LONGITUDINAL BEAM DYNAMICS - TUTORIAL
CAS 2011 - Chios

For the Large Hadron Collider (LHC), the beam gets accelerated in the PS Booster (PSB) and transferred to the Proton Synchrotron (PS). The PS receives proton bunches ($\Delta t_b \approx 150$ ns) during a first magnetic flat-top (i.e. no acceleration) with a kinetic energy of 1.4 GeV. When the PS machine is filled, the proton beam is accelerated up to a final momentum of 26 GeV/c and extracted to the Super Proton Synchrotron (SPS).

1) Calculate the magnetic field in the PS at injection and ejection.
2) At injection, calculate and compare the RF frequency in the PS and in the PSB.
3) What is the synchronous phase in the PS at injection?
4) Assuming that the orbit remains the same during the acceleration, how does the RF frequency change between injection and ejection in the PS?
5) Should a phase jump system be implemented in the PS?
6) At PS injection, the maximum relative momentum spread and the longitudinal emittance are $\Delta p/p = \pm 2.10^{-3}$ and $\varepsilon = \pi \Delta E \Delta t = 0.9$ eV.s. Assuming a perfect elliptic area, calculate the value of the bunch length Δt_b. Compare your result to the value given in the introduction.
7) What is the value of V_{RF} at injection in the PS for a perfect longitudinal matching?
8) From the peak RF voltage found in question 7, calculate the PS synchrotron frequency f_s and the synchrotron tune Q_s at injection. Does it verify $Q_s << 1$?

Numerical values:

$R_{PS} = 100\,\text{m}$	PS radius
$R_{PSB} = R_{PS}/4 = 25$ m	PSB radius
$\hat{V}_{RF}^{PSB} = 8$ kV	Peak RF voltage at the ejection of the PSB
$\alpha_p^{PS} = \gamma_{tr,PS}^{-2} = 0.027$	PS momentum compaction factor
$\alpha_p^{PSB} = \gamma_{tr,PSB}^{-2} = 0.0617$	PSB momentum compaction factor
$\rho_{PS} = 70\,\text{m}$	PS curvature radius