Higgs Modes and Surface Majorana Modes in Topological Superfluid ^{3}He

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Outline

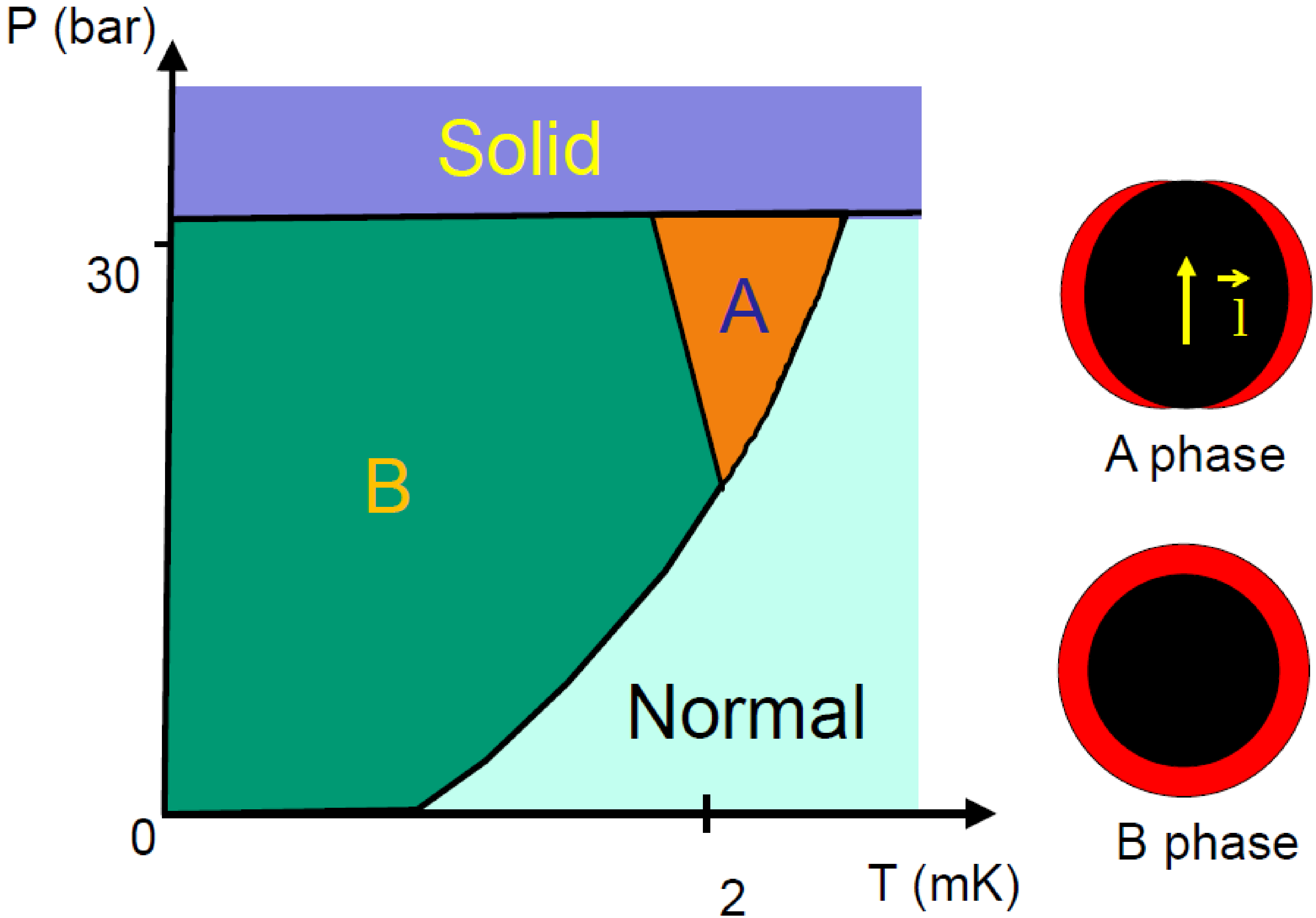
1. Introduction of superfluid ^{3}He

2. Higgs modes in superfluid ^{3}He

3. Transverse sounds in superfluid ^{3}He

4. Majorana modes in superfluid ^{3}He

Phase diagram of spin-triplet p-wave superfluid ^{3}He



Order parameter of spin-triplet p-wave states 1

Order parameter

$$\Delta_{\sigma_1\sigma_2}(\hat{p}) = \Delta \sum_{\alpha} \sum_{i} (i\underset{\substack{\uparrow \\ S=1}}{\sigma_\alpha}\sigma_y)_{\sigma_1\sigma_2} \, A_{\alpha i} \, \underset{\substack{\uparrow \\ L=1}}{\hat{p}_i}$$

$$\alpha = x, y, z$$
$$i = x, y, z$$

A phase

$$A_{\alpha i} = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & i \\ 0 & 0 & 0 \end{pmatrix}$$

B phase

$$A_{\alpha i} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

Order parameter of spin-triplet p-wave states 2

$$\Delta_{\sigma_1\sigma_2}(\hat{p}) = \Delta \sum_{\alpha} \sum_{i} (i\sigma_\alpha \sigma_y)_{\sigma_1\sigma_2} \; A_{\alpha i} \; \hat{p}_i$$

$$= \Delta \sum_{\alpha} (i\sigma_\alpha \sigma_y)_{\sigma_1\sigma_2} d_\alpha$$

$$\hat{d}_\alpha(\hat{p}) \equiv \sum_{i} A_{\alpha i} \; \hat{p}_i$$

$$\Delta = \begin{pmatrix} \Delta_{\uparrow\uparrow} & \Delta_{\uparrow\downarrow} \\ \Delta_{\downarrow\uparrow} & \Delta_{\downarrow\downarrow} \end{pmatrix} = \begin{pmatrix} -d_x + id_y & d_z \\ d_z & d_x + id_y \end{pmatrix}$$

A phase order parameter

$$\left|\mathrm{A\ phase}\right\rangle = \Delta_A\left(p_x + ip_y\right)\left\{\left|\downarrow\downarrow\right\rangle + \left|\uparrow\uparrow\right\rangle\right\}$$

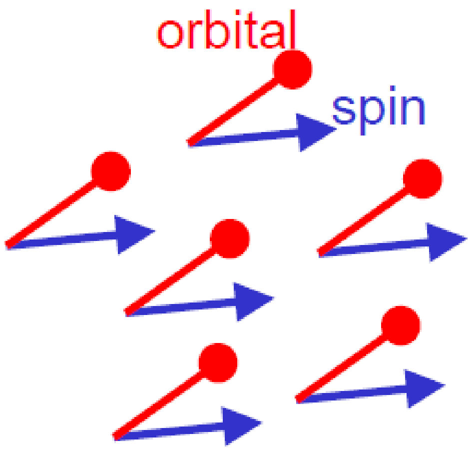
$L_z = +1$ $S_z = 0$

Broken relative gauge-orbit symmetry

Broken time-reversal symmetry → Ikegami (anomalous Hall effect), Sasaki (MRI)

$$\mathrm{G} = \mathrm{SO(3)_S} \times \underbrace{\mathrm{SO(3)_L} \times \mathrm{U(1)}} \times \mathrm{P} \times \mathrm{T} \times \mathrm{C}$$

$$\mathrm{G_A} = \mathrm{U(1)_{S_z}} \times \mathrm{U(1)_{L_z-\varphi}} \times \mathrm{P} \times \cancel{\mathrm{T}} \times \mathrm{C}$$



Vollhardt and Wolfle, The Superfluid Phases of Helium 3

B phase order parameter

$$|\mathrm{B\,phase}\rangle = \Delta_B \{ (p_x + ip_y)|\downarrow\downarrow\rangle + (p_x - ip_y)|\uparrow\uparrow\rangle + p_z|\uparrow\downarrow + \downarrow\uparrow\rangle \}$$

L_z=+1 S_z=-1 L_z=-1 S_z=+1 L_z=0 S_z=0

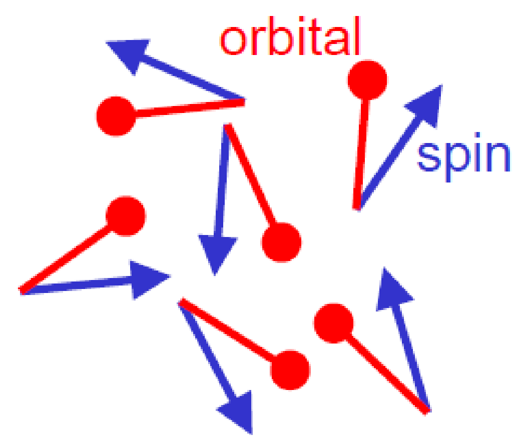
$J = L + S = 0$

Broken relative spin-orbit symmetry

$$G = SO(3)_S \times SO(3)_L \times U(1) \times P \times T \times C$$

$$G_B = SO(3)_{S+L} \times U(1) \times P \times T \times C$$

Generator $J = L + S$



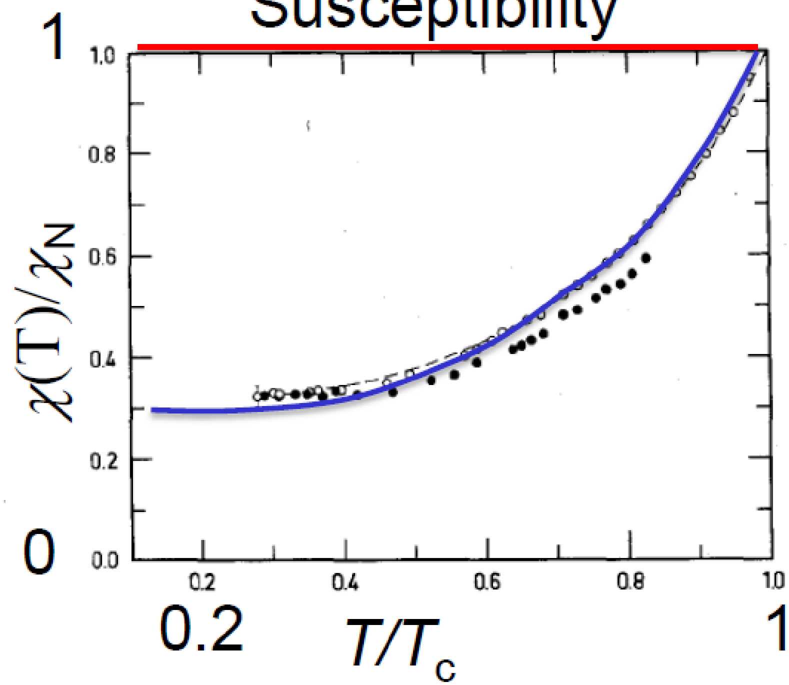
Vollhardt and Wolfle, The Superfluid Phases of Helium 3

CW-NMR

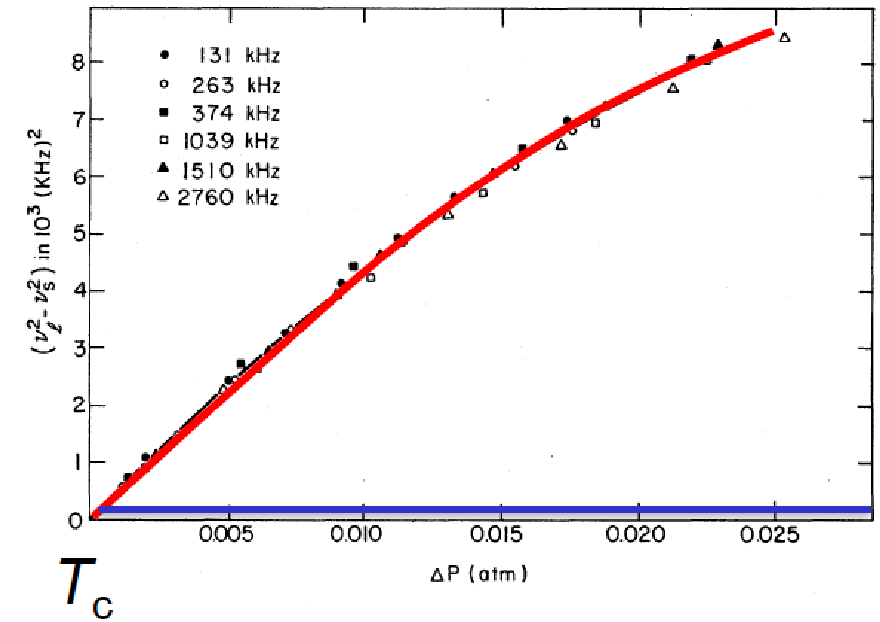
Direct probe of spin states in triplet cooper pairs

$$\omega = \omega_{A,B}(H_0, \Delta(T), \hat{l}, \hat{n}, \beta, \cdots)$$

Susceptibility



Frequency shift



A phase; $\chi = \chi_A = \chi_N$

B phase; $\chi = \chi_B(T)$

A phase; $\omega^2 = \omega_L^2 + \Omega_A^2(T)$

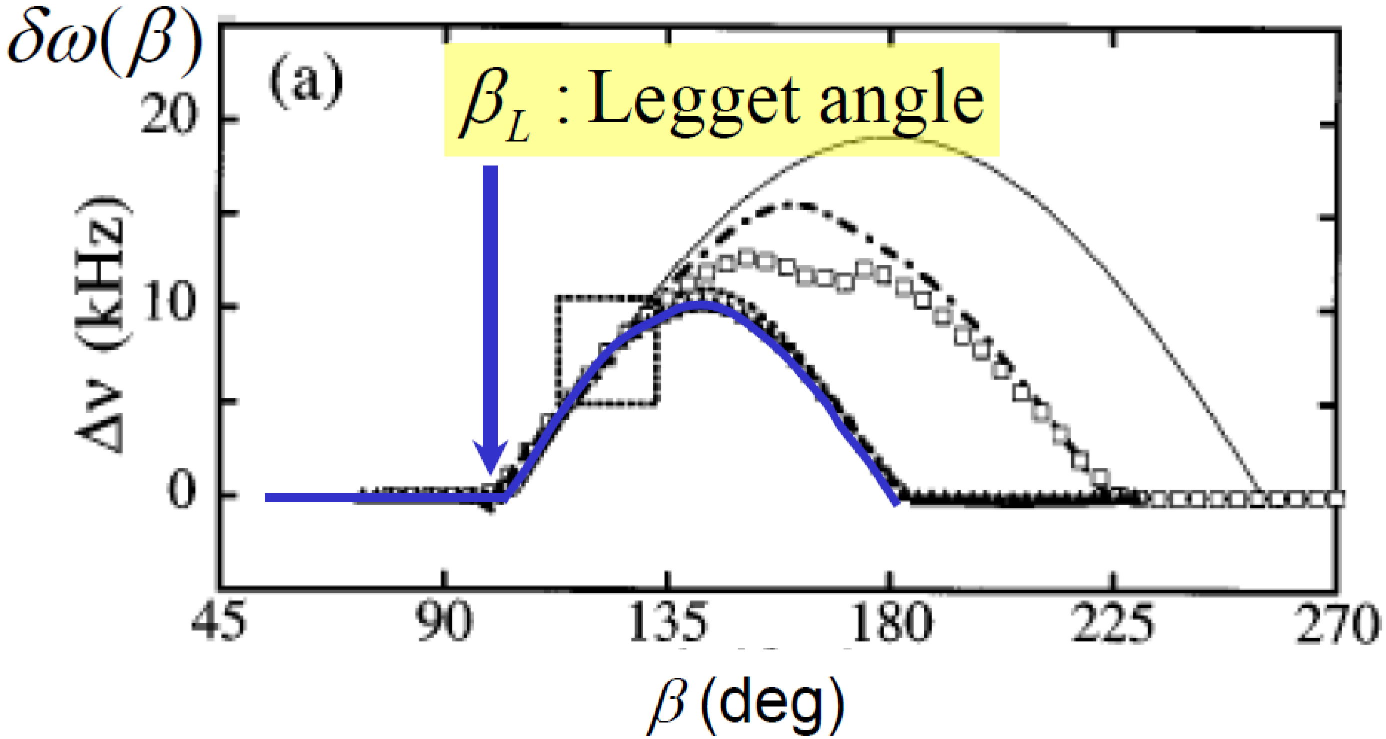
B phase; $\omega^2 = \omega_L^2$

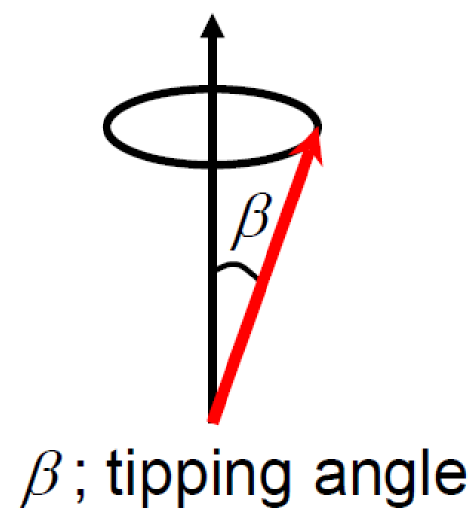
Ahonen *et al*. 1975

Osheroff *et al*. PRL1972

Pulsed NMR

tipping angle dependence of frequency shift in B phase





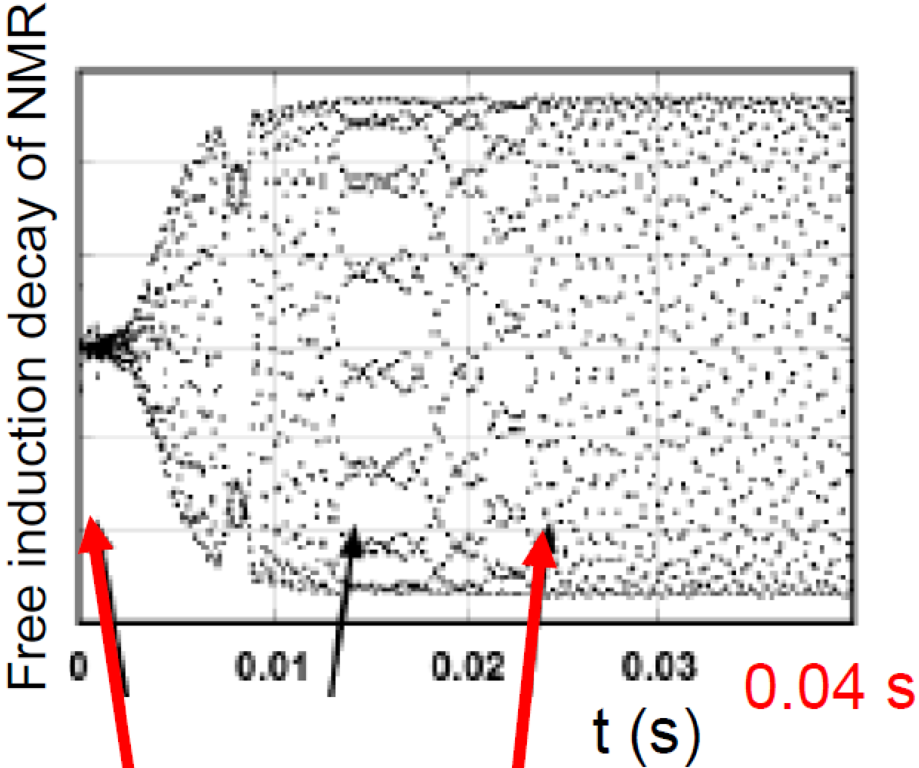
$$0 < \beta < \beta_L = 104^\circ \qquad \beta > \beta_L = 104^\circ$$

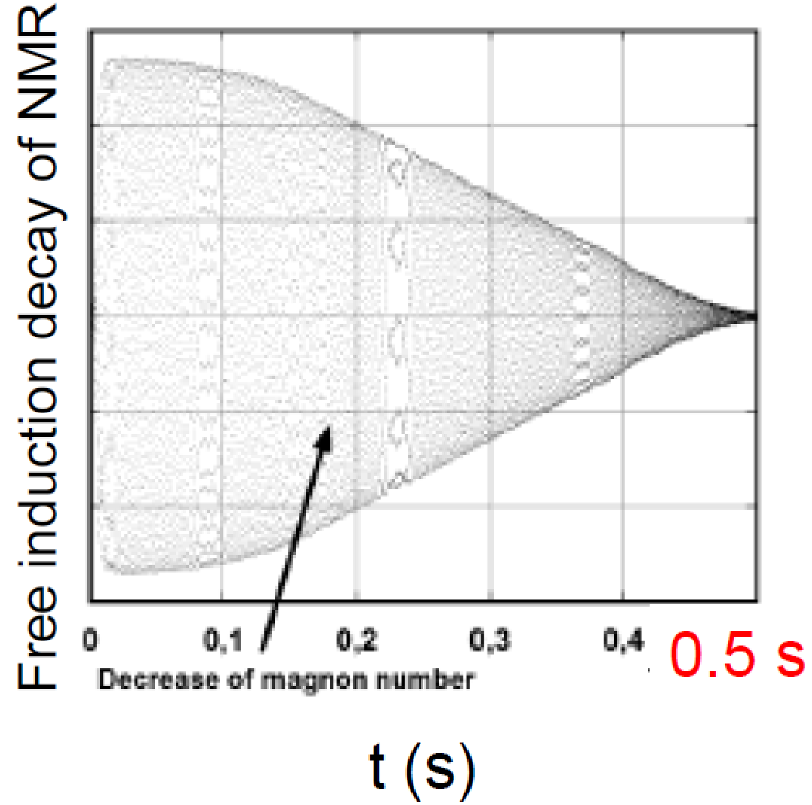
$$(\cos\beta_L = 1/4)$$

$$\delta\omega(\beta) = 0 \qquad \delta\omega(\beta) = -\frac{4}{15}\frac{\Omega_B{}^2}{\omega_L}(1 + 4\cos\beta)$$

Rand *et al*. PRL1996

Magnon BEC 1
(HPD, homogeneous precessing domain)





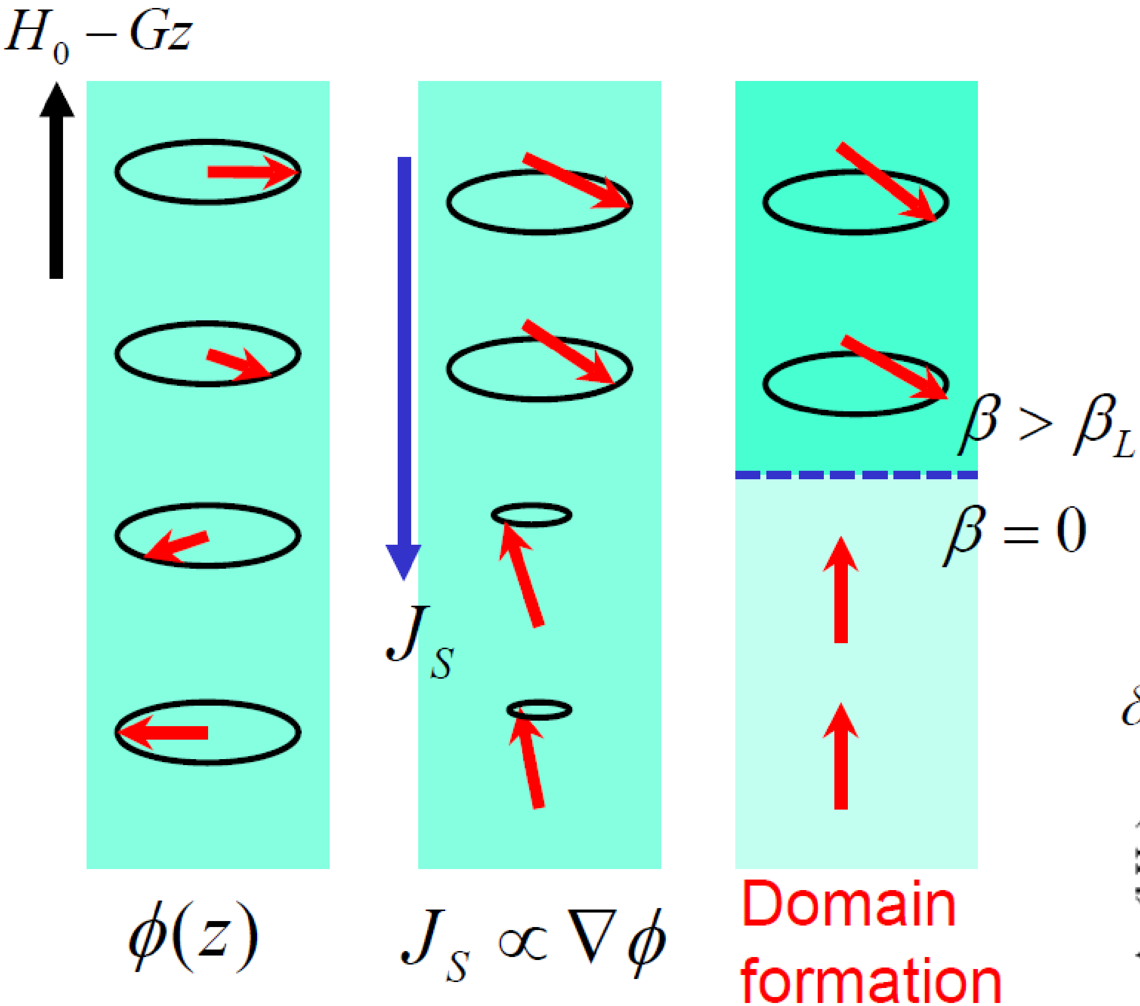
Dephasing;
T_2 in ms

Magnon BEC

$$\hat{N} = \hat{a}_0^+ \hat{a}_0 = \frac{S - \hat{S}_z}{\hbar} = \frac{S(1 - \cos\beta)}{\hbar}$$

Borovik-Romanov *et al*. JETP Lett.1984

Magnon BEC 1
(HPD, homogeneous precessing domain)



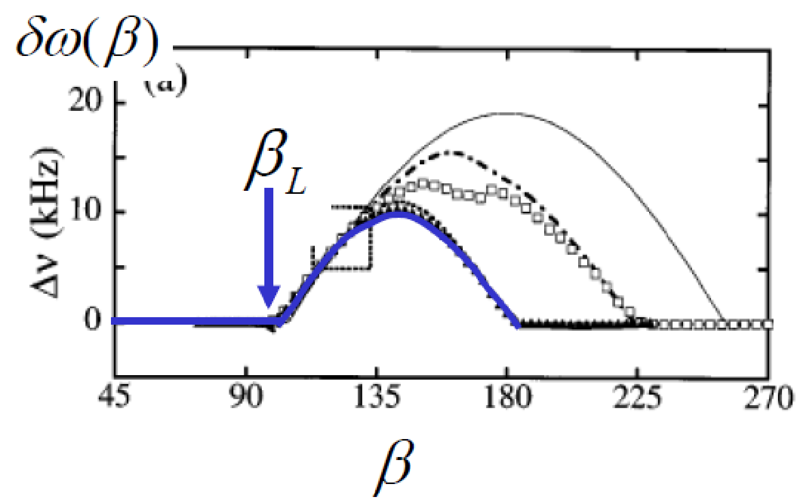
$$\omega(z) = \gamma(H_0 - Gz)$$

$$\phi(z) = \omega(z)t$$

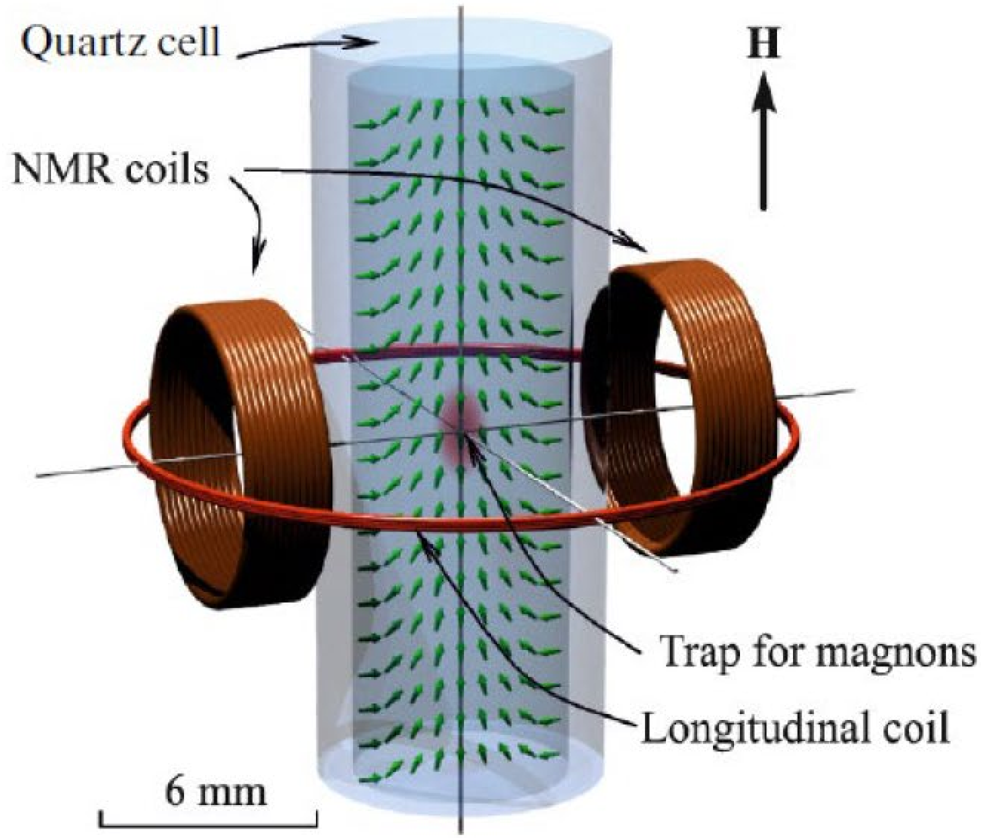
$$\nabla \phi = -\gamma G t$$

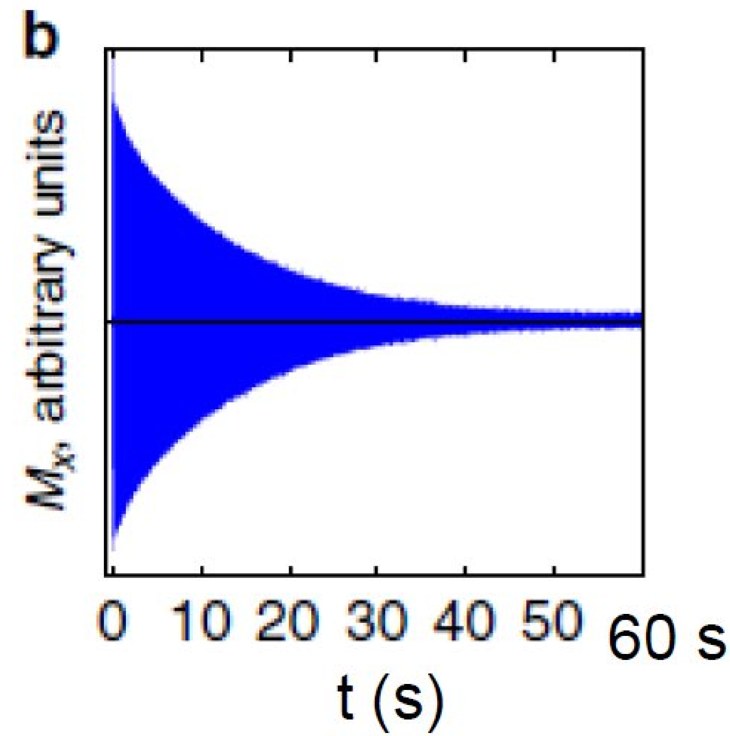
$$J_S \propto \nabla \phi$$

$$\omega(z) = \omega(\beta_L = 104^\circ)$$



Magnon BEC 2
(persistent precessing domain, spin laser, Q-ball, time crystal)





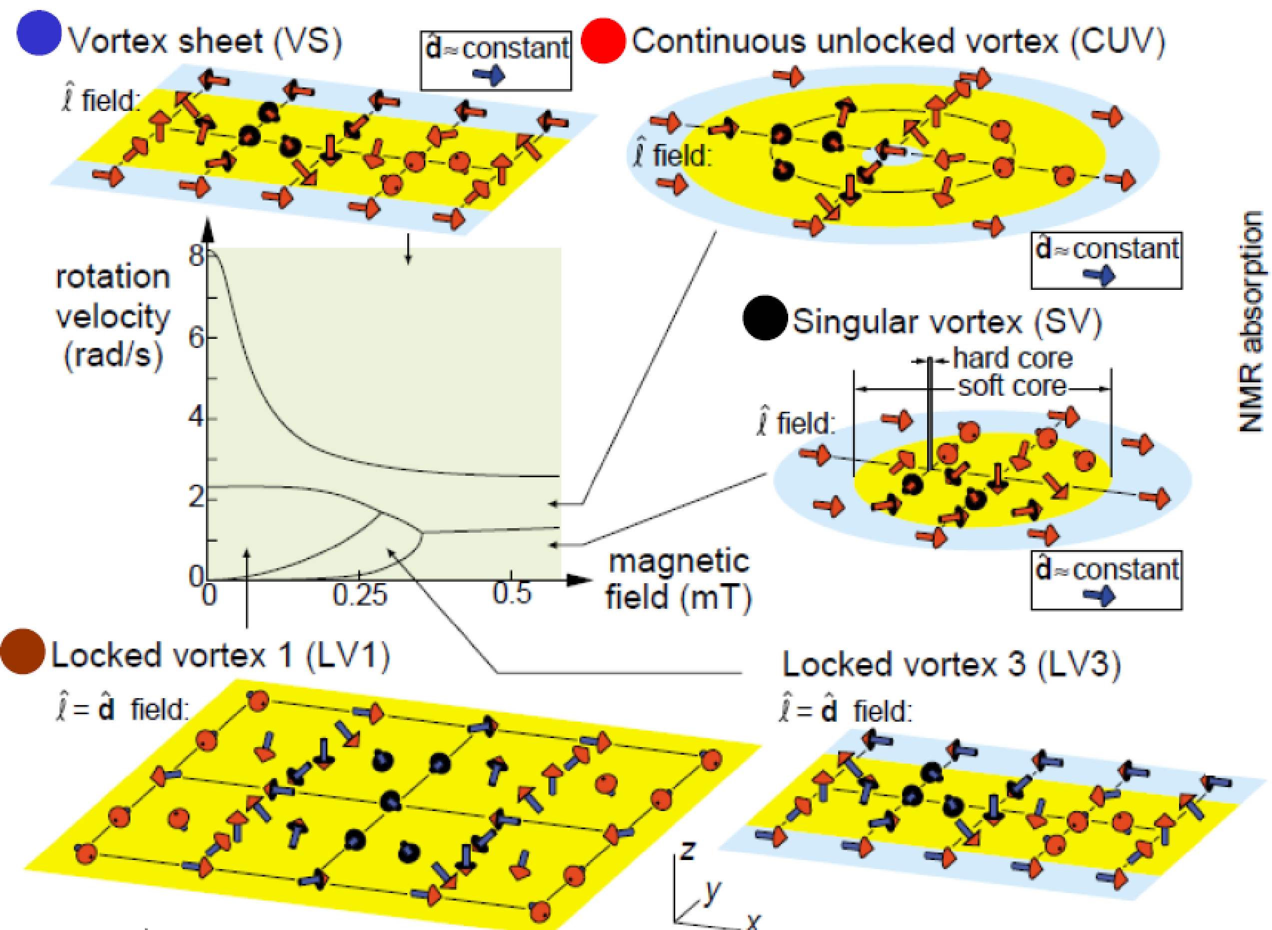
$$\tau_{\max} \approx 30\ \text{min!!!}$$

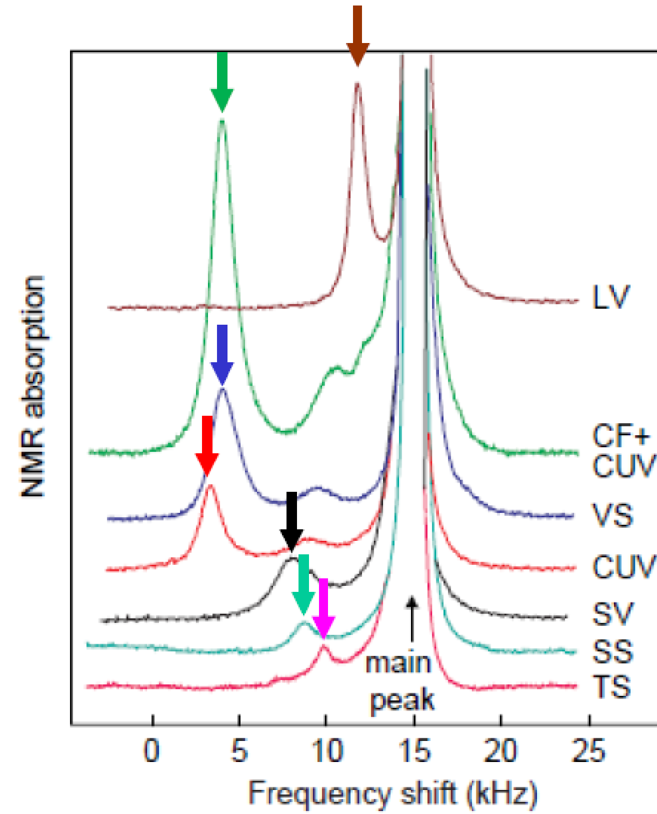
$$S(1-\cos\beta) = \hbar|\Psi|^2$$

$$F - \mu N = \int d^3r \left(\frac{|\nabla\Psi|^2}{2m_s} + (\omega_L(r) - \omega)|\Psi|^2 + F_{SO}(|\Psi|^2) \right)$$

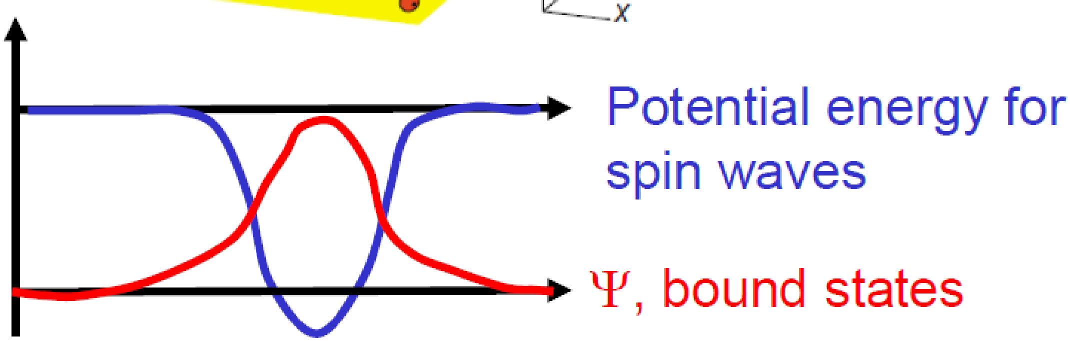
Bunkov *et al*. PRL1992

Various vortices in A phase





Satellite peaks from bound states



Lounasmaa, Thuneberg
Proc. Nat. Acad Sci. 1999

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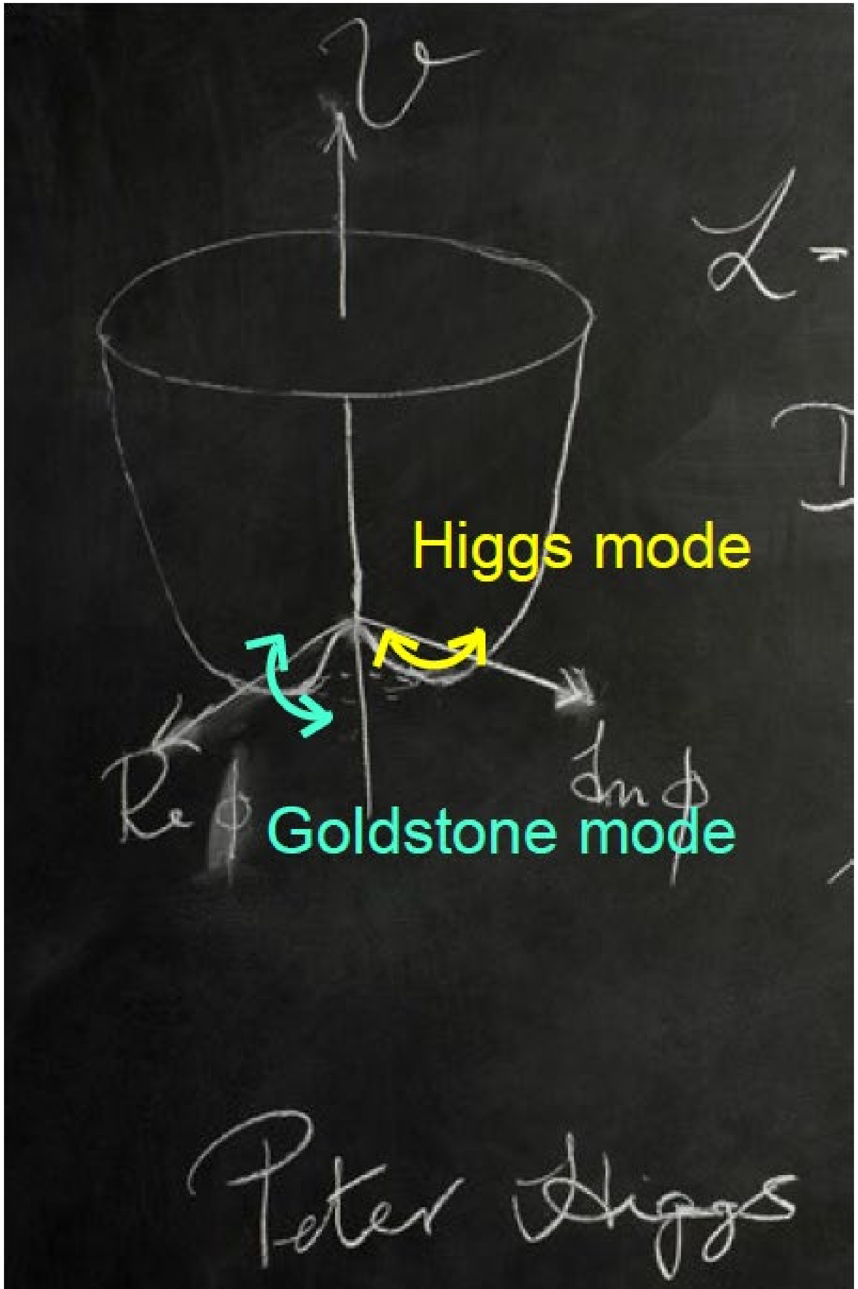
Higgs (amplitude) mode and Goldstone (phase) mode



Goldstone mode; gapless
Higgs mode; gapped

S-wave superconductors,
Matsunaga *et al*., PRL 2013

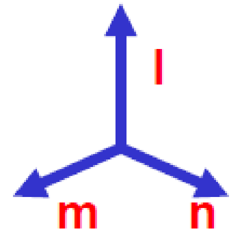
Cold gases in optical traps,
Endres, Fukuhara *et al*., Nature 2012

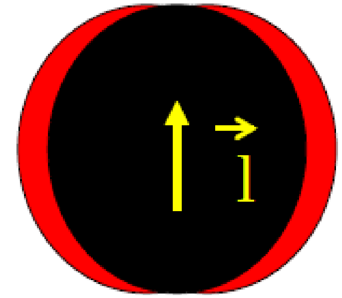
⋮

A phase

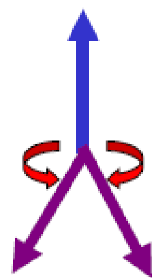
Ground State

$$d_{\mu i} = \Delta_A \begin{array}{c} \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & i \\ 0 & 0 & 0 \end{pmatrix} \end{array} \begin{array}{l} \rightarrow \mathbf{m} \\ \rightarrow \mathbf{n} \\ \end{array}$$





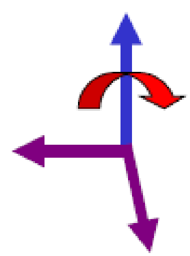
$$d_{\mu i} \propto \begin{pmatrix} 0 & 0 & 1+i\delta d \\ 0 & 0 & i+\delta d \\ 0 & 0 & 0 \end{pmatrix}$$



Clapping mode

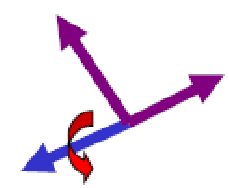
Goldstone modes and Higgs modes (total #18)

$$d_{\mu i} \propto \begin{pmatrix} 0 & 0 & 1+i\delta d \\ 0 & 0 & i-\delta d \\ 0 & 0 & 0 \end{pmatrix}$$



Sound

$$d_{\mu i} \propto \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & i \\ 0 & 0 & i\delta d \end{pmatrix}$$



Flapping mode
or Orbital wave

B phase

Residual symmetry : $G = SO(3)_{S+L} \times T \times C$

Generator $J = S + L$

Eigen state can be classified by (J, J_z, C)

$$d_{\mu i} = \Delta_B \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

Ground state, $J = 0$

$$\delta d_{\mu i} = \sum_{J, J_z, C} D^{C}_{J, J_z} \, t^{J, J_z}_{\mu i}$$

Excited states

(order parameter collective modes)

$J = 0, 1, 2 \quad C = \pm$ Goldstone modes and Higgs modes (total #18)

$J = 2-$ Imaginary Squashing Mode, $\sqrt{12/5}\Delta$ (5 fold degeneracy)

$J = 2+$ Real Squashing Mode, $\sqrt{8/5}\Delta$ (5 fold degeneracy)

Nagai, Prog. Theor. Phys. **54**, 1 (1975)

Maki, J. Low Temp. Phys. **24**, 755 (1976)

Why (longitudinal) ultrasound?

Time scale in normal phase

Life time of quasiparticles $\tau \propto \dfrac{1}{T^2}$

$\omega\tau < 1$ Hydrodynamic; longitudinal first sound

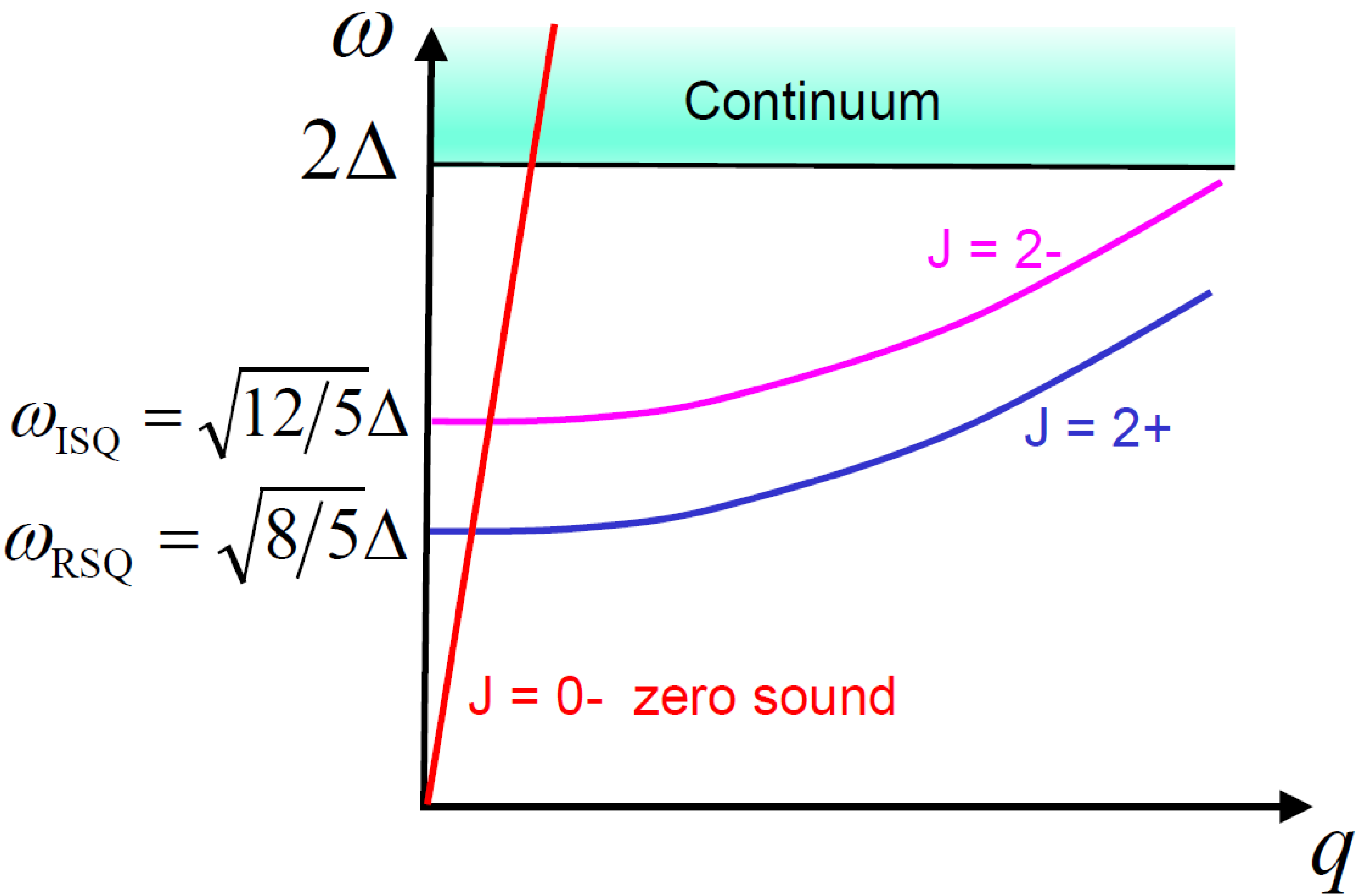
$\omega\tau > 1$ Collisionless; longitudinal zero sound

Energy scale in superfluid phase

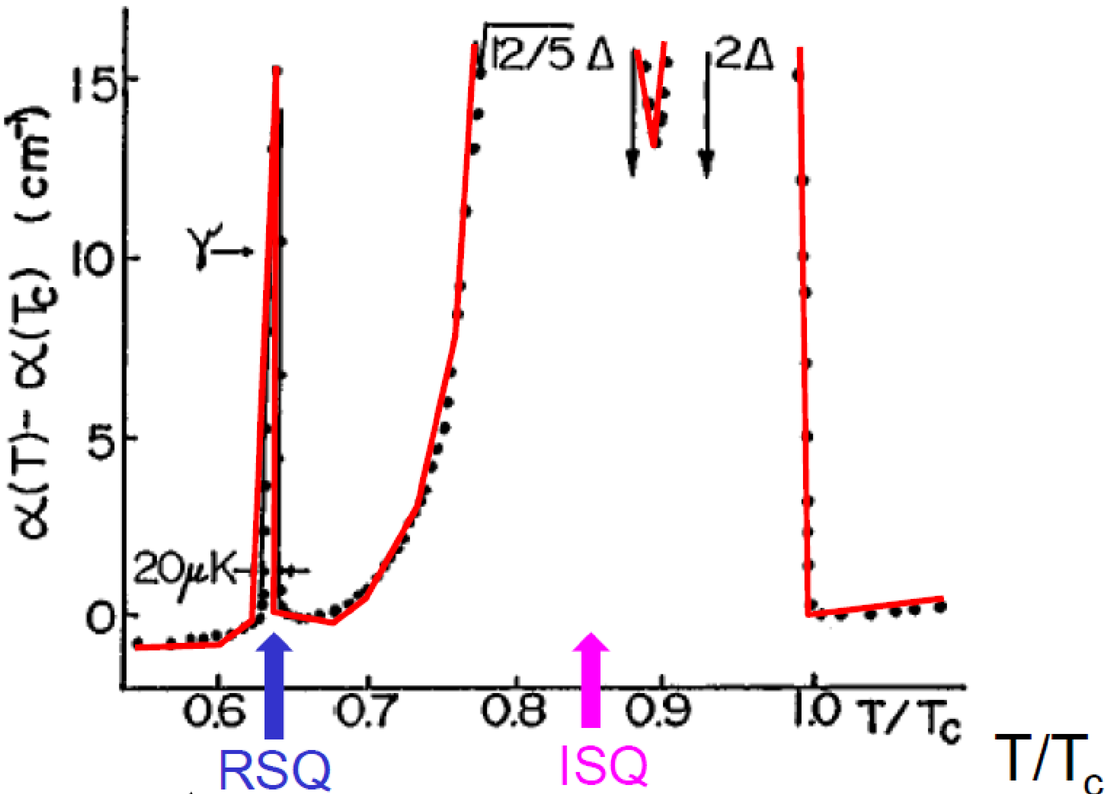
$$2\Delta \approx 100\ \mathrm{MHz}$$

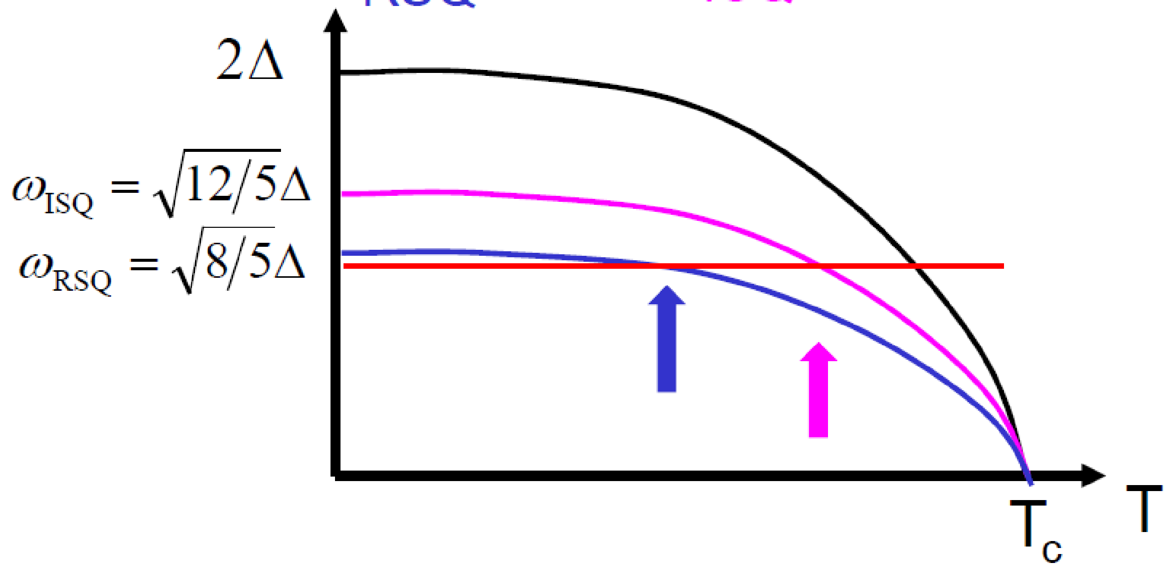
Acoustic spectroscopy of
Higgs modes and Majorana modes
by using ultrasound of 10 ~ 100 MHz

Resonance with Higgs modes in ^{3}He-B

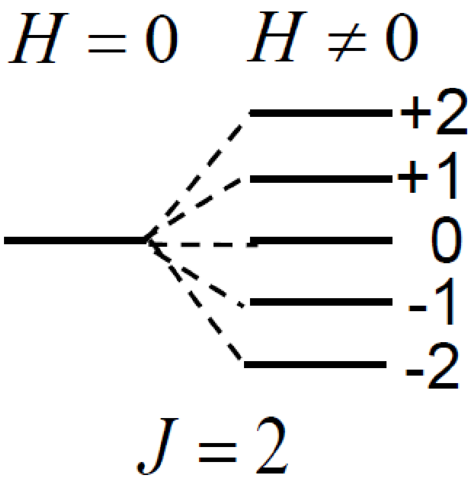


Acoustic spectroscopy of Higgs modes in ^{3}He-B





Zeeman splitting of real squashing modes in ^{3}He-B



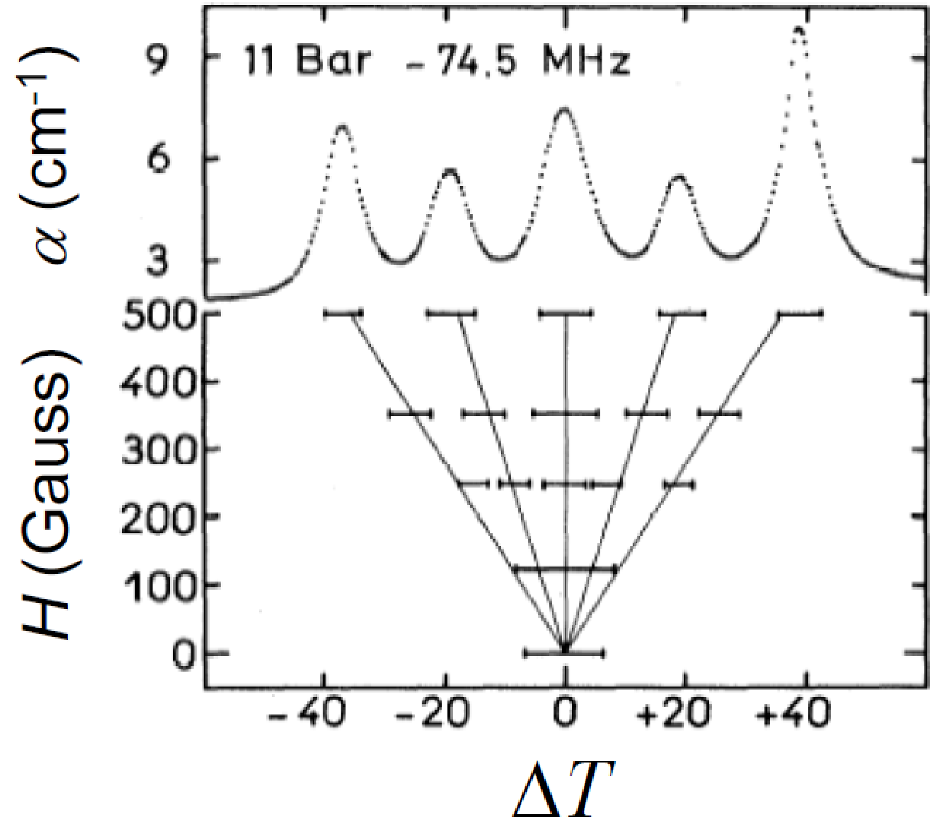


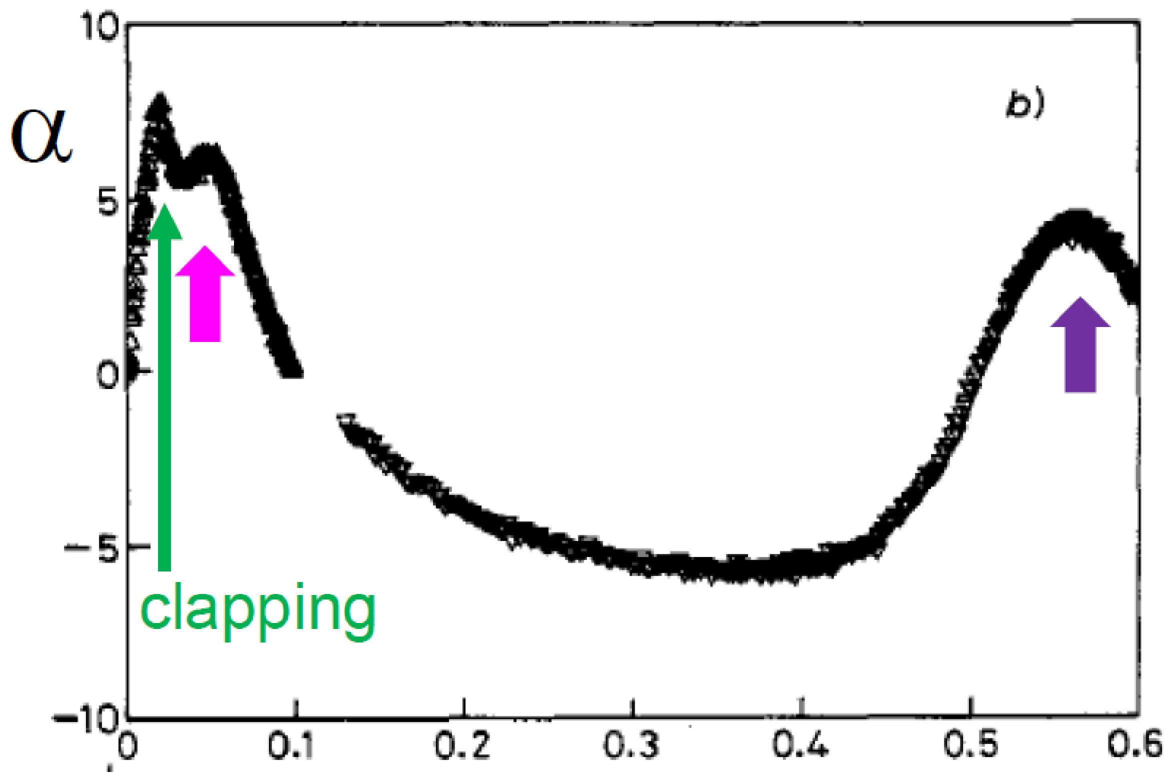
FIG. 2. Linear splitting with transverse magnetic field at 11.0 bars and 74.5 MHz. The bars indicate the half maximum full linewidth of the lines.

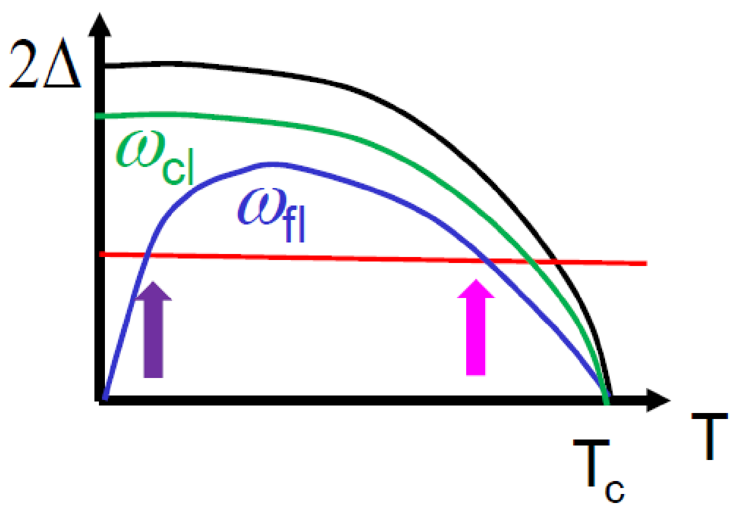
Confirmation of J = 2+

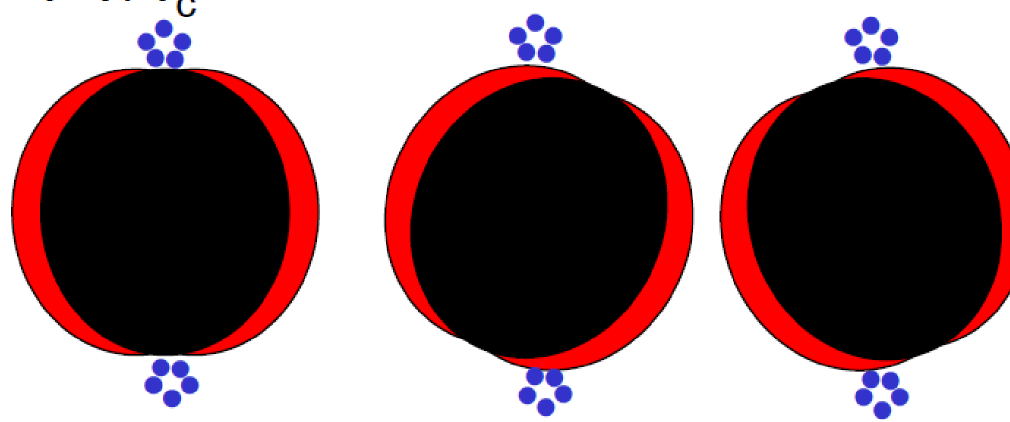
Identification of broken relative spin-orbit symmetry in B phase

Avenel, Varoquaux, Ebisawa, PRL 1980

Reentrant resonance with normal flapping mode in ^{3}He-A







Ling, Saunders, Wojtanowski and Dobbs, EPL 1989

Nambu sum rule

^{3}He-B

$$\overbrace{\omega_{\mathrm{RSQ}}^2 + \omega_{\mathrm{ISQ}}^2}^{\text{collective modes}} = \left(\sqrt{\frac{8}{5}}\Delta\right)^2 + \left(\sqrt{\frac{12}{5}}\Delta\right)^2 = \overbrace{(2\Delta)^2}^{\text{pair creations}} \qquad J = 2$$

$$\omega_{1-}^2 + \omega_{1+}^2 = (0)^2 + (2\Delta)^2 = (2\Delta)^2 \qquad J = 1$$

$$\omega_{0-}^2 + \omega_{0+}^2 = (0)^2 + (2\Delta)^2 = (2\Delta)^2 \qquad J = 0$$

^{3}He-A

$$\omega_{2+}^2 + \omega_{2-}^2 = \left(\sqrt{\frac{8}{3}}\langle\Delta\rangle\right)^2 + \left(\sqrt{\frac{4}{3}}\langle\Delta\rangle\right)^2 = \left(2\langle\Delta\rangle\right)^2$$

2D-^{3}He-A

$$\omega_{2+}^2 + \omega_{2-}^2 = \left(\sqrt{2}\Delta\right)^2 + \left(\sqrt{2}\Delta\right)^2 = (2\Delta)^2$$

s-wave BCS

$$\omega_{\mathrm{Goldstone}}^2 + \omega_{\mathrm{Higgs}}^2 = (0)^2 + (2\Delta)^2 = (2\Delta)^2$$

Supersymmetry?

Nambu, Physica D **15**, 147 (1985)

Volovik and Zubkov, J. Low Temp. Phys. **175**, 486 (2014)

Broken symmetries

3D B-phase

$$G = \boxed{SO(3)_S \times SO(3)_L} \times U(1)$$

$$G_{3D\text{-}B} = \boxed{SO(3)_{S+L}} \times U(1)$$

$$M_{\mathrm{Higgs1}}^2 + M_{\mathrm{Higgs2}}^2 = 4M_{\mathrm{top}}^2$$

M_{Higgs2} = 325 GeV

2D A-phase

$$G = SO(3)_S \times \boxed{SO(2)_L \times U(1)}$$

$$G_{2D\text{-}A} = U(1)_{S_z} \times \boxed{U(1)_{L_z-\varphi}}$$

$$M_{\mathrm{Higgs2}}^2 + M_{\mathrm{Higgs2}}^2 = 4M_{\mathrm{top}}^2$$

M_{Higgs2} = 245 GeV

Standard model

$$G = SU(3) \times \boxed{SU(2)_L \times U(1)}$$

$$G = SU(3) \times \boxed{U(1)_Q}$$

hints of 325 GeV Higgs boson in earlier experiments

* CDF/PUB/EXOTICS/PUBLIC/10603 July 17, 2011
Search for High-Mass Resonances Decaying
into ZZ in $p\bar{p}$ Collisions at $s^{1/2}$ = 1.96TeV

"The invariant masses of four events are clustered around 325 GeV"

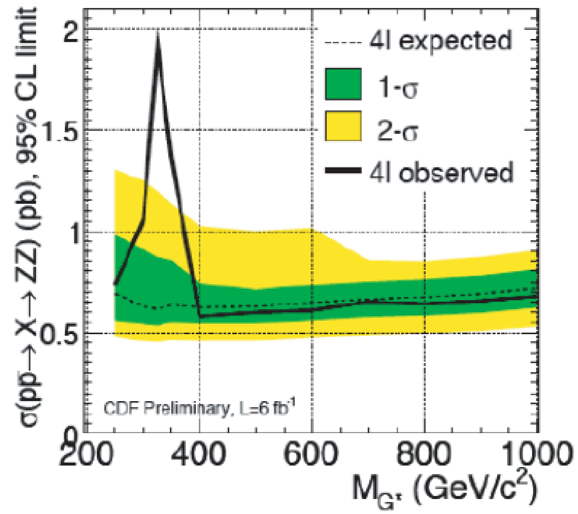
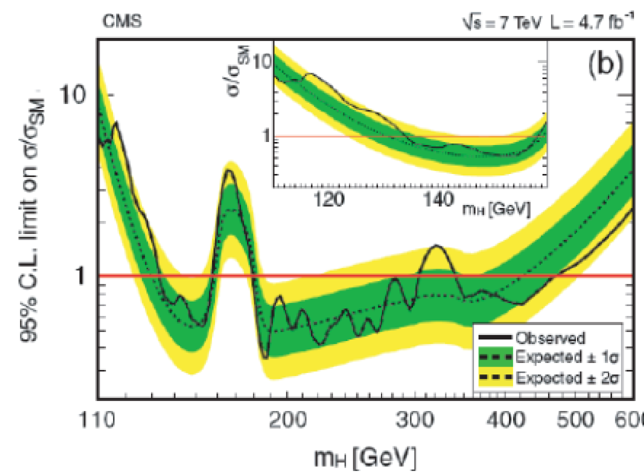


FIG. 13: Expected and observed 95% CL limits on $\sigma(p\bar{p} \to X \to ZZ)$ from the $ZZ \to \ell^+\ell^-\ell^+\ell^-$ channel; the four events with M_{ZZ}=327 GeV/c^2 result in a deviation from the expected limit.

* Search for the Standard Model Higgs Boson
in CMS Collaboration @ LHS
(Compact Muon Solenoid))

PRL 108, 111804 (2012)

"Small excesses of events are observed around masses of 119, 126 & 320 GeV "



Volovik and Zubkov, J. Low Temp. Phys. **175**, 486 (2014)
Volovik, QFS2014

Deviation of the Nambu Sum Rule

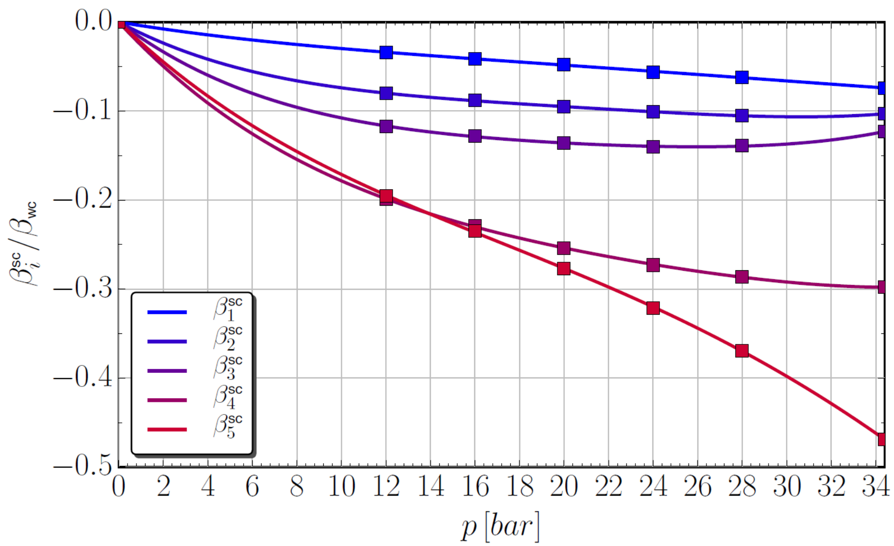


FIG. 1. Strong-coupling corrections ($\beta_i^{sc} \equiv \beta_i - \beta_i^{wc}$) to the GL β parameters interpolated from the results of Ref. [27] (data squares). The β_i^{sc} are extrapolated below $P = 12$ bar to weak coupling ($\beta_i^{sc} = 0$) at $p = 0$ bar.

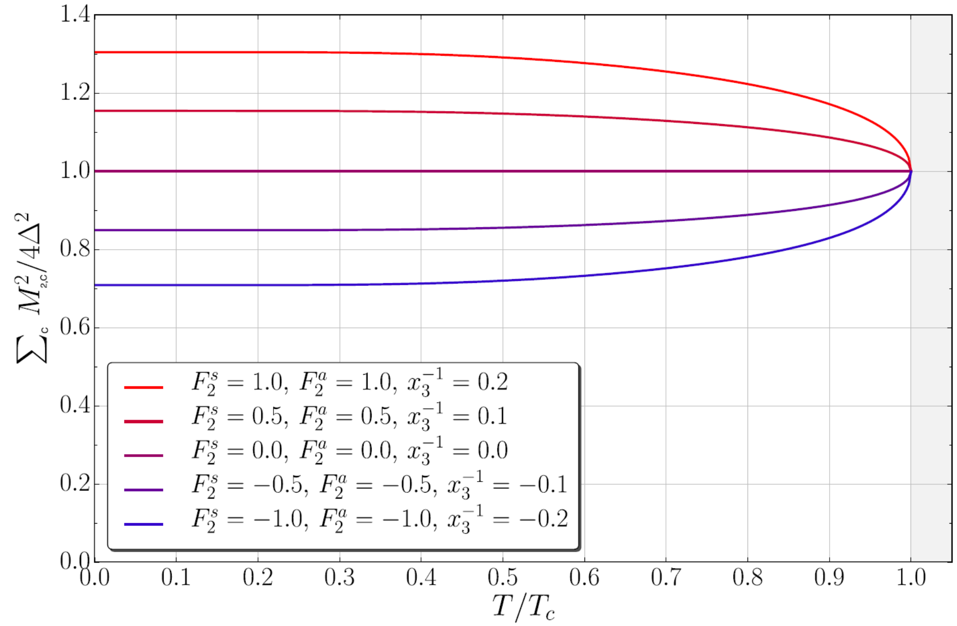
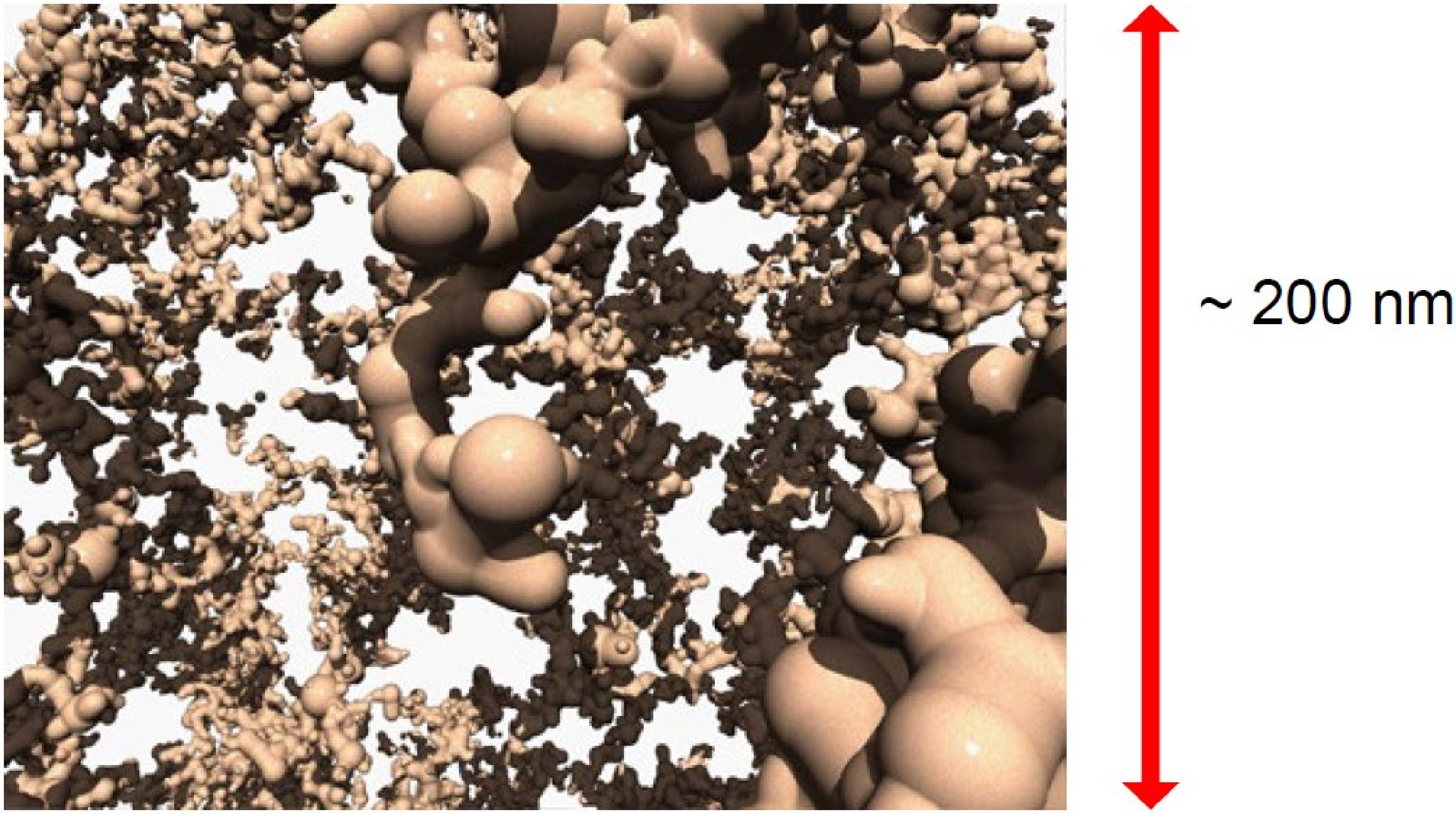


FIG. 4. Deviation of the Nambu sum from polarization corrections to the the $J = 2$ Higgs modes of ^{3}He-B for a range of interactions in both the Landau and Cooper channels.

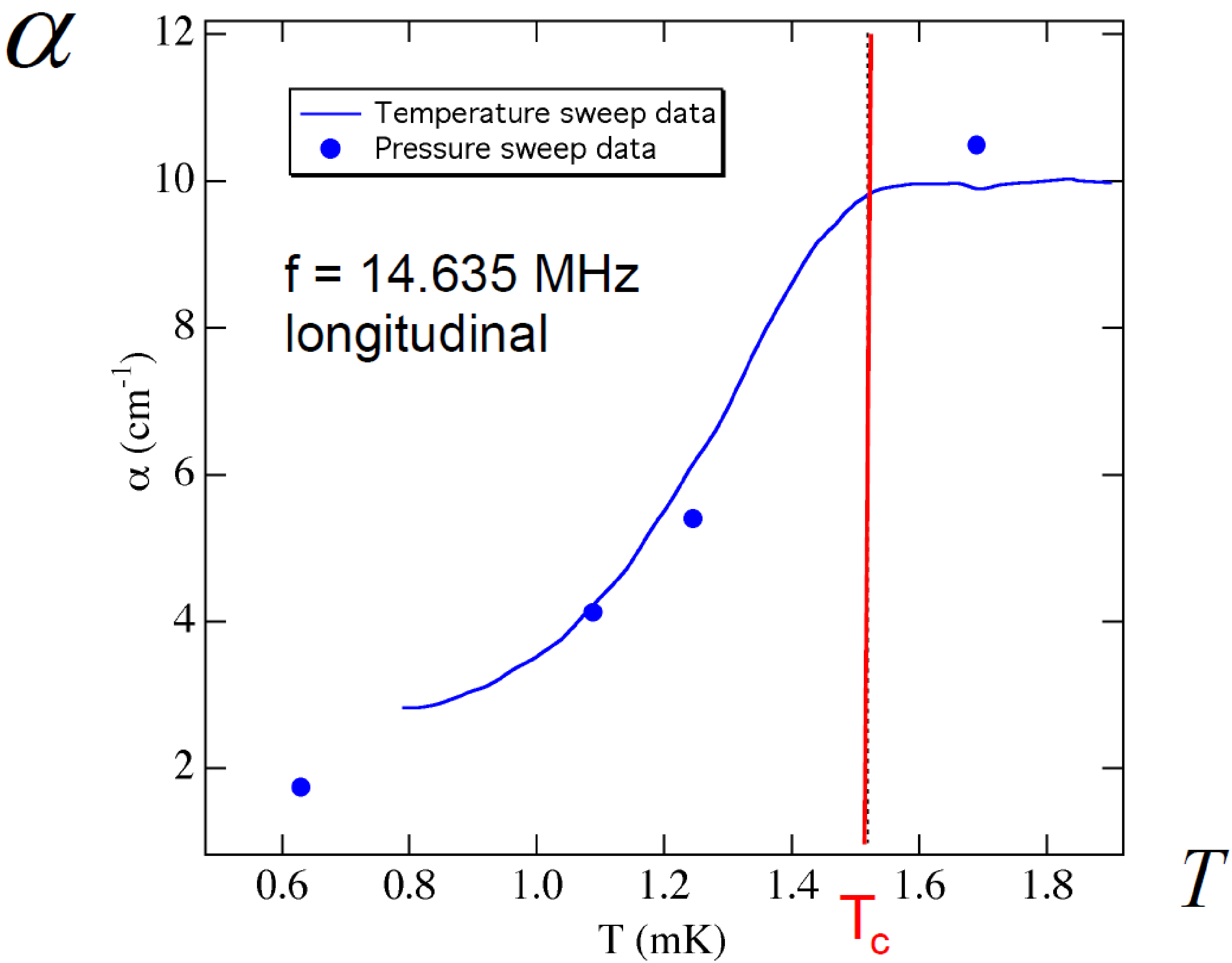
Sauls, Mizushima, PRB 95 094515 (2017)

Looking for Higgs modes of superfluid ^{3}He in aerogel to identify superfluid symmetries



Simulation of 98% porosity silica aerogel which works as impurities for ^{3}He.

No attenuation peak due to pair breaking and Higgs modes in aerogel



$\omega\tau_{\text{impurity}} < 1$; first sound

RN, Halperin, *et al.*, PRL **85**, 4325 (2000)

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Transverse sounds in superfluid ^{3}He

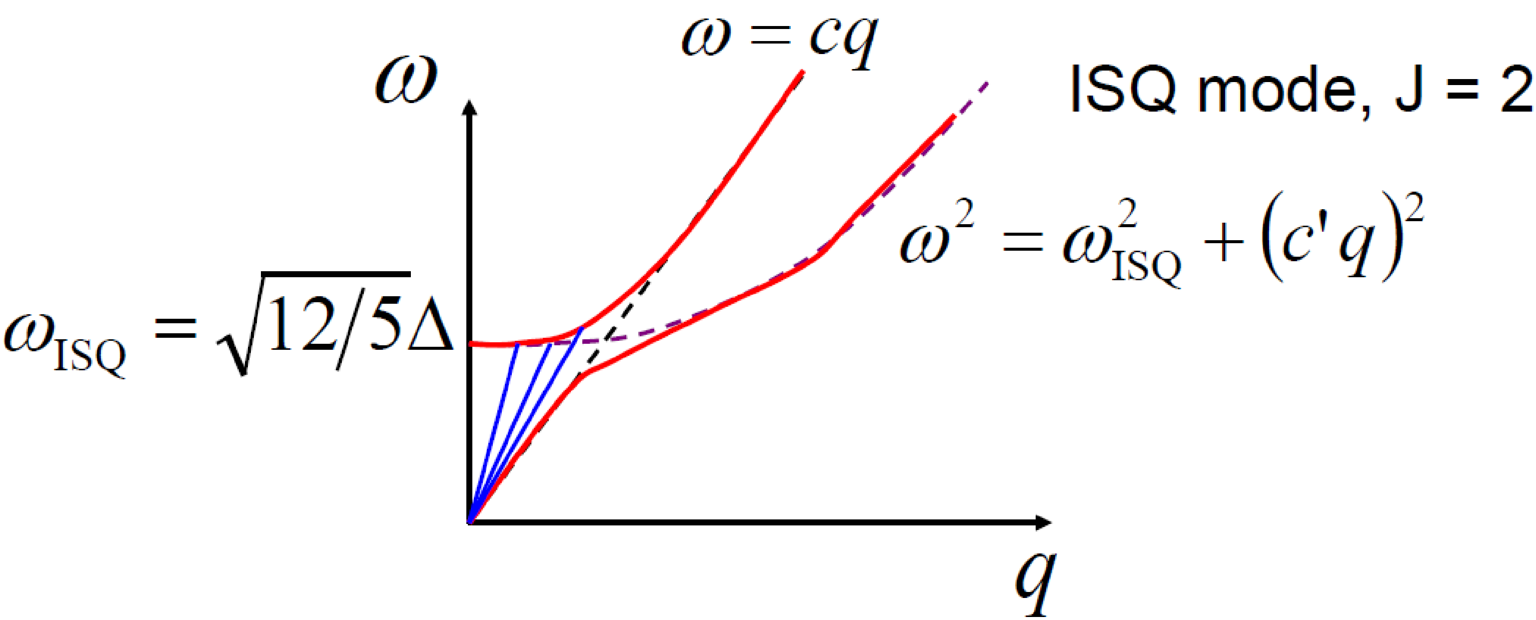
Transverse zero sound should be possible in strongly correlated Fermi liquids,

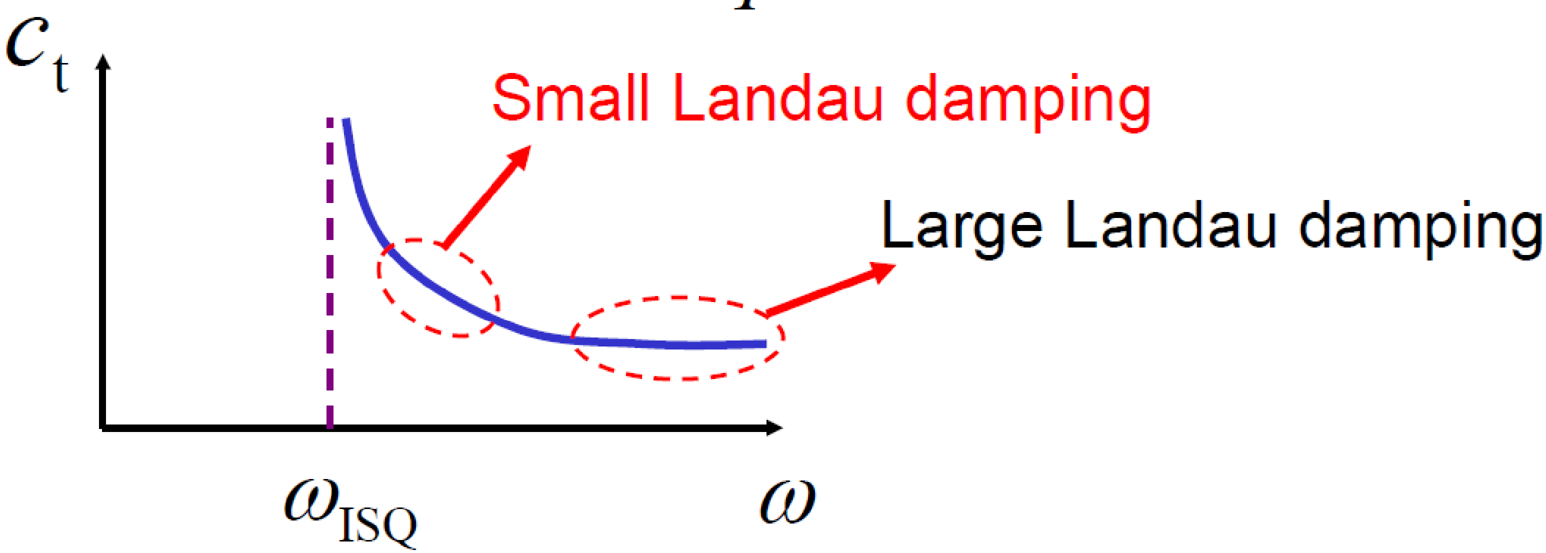
if $$F_1^s + \frac{3F_2^s}{1+F_2^s/5} > 6$$

In normal liquid ^{3}He, experimental results are still elusive due to large Landau damping; $c_t \approx v_F$

For clear observation, $c_t >> v_F$

Transverse zero sound in superfluid ^{3}He-B via off resonant coupling with imaginary squashing modes





Moores and Sauls, J. Low Temp. Phys. 1993

Dispersion Relation of transverse sound in ^{3}He-B

$$\left(\frac{\omega}{q v_{\mathrm{F}}}\right)^2 = \left(\frac{c_{\mathrm{t}}}{v_{\mathrm{F}}}\right)^2 \rho_n + \frac{2}{5}\left(\frac{c_{\mathrm{t}}}{v_{\mathrm{F}}}\right)^2 \rho_s \left\{\frac{\omega^2}{(\omega + i\Gamma)^2 - \omega_{\mathrm{ISQ}}^2}\right\}$$

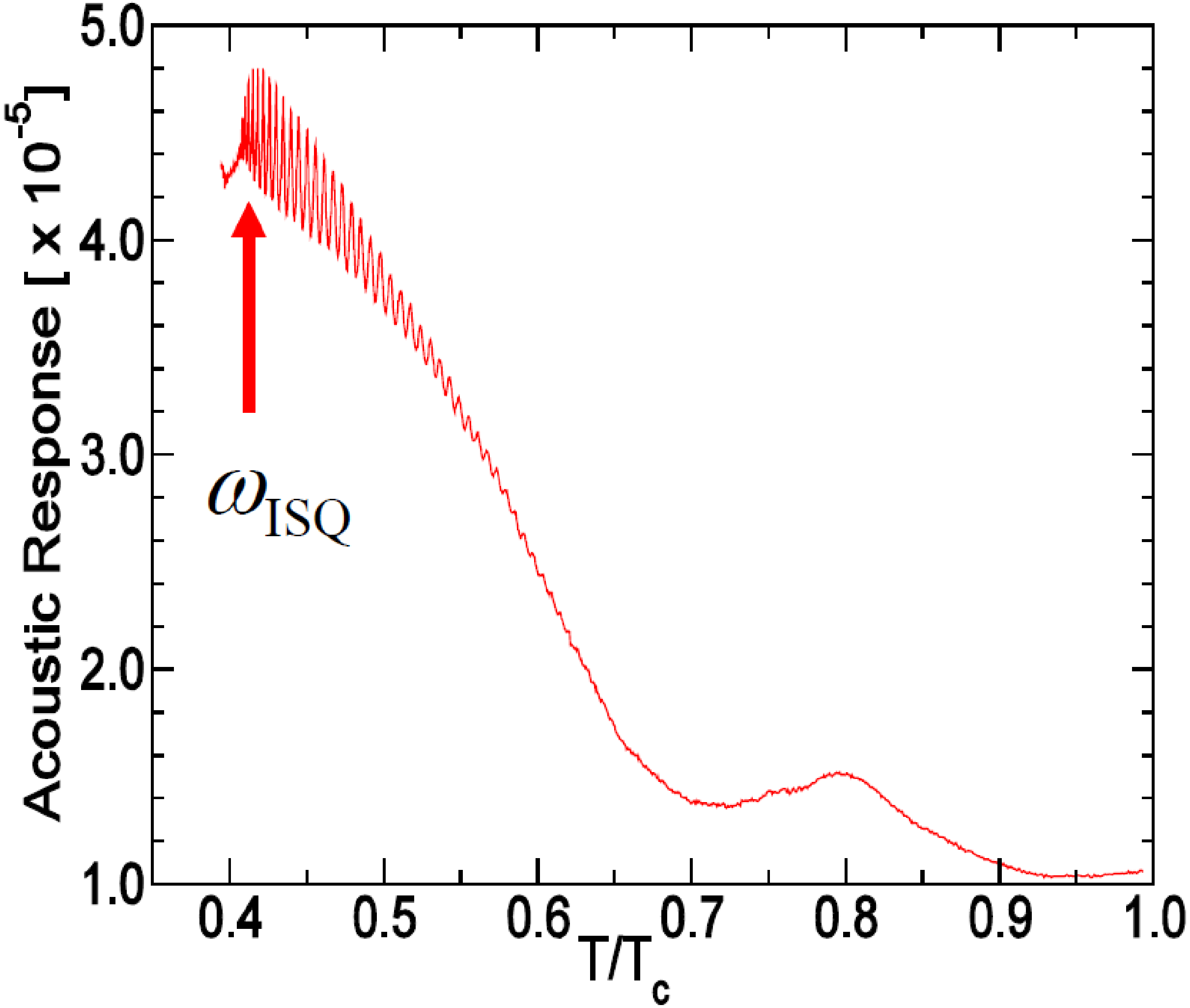
quasiparticle stiffness

condensate stiffness

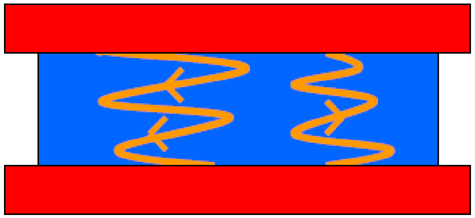
$$\omega \to \omega_{\mathrm{ISQ}} \qquad c_{\mathrm{t}} \to \infty$$

Moores and Sauls, J. Low Temp. Phys. 1993

Propagation of transverse zero sound in superfluid ^{3}He-B



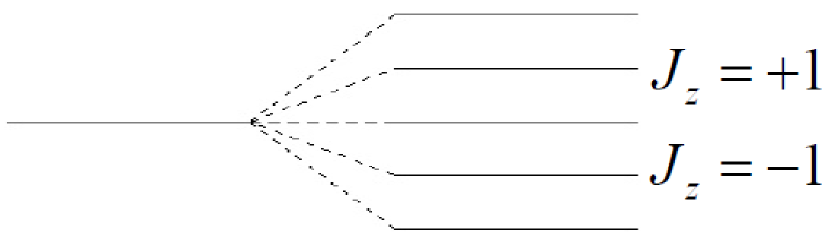
Acoustic cavity



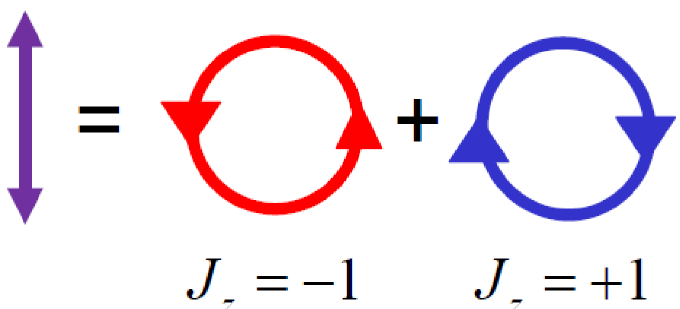
Lee, Haard, Sauls, Halperin, Nature **400**, 431 (1999)

Acoustic Faraday rotation of transverse sound

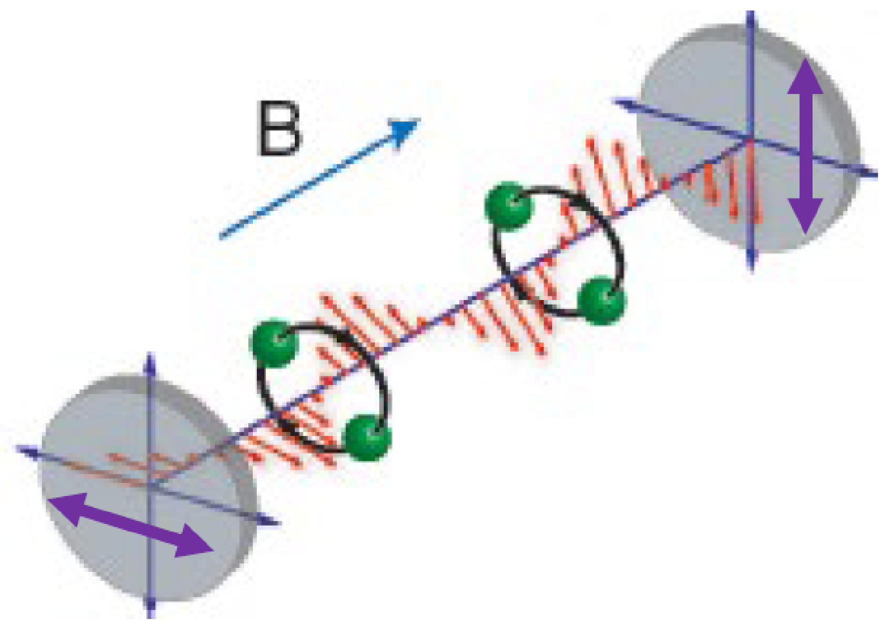
$$\left(\frac{\omega}{qv_{\mathrm{F}}}\right)^2 = \left(\frac{c_{\mathrm{t}}}{v_{\mathrm{F}}}\right)^2 \rho_n + \frac{2}{5}\left(\frac{c_{\mathrm{t}}}{v_{\mathrm{F}}}\right)^2 \rho_s \left\{\frac{\omega^2}{(\omega + i\Gamma)^2 - \omega_{\mathrm{ISQ}}^2}\right\}$$



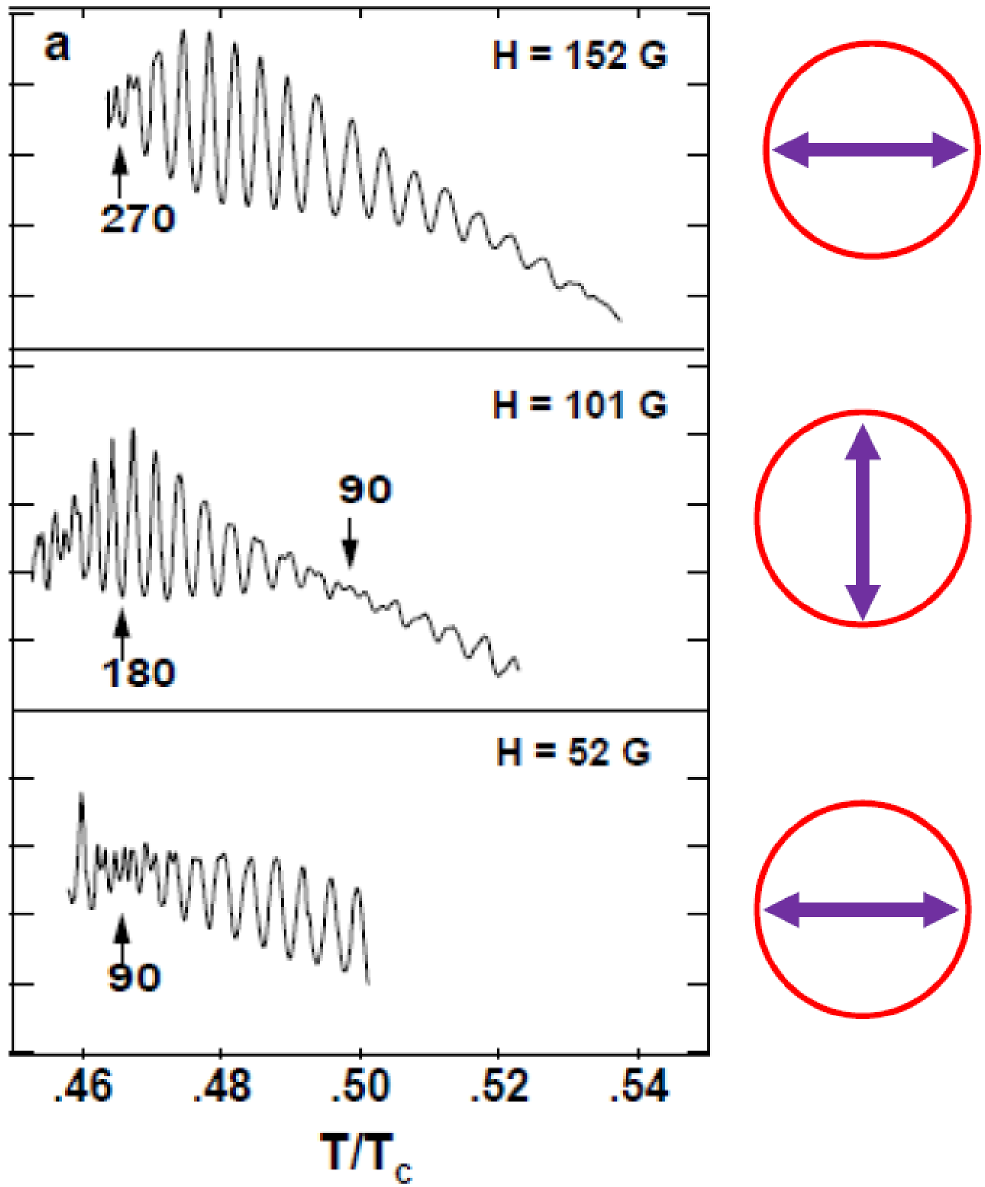
$$\omega_{\mathrm{ISQ}} = \sqrt{\frac{12}{5}\Delta + g\gamma J_z H}$$



$$c_+ > c_-$$



Faraday rotations in the transverse waves



Rotation period → g-factor → f-wave interaction

Y. Lee *et al.*, Nature **400**, 431 (1999)

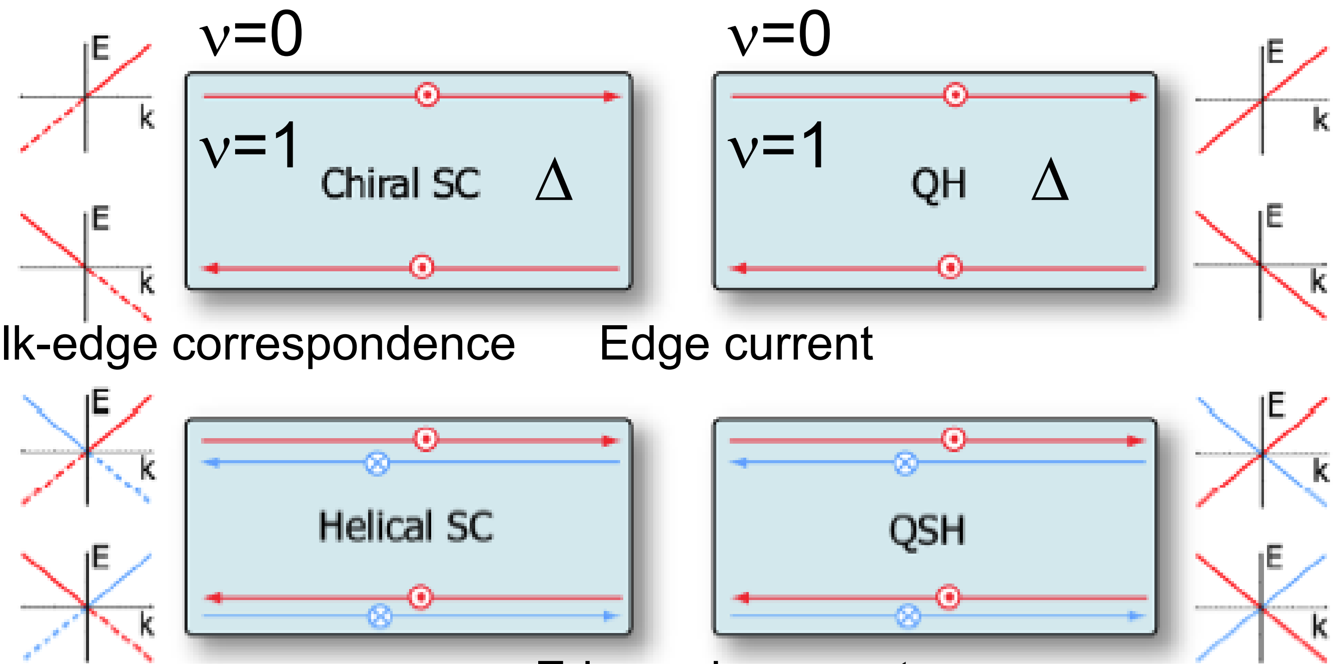
Outline

1. Introduction of superfluid ^{3}He
2. Higgs modes in superfluid ^{3}He
3. Transverse sounds in superfluid ^{3}He
4. Majorana modes in superfluid ^{3}He

Topological superfluids | Topological insulators

T-breaking

2d-A、Sr_2RuO_4



Quantum Hall

Bulk-edge correspondence

Edge current

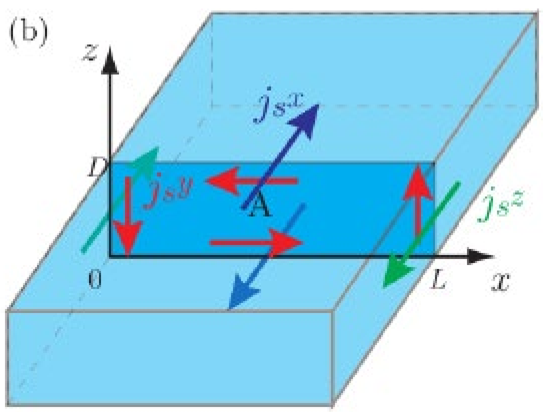
T-invariant

Quantum spin Hall

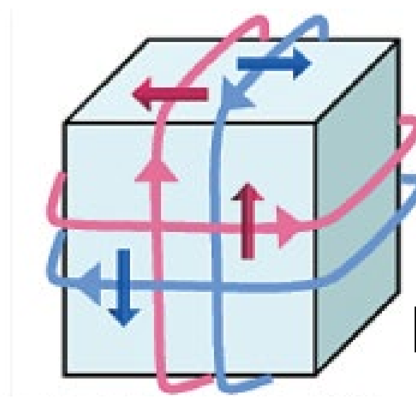
Edge spin current

3d

^{3}He-B



Majorana fermions



Dirac fermions

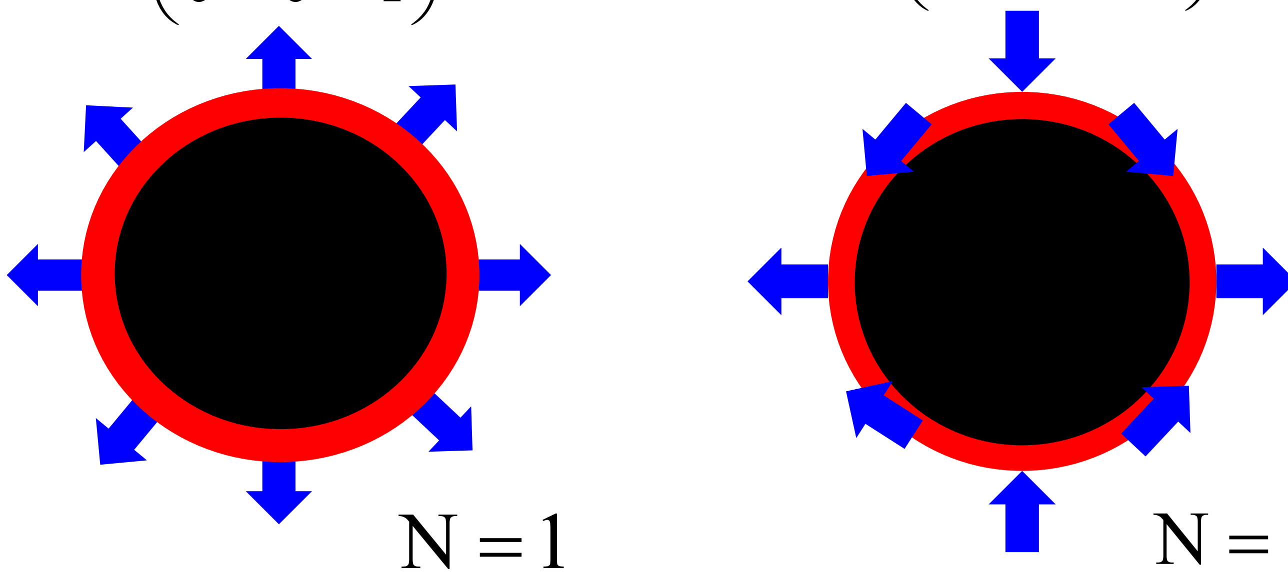
X. L. Qi, S.C. Zhang, PRL 2009

Tsutsumi, *et al.*, JPSJ, PRB 2010

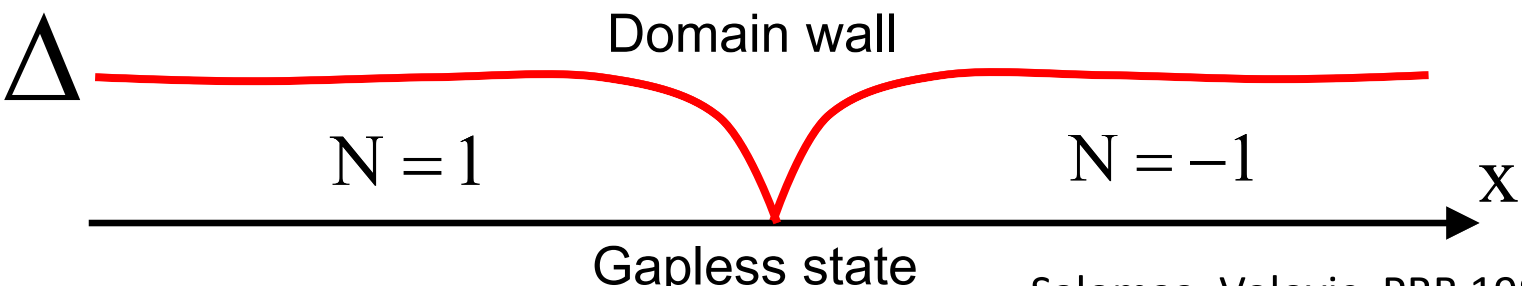
Topology of B phase

$$A_{\mu i} = \Delta_B \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \qquad A_{\mu i} = \Delta_B \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{pmatrix}$$

$$\hat{d}_\mu = A_{\mu i}\hat{p}_i$$



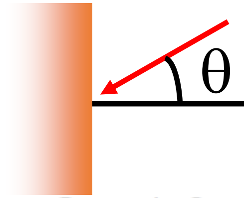
Bulk-edge correspondence

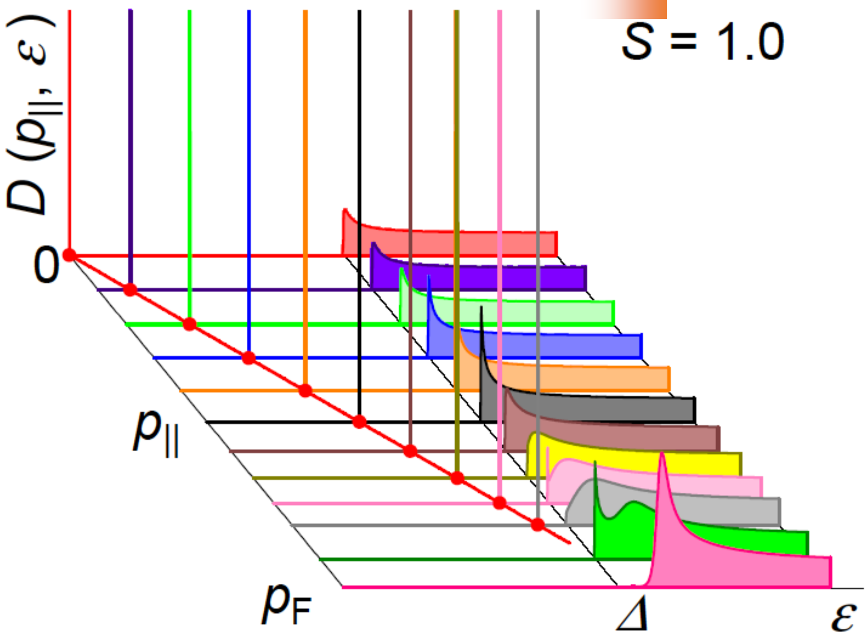


Salomaa, Volovic, PRB 1988
Mizushima, Butsurigakkaishi 2017

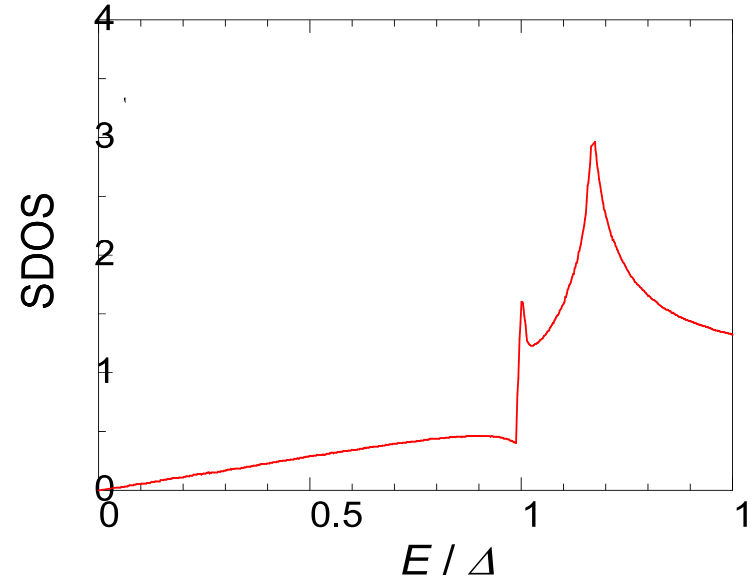
Theoretically calculated SDOS in BW state on specular surface

angle resolved





angle averaged (Natato 1998)



$$N(E) \propto E$$

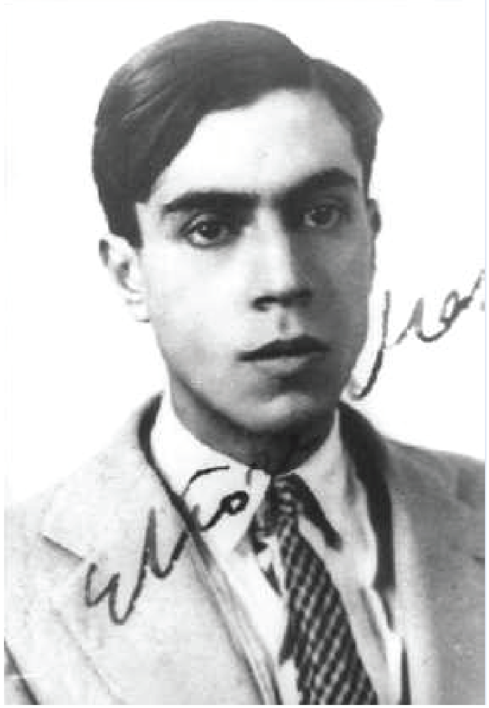
$$E = \Delta_{//} \sin\theta = c_{//} p_{//}$$

No sharp peak at zero energy but a broad SABS band appears within the bulk energy gap Δ.

“Dirac” cone on ^{3}He-B

Majorana fermions

originally predicted as a possible candidate of neutrinos. (1937)



Majorana fermions

have linear dispersion

$$E = cp$$

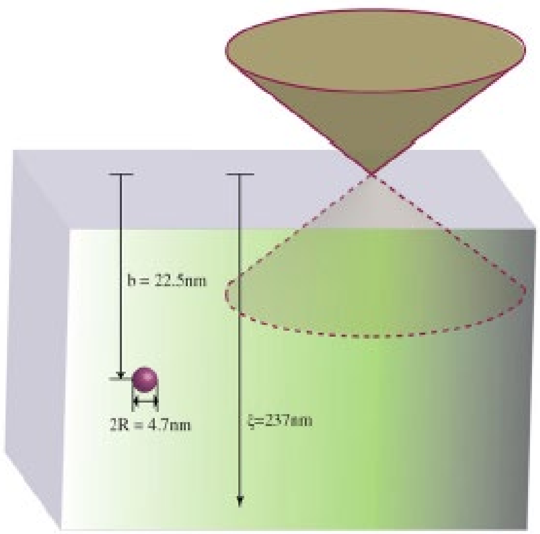
and

equivalence of particles and antiparticles

$$\Psi = \Psi^{+}$$

Majorana surface state in ^{3}He-B

"Majorana cone"

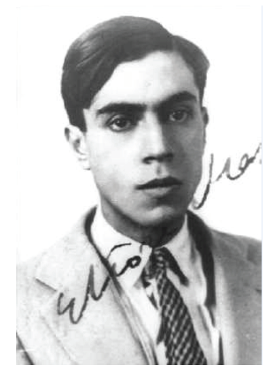


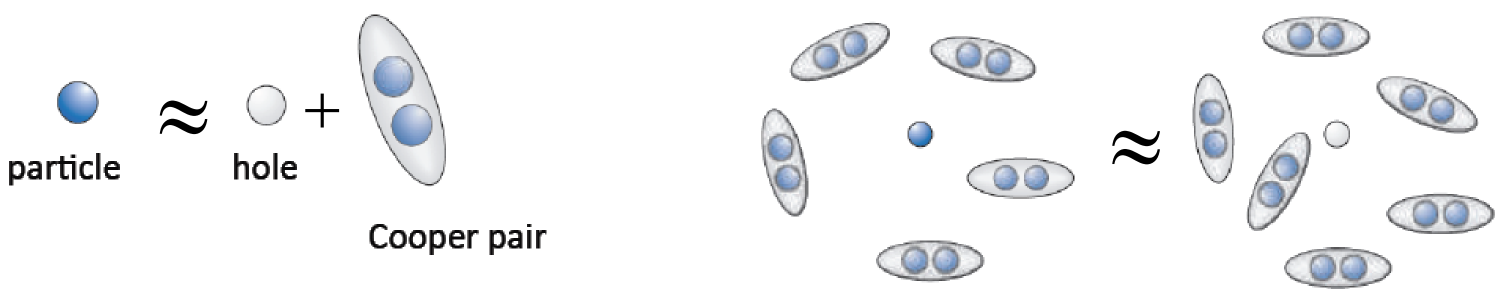
Chung and Zhan, PRL09

$$\psi = \psi^{+}$$

particel = anti-particel

SABS: Majorana Fermion



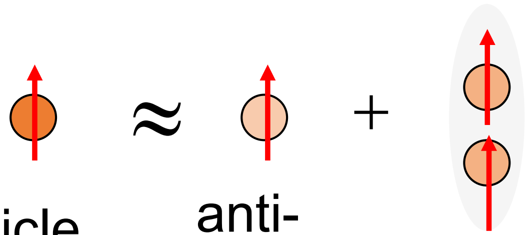


Wilczek, Nat. Phys. 5, 614 (2009)

A. P. Schnyder, S. Ryu, A. Furusaki, and A. W. W. Ludwig, Phys. Rev. B 78, 195125 2008

R. Roy, arXiv:0803.2868v1, 19 Mar 2008

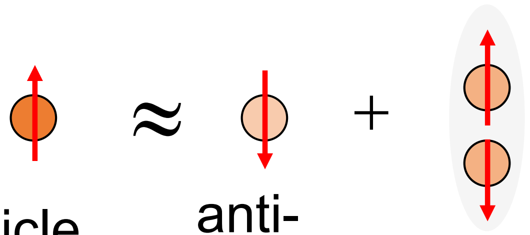
Spin triplet



particle ≈ anti-particle + Cooper pair

Majorana

Spin singlet



particle ≈ anti-particle + Cooper pair

$$\gamma^{+}_{\uparrow\, E>0} \neq \gamma_{\downarrow\, -E<0}$$

Not Majorana

	Bulk order parameters	Surface states
(Triplet) Superconductors	Difficult to determine (complicated by charge, multi-band, defects, impurities, magnetism, …)	Tunneling conductance
Spin-triplet p-wave Superfluid ^3He	Well established (simple, clean,…)	No good surface probe (until recently)

^3He: Good testing ground for topological quantum physics!!

Acoustic impedance measurements

Transverse acoustic impedance of AC-cut quartz in liquid ^{3}He

$$Z = \frac{\Pi_{xz}}{u_x} = Z' + iZ''$$

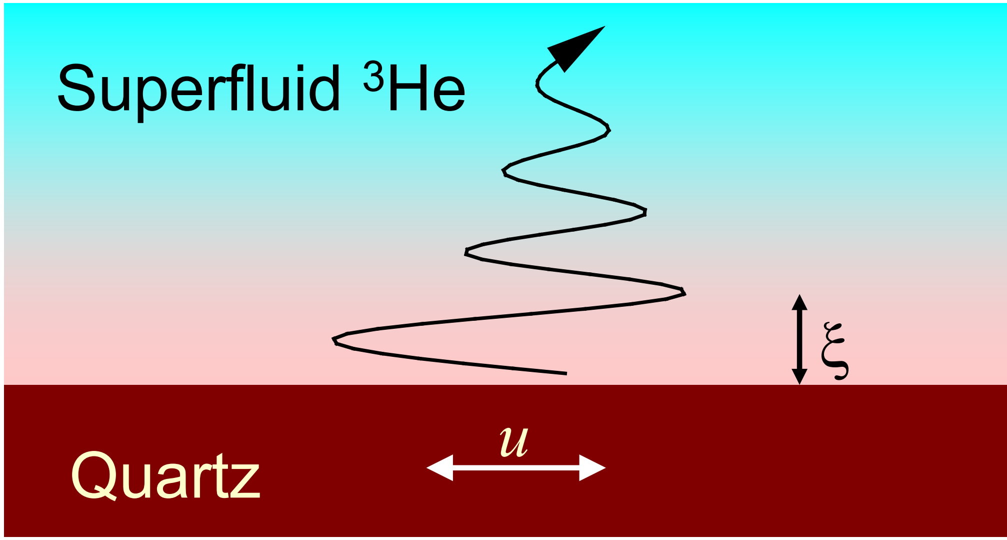
Π_{xz} Stress tensor of liquid on quartz

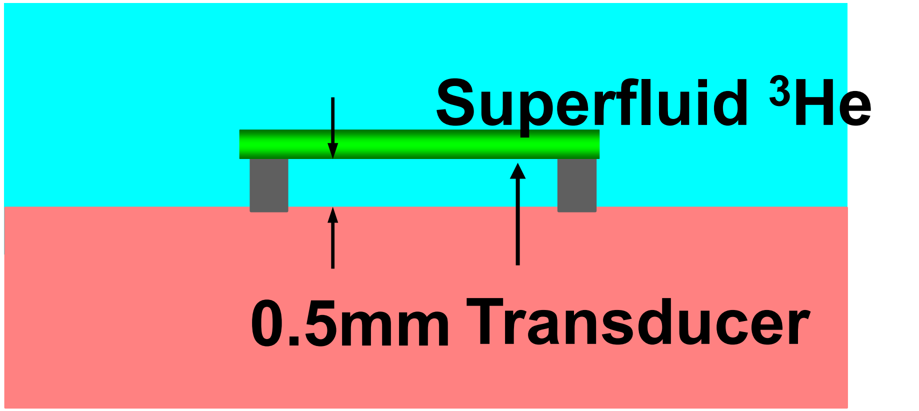
u_x Oscillation velocity

$$Z' - Z'_0 = \frac{1}{4} n\pi Z_q \left(\frac{1}{Q} - \frac{1}{Q_0} \right)$$

$$Z'' - Z''_0 = \frac{1}{2} n\pi Z_q \frac{f - f_0}{f_0}$$

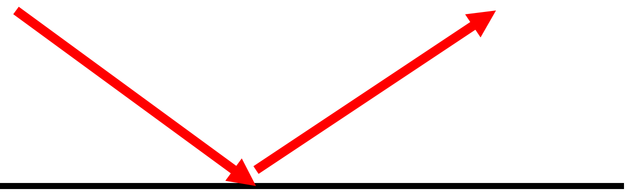
$$Z_q = \rho_q c_q$$

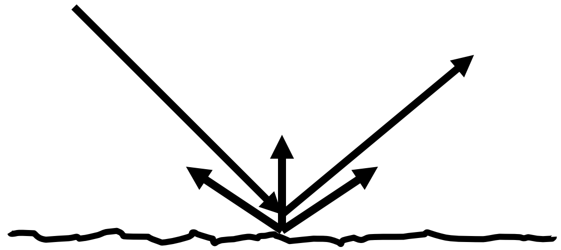


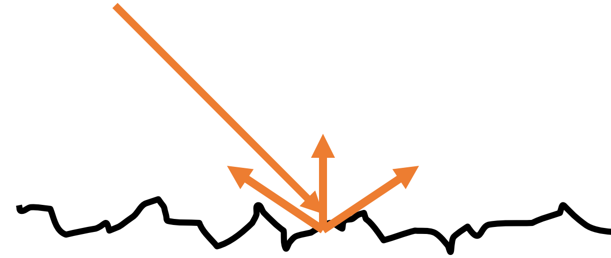


Quasiparticles scattering off a wall

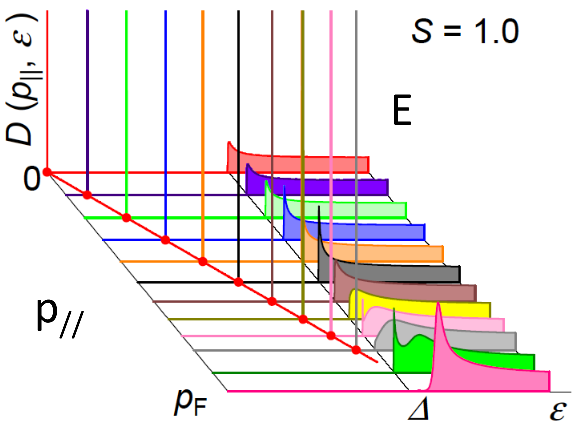
Specular limit | Partially specular | Diffusive limit



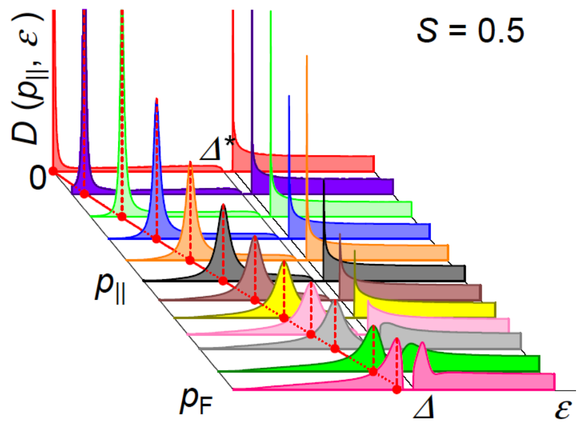




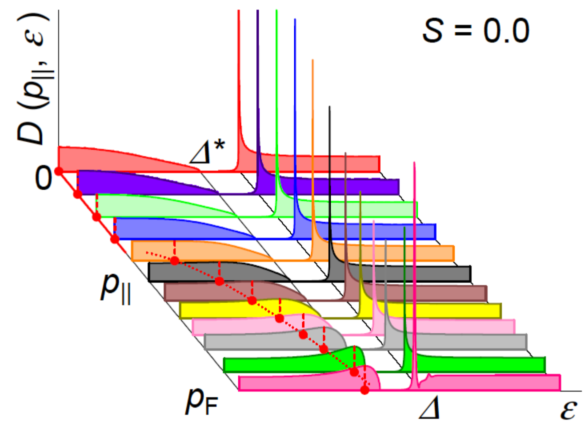
S = 1 | 1 > S > 0 | S = 0



theory



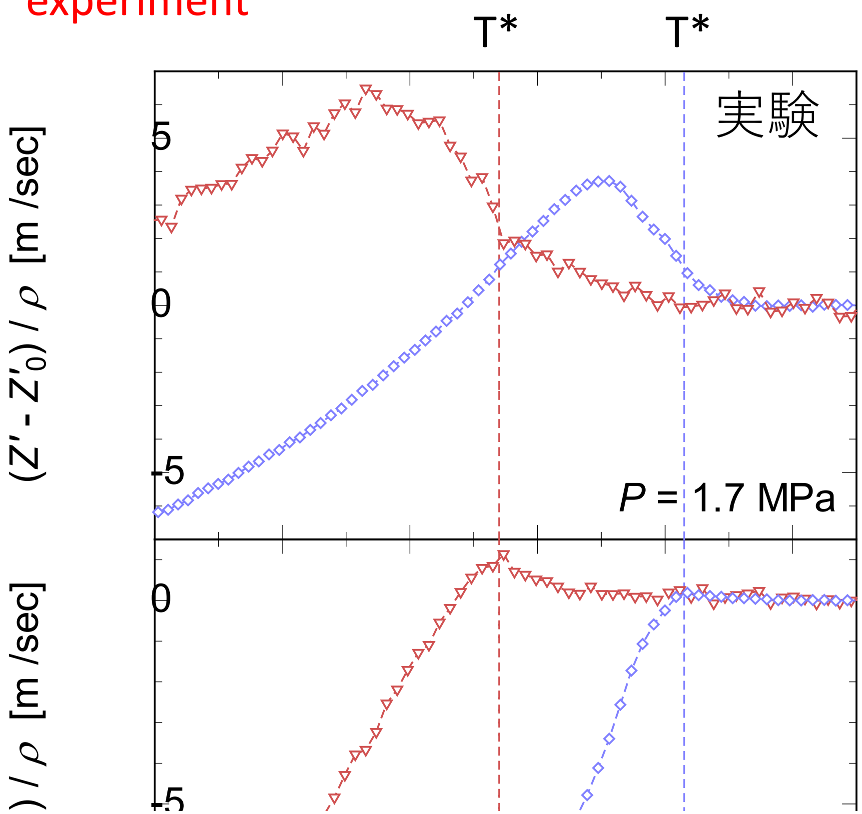
S = 0.5



S can be controlled continuously by thin ^4He layers on a wall.

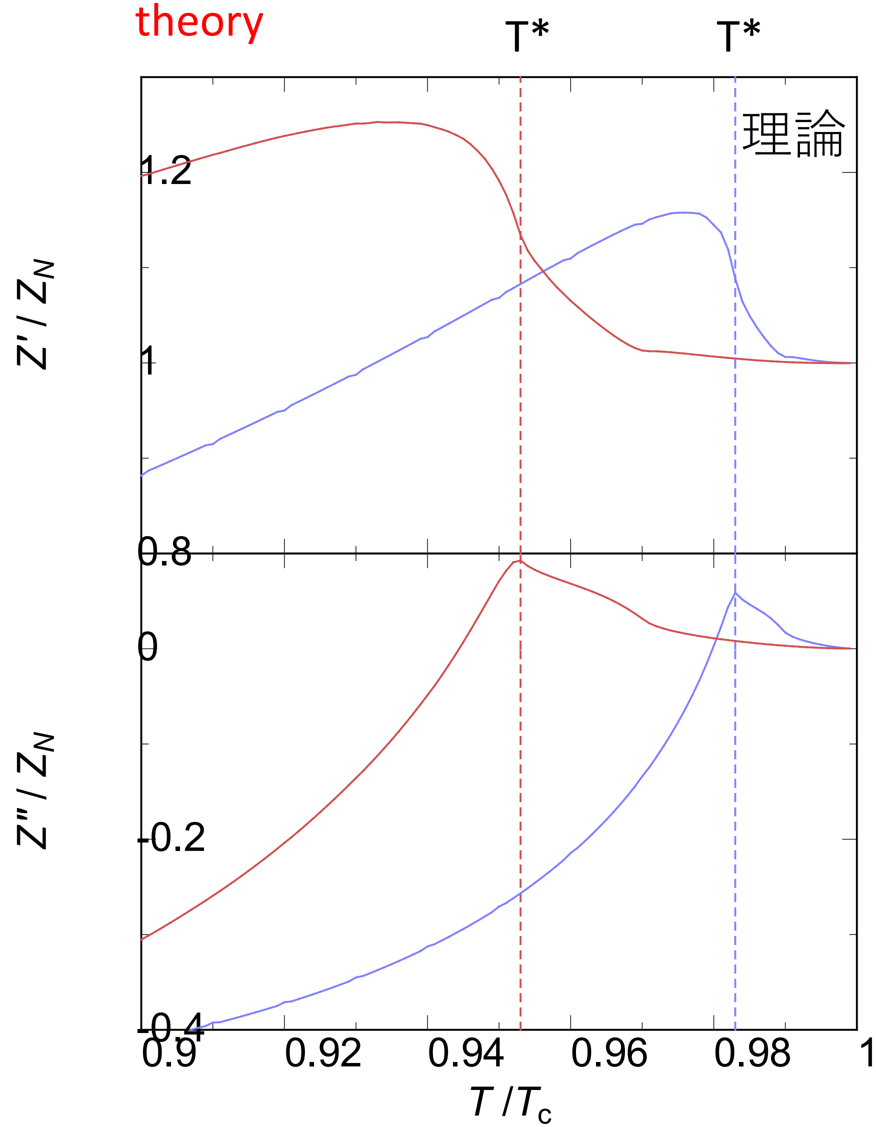
Z(T) at S = 0

experiment



28 MHz
46 MHz

theory

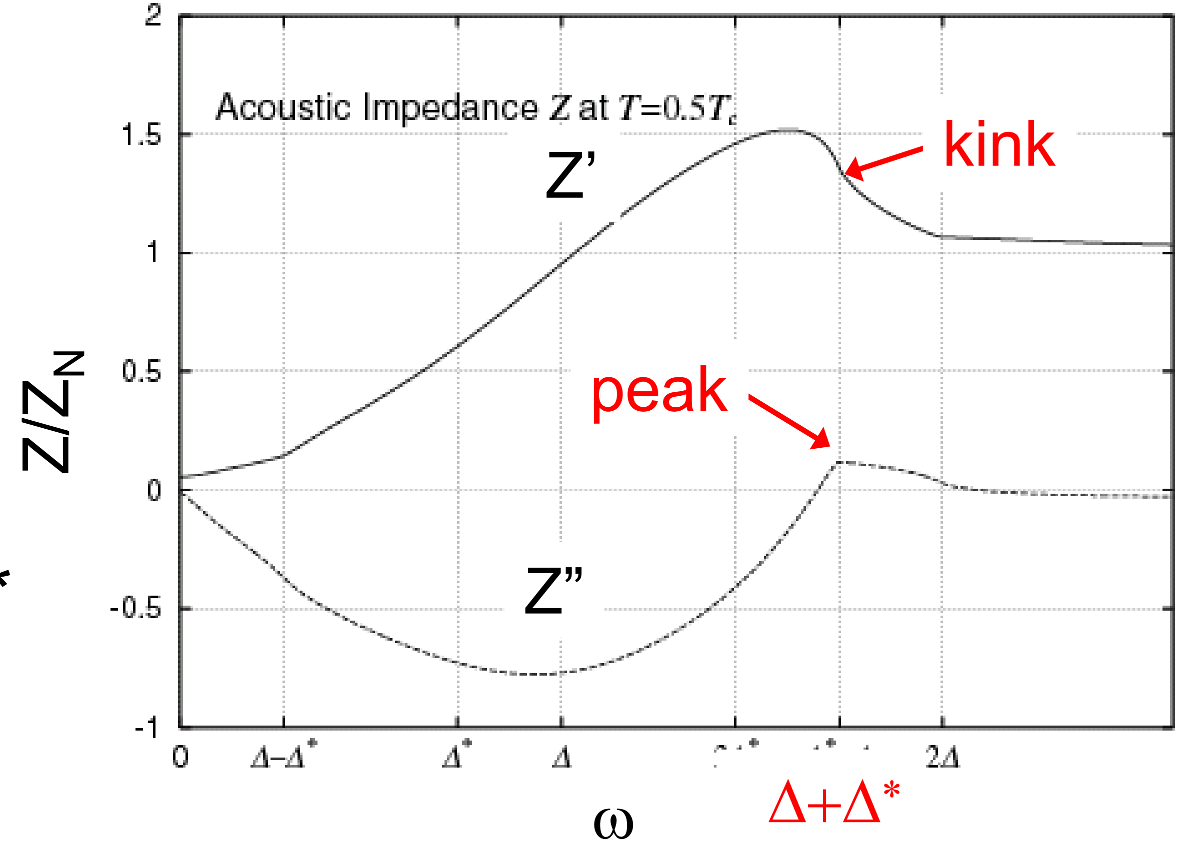


Aoki *et al.*, Phys. Rev. Lett. **95**, 075301 (2005)

Z(ω) theory with SABS at S = 0

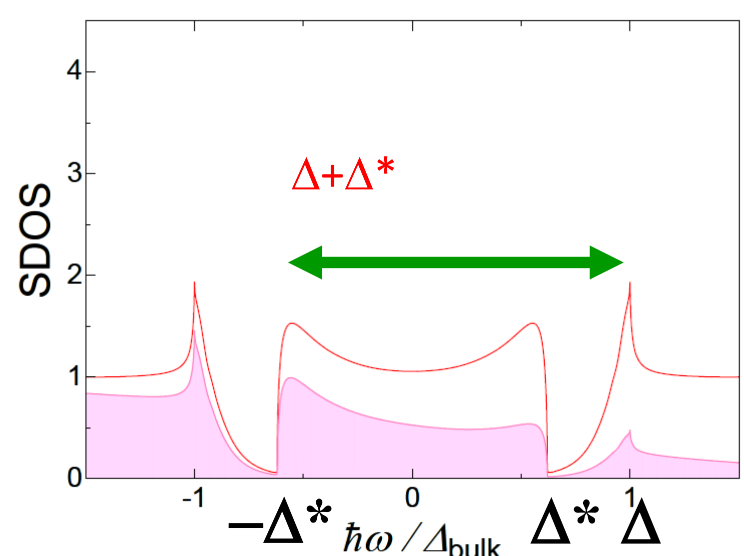
Kink and peak are singularities when $\hbar\omega = \Delta + \Delta^*$

$$\int_0^{\hbar\omega} d\varepsilon N(\varepsilon) N(\hbar\omega - \varepsilon)$$



First experimental confirmation of the sub-gap structure.

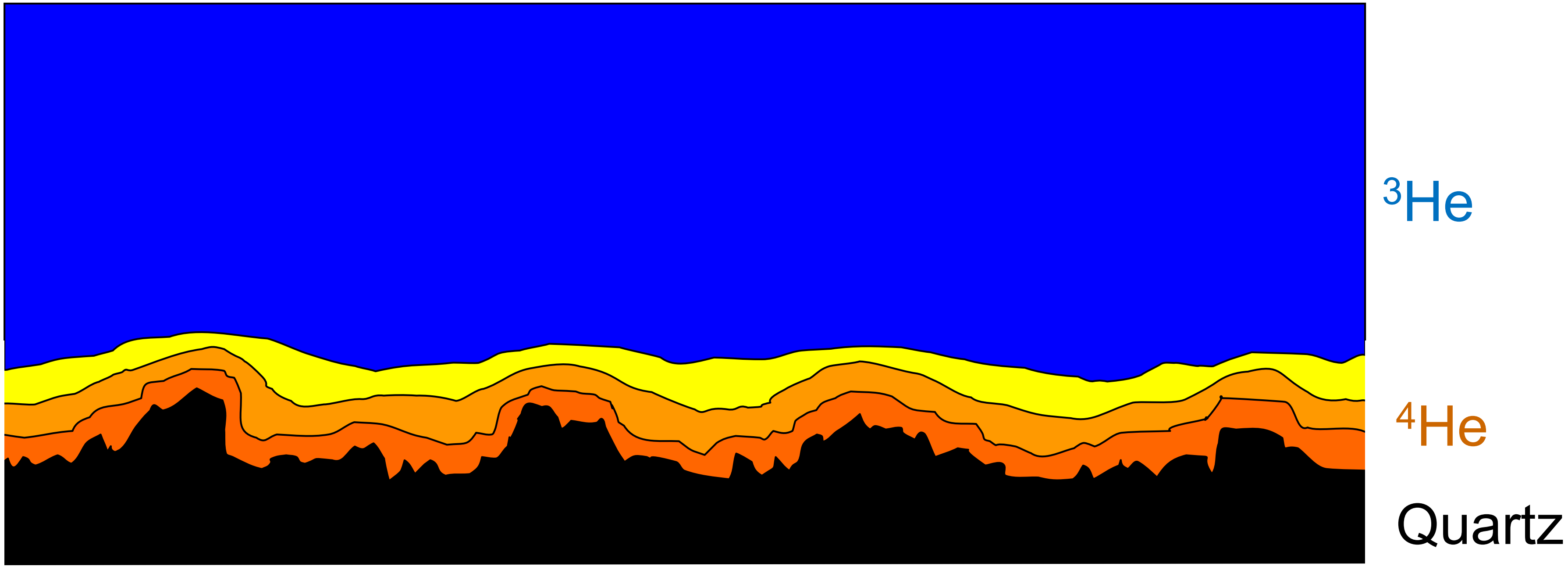
Aoki *et al*., Phys. Rev. Lett. **95**, 075301 (2005)



S = 0

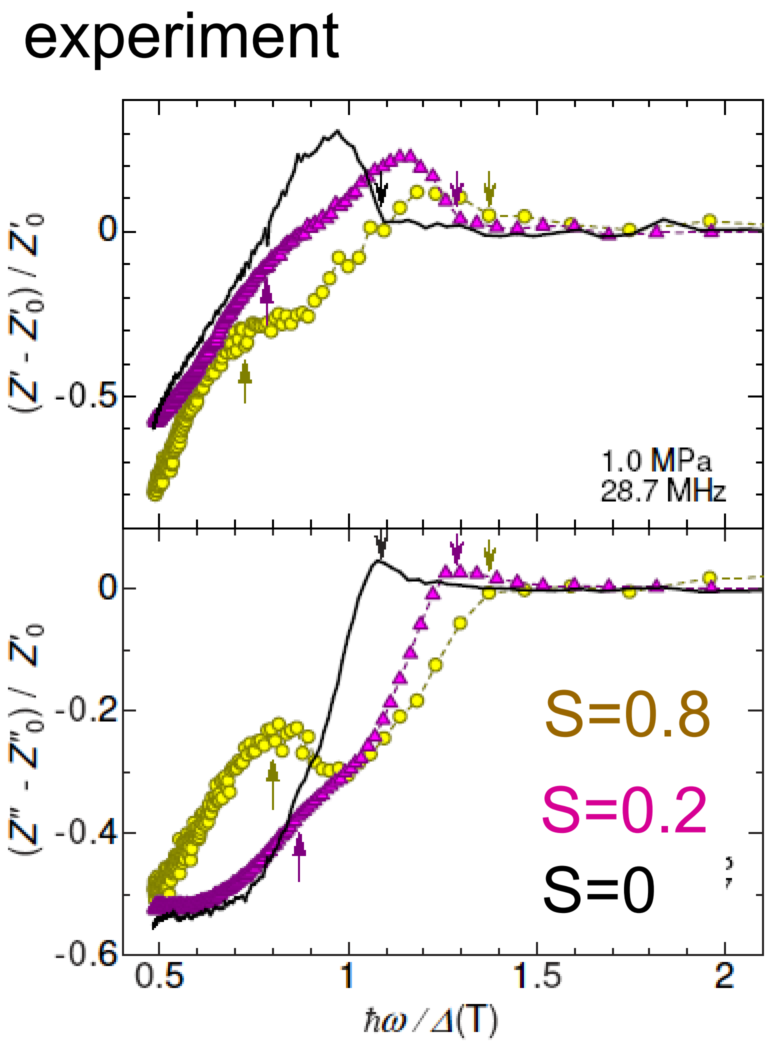
Higher specularity wall; $0 < S < 1$

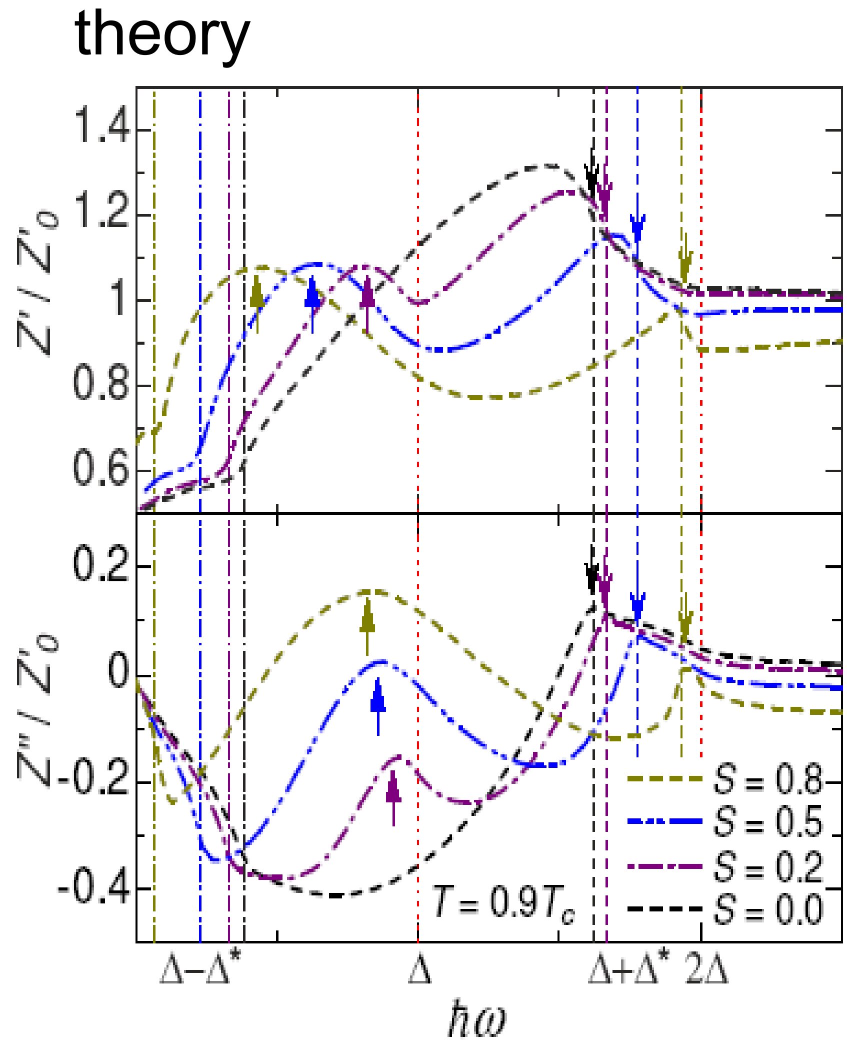
Coat a wall with ^4He layers



schematics

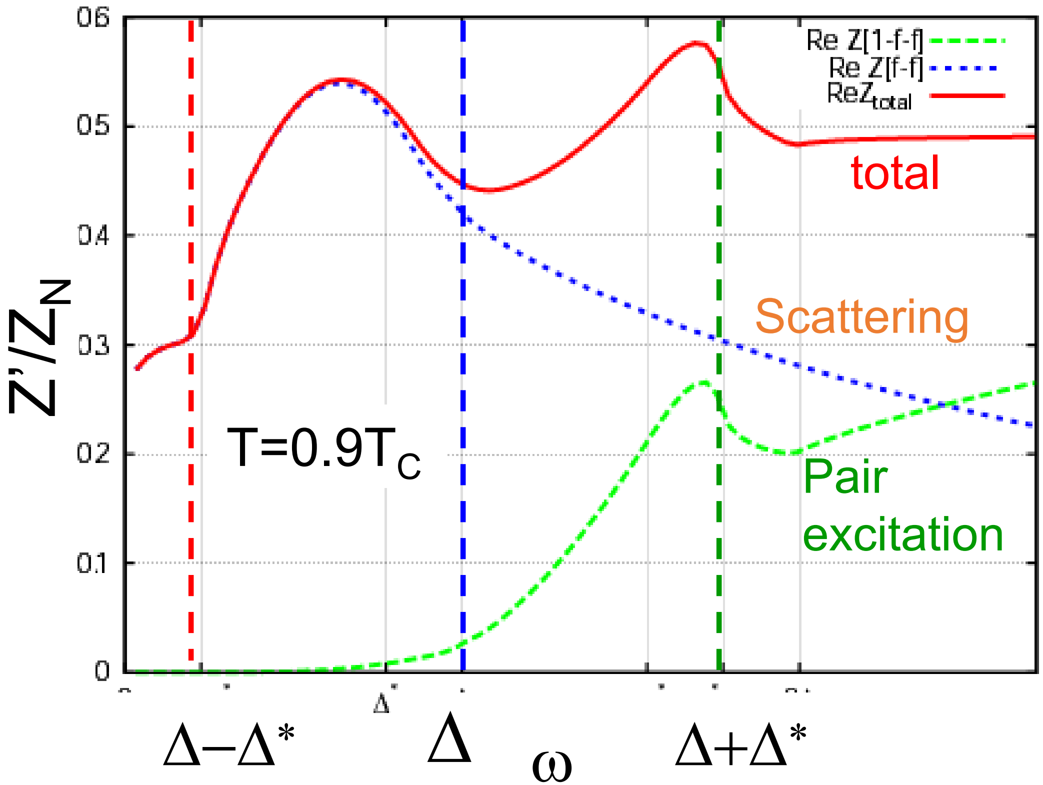
Z(ω/Δ) at S > 0

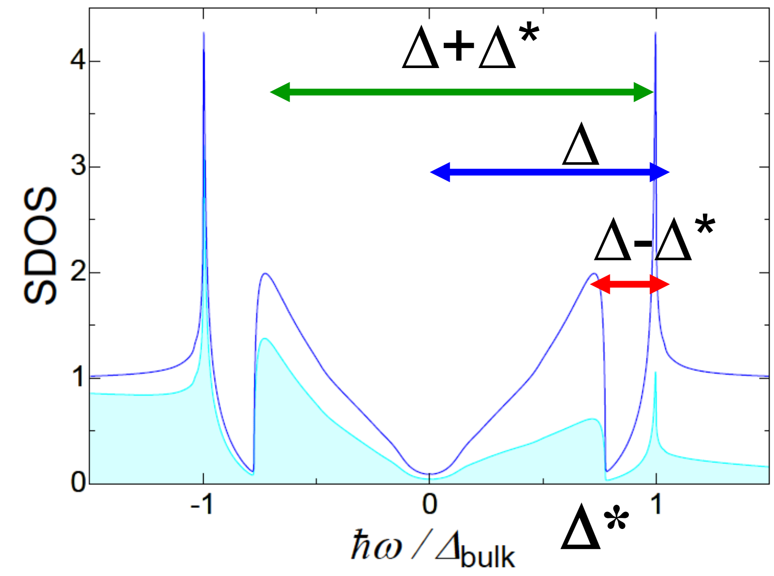


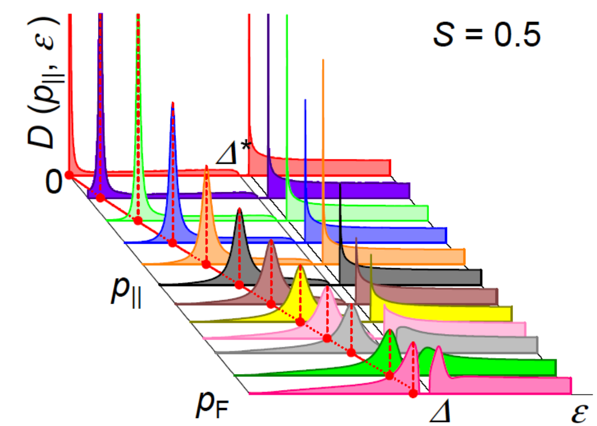


Murakawa *et al*., Phys. Rev. Lett. **103**, 155301 (2009)
Murakawa *et al*., J. Phys. Soc. Jpn. **80**, 013602 (2011), Paper award

Z(ω) theory by Nagato *et al.* for S = 0.5





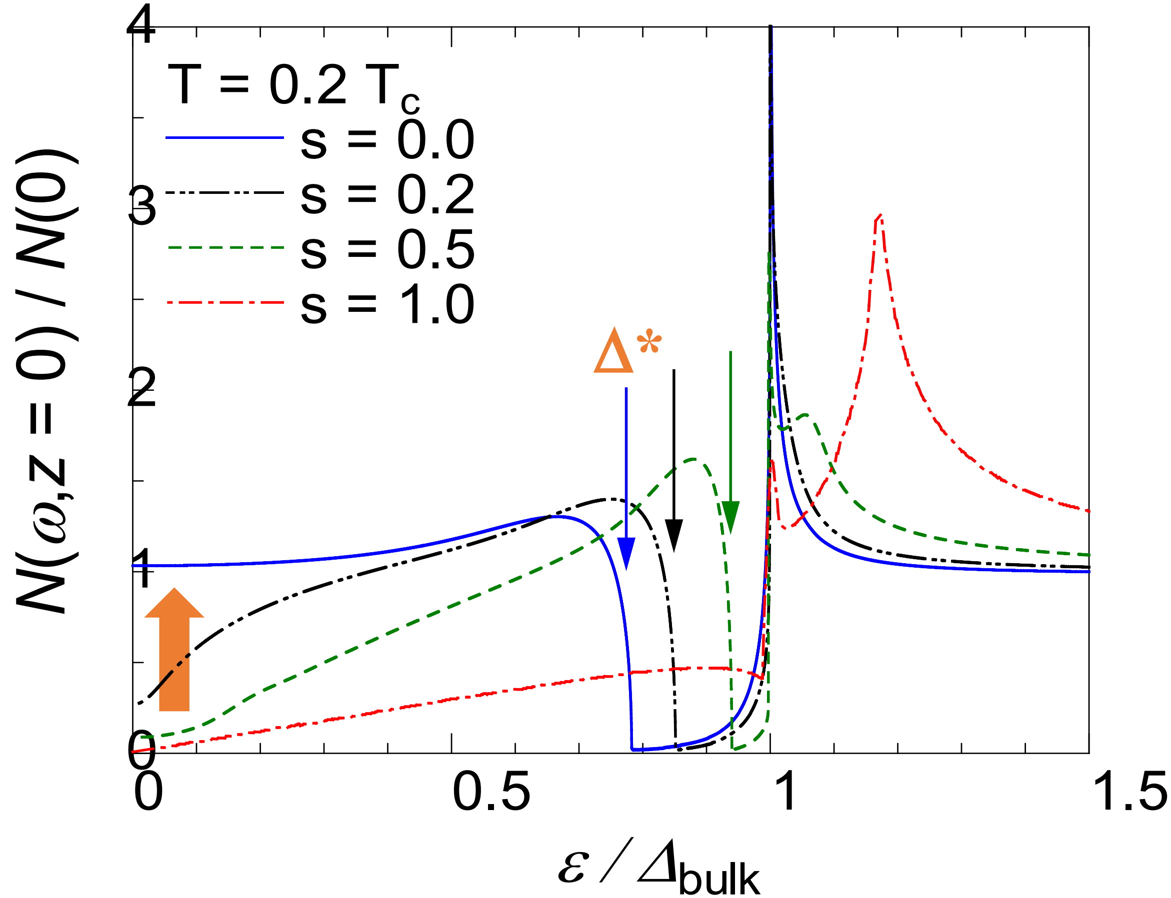


^{3}He-B is truly a topological superfluid showing the bulk-edge correspondence at S >> 0.

Two peaks in Z(ω) due to the formation of Majorana cone.

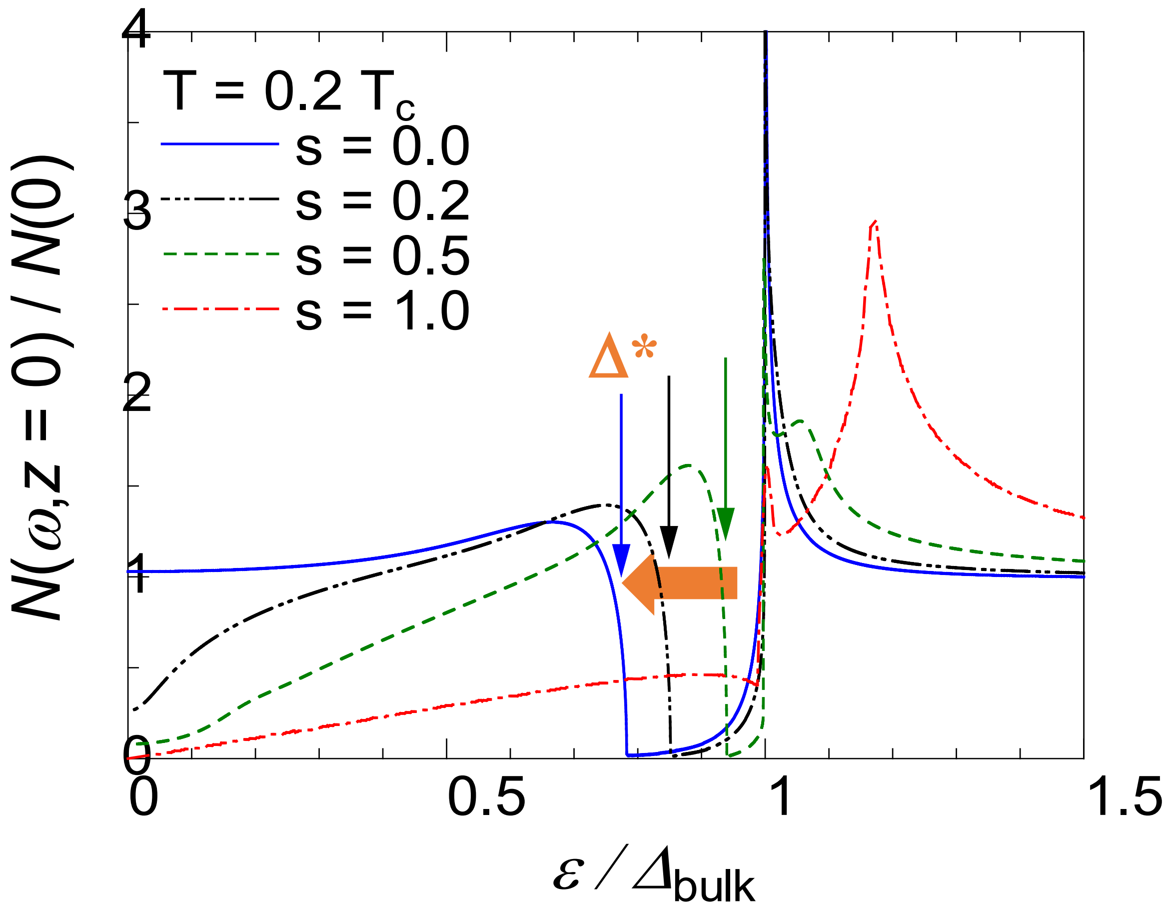
What is a topological nature of the surface states in our measurements?

Usually, particles are localized by the disorder and gap appears at zero energy.
However, topological surface states are not gappable in the presence of disorder.



What is a Majorana nature of the surface states in our measurements?

Anomalous scattering of the Majorana fermions makes in Δ^* in the presence of disorder.



Δ^* is formed due to $\Psi=\Psi^\dagger$. Nagato *et al.*

In the presence of the roughness, $p_{//}$ is no longer an eigenstate.

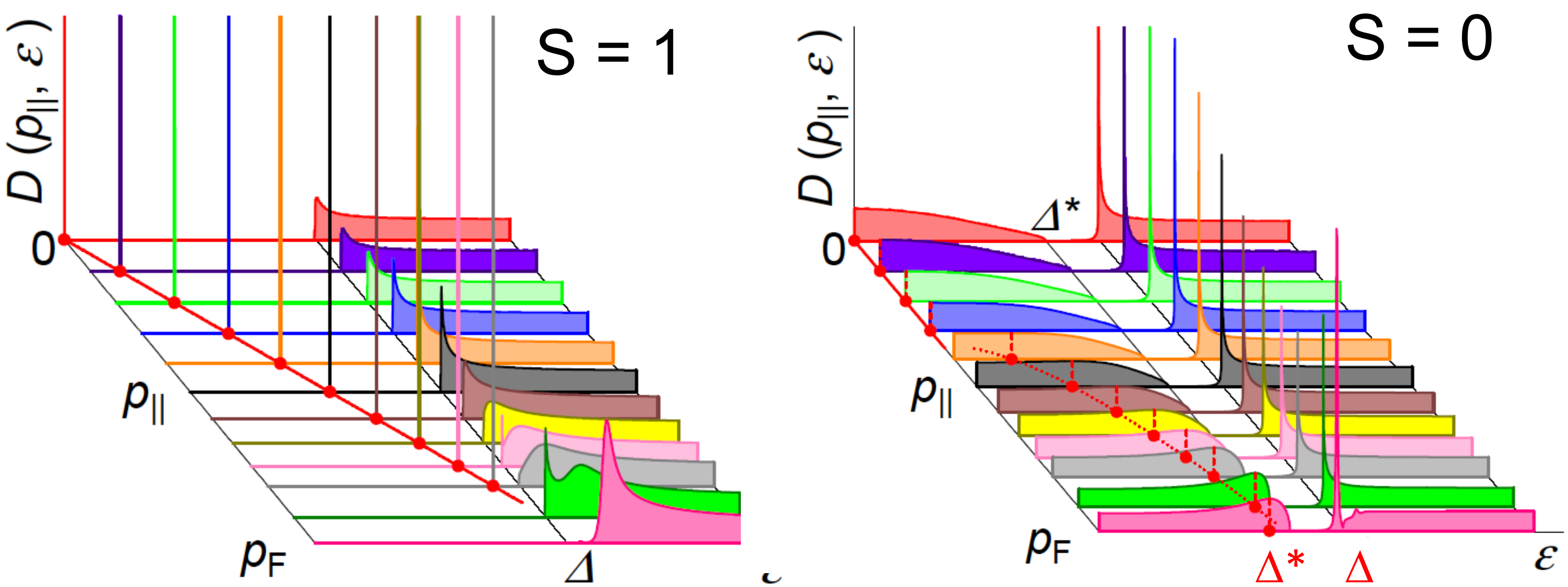
Scattering results in the broadening and energy shift.

Scattering between continuum above Δ and SABS results in strong label repulsion due to $\Psi=\Psi^\dagger$.

No states between Δ and Δ^*.

Majorana nature.

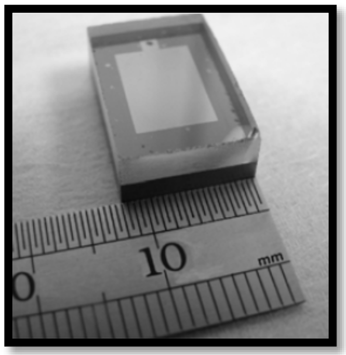
$$\epsilon - \epsilon_n^{(0)} = \sum_m \frac{|V_{nm}|^2}{\epsilon - \epsilon_m^{(0)}}$$



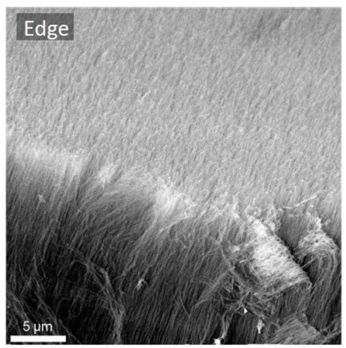
Theorists' dreams are in reality due to experimental developments.

New states; stripe, distorted, glass, polar, p+f state, 2D topological phases, …
New excitations; Higgs, Majoranas, HQV, magnon BEC, defects, orbital wave, …

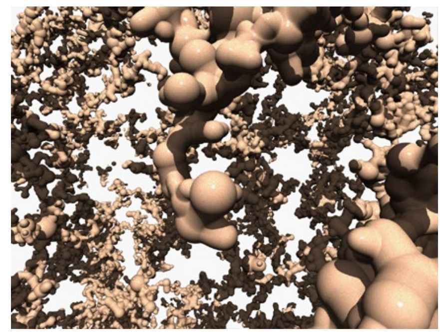
New geometries; slabs, tubes, disorder, anisotropy, periodicity, specularity, …

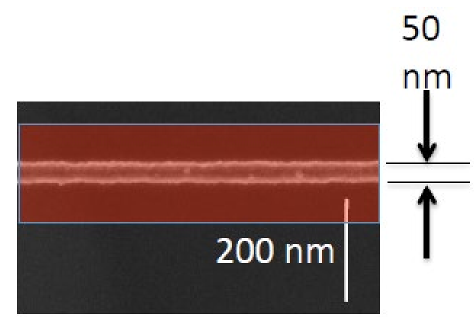


Slab, RHUL



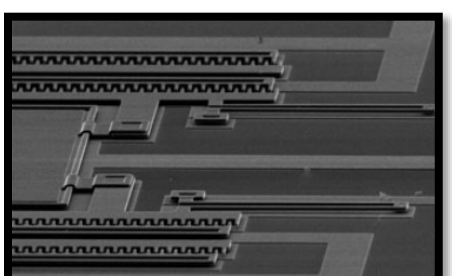
Nematic aerogel, Kapitza

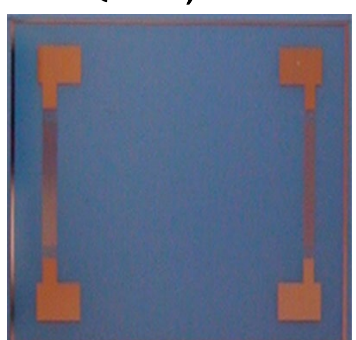

Aerogel, Northwestern

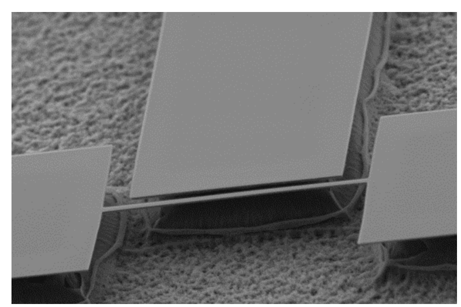


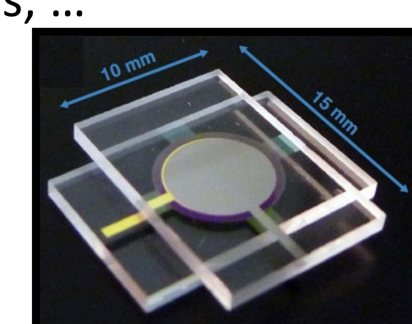
Beam, Cornell

New detections; sensitive and local NMR, NEMS, SAW, superfluid resonator, gyroscope, Josephson, SHeQUID, ion mobility, Andreev reflections, …


MEMS, Florida


SAW, Tokyo Tech.


NEMS, Grenoble



Resonator, Alberta

Superfluid transition of ^{3}He by irradiation of neutrons or gamma-ray

Baked Alaska model; ballistic expansion of quasiparticles for A-B transition
(Leggett, Osheroff, Fukuyama)

Aurora de Venice model; diffusive expansion of quasiparticles for A-B transition
(Bunkov)

Kibble-Zurek model; diffusive expansion of quasiparticles for vortex formation
(Helsinki, Lancaster, Grenoble)

Summary

We have investigated surface Majorana states of ^{3}He-B by acoustic impedance measurements.
Boundary condition dependence of SDOS revealed topological stability of the surface zero modes
and
anomalous scattering of Majorana fermions in the presence of disorder.

Review; Y. Okuda and R. Nomura, J. Phys.: Cond. Matt. **24**, 343201 (2012)

Future plans;
topological phase transitions in magnetic field,
Majorana Ising spin,
Nano topological superfluids…