The Theory Behind SNEWPY

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The Big Picture

- To compare with an observed SN neutrino signal (and design detectors that detect SN neutrinos well) we want to calculate the neutrino signal from supernova simulations
- Most simulations of supernova do not include the flavor mixing of the neutrinos.
- What they provide are the outgoing neutrino spectra Φ_α typically with $\alpha = \{e,x\}$ at the edge of the simulation volume.
  - In what follows we will model the neutrinos as all coming from a neutrinobulb – a hard neutrinosphere defined by the radius R_ν.
  - Apart from flavor mixing, we will assume these spectra are unchanged since emission from the neutrinosphere R_ν.
- What we want are the fluxes F_α of the neutrinos at Earth for a SN at a distance d and including flavor transformation and decoherence.

SNEWPY

- SNEWPY is a software package written in python that:
  - generates a (time series of) neutrino fluence from a supernova simulation at a given distance after convolving it with a prescription for the flavor mixing.
  - runs the fluence(s) through SNOwGLoBES to compute the event rates in neutrino detectors
  - collates the output from SNOwGLoBES into the observable channels.
- It does not replace SNOwGLoBES, it complements it.

- The neutrino fluxes at Earth can be arranged into a vector

$$F_F(r_\oplus) = \begin{pmatrix} F_e(r_\oplus) \\ F_\mu(r_\oplus) \\ F_\tau(r_\oplus) \end{pmatrix} \qquad \bar{F}_F(r_\oplus) = \begin{pmatrix} \bar{F}_e(r_\oplus) \\ \bar{F}_\mu(r_\oplus) \\ \bar{F}_\tau(r_\oplus) \end{pmatrix}$$

- We denote by a Greek index a generic flavor basis state.
- The flavor fluxes are a linear combination of the fluxes of the mass states which can also be arrange in a vector.

$$F_M(r_\oplus) = \begin{pmatrix} F_1(r_\oplus) \\ F_2(r_\oplus) \\ F_3(r_\oplus) \end{pmatrix} \qquad \bar{F}_M(r_\oplus) = \begin{pmatrix} \bar{F}_1(r_\oplus) \\ \bar{F}_2(r_\oplus) \\ \bar{F}_3(r_\oplus) \end{pmatrix}$$

- We denote by a Latin index a generic mass state.

- The two vectors are related by a matrix D

$$F_F(r_\oplus) = D\, F_M(r_\oplus) \qquad \bar{F}_F(r_\oplus) = D\, \bar{F}_M(r_\oplus)$$

- Ignoring Earth matter effects, the elements of D are the square magnitudes of the vacuum mixing matrix U_V.

$$D_{\alpha i} = \left| U_{V,\alpha i} \right|^2$$

- The components of the flux vector are the diagonal elements of the flux matrix $\mathscr{F}$

$$\mathscr{F} = c \int d\Omega\, \rho \cos\theta$$

- with ρ the density matrix.
- The diagonal elements of $\mathscr{F}$ are the coherences which disappear on the trip to Earth.

- The decoherence can be taken into account mathematically by

$$F_M = \sum_j |\nu_j\rangle\langle\nu_j|\mathscr{F}_M|\nu_j\rangle$$

$$\bar{F}_M = \sum_j |\bar{\nu}_j\rangle\langle\bar{\nu}_j|\bar{\mathscr{F}}_M|\bar{\nu}_j\rangle$$

- where $|\nu_j\rangle$ and $|\bar{\nu}_j\rangle$ are matter basis vectors
- The flux matrix is written as

$$\mathscr{F}_M(r_\oplus) = \frac{1}{4\pi d^2} \Phi_M(r_\oplus)$$

- with Φ_M the spectral matrix in the matter basis at Earth

- The flavor basis spectra at the neutrino sphere are assumed to be pure diagonal

$$\Phi_F(R_\nu) = \begin{pmatrix} \Phi_e(R_\nu) & 0 & 0 \\ 0 & \Phi_\mu(R_\nu) & 0 \\ 0 & 0 & \Phi_\tau(R_\nu) \end{pmatrix}$$

$$\Phi_F(R_\nu) = \sum_\beta |\nu_\beta\rangle\langle\nu_\beta| \Phi_\beta(R_\nu)$$

$$\bar{\Phi}_F(R_\nu) = \begin{pmatrix} \bar{\Phi}_e(R_\nu) & 0 & 0 \\ 0 & \bar{\Phi}_\mu(R_\nu) & 0 \\ 0 & 0 & \bar{\Phi}_\tau(R_\nu) \end{pmatrix}$$

$$\bar{\Phi}_F(R_\nu) = \sum_\beta |\bar{\nu}_\beta\rangle\langle\bar{\nu}_\beta| \bar{\Phi}_\beta(R_\nu)$$

- The spectra at Earth is related to the spectra at the neutrinosphere

$$\Phi_M(r_\oplus) = S_M \Phi_M(R_\nu) S_M^\dagger$$

$$\bar{\Phi}_M(r_\oplus) = \bar{S}_M \bar{\Phi}_M(R_\nu) \bar{S}_M^\dagger$$

- with S_M and $\bar{S}_M$ the evolution matrices in the matter basis.
- The matter basis spectra at the neutrinosphere are related to the spectra of the flavor states

$$\Phi_M(R_\nu) = U^\dagger \Phi_F(R_\nu) U$$

$$\bar{\Phi}_M(R_\nu) = \bar{U}^\dagger \bar{\Phi}_F(R_\nu) \bar{U}$$

- with the U's the matter mixing matrix at the neutrinosphere.

- Putting it all together we get the final equation

$$F_F = \frac{1}{4\pi d^2} D \sum_{i,\beta} \Phi_\beta \left|\nu_i\right\rangle \left| \left\langle \nu_i \right| S_M U^\dagger \left| \nu_\beta \right\rangle \right|^2$$

$$\bar{F}_F = \frac{1}{4\pi d^2} D \sum_{i,\beta} \bar{\Phi}_\beta \left|\bar{\nu}_i\right\rangle \left| \left\langle \nu_i \right| \bar{S}_M \bar{U}^\dagger \left| \bar{\nu}_\beta \right\rangle \right|^2$$

- The flavor fluxes at Earth are linear combinations of the spectra at the neutrinosphere.

Transition probabilities

- The relationship between fluxes at Earth and the intial spectra can be simplified by introducing the transition probabilities.
- Assuming the spectra of μ and τ are identical and denoting either as the x flavor, these probabilities are defined to be

$$F_e(r_\oplus)=\frac{1}{4\pi d^2}\left(p_{ee}\Phi_e(R_\nu)+p_{ex}\Phi_x(R_\nu)\right)$$

$$F_x(r_\oplus)=\frac{1}{4\pi d^2}\left(p_{xe}\Phi_e(R_\nu)+p_{xx}\Phi_x(R_\nu)\right)$$

$$\bar{F}_e(r_\oplus)=\frac{1}{4\pi d^2}\left(\bar{p}_{ee}\bar{\Phi}_e(R_\nu)+\bar{p}_{ex}\bar{\Phi}_x(R_\nu)\right)$$

$$\bar{F}_x(r_\oplus)=\frac{1}{4\pi d^2}\left(\bar{p}_{xe}\bar{\Phi}_e(R_\nu)+\bar{p}_{xx}\bar{\Phi}_x(R_\nu)\right)$$

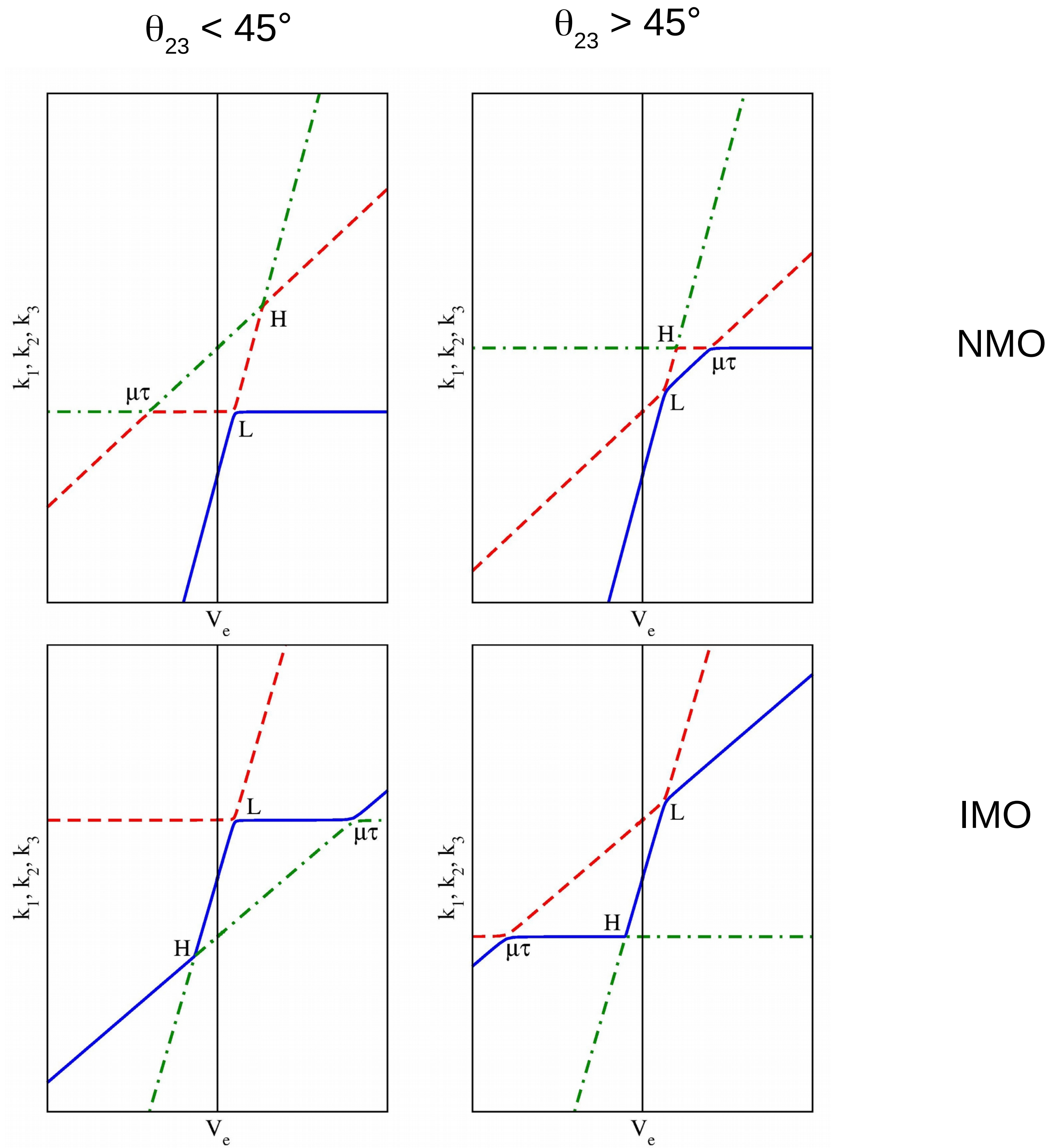
- There are relationships between the p's.

3 Flavors

- For three flavors it turns out we need just the elements of the first row of the D matrix

$$D_{e1} = \cos^2\theta_{12}\cos^2\theta_{13}$$
$$D_{e2} = \sin^2\theta_{12}\cos^2\theta_{13}$$
$$D_{e3} = \sin^2\theta_{13}$$

- To compute the matter mixing matrices we need to construct the neutrino Hamiltonian H at the neutrinosphere.
- The eigenvalues of H are k_i.
- The mixing matrix is the matrix which diagonalizes H.
  - formulae exist e.g. Kneller & McLaughlin, PRD, **80** 053002 (2009)



θ23 < 45°
θ23 > 45°
NMO
IMO
k1, k2, k3
Ve
H
L
μτ

- For the NMO the mixing matrices at the neutrinosphere are

$$U = \begin{pmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix} \qquad \bar{U} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

- while for the IMO the mixing matrices at the neutrinosphere are

$$U = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix} \qquad \bar{U} = \begin{pmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix}$$

Adiabtic Propagation

- Let's consider the case of adiabatic propagation.
- In this case the S matrices are diagonal in the matter basis.

$$S_M = \begin{pmatrix} e^{i\chi_{11}} & 0 & 0 \\ 0 & e^{i\chi_{22}} & 0 \\ 0 & 0 & e^{i\chi_{33}} \end{pmatrix} \qquad \bar{S}_M = \begin{pmatrix} e^{i\bar{\chi}_{11}} & 0 & 0 \\ 0 & e^{i\bar{\chi}_{22}} & 0 \\ 0 & 0 & e^{i\bar{\chi}_{33}} \end{pmatrix}$$

- with the χ's phases which will turn out not to appear in the final formulae.

- Inserting all the terms we find for the NMO

$$p_{ee} = D_{e3} \qquad p_{ex} = 1 - p_{ee} \qquad \bar{p}_{ee} = D_{e2} \qquad \bar{p}_{ex} = 1 - \bar{p}_{ee}$$

$$p_{xe} = \frac{(1 + p_{ee})}{2} \qquad p_{xx} = \frac{(1 - p_{ee})}{2} \qquad \bar{p}_{xe} = \frac{(1 + \bar{p}_{ee})}{2} \qquad \bar{p}_{xx} = \frac{(1 - \bar{p}_{ee})}{2}$$

- while for the NMO

$$p_{ee} = D_{e2} \qquad p_{ex} = 1 - p_{ee} \qquad \bar{p}_{ee} = D_{e3} \qquad \bar{p}_{ex} = 1 - \bar{p}_{ee}$$

$$p_{xe} = \frac{(1 + p_{ee})}{2} \qquad p_{xx} = \frac{(1 - p_{ee})}{2} \qquad \bar{p}_{xe} = \frac{(1 + \bar{p}_{ee})}{2} \qquad \bar{p}_{xx} = \frac{(1 - \bar{p}_{ee})}{2}$$

Nonadiabtic Propagation

- Let's consider a second example: nonadiabatic propagation with complete swapping at the H resonance
- The S matrices are not diagonal in the matter basis.
- For the NMO the S matrix becomes ($\bar{S}$ is adiabatic)

$$S = \begin{pmatrix} e^{i\chi_{11}} & 0 & 0 \\ 0 & 0 & e^{i\chi_{23}} \\ 0 & e^{i\chi_{32}} & 0 \end{pmatrix}$$

- while for the IMO $\bar{S}$ changes to (S is adiabatic)

$$\bar{S} = \begin{pmatrix} 0 & 0 & e^{i\bar{\chi}_{13}} \\ 0 & e^{i\bar{\chi}_{22}} & 0 \\ e^{i\bar{\chi}_{31}} & 0 & 0 \end{pmatrix}$$

- In this scenario we find for the NMO

$$p_{ee}=D_{e2} \qquad p_{ex}=1-p_{ee} \qquad \bar{p}_{ee}=D_{e1} \qquad \bar{p}_{ex}=1-\bar{p}_{ee}$$

$$p_{xe}=\frac{(1+p_{ee})}{2} \qquad p_{xx}=\frac{(1-p_{ee})}{2} \qquad \bar{p}_{xe}=\frac{(1+\bar{p}_{ee})}{2} \qquad \bar{p}_{xx}=\frac{(1-\bar{p}_{ee})}{2}$$

- and for the IMO the transition probabilities end up being the same as the NMO
  - nonadiabatic evolution cannot distinguish the mass ordering

$$p_{ee}=D_{e2} \qquad p_{ex}=1-p_{ee} \qquad \bar{p}_{ee}=D_{e1} \qquad \bar{p}_{ex}=1-\bar{p}_{ee}$$

$$p_{xe}=\frac{(1+p_{ee})}{2} \qquad p_{xx}=\frac{(1-p_{ee})}{2} \qquad \bar{p}_{xe}=\frac{(1+\bar{p}_{ee})}{2} \qquad \bar{p}_{xx}=\frac{(1-\bar{p}_{ee})}{2}$$

- SNEWPY contains many more cases for the neutrino flavor evolution including neutrino decay and four-neutrino cases.
  - All the probabilities are static in time: the neutrino decay is energy dependent but all others are not.

- Future extensions will add time and energy dependent transition probabilities – e.g. for shock wave effects or Earth matter.

Any questions?