Oscillation tomography study of Earth's composition and density with atmospheric neutrinos

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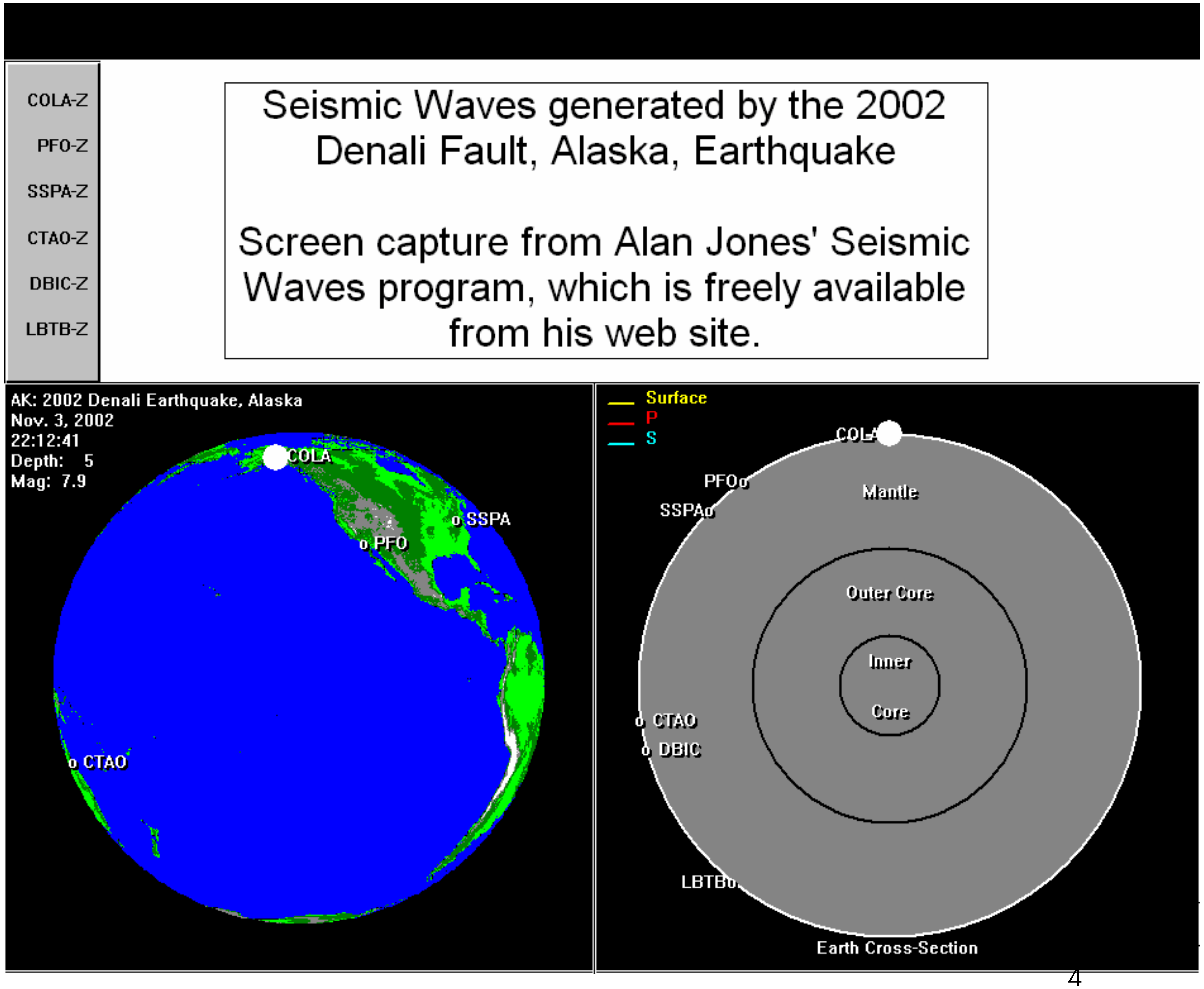
Introduction

Why study Earth’s Interior?

Knowledge of the composition and inner structure is essential for understanding basic geological phenomena, such as volcanology, earthquakes, plate tectonics, and mountain building.

Among other applications are the extraction of minerals and the location of oil fields.

Mains sources of information



Seismic Waves generated by the 2002 Denali Fault, Alaska, Earthquake

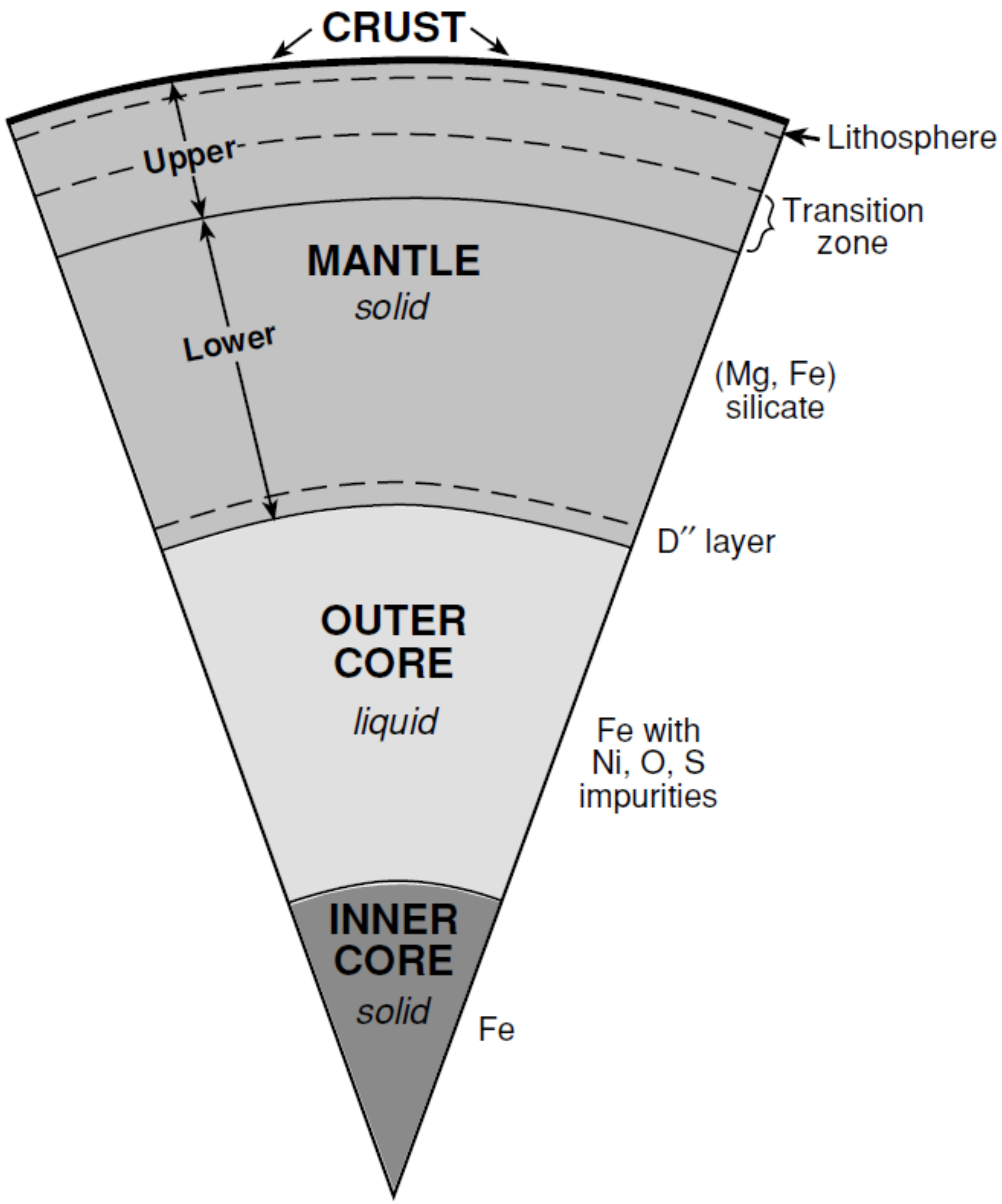
Screen capture from Alan Jones' Seismic Waves program, which is freely available from his web site.

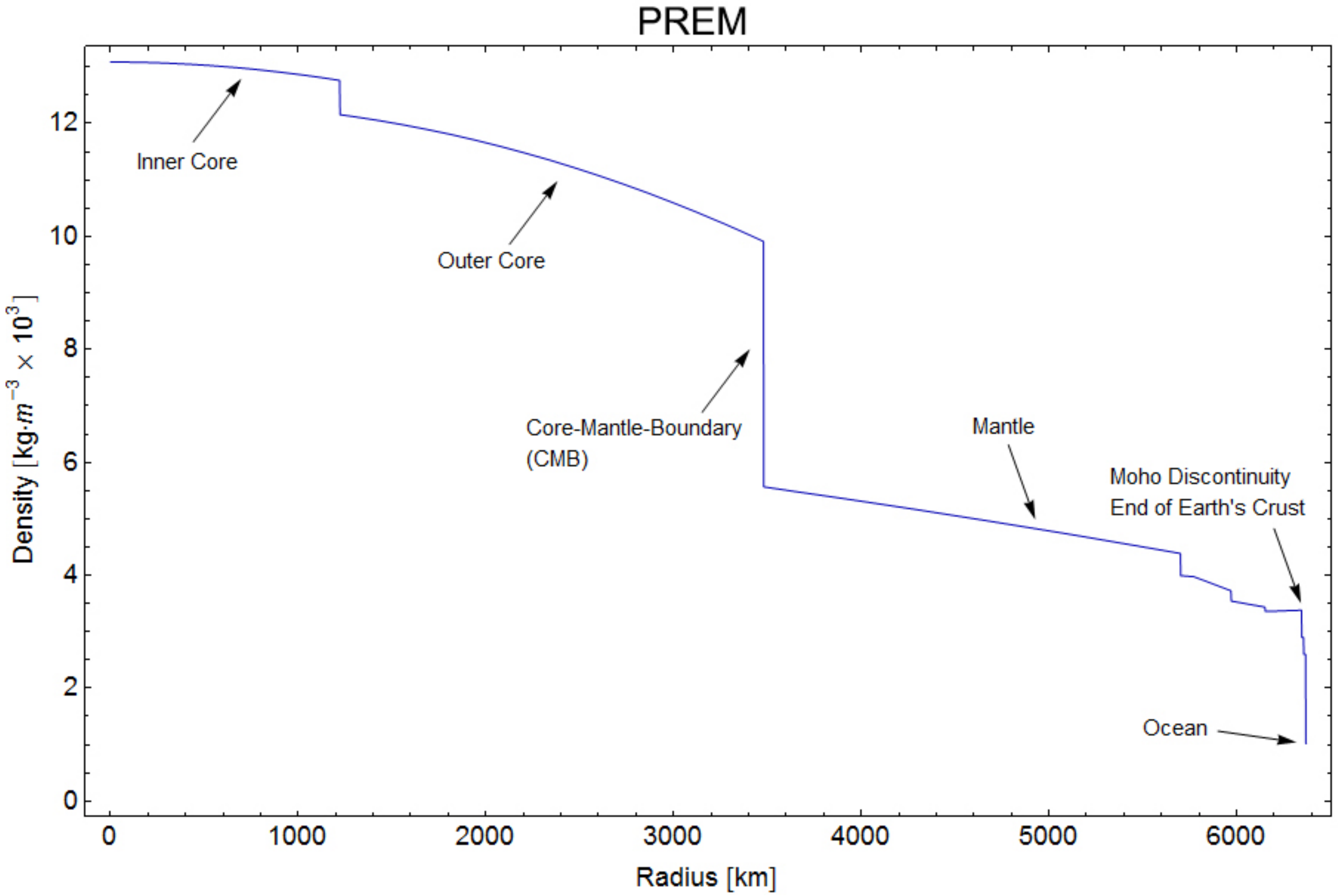
AK: 2002 Denali Earthquake, Alaska
Nov. 3, 2002
22:12:41
Depth: 5
Mag: 7.9

Kola Superdeep Borehole

Earth Model

Earth’s inner structure





Main Layers

Crust (0 - Moho | $\rho = (2.60 - 2.90)$ g/cm^3)

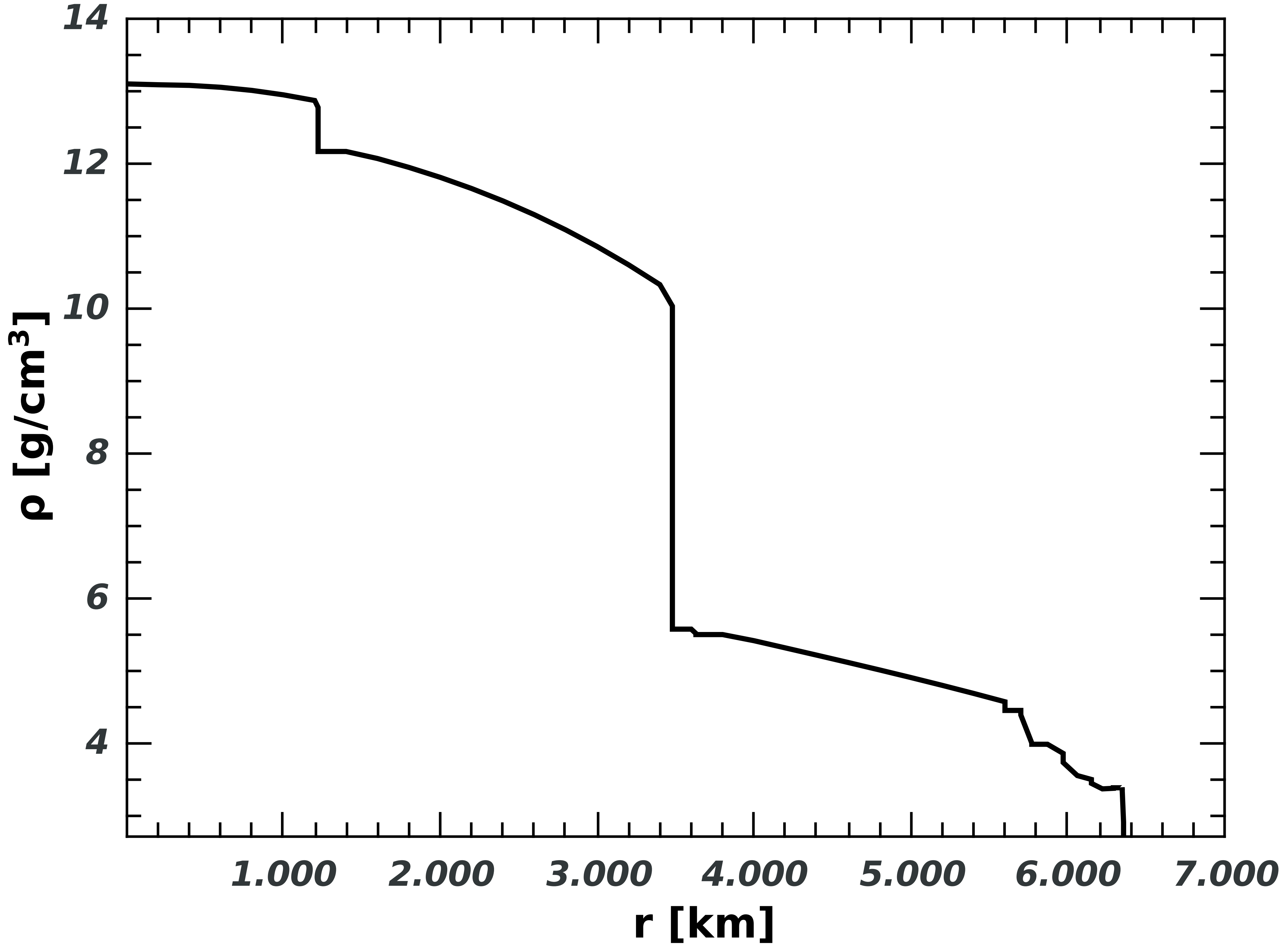
Upper Mantle (Moho - 670 km | $\rho = (3.38 - 3.99)$ g/cm^3)

Lower Mantle (670 km - 2891 km | $\rho = (4.38 - 5.56)$ g/cm^3)

Outer Core (2891 km - 5150 km | $\rho = (9.90 - 12.16)$ g/cm^3)

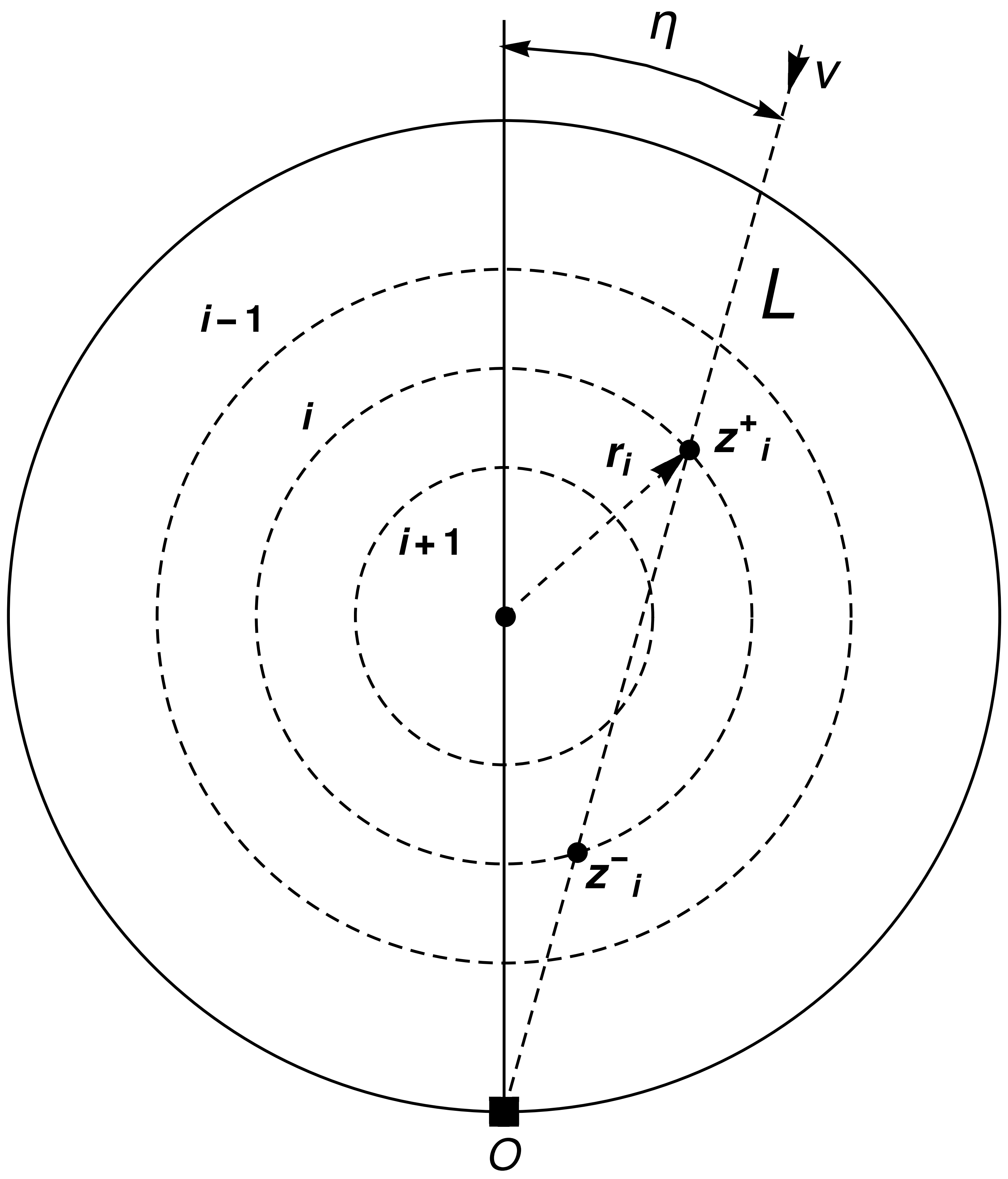
Inner Core (5150 km - 6371 km | $\rho = (12.76 - 13.08)$ g/cm^3)

Our Model



55-layer approximation of the PREM profile

Our work is the only one in the literature that considers both restrictions.



$$M_\oplus = \frac{4}{3}\pi \sum_{i=1}^{55} \rho_i (R_{i+1}^3 - R_i^3) = M_{\mathrm{IC}} + M_{\mathrm{OC}} + M_{\mathrm{M}_1} + M_{\mathrm{M}_2} + M_{\mathrm{C}} = 5.9724 \times 10^{27}\ \mathrm{g}$$

$$(1)$$

$$I_\oplus = \frac{8}{15}\pi \sum_{i=1}^{55} \rho_i (R_{i+1}^5 - R_i^5) = I_{IC} + I_{\mathrm{OC}} + I_{\mathrm{M}_1} + I_{\mathrm{M}_2} + I_{\mathrm{C}} = 5.9724 \times 10^{44}\ \mathrm{g\,cm}^2$$

Table 1 Compositions of the main Earth layers

Layer	n^0 of shells	R_{inf} - R_{sup} [km]	Z/A
Inner core	7	0–1221.5	0.4691
Outer core	13	1221.5–3480	0.4691
Lower mantle	N_{M_1}—21	3480—R_{M_1}	0.4954
Upper mantle	49-N_{M_1}	R_{M_1}—6346	0.4954
Crust	6346–6371	6346–6371	0.4956

Our Model

To modify the densities of the outer core and the lower and upper mantle, we multiply the densities of all shells within each of these layers by the respective rescaling factor, $f_{\rm OC}$, $f_{\rm M_1}$ and, $f_{\rm M_2}$.This is done in such a way that neither $M_\oplus$ nor $I_\oplus$changes.

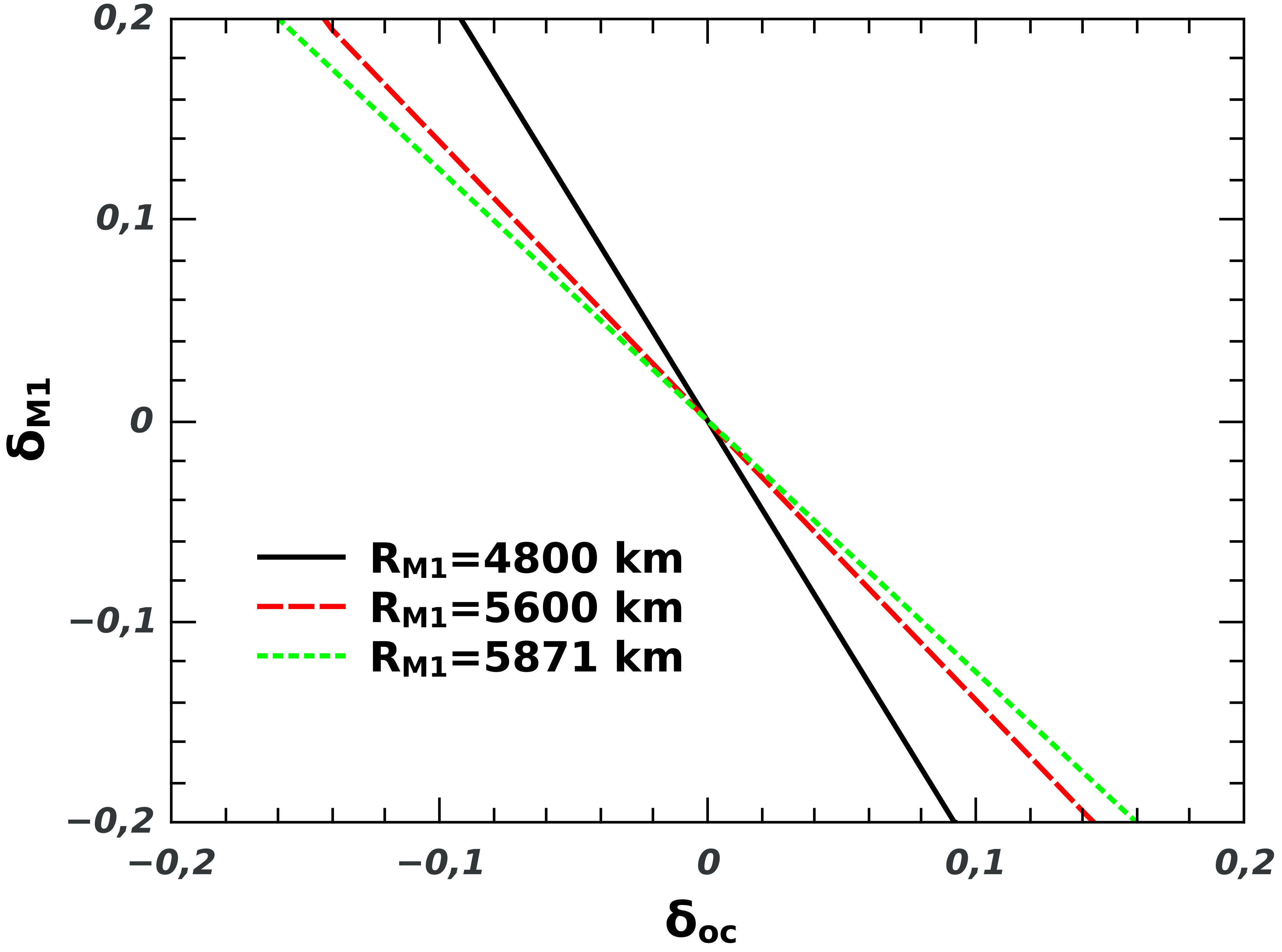
$$\begin{aligned} M_\oplus &= M_{\rm IC} + f_{\rm OC} M_{\rm OC} + f_{\rm M_1} M_{\rm M_1} + f_{\rm M_2} M_{\rm M_2} + M_{\rm C} \\ I_\oplus &= I_{IC} + f_{\rm OC} I_{\rm OC} + f_{\rm M_1} I_{\rm M_1} + f_{\rm M_2} I_{\rm M_2} + I_{\rm C} \end{aligned} \tag{2}$$

Equating Eqs. (1) and (2), we obtain the following homogeneous system of linear equations:

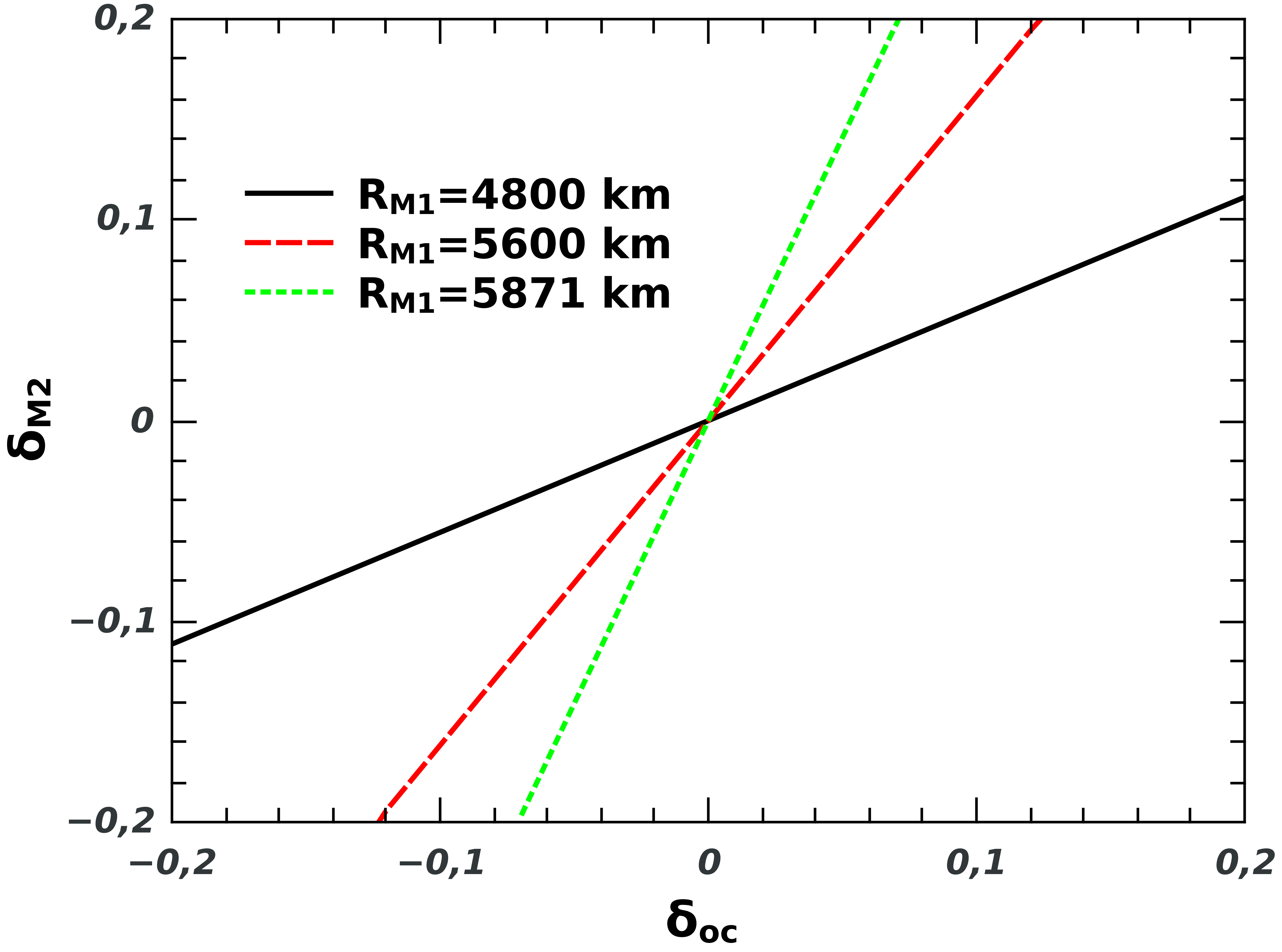
$$\begin{aligned} \delta_{\rm OC} M_{\rm OC} + \delta_{\rm M_1} M_{\rm M_1} + \delta_{\rm M_2} M_{\rm M_2} &= 0, \\ \delta_{\rm OC} I_{\rm OC} + \delta_{\rm M_1} I_{\rm M_1} + \delta_{\rm M_2} I_{\rm M_2} &= 0, \end{aligned} \tag{3}$$

Where $\delta_{\rm OC} = f_{\rm OC} - 1$, $\delta_{\rm M_1} = f_{\rm M_1} - 1$, and $\delta_{\rm M_2} = f_{\rm M_2} - 1$ are the relative changes of the densities in the outer core, lower mantle, and upper mantle, respectively.

Lower Mantle



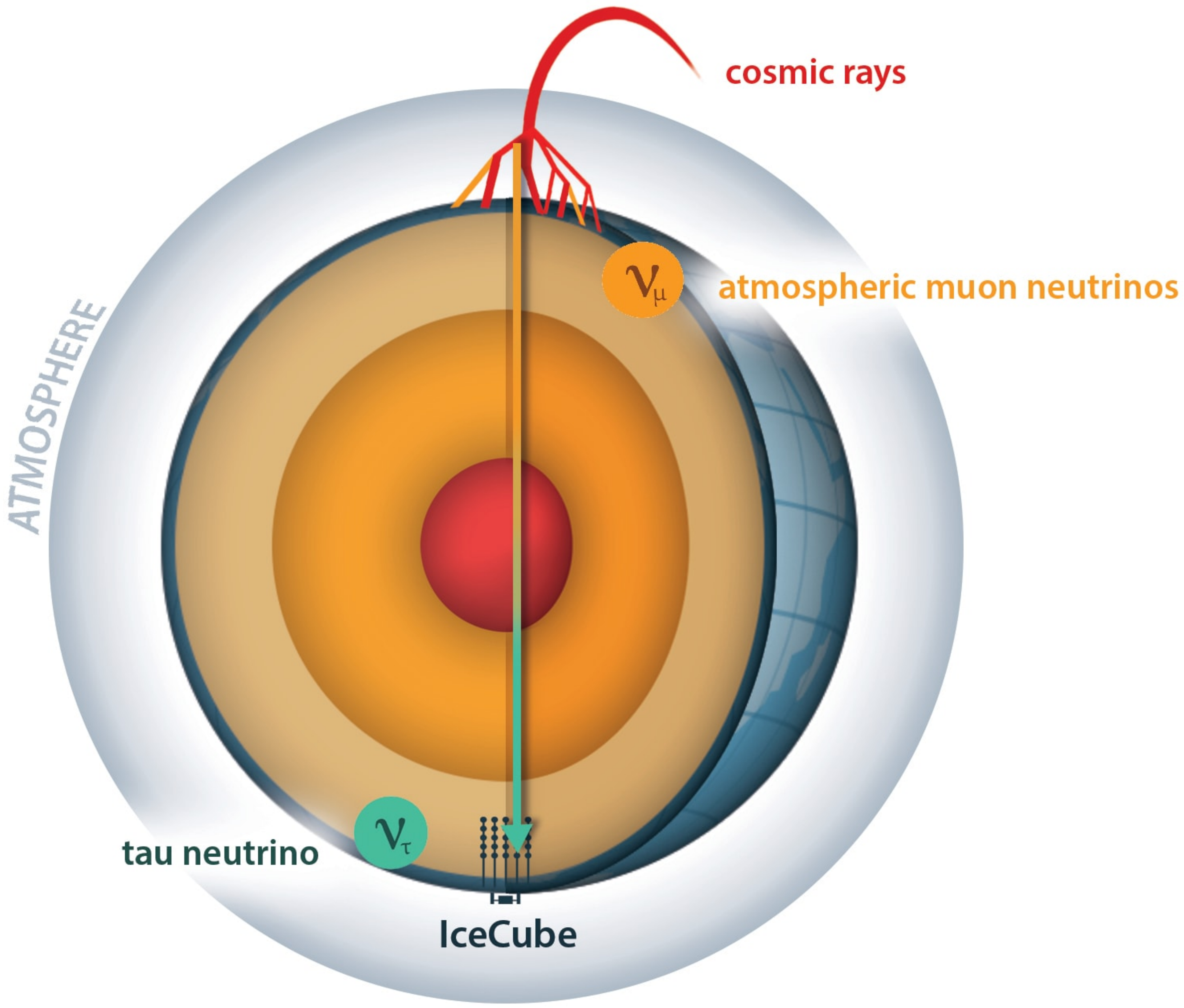
Upper Mantle



The value of the radius R_{M_1} set the position of the boundary between the regions M_1 and M_2 and by varying it we can change the number of shells within each of these layers. This in turn modifies the values of $M_{M_{1,2}}$ and $I_{I_{1,2}}$ and makes the quantities $\delta_{1,2}$ dependent on R_{M_1}. The above graphs show the relative changes in the densities of layers M_1 and M_2 as a function of the relative change in the density of the outer core for three different positions of the boundary between M_1 and M_2.

Atmospheric neutrino oscillations

Neutrino oscillation in matter

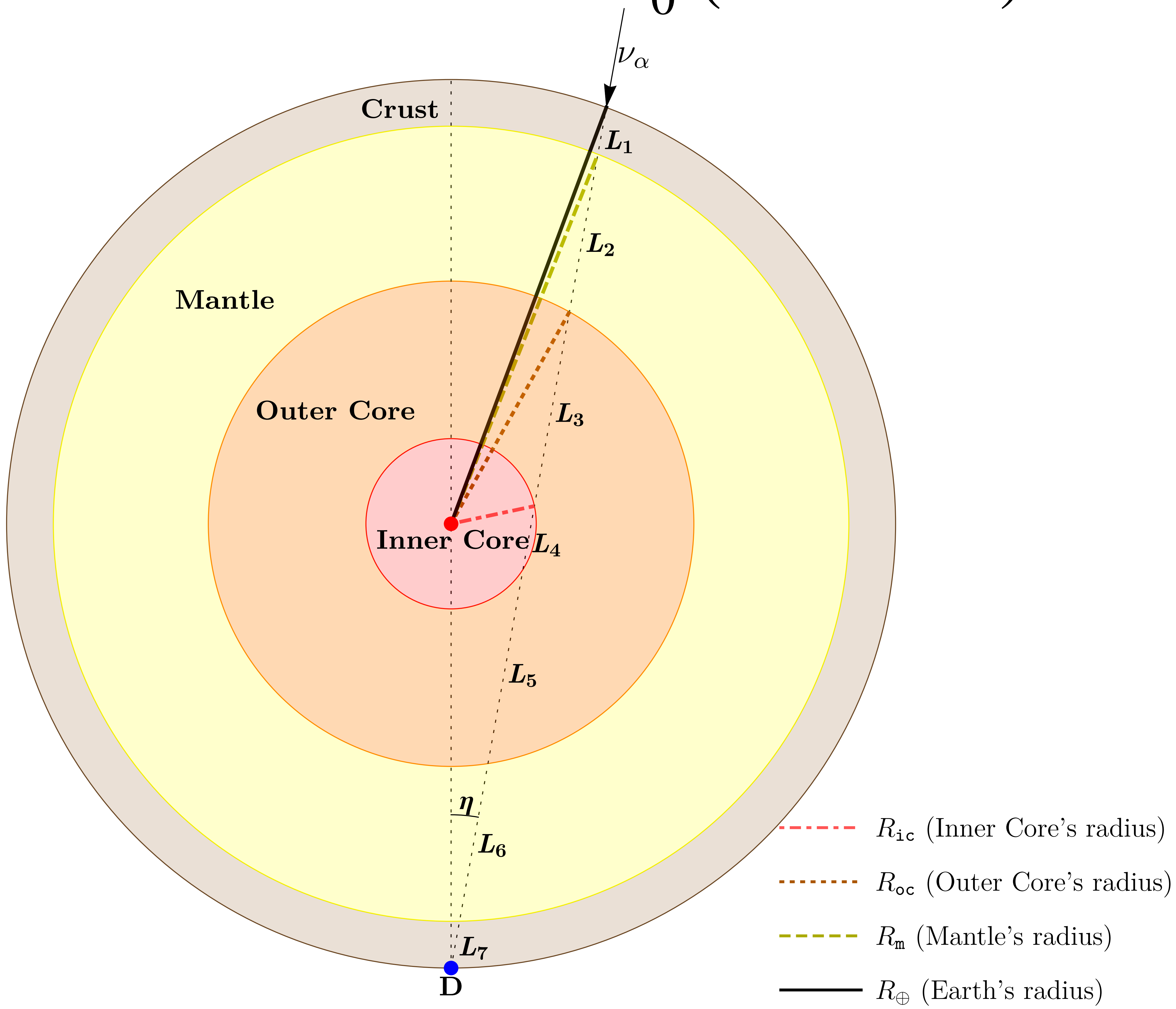


When neutrinos propagate in a medium, the coherent forward scattering with electrons is different for ν_e and $\nu_{\mu,\tau}$, resulting in different refraction indexes for the electron neutrino and the other flavors.

Consequently, neutrino oscillations can be modified in matter compared to oscillations in vacuum, and new resonance enhancement effects appear.

These effects are sensitive to the ***density*** and ***composition*** of the medium, and we will take advantage of this to examine the inner parts of our planet through the oscillation of atmospheric neutrinos.

Let a neutrino ν_α that enters the solid terrestrial matter at time t_0. At any time $t > t_0$ the state of the system $|\psi(t)\rangle$ can be expressed as $|\psi(t)\rangle = \hat{\mathscr{U}}(t, t_0)|\psi(t_0)\rangle$, where $|\psi(t_0)\rangle = |\nu_\alpha\rangle$ and $\hat{\mathscr{U}}(t, t_0)$ is the evolution operator. The probability of having a neutrino of flavor β inside the Earth, at a distance $\ell \simeq t - t_0$ $(\hbar = c = 1)$ from the entry point, is



$$\boxed{P_{\nu_\alpha \to \nu_\beta}(\ell) = |\hat{\mathcal{U}}_{\beta\alpha}(\ell)|^2} \tag{4}$$

$$\boxed{i\frac{\mathrm{d}}{\mathrm{d}\ell}\hat{\mathcal{U}}(\ell) = \hat{H}(\ell)\hat{\mathcal{U}}(\ell)} \tag{5}$$

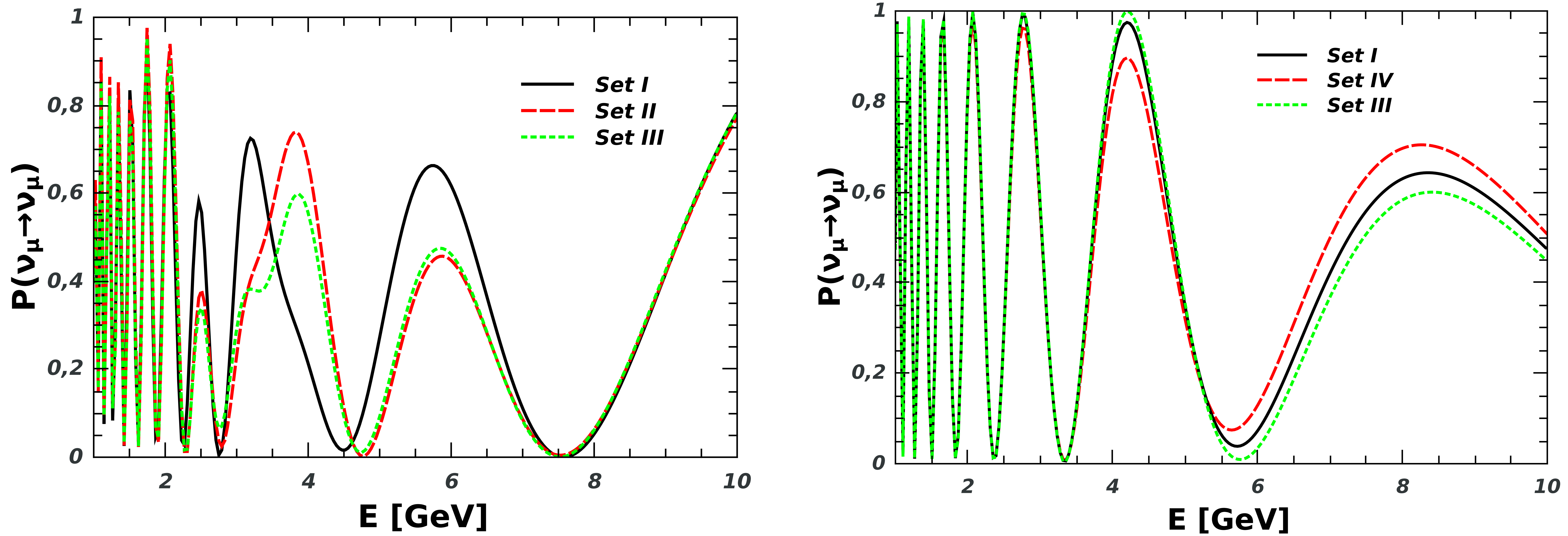
$$\boxed{\hat{H}(\ell) = U H_0 U^\dagger + V_{\mathrm{CC}}(\ell) Y} \tag{6}$$

$$\boxed{U = \mathcal{O}_{23}\Gamma\mathcal{O}_{13}\mathcal{O}_{12}} \tag{7}$$

$$\boxed{V_{\mathrm{CC}} = \sqrt{2}G_{\mathrm{F}} n_e(\ell) = \sqrt{2}G_{\mathrm{F}}\frac{\rho(\ell)}{m_u}\frac{Z}{A}(\ell)} \tag{8}$$

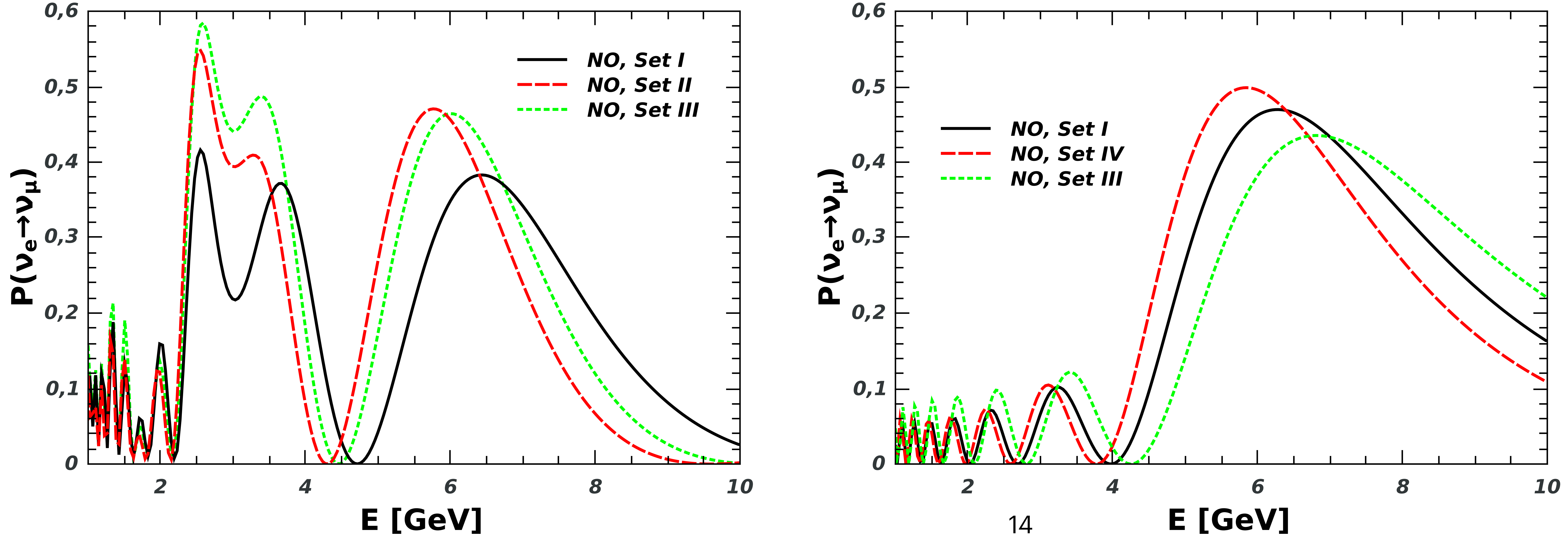
$$H_0 = \mathrm{diag}(0, \Delta_{21}, \Delta_{31}) \qquad Y = \mathrm{diag}(1, 0, 0)$$

Neutrino events and test of Earth‘s composition



Muon survival probability for a nadir angle of 20º and 50º

Set	f_{oc}	$f_{(Z/A)_{oc}}$	$f_{(Z/A)_{M_1}}$
I	1	1	1
II	1	1.01	1
III	1.01	1	1
IV	1	1	1.01



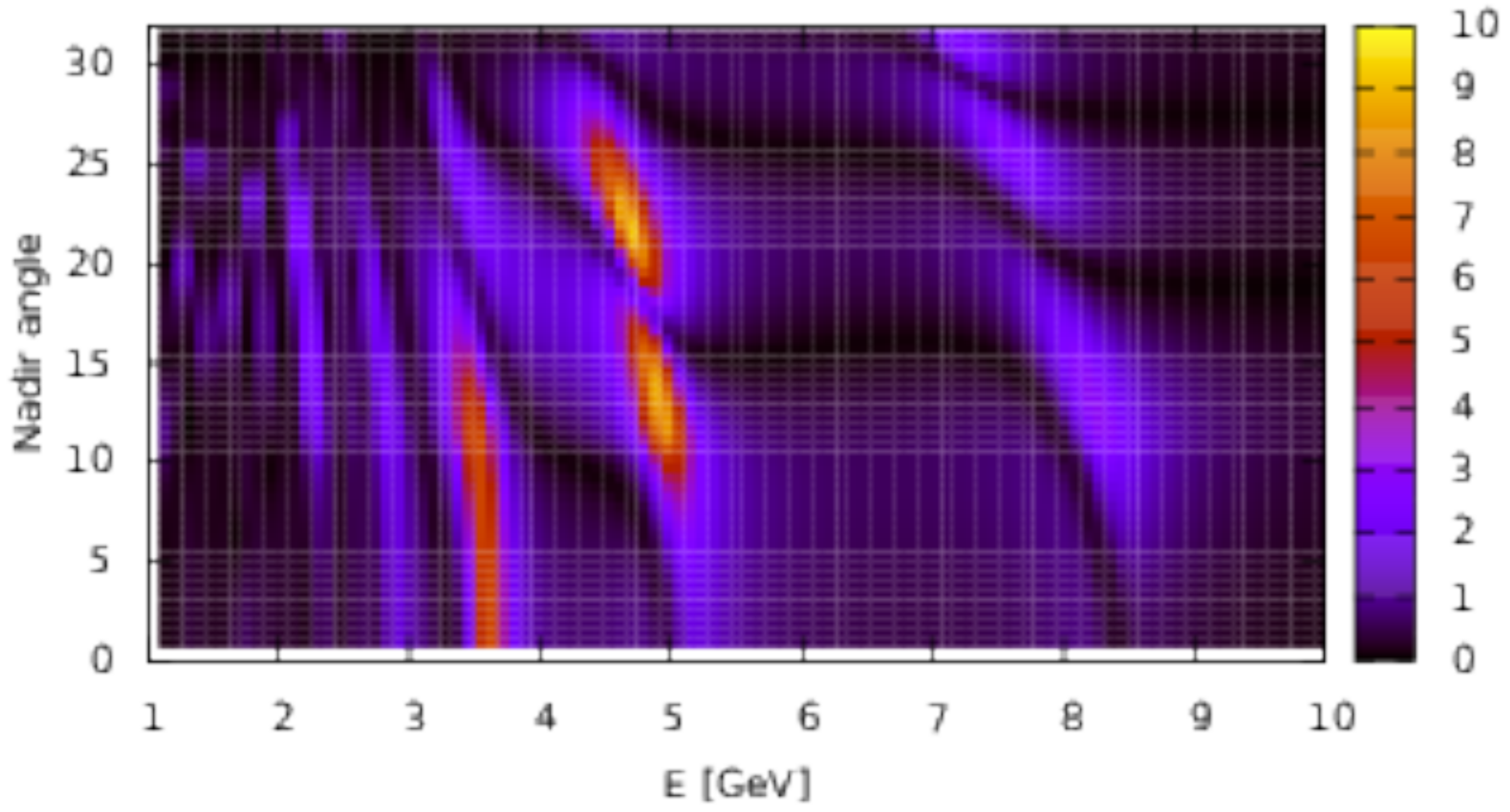
Electron to muon neutrino conversion probability for a nadir angle of 20º and 50º

Sensitivity zones

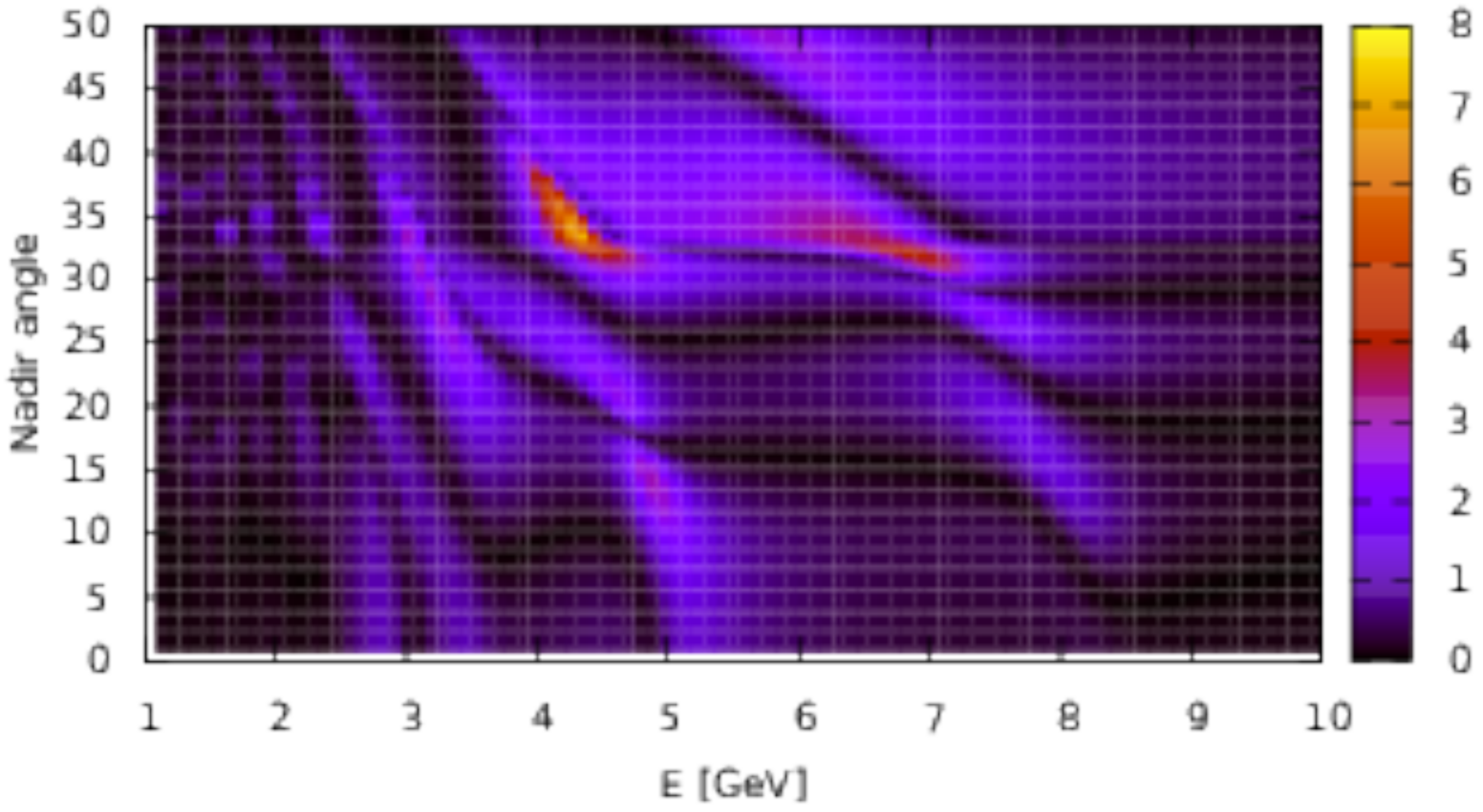
$$\begin{aligned}\mathcal{N}_\mu = n_N T \int_{E_{\min}}^{E_{\max}} \mathrm{d}E \int_{\eta_{\min}}^{\eta_{\max}} \mathrm{d}\Omega \Bigg[&\sigma^{cc}_{\nu_\mu}(E)\left(P_{\nu_\mu\to\nu_\mu}\frac{d\Phi^{\nu_\mu}}{dE} + P_{\nu_e\to\nu_\mu}\frac{d\Phi^{\nu_e}}{dE}\right) + \sigma^{cc}_{\bar\nu_\mu}(E)\left(P_{\bar\nu_\mu\to\bar\nu_\mu}\frac{d\Phi^{\bar\nu_\mu}}{dE} + P_{\bar\nu_e\to\bar\nu_\mu}\frac{d\Phi^{\bar\nu_e}}{dE}\right) + \\ &\sigma^{cc}_{\nu_\tau}(E) Br_{\tau\to\mu}\left(P_{\nu_\mu\to\nu_\tau}\frac{d\Phi^{\nu_\mu}}{dE} + P_{\nu_e\to\nu_\tau}\frac{d\Phi^{\nu_e}}{dE}\right) + \sigma^{cc}_{\bar\nu_\tau}(E) Br_{\bar\tau\to\bar\mu}\left(P_{\bar\nu_\mu\to\bar\nu_\tau}\frac{d\Phi^{\bar\nu_\mu}}{dE} + P_{\bar\nu_e\to\bar\nu_\tau}\frac{d\Phi^{\bar\nu_e}}{dE}\right)\Bigg]\end{aligned}$$

$$\Upsilon(E,\eta) \equiv \left|1 - \frac{\mathcal{N}_\mu^{(Z/A)}}{\mathcal{N}_\mu^{0}}\right| \times 100$$

Normal ordering. 1% Change composition in the outer core

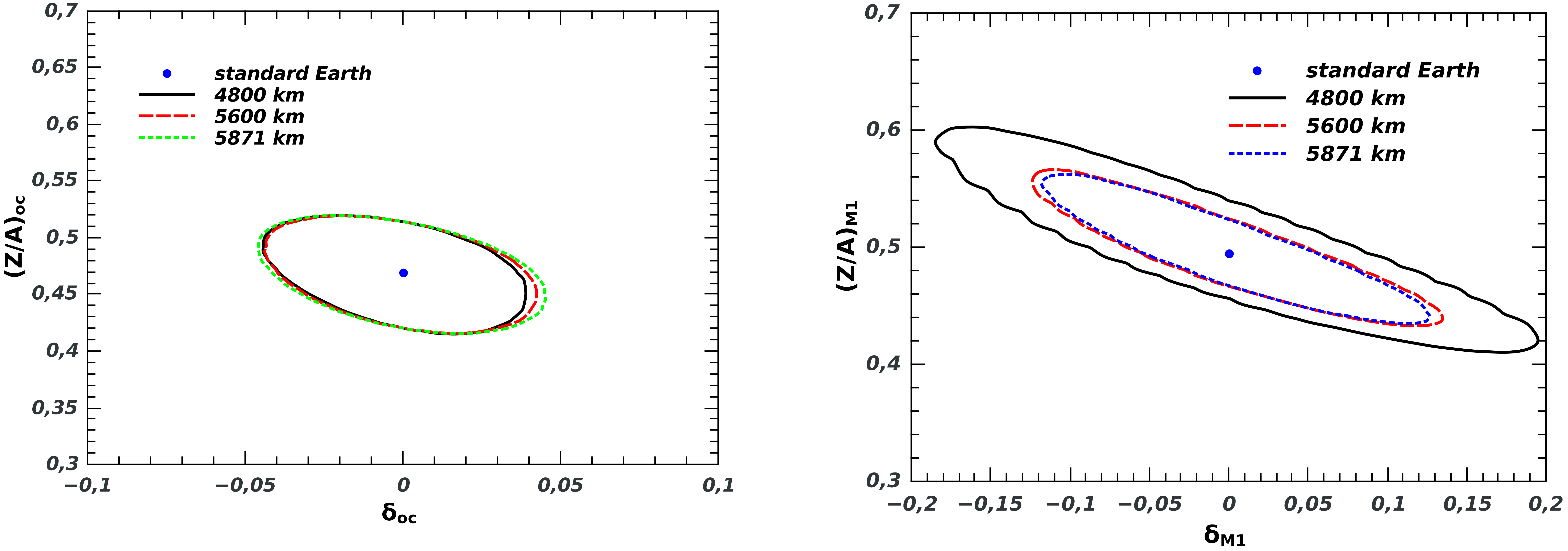


Inverse Ordering. 1% Change composition in the lower mantle



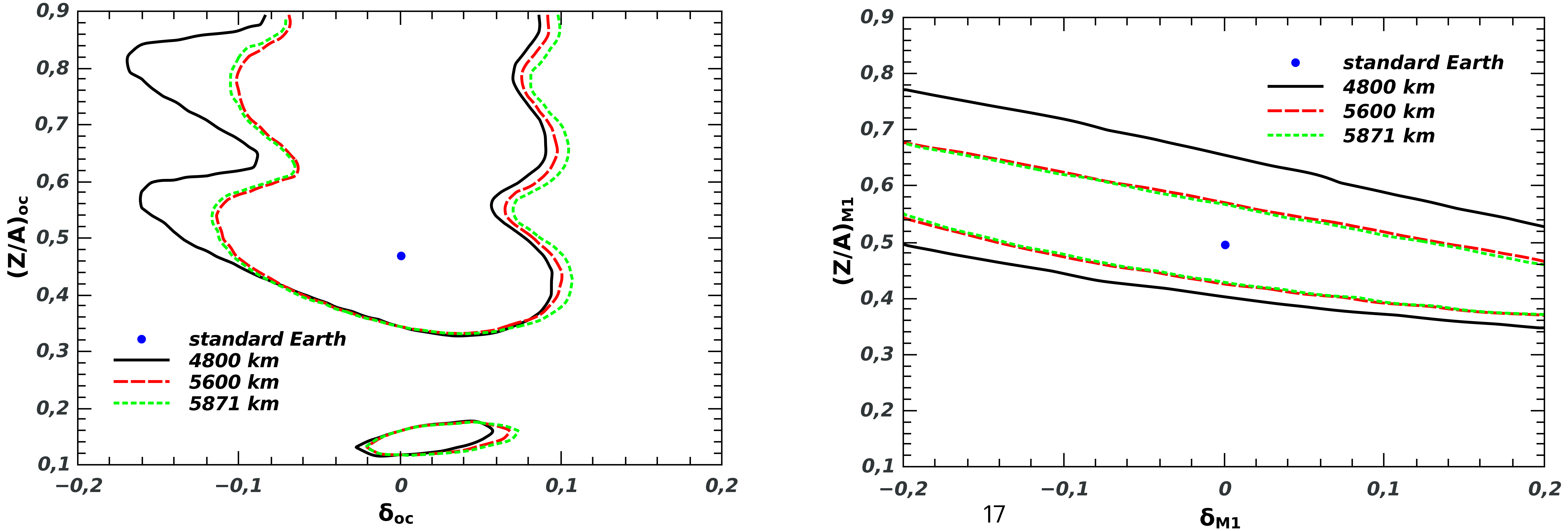
Level sets of the function $\Upsilon(E,\eta)$ for 1% change in the composition of the outer core

Results and final comments

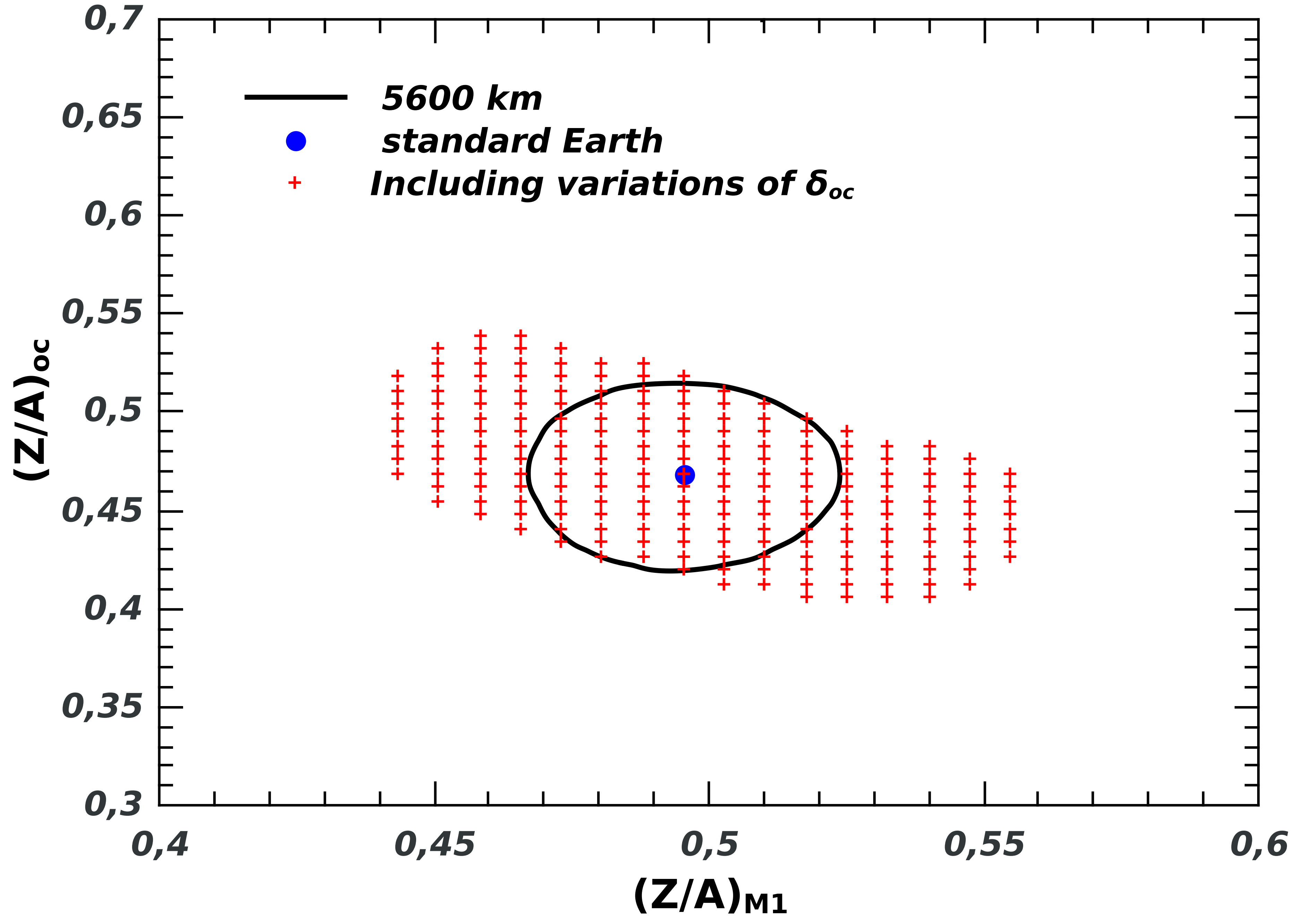


Normal Ordering

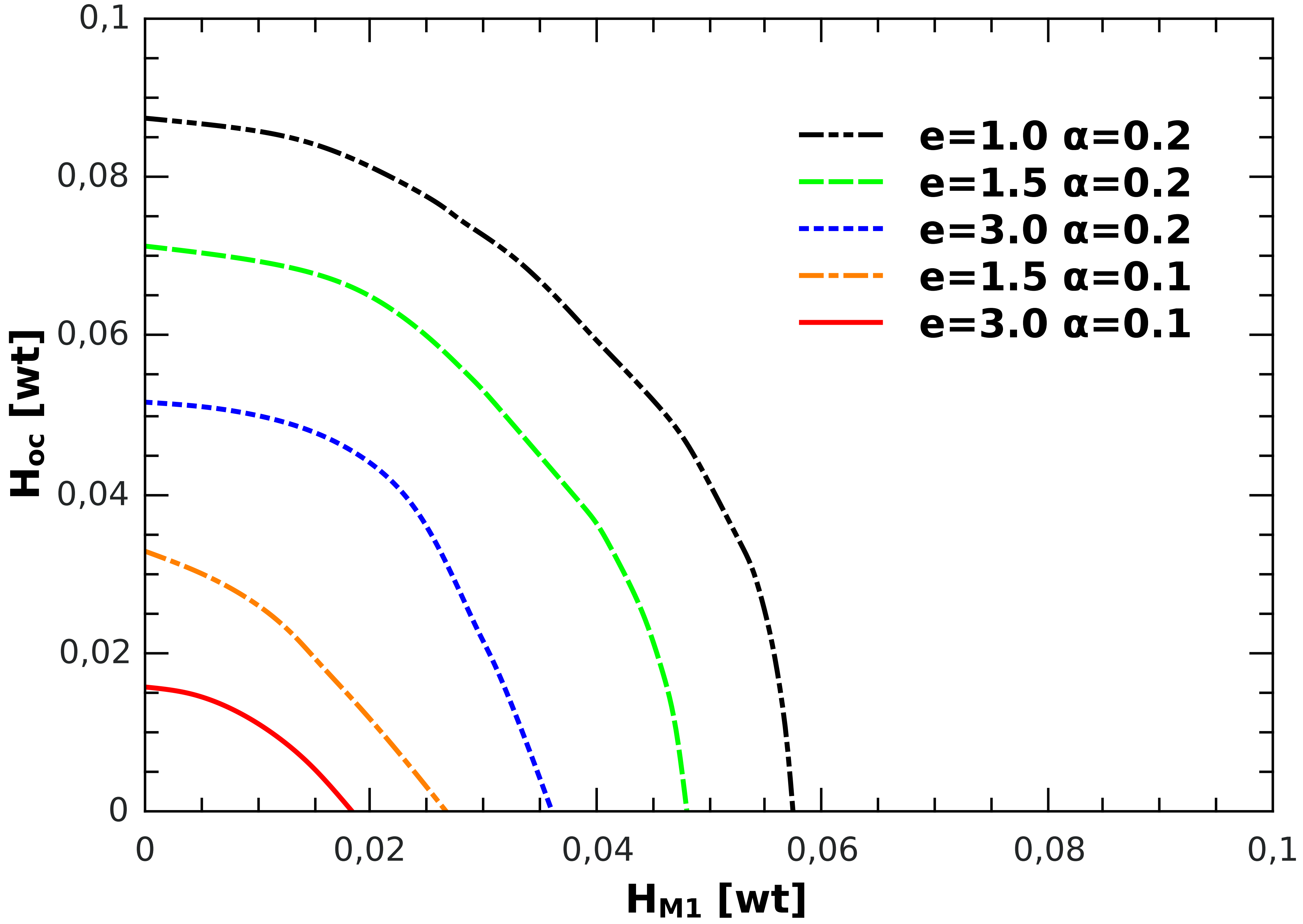
Expected 1 σ regions for combined composition and density measures in the outer core and lower mantle for three different values of the lower mantle radii



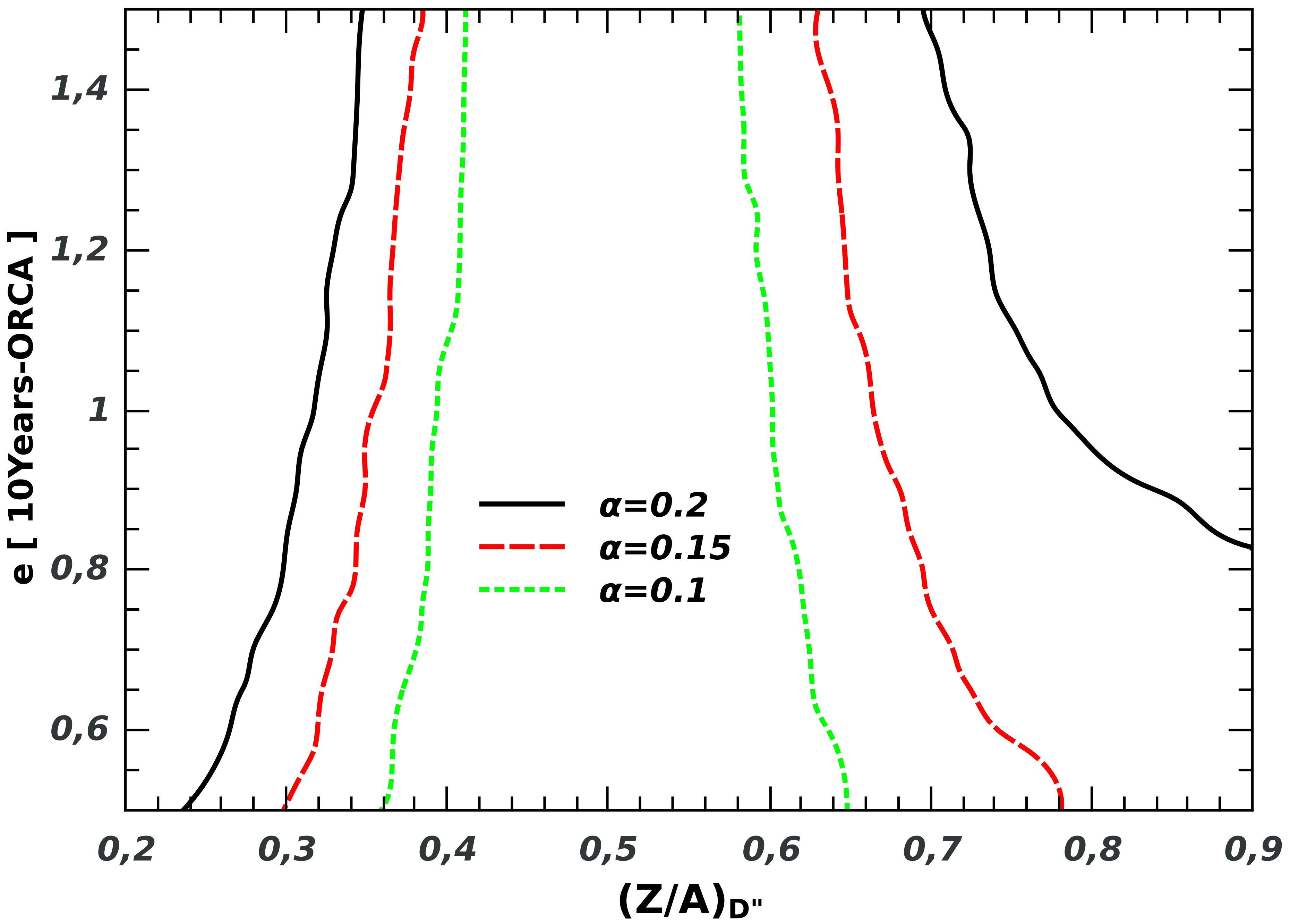
Inverted Ordering



Fraction of hydrogen in the outer core and the lower mantle for extra exposition e and resolution α.

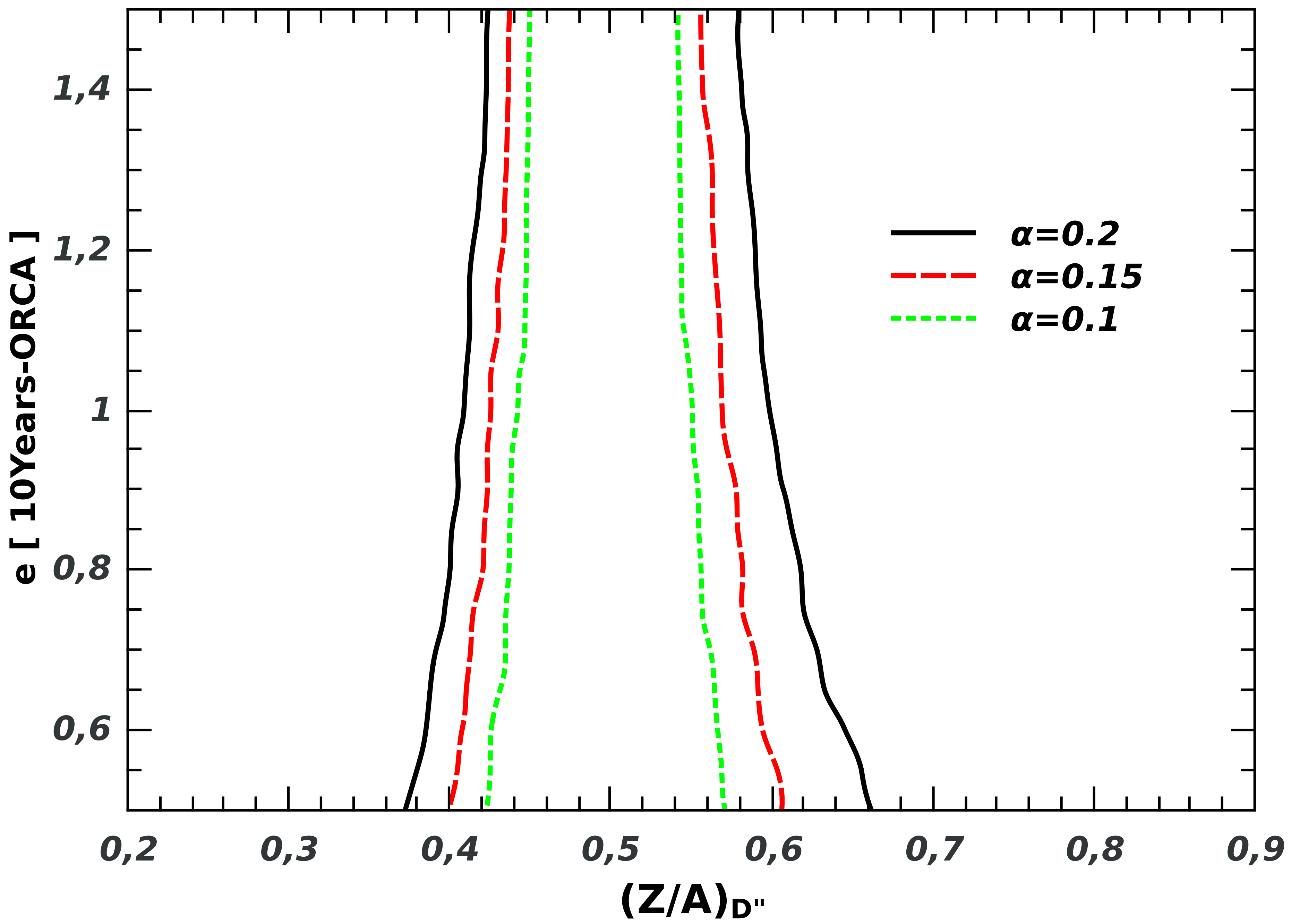


D'' layer



200 km

500 km



Conclusions

- We have studied the possibility of conducting an oscillation tomography of the Earth based on the matter effects on the flavor oscillations of atmospheric neutrinos.
- Using the μ-like events in a generic large Cherenkov detector as physical observables and making a Monte Carlo simulation of the energy and azimuthal angle distribution of these events, we tested possible variants with respect to a geophysical reference model with the densities as given by PREM and a different composition in the outer core and lower mantle.
- As seen in the previous graphs, an experiment like ORCA has somewhat limited potential to reveal the Earth's non-standard composition and density. This worsens at 2 and 3 σ and for the inverted ordering.
- We paid particular attention to the D'' region. Since the thickness of this remote interface between the rocky mantle and the iron core is relatively thin, changes in density and composition have little effect on our observable.
- Clarifying other questions about neutrinos' properties can improve this spectroscopy technique. The most favorable scenario is when we have a normal mass ordering and $\theta_{23} = 45^\circ$.

Thank you!