# Physical acceptability of anisotropic compact objects

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COMPLEJO ASTRONÓMICO
Grupo Halley

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Latin American alliance for Capacity buildiNG in Advanced physics
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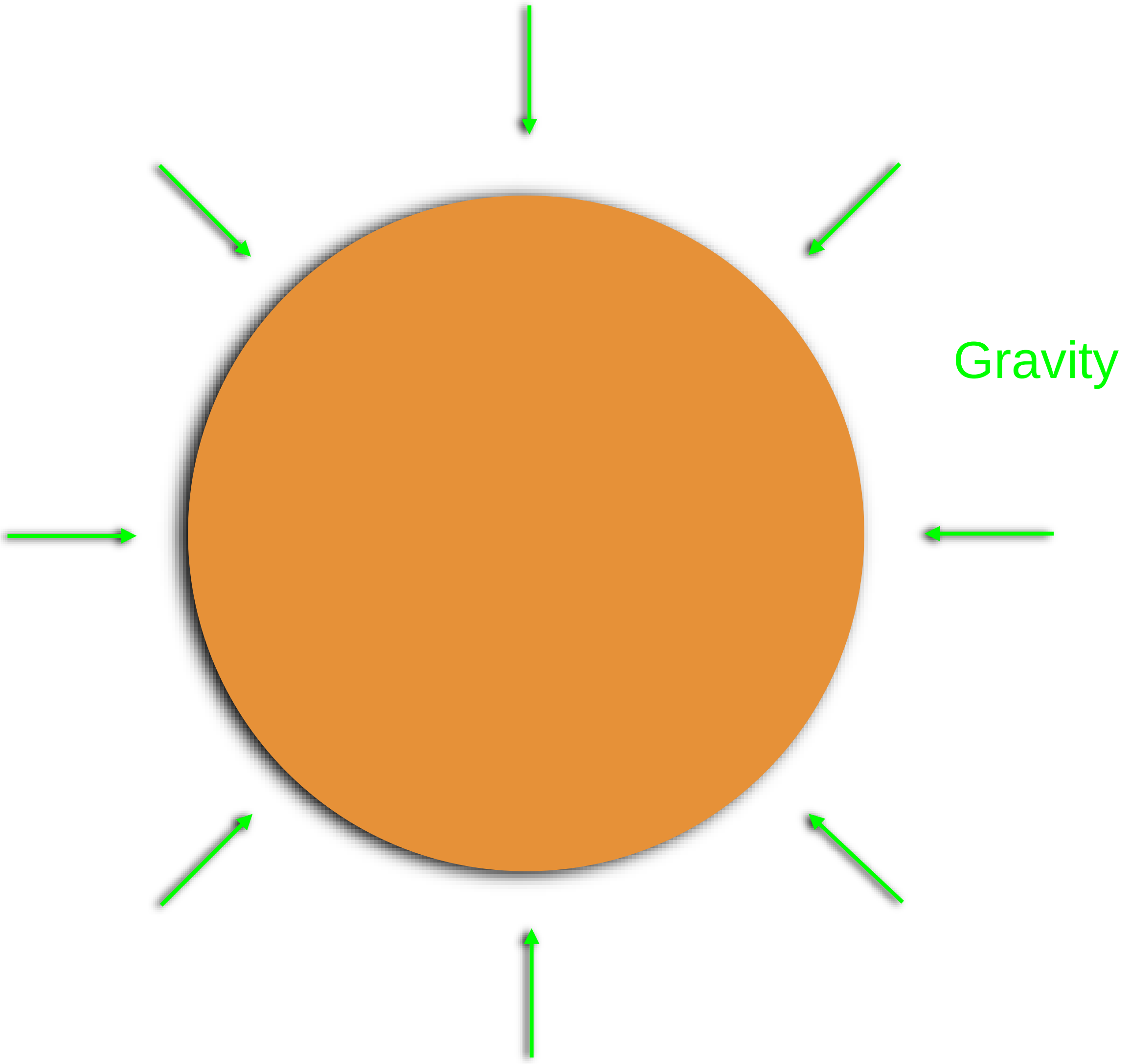
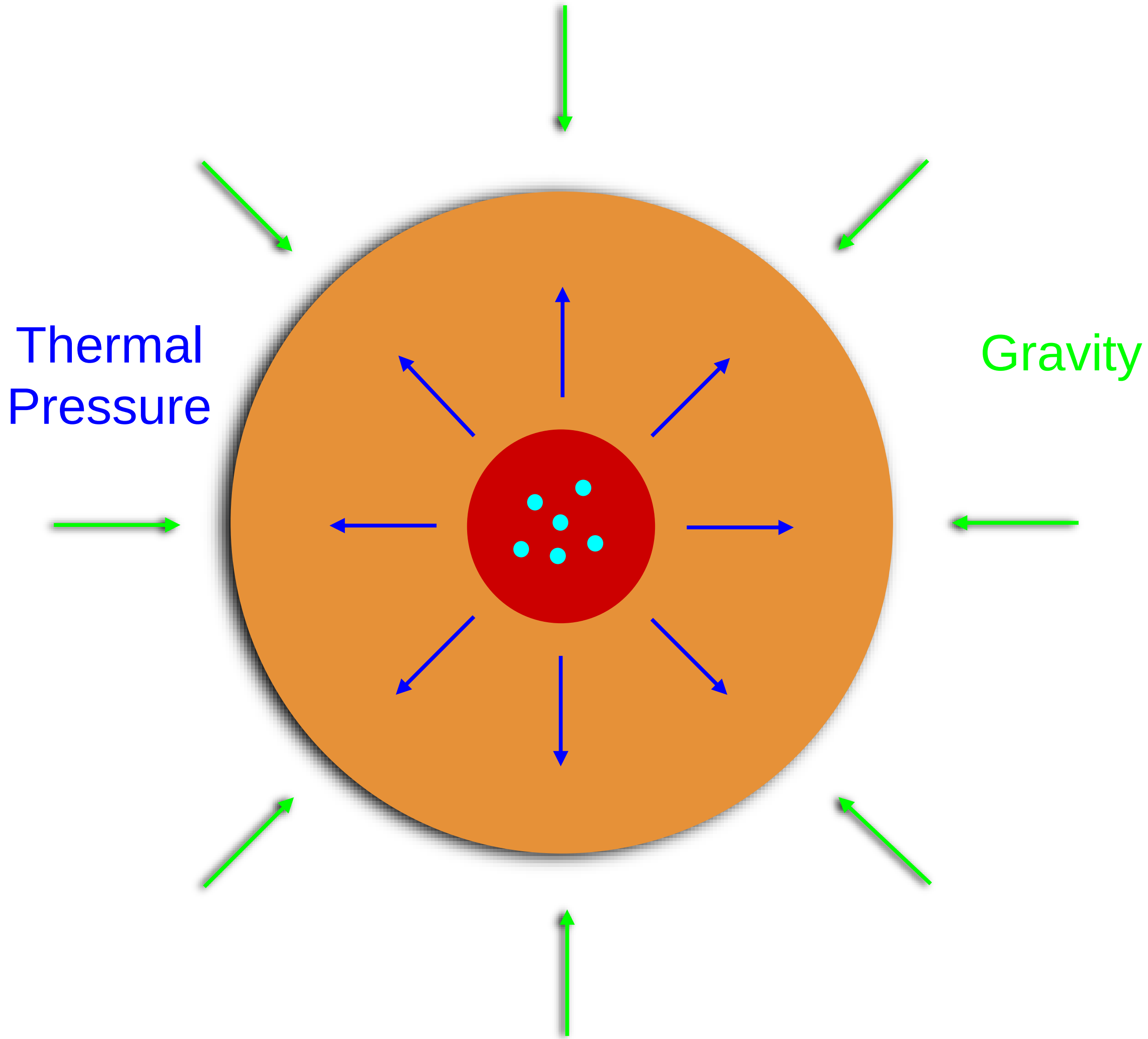
# Introduction: neutron stars

Stars



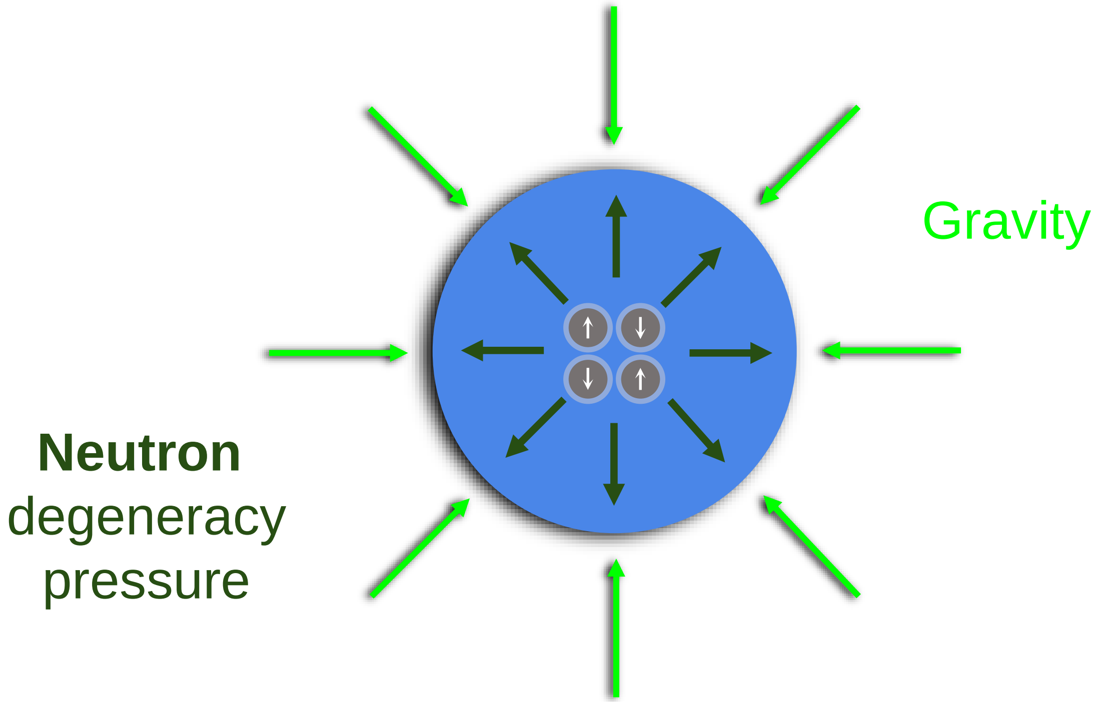
# Introduction: neutron stars

Stars



# Introduction: neutron stars

Neutron stars



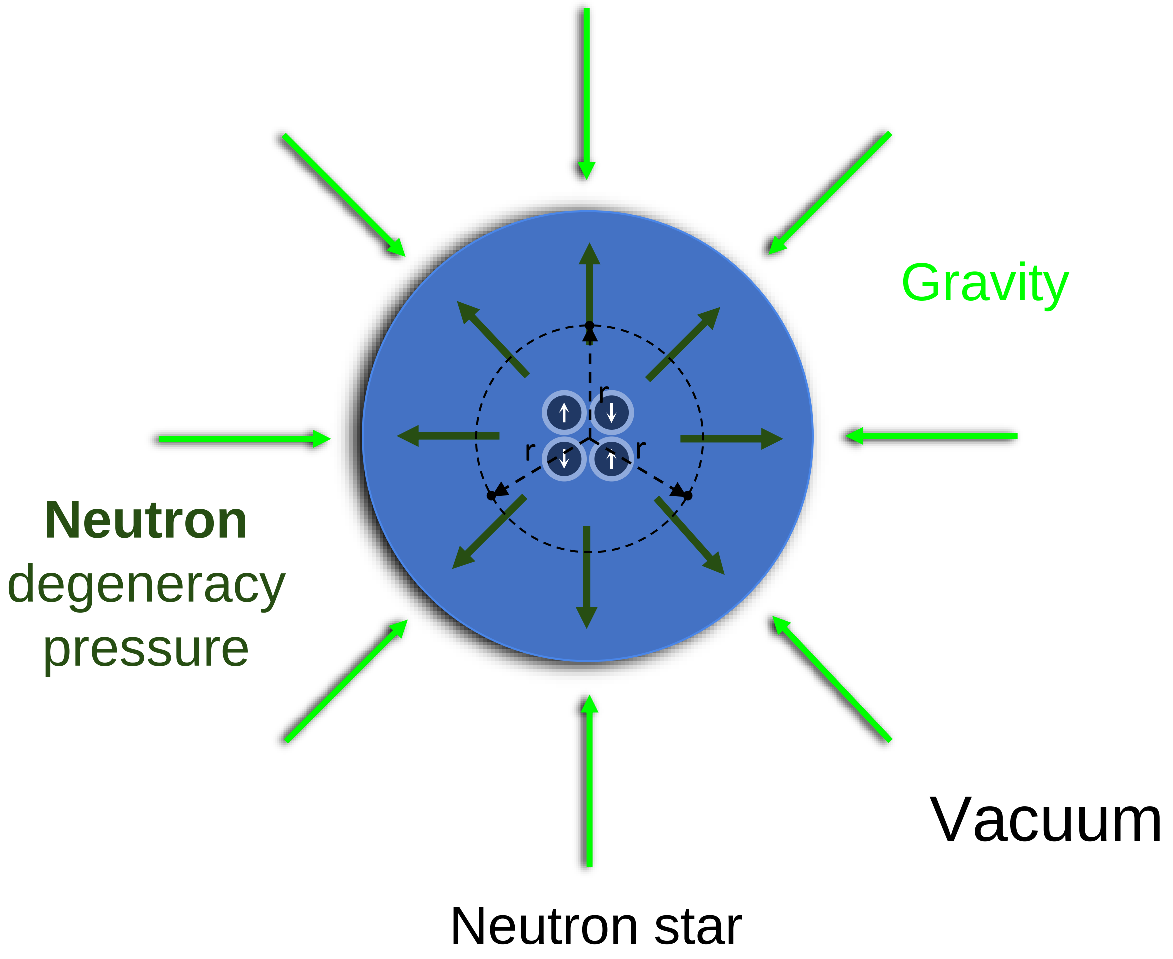

- Stellar remnants

- Compact objects
  - Radius: 8 - 15 km
  - Mass: 1.4 - 2.35 $M_\odot$
  - Density: $10^{18}$ kg/m$^3$

R.W. Romani, et al., Astrophys. J. Lett., 2022

- General Relativity
  - Strong fields
  - Relativistic velocities
  - Mass-energy equivalence

# Modeling: simplifications




- Self-gravitating matter
- Hydrostatic equilibrium
- Spherical symmetry
- Anisotropic fluid

Equations of state are needed to close the system

# Modeling: structure equations

Structure equations

$$\begin{cases} \dfrac{\mathrm{d}P}{\mathrm{d}r} = -\dfrac{(\rho + P)(m + 4\pi r^3 P)}{r(r-2m)} + \dfrac{2(P_\perp - P)}{r} \\ \dfrac{\mathrm{d}m}{\mathrm{d}r} = 4\pi r^2 \rho \end{cases}$$

Geometrized units

$$c = G = 1$$

Unknowns: $P,\ P_\perp,\ \rho,\ m$

# Modeling: structure equations

Structure equations

$$\begin{cases} \dfrac{\mathrm{d}P}{\mathrm{d}r} = -\dfrac{(\rho + P)\left(m + 4\pi r^3 P\right)}{r\,(r - 2m)} + \dfrac{2\left(P_\perp - P\right)}{r} \\[2ex] \dfrac{\mathrm{d}m}{\mathrm{d}r} = 4\pi r^2 \rho \end{cases}$$

Geometrized units

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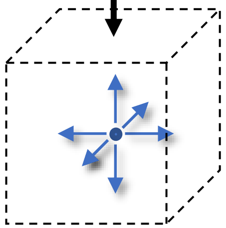
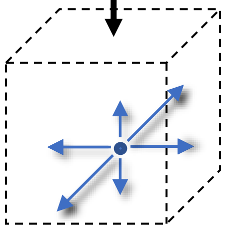
Unknowns: $P,\ P_\perp,\ \rho,\ m$

Isotropic fluid



Anisotropic fluid



# Modeling: structure equations

Structure equations

$$\begin{cases} \dfrac{\mathrm{d}P}{\mathrm{d}r} = -\dfrac{(\rho + P)\,(m + 4\pi r^3 P)}{r\,(r - 2m)} + \dfrac{2\,(P_\perp - P)}{r} \\ \dfrac{\mathrm{d}m}{\mathrm{d}r} = 4\pi r^2 \rho \end{cases}$$

Geometrized units

$$c = G = 1$$

Unknowns: $P,\ P_\perp,\ \rho,\ m$

Usually, equations of state are used to close the system

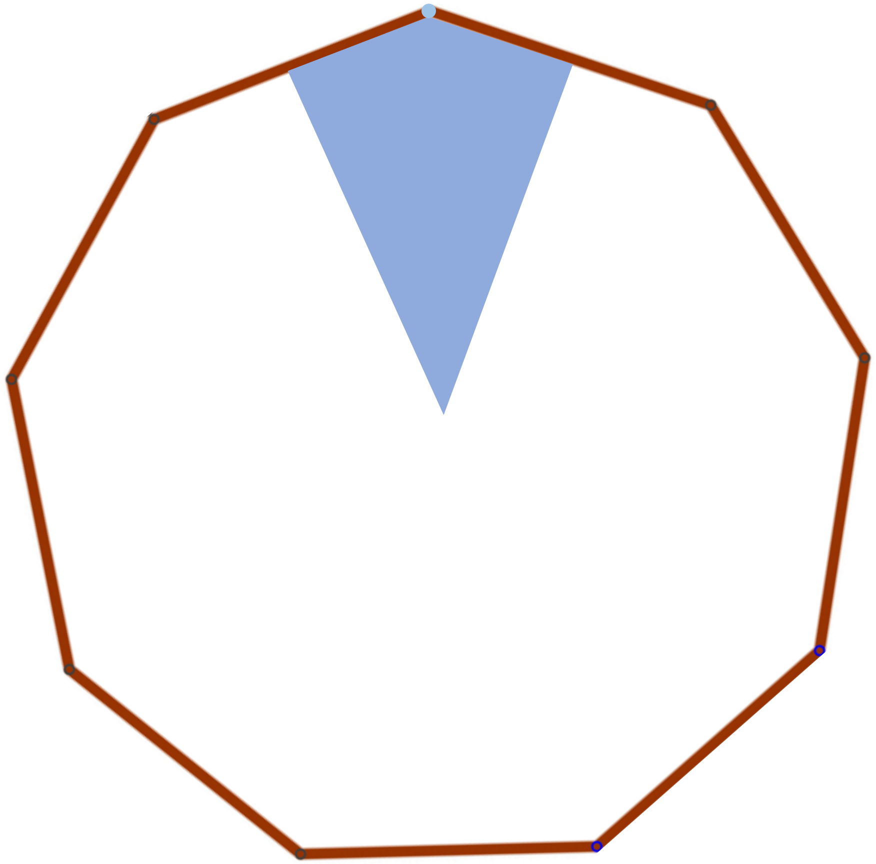
Neutron star central density: ~$10^{18}$ kg/m$^3$ $>$ Nuclear density: ~$10^{17}$ kg/m$^3$

¡Equation of state that describes ultradense matter is unknown!

# Acceptability conditions

Solutions will be physically acceptable if they fulfill the following conditions

C1: $2m/r < 1$



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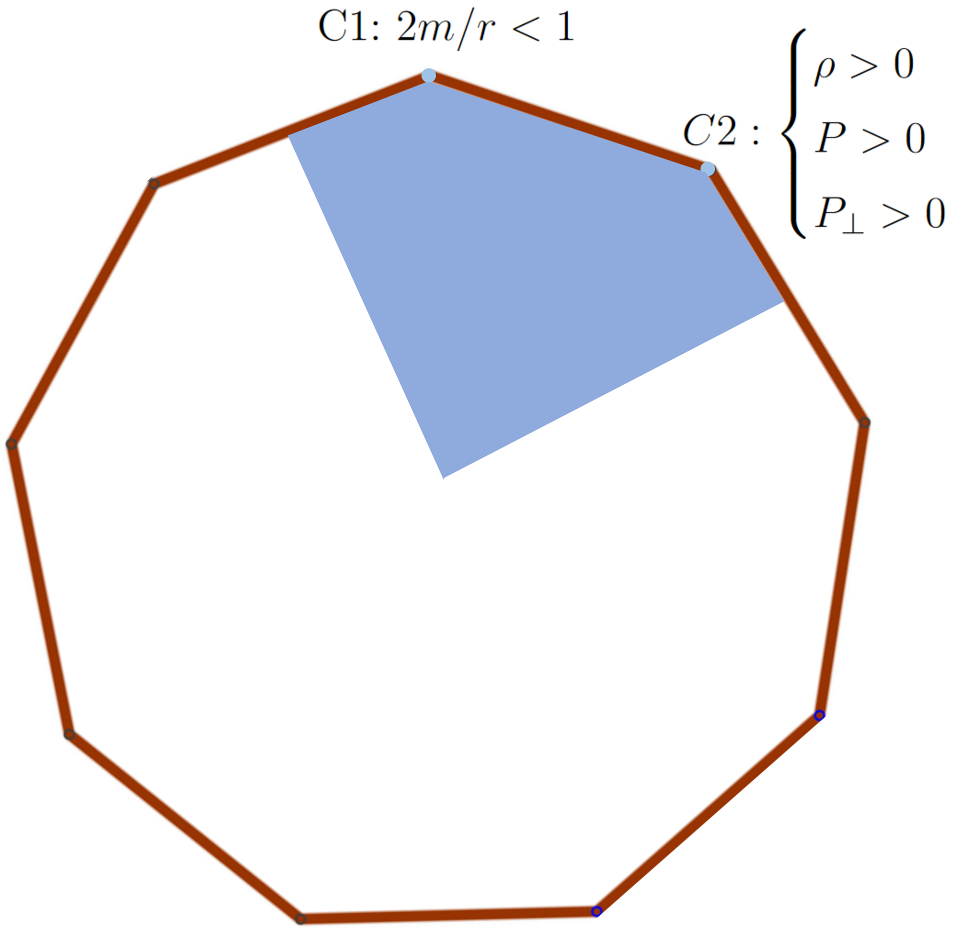

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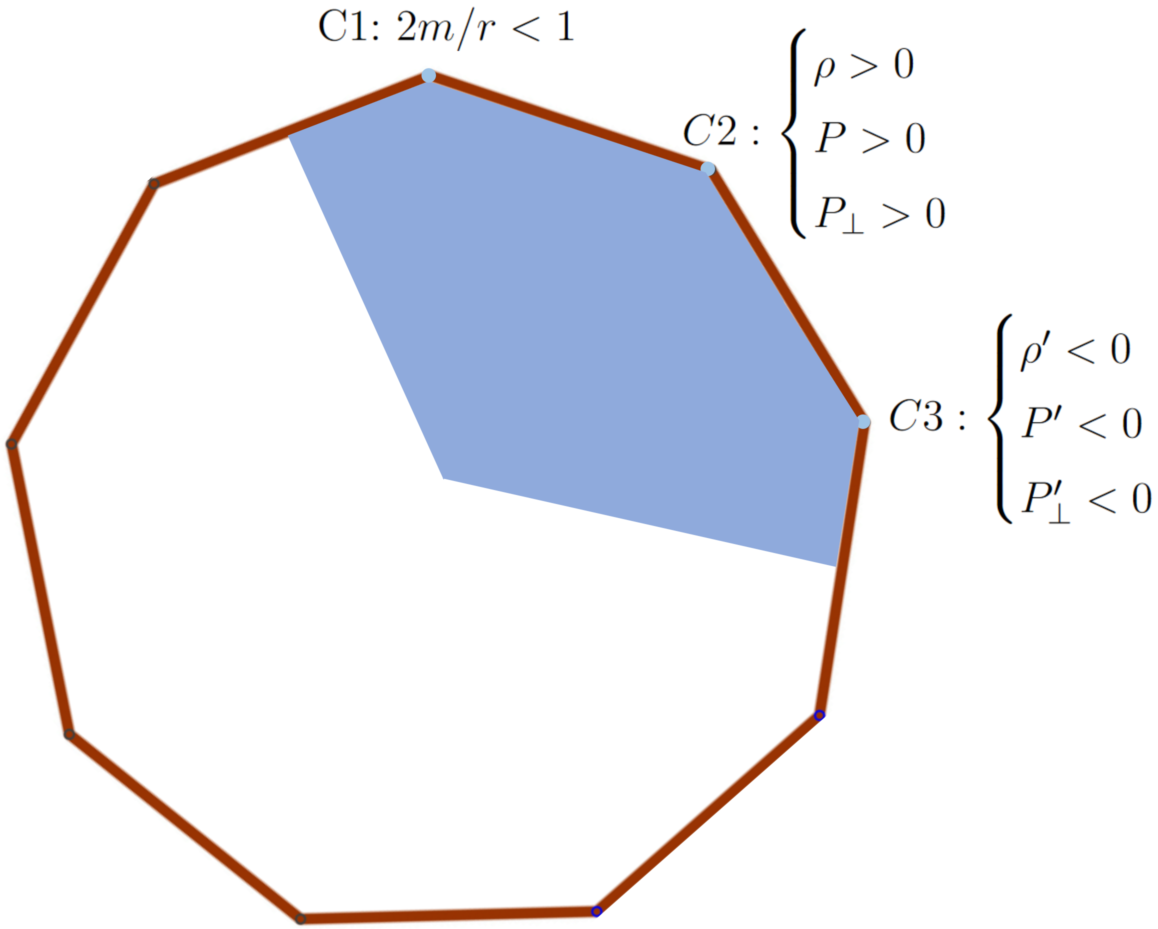

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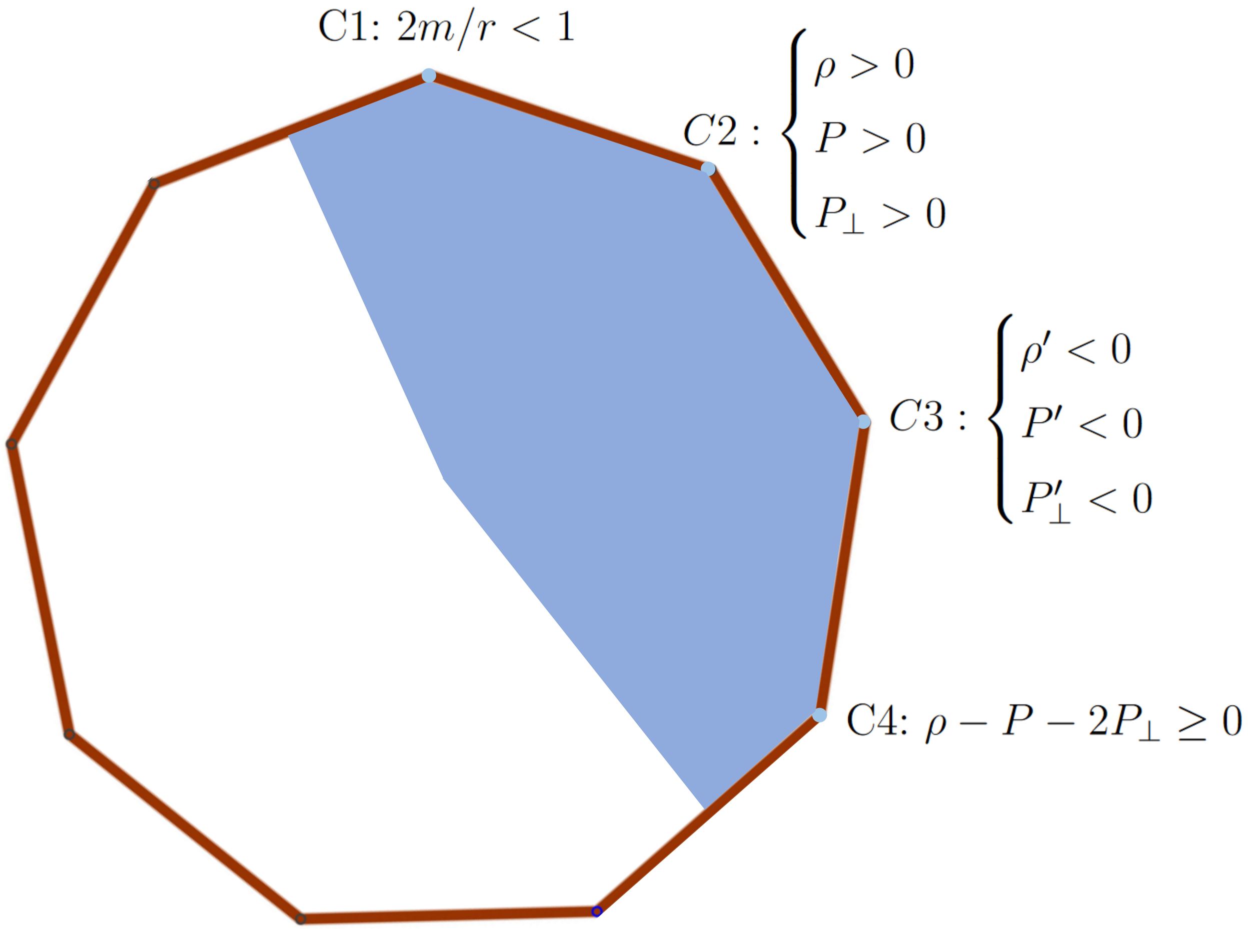

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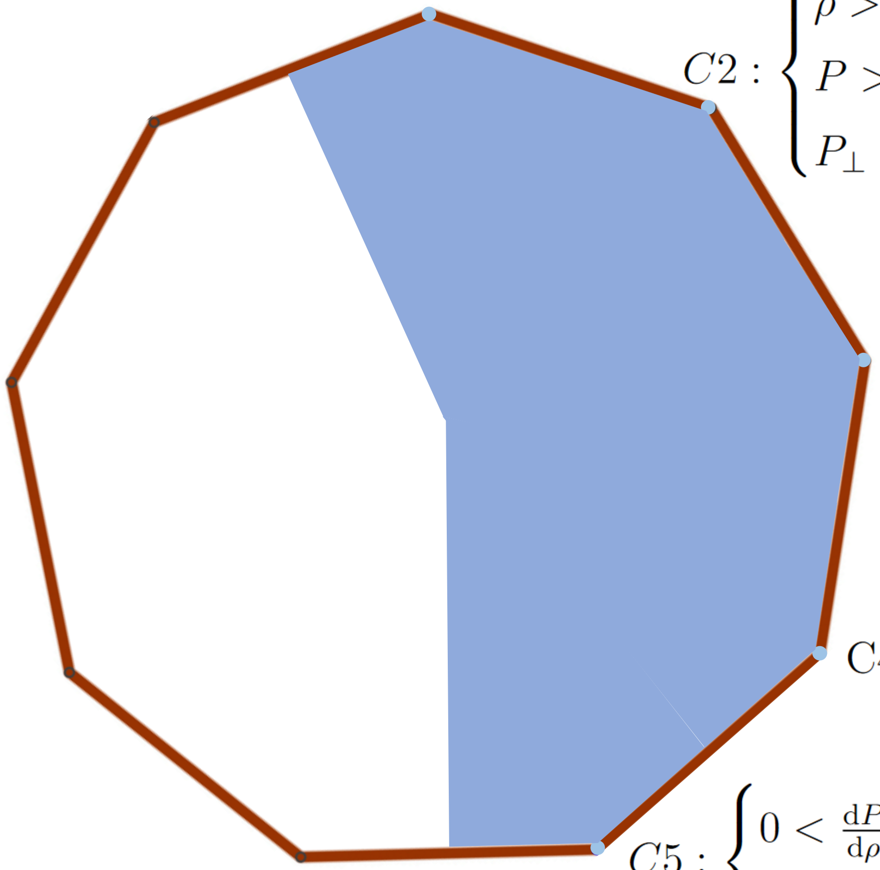

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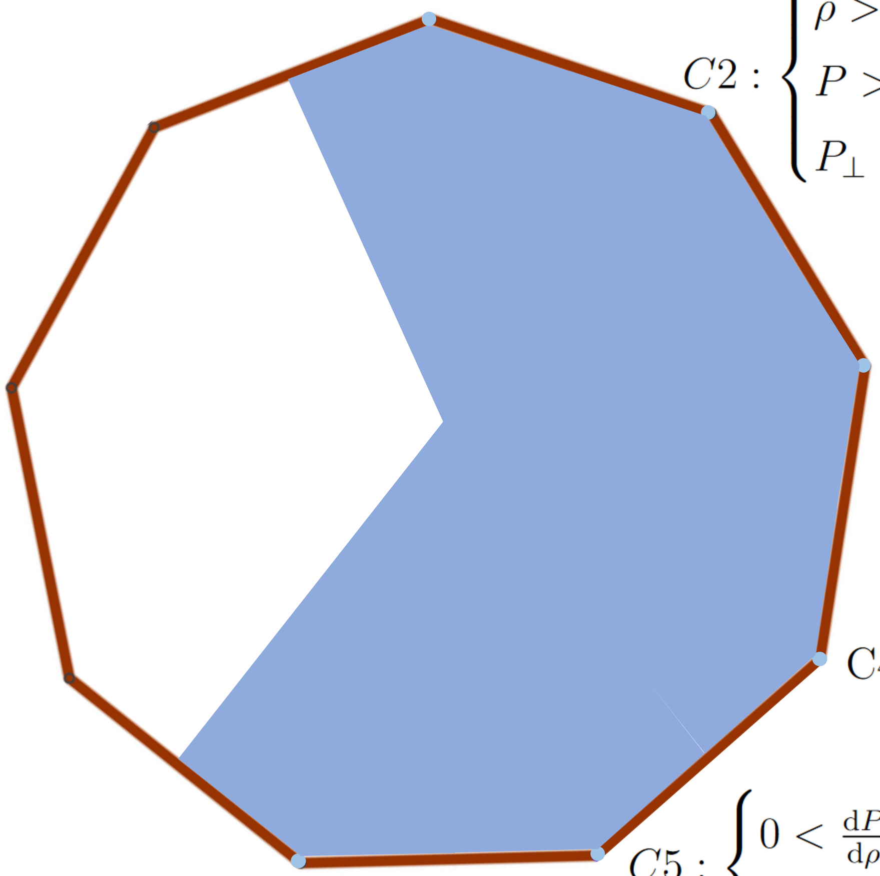

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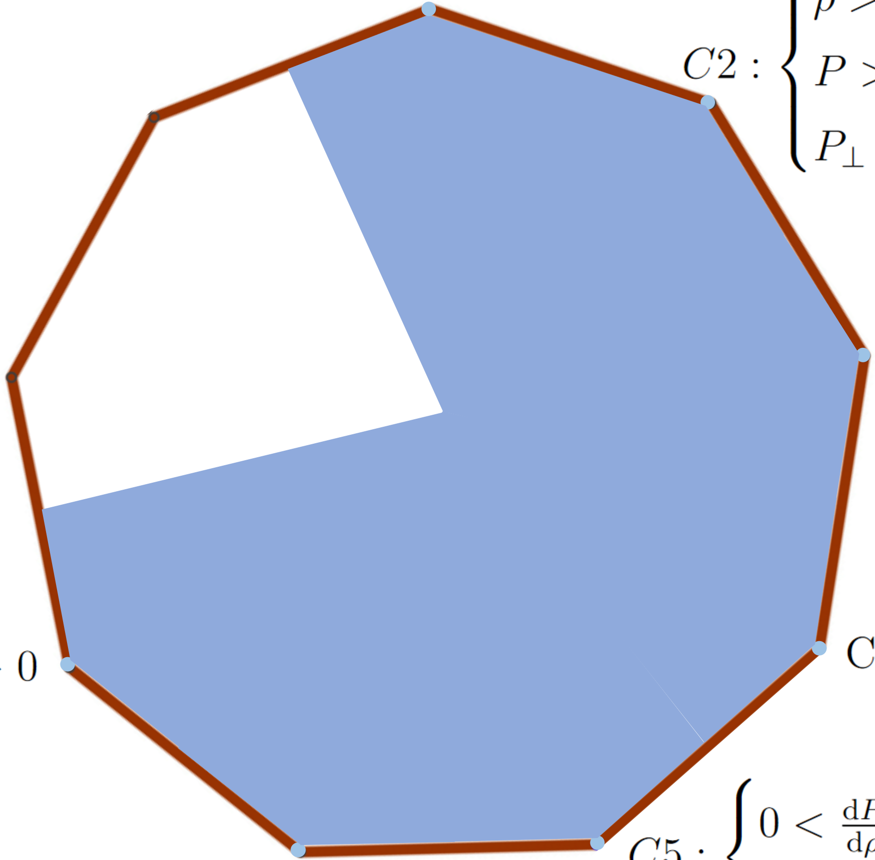

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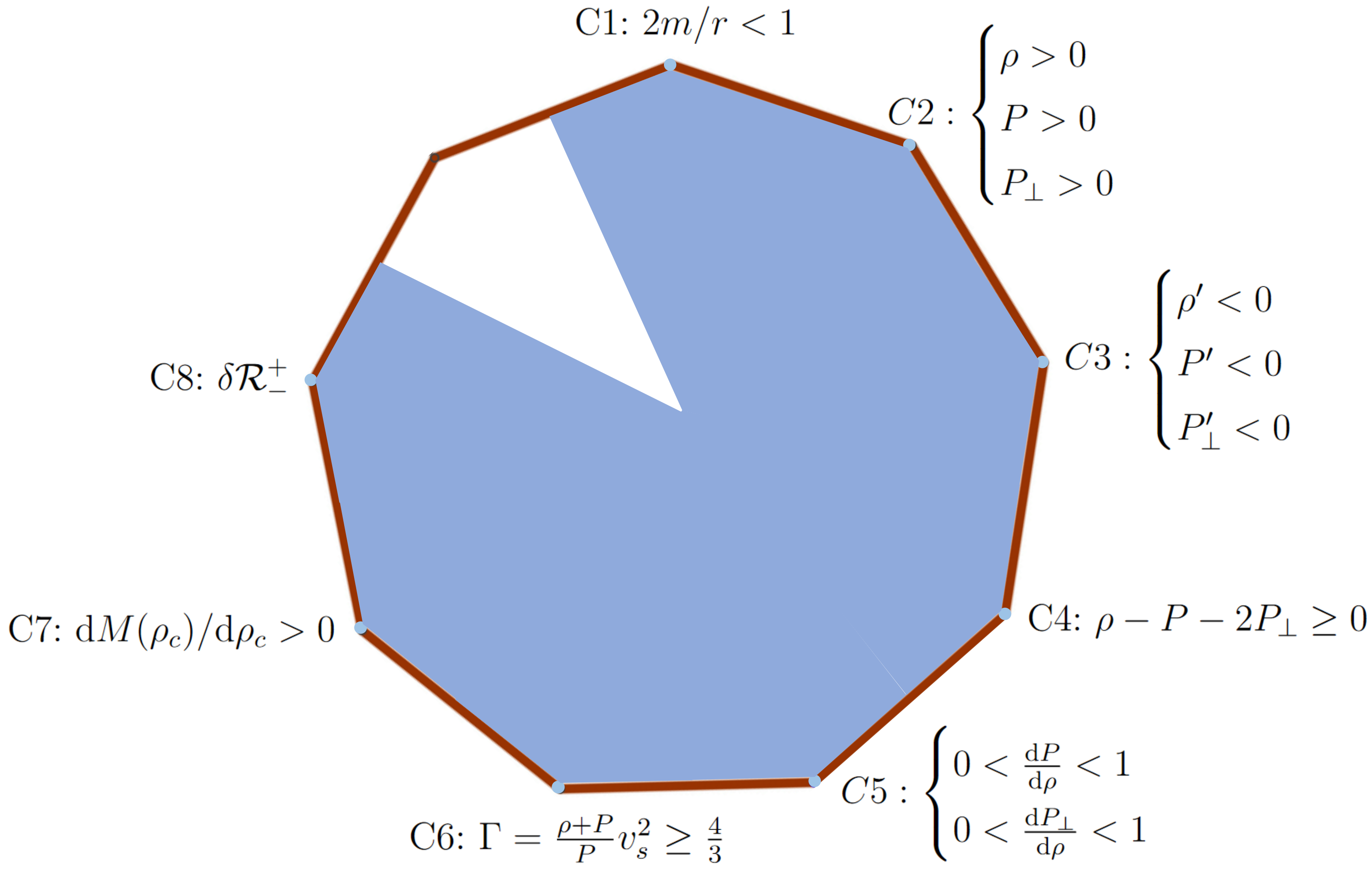

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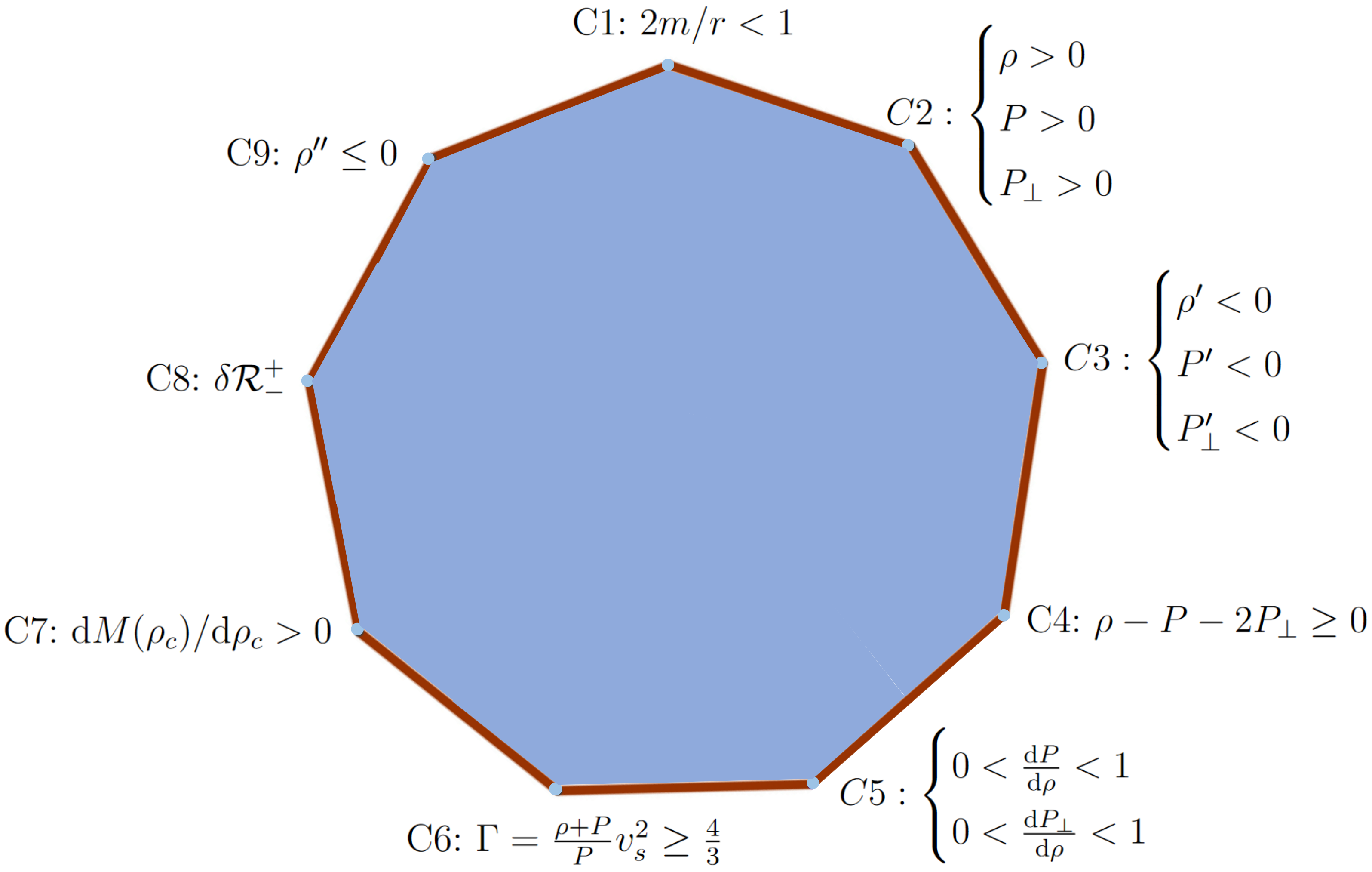


# First work

Barotropic equation of state + ansatz for anisotropy

# First work: system of equations

Structure equations

$$\begin{cases} \dfrac{\mathrm{d}P}{\mathrm{d}r} = -\dfrac{(\rho + P)\,(m + 4\pi r^3 P)}{r\,(r - 2m)} + \dfrac{2\,(P_\perp - P)}{r} \\ \dfrac{\mathrm{d}m}{\mathrm{d}r} = 4\pi r^2 \rho \end{cases}$$

Is proposed: $P = \kappa\rho^{1+\frac{1}{n}} + \alpha\rho - \beta$ generalized polytropic equation of state

Is chosen: $P_\perp - P = Cr(\rho + P)\left[\dfrac{m + 4\pi r^3 P}{r(r - 2m)}\right]$, therefore

$$\frac{\mathrm{d}P}{\mathrm{d}r} = -h\frac{(\rho + P)(m + 4\pi r^3 P)}{r(r - 2m)}$$ ; where $h = 1 - 2C$

# First work: numerical integration

Structure equations

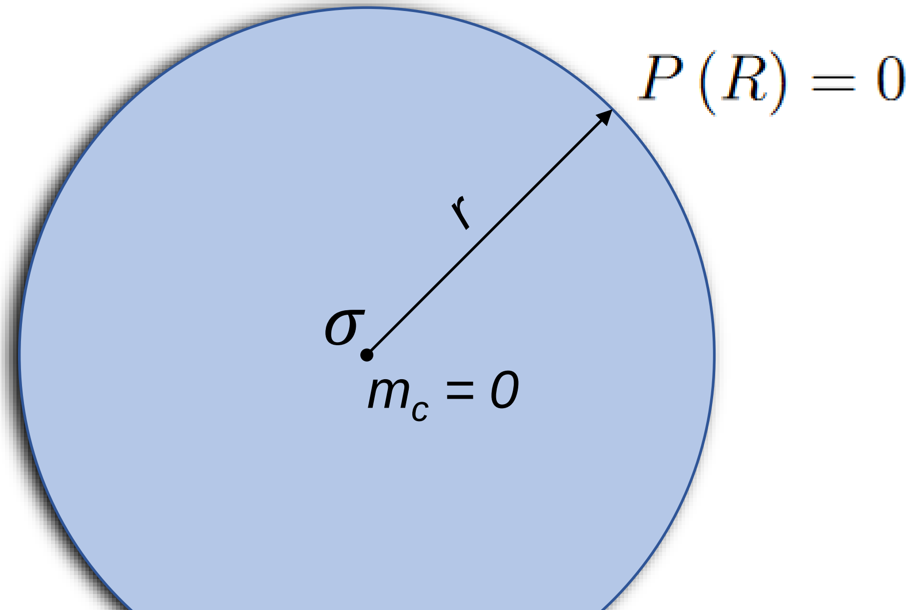
$$\begin{cases} \dfrac{\mathrm{d}P}{\mathrm{d}r} = -h\dfrac{(\rho + P)(m + 4\pi r^3 P)}{r(r-2m)} \\ \dfrac{\mathrm{d}m}{\mathrm{d}r} = 4\pi r^2 \rho \end{cases}$$

+

Equation of state

$$P = \kappa \rho^{1+\frac{1}{n}} + \alpha\rho - \beta$$

The system is integrated numerically with initial conditions parameterized by $\sigma = P_c/\rho_c$, until reaching the condition $P\,(R) = 0$.

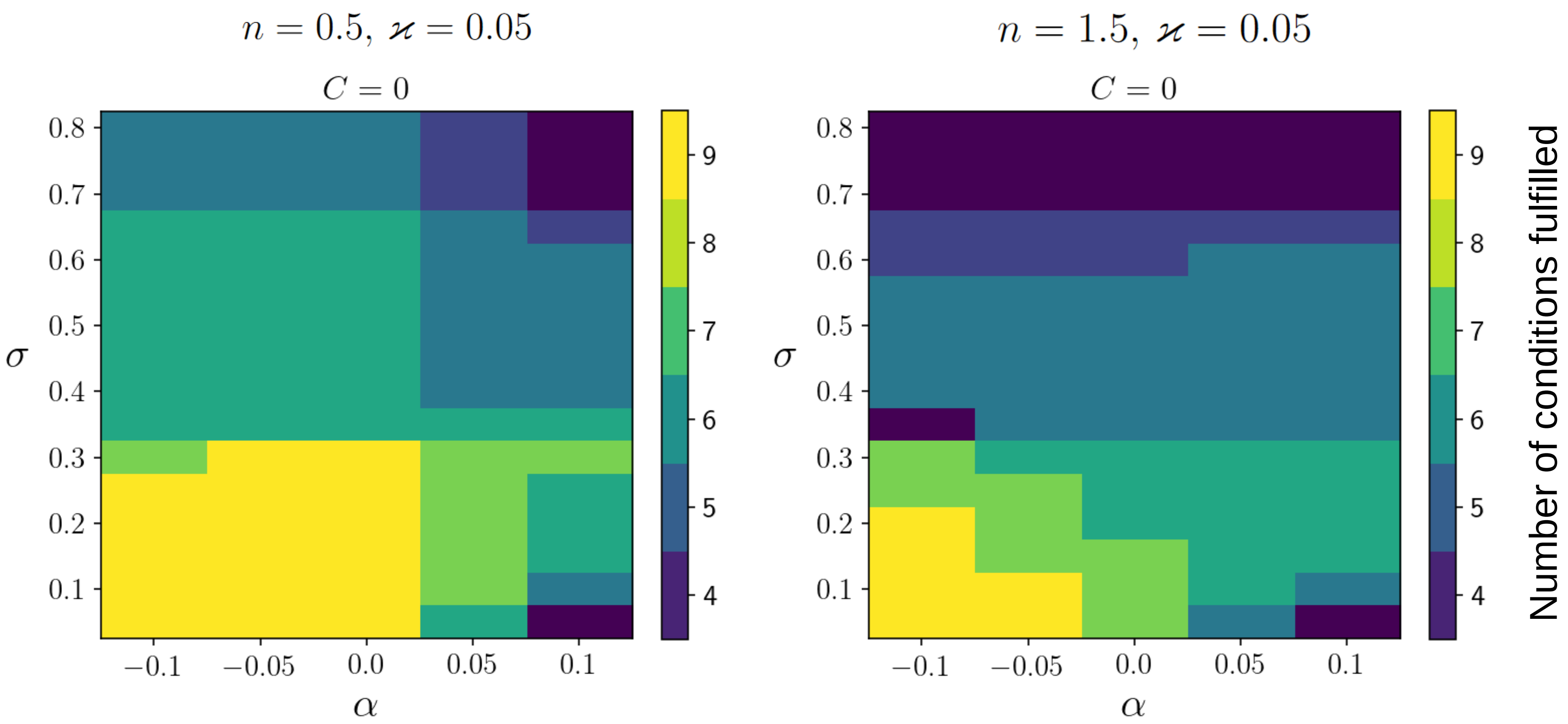



## Parmeters

- Variation of *n* describes a wide range of materials
- σ indicates how relevant the relativistic regime is
- The coefficient of the linear term, α, is closely related to the speed of sound
- C quantifies the amount of anisotropy
- $\varkappa = \rho_b/\rho_c$ the density drop from the centre to the surface

# First work: parameter space

$$P = \kappa \rho^{1+\frac{1}{n}} + \alpha \rho - \beta$$




$$\sigma = \frac{P_c}{\rho_c}$$

$$P_\perp - P = Cr(\rho + P)\left[\frac{m + 4\pi r^3 P}{r(r-2m)}\right]$$

$$\varkappa = \rho_b / \rho_c$$

# First work: neutron stars candidates

| | Object | | | |
|---|---|---|---|---|
| Input parameters | J0737-3039 $n = 0.50$ | J1518+4904 $n = 1.00$ | GMn075 $n = 0.75$ | PMn075 $n = 0.75$ |
| $C$ | 0.09 | 0.125 | 0.05 | 0.05 |
| $\alpha$ | -0.01 | 0.01 | -0.01 | 0.0 |
| $\varkappa$ | 0.05 | 0.15 | 0.17 | 0.0 |
| $\sigma$ | 0.10 | 0.15 | 0.18 | 0.18 |
| $\rho_c \times 10^{15}$ (g/cm$^3$) | 0.66 | 1.79 | 1.41 | 1.41 |
| Output parameters | | | | |
| $M$ ($M_\odot$) | 1.33 | 1.56 | 1.50 | 1.56 |
| $R$ (km) | 11.49 | 9.88 | 10.0 | 10.9 |
| $2\mathcal{C}_\star$ | 0.34 | 0.47 | 0.44 | 0.42 |
| $\rho_b \times 10^{14}$ (g/cm$^3$) | 0.33 | 2.69 | 2.4 | 0.0 |

# Second work

Known density profile + 4 different ansatz for anisotropy

# Second work: system of equations

Structure equations

$$\begin{cases} \dfrac{\mathrm{d}P}{\mathrm{d}r} = -\dfrac{(\rho + P)\left(m + 4\pi r^3 P\right)}{r\,(r - 2m)} + \dfrac{2\Delta}{r} \\ \dfrac{\mathrm{d}m}{\mathrm{d}r} = 4\pi r^2 \rho \end{cases}$$

$$\Delta = P_\perp - P$$

Is proposed: $\rho(r) = \rho_c\left(1 - \alpha r^2\right)$ Known density profile

Thus, the mass function is also known

$$m(r) = \int_0^R 4\pi r^2 \rho\,(r)\,\mathrm{d}r$$

# Second work: strategies to introduce anisotropy

$$\Delta = P_\perp - P$$

1. Anisotropy proportional tothe gravitational force

$$\Delta = C_1 \frac{(\rho + P)\,(m + 4\pi r^3 P)}{r - 2m}$$

M. Cosenza, et al., J. Math. Phys., 1981

2. Quasi-local equation of state

$$\Delta = C_2 P \mu\,, \quad \mu = 2m/r$$

D. Doneva, et al., Phys. Rev. D, 2012

3. Anisotropy proportional to the Pressure gradient

$$\Delta = -C_4 f\,(\rho)\, k^\mu \nabla_\mu P$$

G. Raposo, et al., Phys. Rev. D, 2019

4. Complexity factor

$$\Delta = -\frac{1}{2r^3}\int_0^r \tilde{r}^3 \rho' \mathrm{d}\tilde{r}$$

L. Herrera, Phys. Rev. D, 2018

# Second work: numerical integration

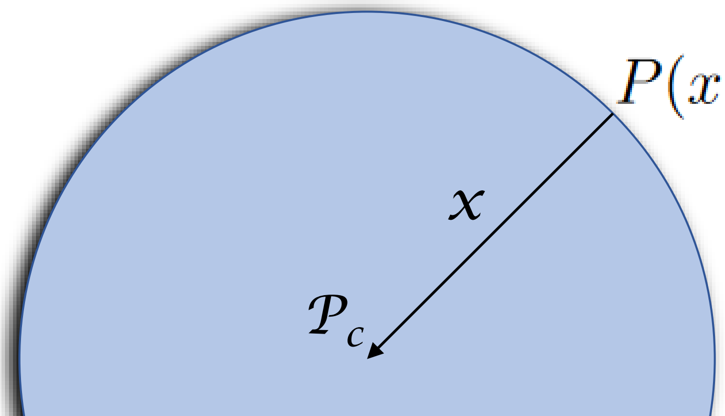
Dimensionless equation (change of variables):

$$m = R\tilde{m}\,, \quad P = \frac{1}{R^2}\tilde{P}\,, \quad P_\perp = \frac{1}{R^2}\tilde{P}_\perp\,, \quad \rho = \frac{1}{R^2}\tilde{\rho}\,, \quad r = Rx\,.$$

$$\Longrightarrow \quad \frac{\mathrm{d}\tilde{P}}{\mathrm{d}x} = -\frac{\left(\tilde{\rho}+\tilde{P}\right)\left(\tilde{m}+4\pi\tilde{P}x^3\right)}{x\left(x-2\tilde{m}\right)} + \frac{2\left(\tilde{P}_\perp - \tilde{P}\right)}{x}$$

$$\tilde{\rho} = \tilde{\rho}_c\left(1-\alpha x^2\right) = \tilde{\rho}_c\left[1-\left(1-\varkappa\right)x^2\right]\ ;\ \varkappa = \frac{\rho_b}{\rho_c} \qquad \Rightarrow \qquad \tilde{m} = 4\pi\tilde{\rho}_c\left[\frac{x^3}{3} - \left(1-\varkappa\right)\frac{x^5}{5}\right]$$

Initial condition and integration space

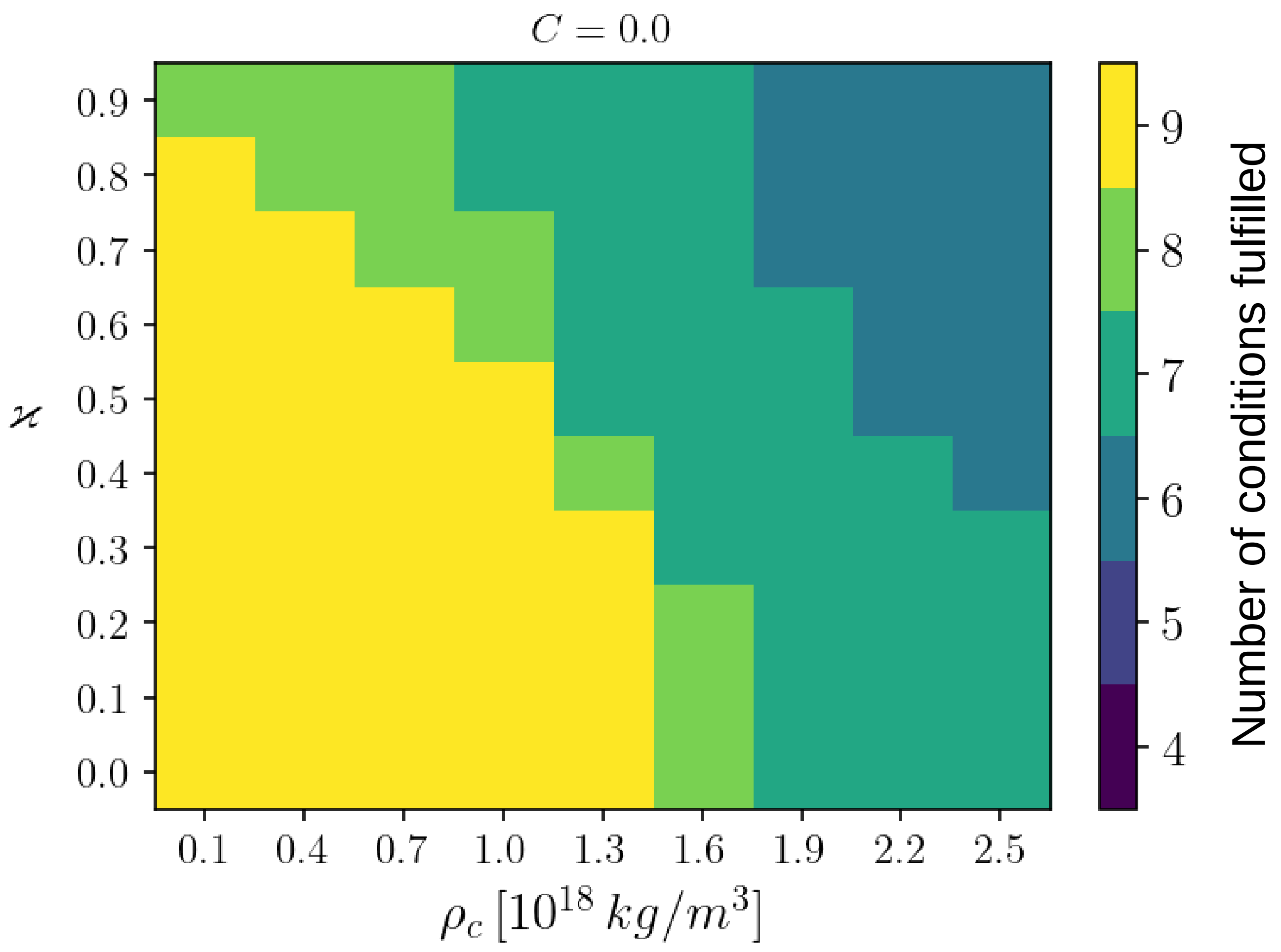



$$P(x=1) = 0$$

$$x = \left[1, 10^{-15}\right]$$

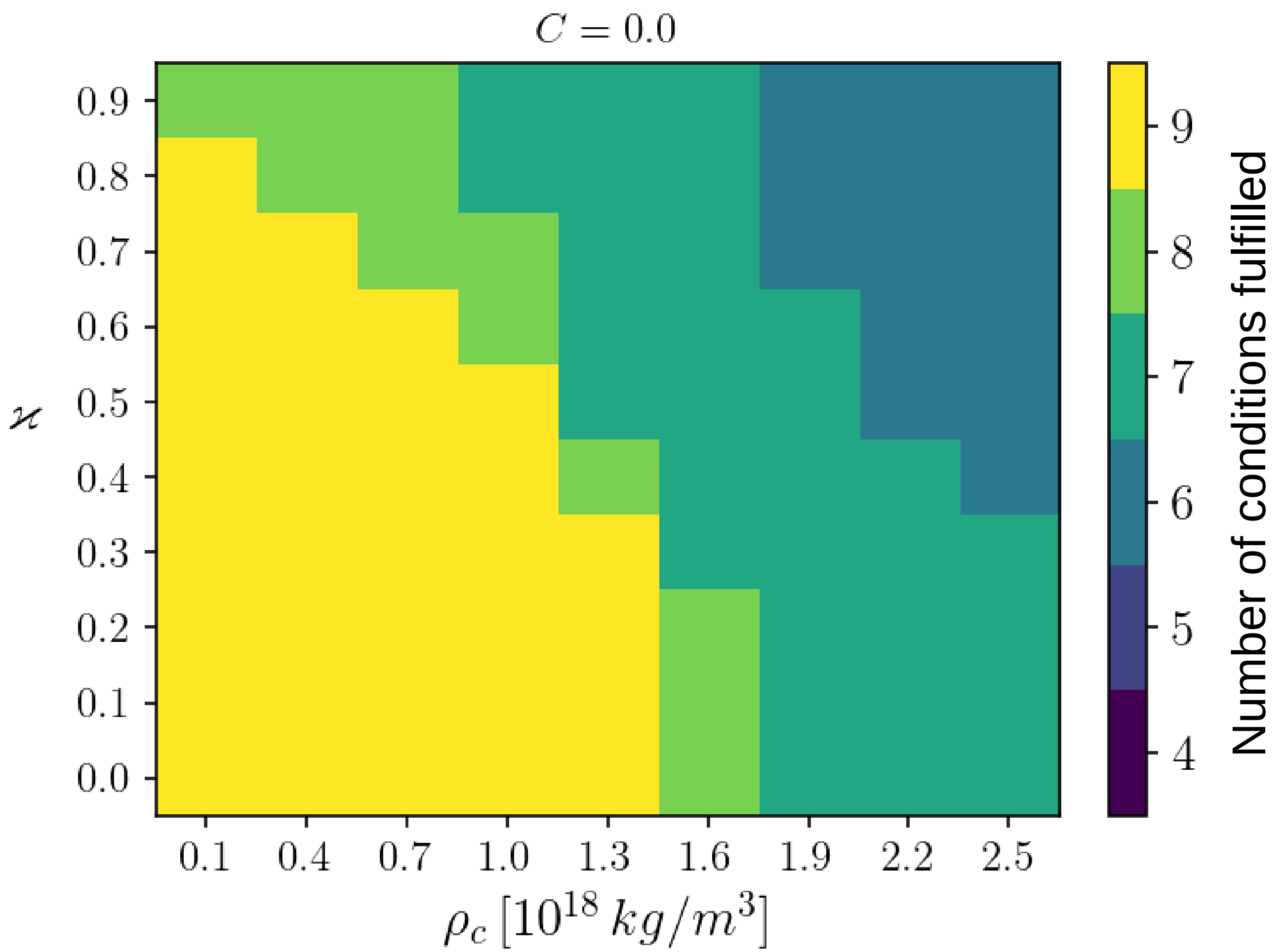
# Second work: parameter space (I)

$$\Delta = C_1 \frac{(\rho + P)(m + 4\pi r^3 P)}{r - 2m}$$



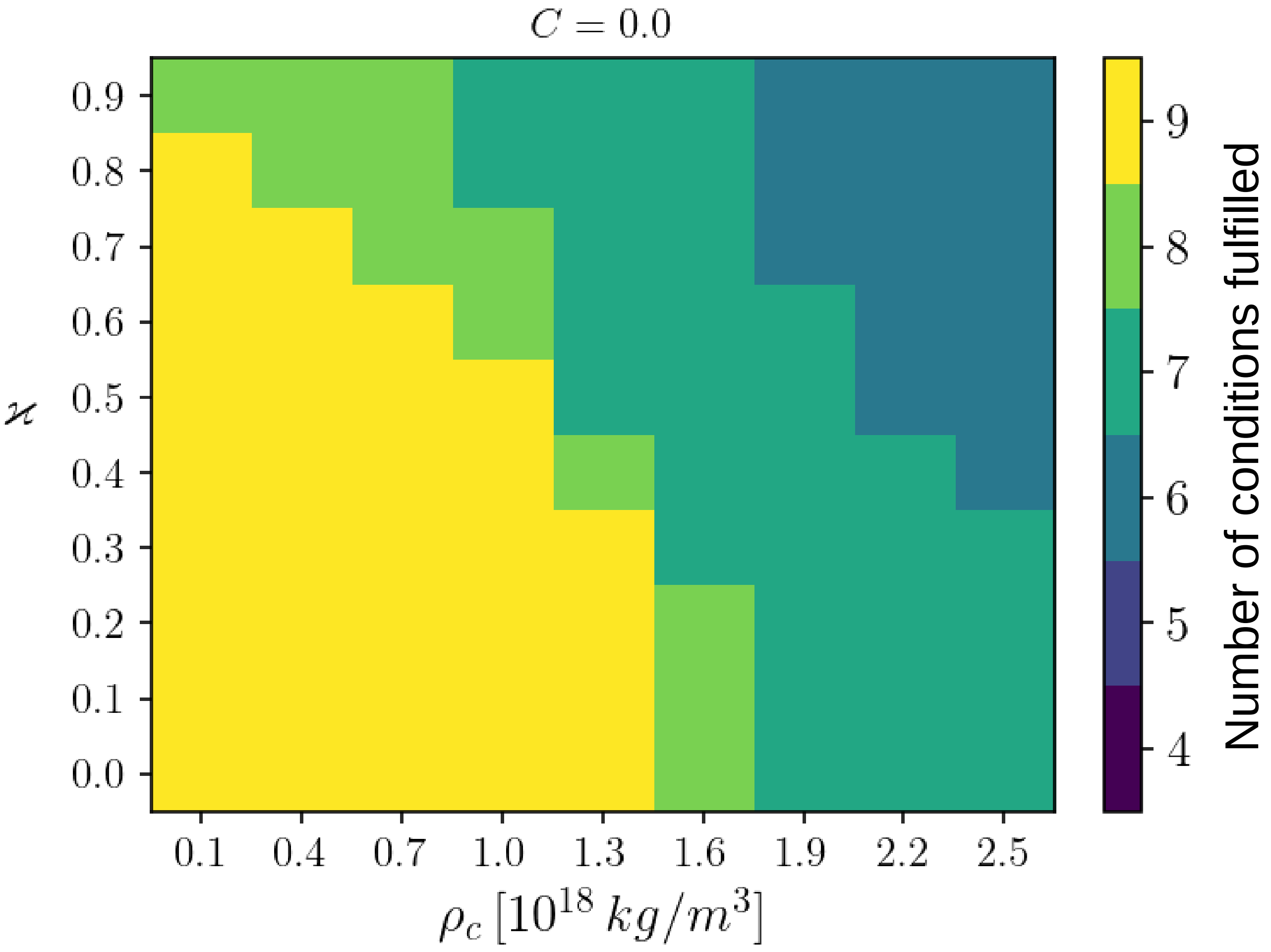
# Second work: parameter space (II)

$$\Delta = C_2 P \mu \,, \quad \mu = 2m/r$$



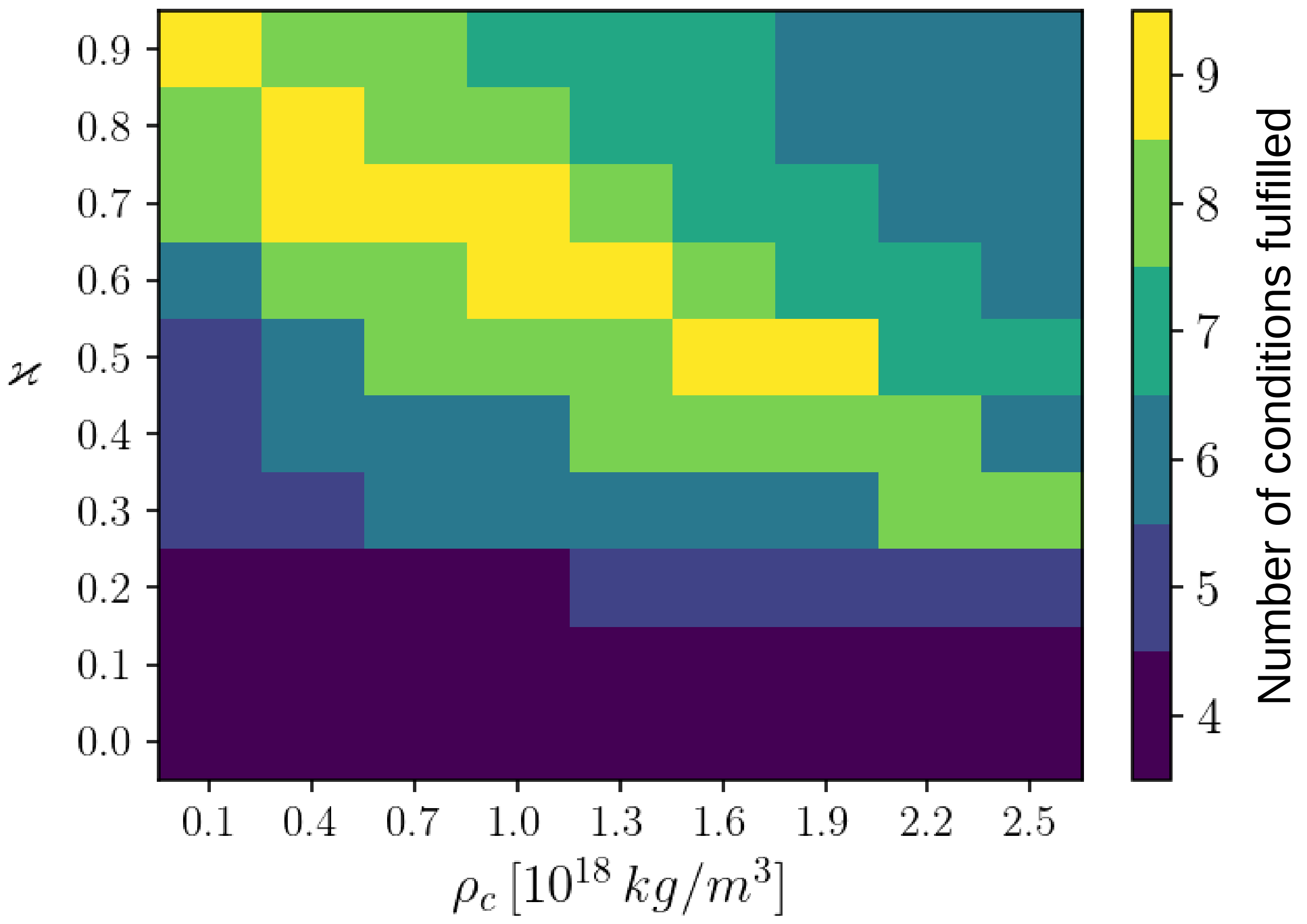
# Second work: parameter space (III)

$$\Delta = -C_4 f\left(\rho\right) k^{\mu} \nabla_{\mu} P \qquad f\left(\rho\right) = \rho$$



# Second work: parameter space (IV)

$$\Delta = -\frac{1}{2r^3}\int_0^r \tilde{r}^3 \rho' \mathrm{d}\tilde{r}$$



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# Questions?

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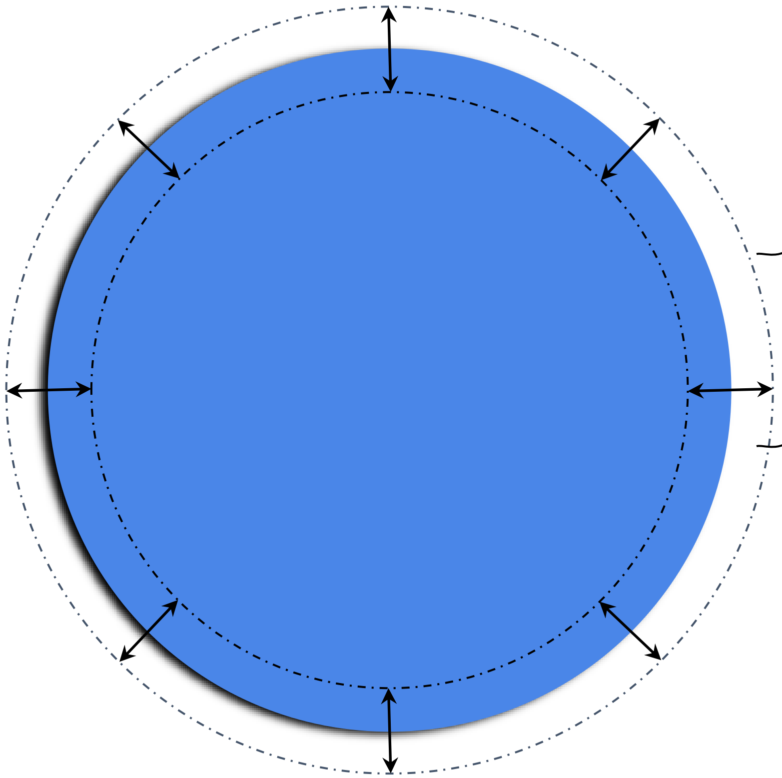
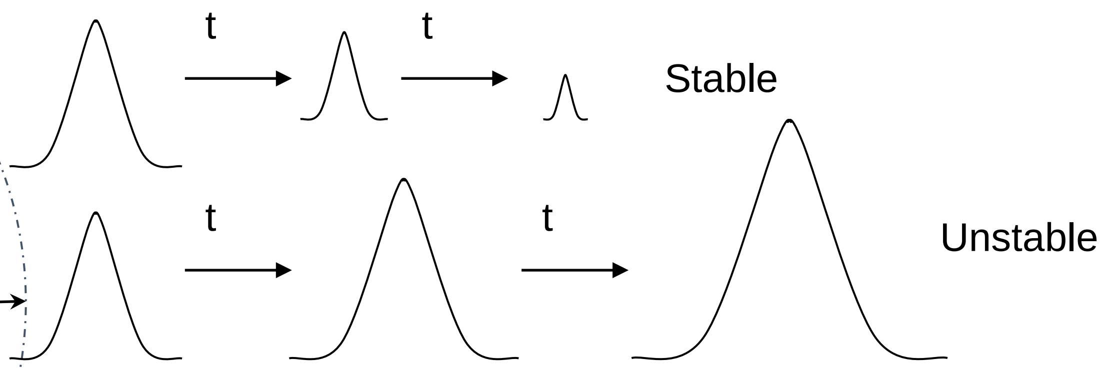
# Stability

Radial perturbations



S. Chandrasekhar, Phys. Rev. Lett., 1964.



# Stability

Hydrostatic Equilibrium Equation = 0

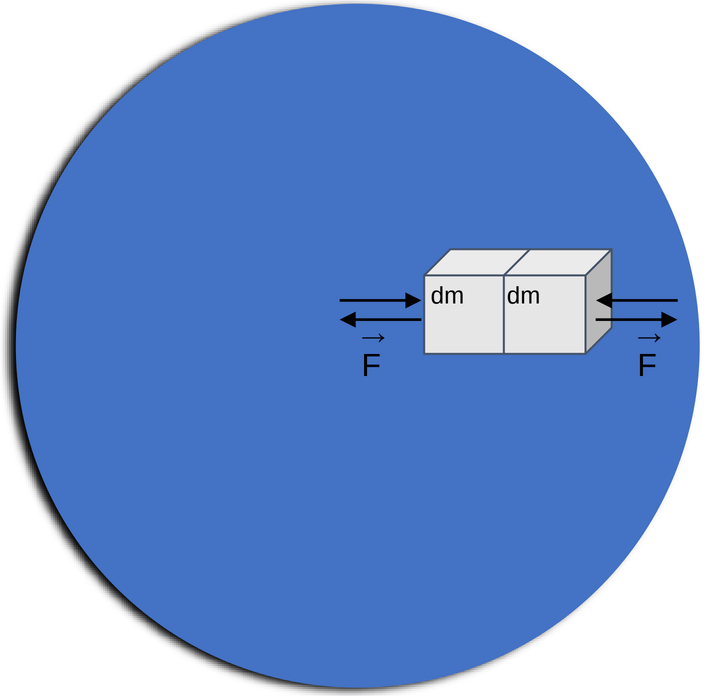
↓

Perturbed Hydrostatic Equilibrium Equation

↓

Radial force distribution just after the perturbation

Cracking

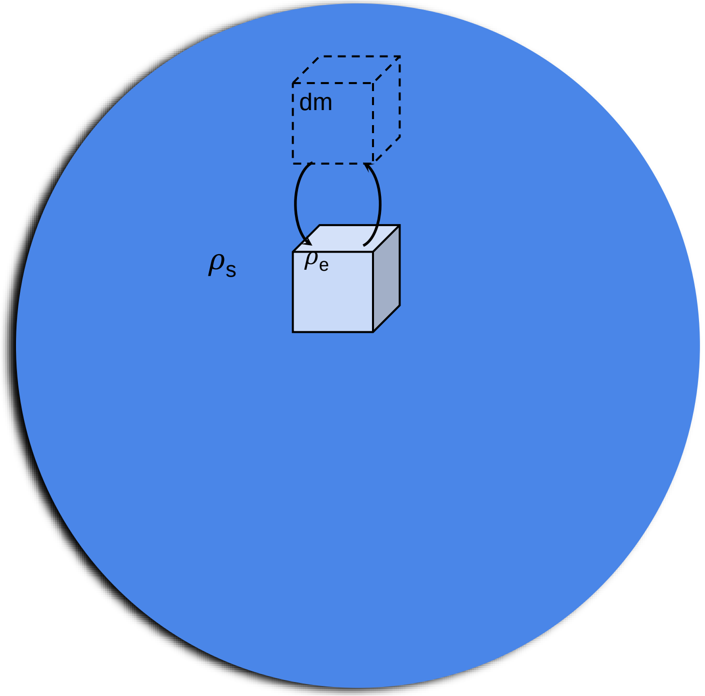



L. Herrera, Physics Letters A., 1992.

# Stability

Convective stability

Archimedes' principle




H. Bondi, Proc. R. Soc. London, Ser. A., 1964.

- $\rho_e < \rho_s$ Stable
- $\rho_e = \rho_s$ Metastable
- $\rho_e > \rho_s$ Unstable