi(0, t_0)$$

$$- 2 \int_t^{t_0} \left\{ \frac{\partial}{\partial t} \Phi(r\hat{n}, t) \right\}_{r=s(t)} dt - \frac{a(t)}{c} \partial_r v^N(r\hat{n}, t) \Big]$$

Luminosity distance

- The luminosity distance at first order is

$$d_L(z,\delta z) = (1+z+\delta z)r \\ + (1+z)\left[\frac{1}{2}(\Phi(r\hat{n},t) - \Phi(0,t_0))r - \int_0^r \Phi(r\hat{n},t_0)dr\right] + \mathcal{O}(\Phi^2)$$

Perturbative solution

- We separated the potential in temporal and radial parts:

$$\Phi(\tau, r) = F(\tau)G(r)$$

$$F(\tau) = \Phi_0 + f_1\tau + f_2\tau^2 + f_3\tau^3 + f_4\tau^4 + \mathcal{O}\left(\tau^5\right)$$

$$G(r) = 1 + g_2 r^2 + g_4 r^4 + \mathcal{O}\left(r^6\right)$$

- The i-i component of Einstein's equations gives and equation for the temporal part:

$$F(t)\left(3H^2(t) + 2\dot{H}(t)\right) + 4H(t)\dot{F}(t) + \ddot{F}(t) = 0\,.$$

Perturbative solution

- Using the big scale expansion for the scale factor, solutions for the temporal function are

$$f_2 = \frac{1}{2}\left(-\Phi_0 H_0^{\mathrm{Pl}^2} - 4 f_1 H_0^{\mathrm{Pl}} + 2\Phi_0 H_0^{\mathrm{Pl}^2} q_0^{\mathrm{Pl}}\right) ,$$

$$f_3 = \frac{1}{6}\left(6\Phi_0 H_0^{\mathrm{Pl}^3} + 19 f_1 H_0^{\mathrm{Pl}^2} - 2\Phi_0 H_0^{\mathrm{Pl}^3} j_0^{\mathrm{Pl}} - 8\Phi_0 H_0^{\mathrm{Pl}^3} q_0^{\mathrm{Pl}} + 6 f_1 H_0^{\mathrm{Pl}^2} q_0^{\mathrm{Pl}}\right) ,$$

$$f_4 = \frac{1}{24}\left(10\Phi_0 H_0^{\mathrm{Pl}^4} j_0^{\mathrm{Pl}} + 20\Phi_0 H_0^{\mathrm{PL}^4} q_0^{\mathrm{Pl}^2} + 30\Phi_0 H_0^{\mathrm{Pl}^4} q_0^{\mathrm{Pl}} - 37\Phi_0 H_0^{\mathrm{Pl}^4} - 8 f_1 H_0^{\mathrm{Pl}^3} j_0^{\mathrm{Pl}} - 76 f_1 H_0^{\mathrm{Pl}^3} q_0^{\mathrm{Pl}} - 108 f_1 H_0^{\mathrm{Pl}^3}\right) .$$

Luminosity Distance Expansion

- We expanded the luminosity distance

$$\mathrm{d}_L(z) = \frac{cz}{H_0^{\mathrm{Pl}}}\left(\mathcal{D}_L^0 + \mathcal{D}_L^1 z + \mathcal{D}_L^2 z^2 + O(z)^3\right)$$

$$\mathcal{D}_L^0 = 1 + \Phi_0 + \frac{f_1}{H_0^{\mathrm{Pl}}} - \frac{4c^2\Phi_0 g_2}{3{H_0^{\mathrm{Pl}}}^2} - \frac{4c^2 f_1 g_2}{3{H_0^{\mathrm{Pl}}}^3}$$

$$\mathcal{D}_L^1 = -\frac{3\Phi_0 q_0^{\mathrm{Pl}}}{2} + \Phi_0 - \frac{q_0^{\mathrm{Pl}}}{2} + \frac{1}{2} - \frac{3f_1 q_0^{\mathrm{Pl}}}{2H_0^{\mathrm{Pl}}} + \frac{f_1}{2H_0^{\mathrm{Pl}}}$$
$$- \frac{\Phi_0 c^2 g_2}{\left(H_0^{\mathrm{Pl}}\right)^2} + \frac{6c^2\Phi_0 g_2 q_0^{\mathrm{Pl}}}{{H_0^{\mathrm{Pl}}}^2} + \frac{14c^2 f_1 g_2 q_0^{\mathrm{Pl}}}{3{H_0^{\mathrm{Pl}}}^3} - \frac{10c^2 f_1 g_2}{3{H_0^{\mathrm{Pl}}}^3}$$

Luminosity Distance Expansion

- We expanded the luminosity distance

$$\mathrm{d}_L(z) = \frac{cz}{H_0^{\mathrm{Pl}}}\left(\mathcal{D}_L^0 + \mathcal{D}_L^1 z + \mathcal{D}_L^2 z^2 + O(z)^3\right)$$

$$\mathcal{D}_L^2 = -\frac{\Phi_0 j_0^{\mathrm{Pl}}}{2} + \frac{5\Phi_0 {q_0^{\mathrm{Pl}}}^2}{2} - \frac{\Phi_0}{4} - \frac{j_0^{\mathrm{Pl}}}{6} + \frac{{q_0^{\mathrm{Pl}}}^2}{2} + \frac{q_0^{\mathrm{Pl}}}{6} - \frac{1}{6} - \frac{2f_1 j_0^{\mathrm{Pl}}}{3H_0^{\mathrm{Pl}}}$$

$$+\frac{5f_1 {q_0^{\mathrm{Pl}}}^2}{2H_0^{\mathrm{Pl}}} - \frac{7f_1}{12H_0^{\mathrm{Pl}}} + \frac{26c^2\Phi_0 g_2 j_0^{\mathrm{Pl}}}{9{H_0^{\mathrm{Pl}}}^2} - \frac{18c^2\Phi_0 g_2 {q_0^{\mathrm{Pl}}}^2}{{H_0^{\mathrm{Pl}}}^2} + \frac{16c^2\Phi_0 g_2 q_0^{\mathrm{Pl}}}{9{H_0^{\mathrm{Pl}}}^2} - \frac{5c^2\Phi_0 g_2}{18{H_0^{\mathrm{Pl}}}^2}$$

$$-\frac{31c^2 f_1 g_2}{9{H_0^{\mathrm{Pl}}}^3} + \frac{20c^2 f_1 g_2 j_0^{\mathrm{Pl}}}{9{H_0^{\mathrm{Pl}}}^3} - \frac{38c^2 f_1 g_2 {q_0^{\mathrm{Pl}}}^2}{3{H_0^{\mathrm{Pl}}}^3} + \frac{58c^2 f_1 g_2 q_0^{\mathrm{Pl}}}{9{H_0^{\mathrm{Pl}}}^3} - \frac{8c^4\Phi_0 g_4}{3{H_0^{\mathrm{Pl}}}^4} - \frac{8c^4 f_1 g_4}{3{H_0^{\mathrm{Pl}}}^5}$$

Luminosity Distance Expansion

- We can compare with standard local luminosity distance

$$\mathrm{d}_L^{\mathrm{Std}}(z) = \frac{cz}{H_0^{\mathrm{Lo}}} \left(\mathcal{D}_L^{0,\mathrm{Std}} + \mathcal{D}_L^{1,\mathrm{Std}} z + \mathcal{D}_L^{2,\mathrm{Std}} z^2 + O(z)^3 \right)$$

$$\mathcal{D}_L^{0,\mathrm{Std}} = 1\,,$$

$$\mathcal{D}_L^{1,\mathrm{Std}} = -\frac{1}{2}(-1 + q_0^{\mathrm{Lo}})\,,$$

$$\mathcal{D}_L^{2,\mathrm{Std}} = -\frac{1}{6}(1 - q_0^{\mathrm{Lo}} - 3{q_0^{\mathrm{Lo}}}^2 + j_0^{\mathrm{Lo}})\,.$$

First results and conclusions

- Planck Data

$$H_0^{\rm Pl} = (67.4 \pm 0.5)\, km/s/Mpc \qquad q_0^{\rm Pl} = -0.5275 \qquad j_0^{\rm Pl} = 1$$

- Local Data

$$H_0^{\rm Lo} = (73,24 \pm 1.74)\, km/s/Mpc$$

- Our parameters are

$$q_0^{\rm Lo}(\Phi_0\,, f_1\,, g_2\,, g_4) \quad and \quad j_0^{\rm Lo}(\Phi_0\,, f_1\,, g_2\,, g_4)$$

First results and conclusions

- If we impose a local void of

$$\Omega_{m,\mathrm{void}} = (-0.30 \pm 0.15)\Omega_m$$

- There is no solution compatible with ΛCDM model.

- We are exploring the space of parameters for compatible solutions.

Final conclusions

- Local density perturbations could reconcile the Hubble tension between local and cosmological measurements of the Universe expansion.
- The temporal evolution is important because from redshift z=0.15 until now elapse approx 2 Gyrs. This is enough time for photons to feel the temporal evolution of the potential due to the local void.
- When constraining local cosmological parameters with Planck results, we found that neither Λ CDM nor $\Lambda(\omega)$ CDM could solve the Hubble tension.

Thank you!