Beam Cooling

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Beam Cooling

Introduction

1. **Electron Cooling**
2. **Ionization Cooling**
3. **Laser Cooling**
4. **Stochastic Cooling**

Beam Cooling

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Beam cooling is synonymous for a reduction of beam temperature

Temperature is equivalent to terms as

phase space volume, emittance and momentum spread

Beam Cooling processes are not following Liouville's Theorem:

`in a system where the particle motion is controlled by external conservative forces the phase space density is conserved´

(This neglect interactions between beam particles.)

Beam cooling techniques are non-Liouvillean processes which violate the assumption of a conservative force.

e.g. interaction of the beam particles with other particles (electrons, photons)

Cooling Force

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Generic (simplest case of a) Cooling Force:

$$F_{x,y,s} = -\alpha_{x,y,s} v_{x,y,s}$$

non conservative, cannot be described by a Hamiltonian

For a 2D subspace distribution function $f(z, z', t)$

$$F_z = -\alpha_z v_z \quad z = x, y, s$$

$$\frac{df(z, z', t)}{dt} = -\lambda_z f(z, z', t)$$ λ_z cooling (damping) rate

in a circular accelerator:

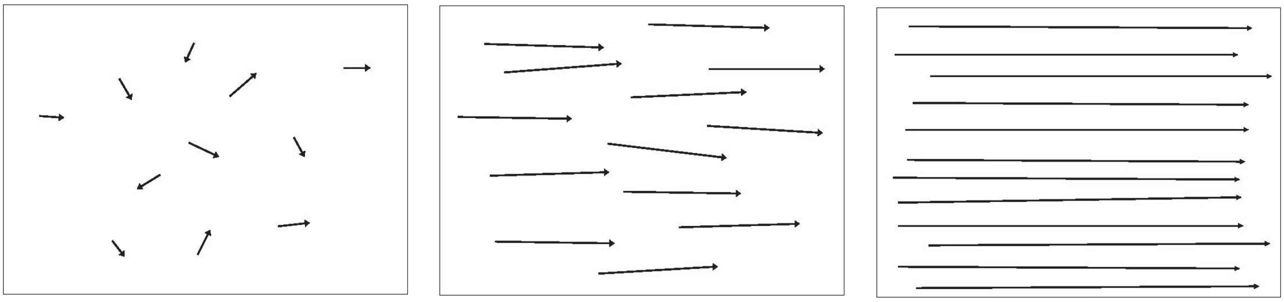
Transverse (emittance) cooling rate $\epsilon_{x,y}(t_0 + t) = \epsilon_{x,y}(t_0)\ e^{-\lambda_{x,y} t}$

Longitudinal (momentum spread) cooling rate $\frac{\delta p_\parallel}{p_0}(t_0 + t) = \frac{\delta p_\parallel}{p_0}(t_0)\ e^{-\lambda_\parallel t}$

Beam Temperature

Where does the beam temperature originate from?

The beam particles are generated in a "hot" source



at rest (source) **low energy** **high energy**

In a standard accelerator the beam temperature is not reduced
(thermal motion is superimposed the average motion after acceleration)

but: many processes can heat up the beam

e.g. heating by mismatch, space charge, intrabeam scattering, internal targets, residual gas

Beam Temperature Definition

Longitudinal beam temperature

$$\frac{1}{2}k_B T_\parallel = \frac{1}{2}mv_\parallel^2 = \frac{1}{2}mc^2\beta^2(\frac{\delta p_\parallel}{p})^2$$

Transverse beam temperature

$$\frac{1}{2}k_B T_\perp = \frac{1}{2}mv_\perp^2 = \frac{1}{2}mc^2\beta^2\gamma^2\theta_\perp^2 \qquad \theta_\perp = \frac{v_\perp}{\beta c}, \quad \theta_\perp(s) = \sqrt{\frac{\epsilon}{\beta_\perp(s)}}$$

Distribution function

$$f(v_\perp, v_\parallel) \propto \exp(-\frac{mv_\perp^2}{2k_B T_\perp} - \frac{mv_\parallel^2}{2k_B T_\parallel})$$

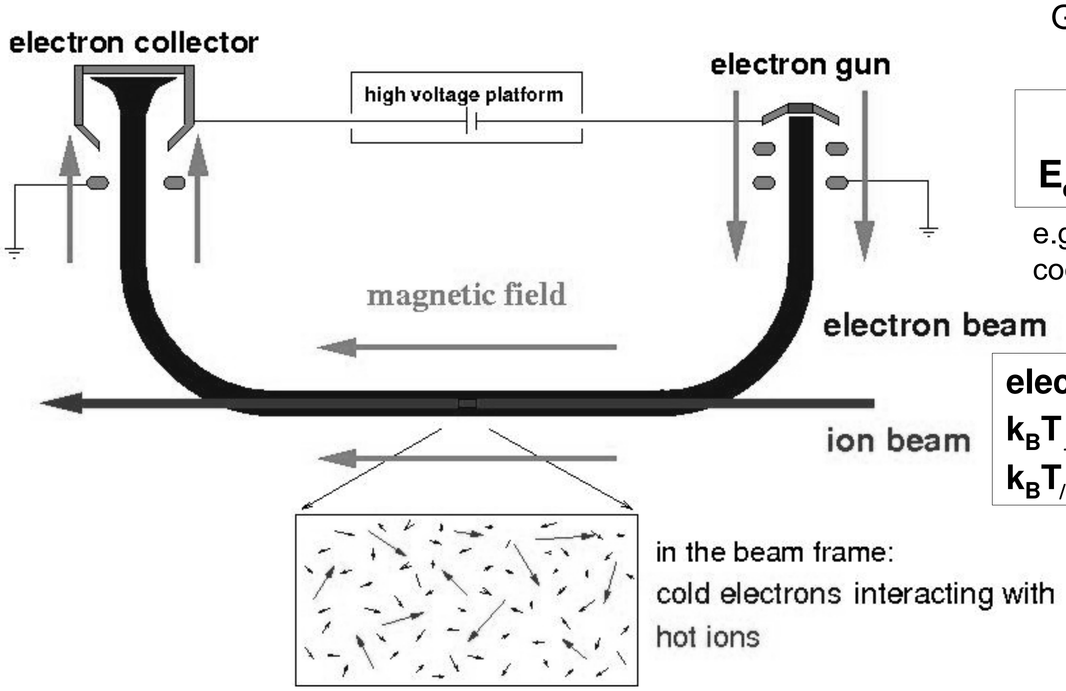
Particle beams can be anisotropic: $k_B T_\parallel \neq k_B T_\perp$
e.g. due to laser cooling or distribution of electron beam

Don't confuse: beam energy $\leftrightarrow$ beam temperature
(e.g. a beam of energy 100 GeV can have a temperature of 1 eV)

Benefits of Beam Cooling

- Improved beam quality
  - Precision experiments
  - Luminosity increase
- Compensation of heating
  - Experiments with internal target
  - Colliding beams
- Intensity increase by accumulation
  - Weak beams from source can be increased
  - Secondary beams (antiprotons, rare isotopes)

1. Electron Cooling



$v_{e//} = v_{i//}$

$E_e = m_e/M_i \; E_i$

e.g. :220 keV electrons
cool 400 MeV/u ions

electron temperature
$k_B T_\perp \approx$ 0.1 eV
$k_B T_{//} \approx$ 0.1 - 1 meV

superposition of a cold intense electron beam with the same velocity

momentum transfer by Coulomb collisions

cooling force results from energy loss in the co-moving gas of free electrons

Simple Derivation of Electron Cooling Force

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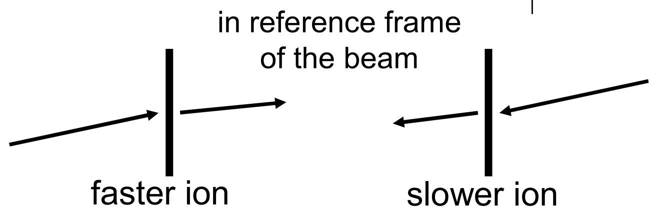
Analogy: energy loss in matter
(electrons in the shell)



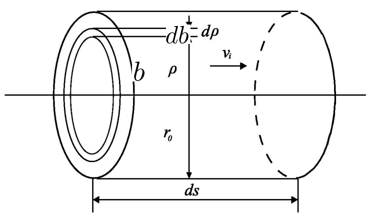
Rutherford scattering: $2\ tan(\frac{\theta}{2}) = \frac{2Z_1Z_2e^2}{4\pi\epsilon_0 \Delta p v b} \quad Z_1 = Q\ (ion),\ Z_2 = -1\ (electron)$

Energy transfer: $\Delta E(b) = \frac{(\Delta p)^2}{2m_e} \simeq \frac{2Q^2e^4}{(4\pi\epsilon_0)^2 m_e v^2}\frac{1}{b^2} \quad (for\ \ b \gg b_{min})$

Minimum impact parameter: $b_{min} = \frac{Qe^2}{(4\pi\epsilon_0)^2 m_e v^2}$ from: $\Delta E(b_{min}) = \Delta E_{max} \simeq m_e v^2$

Energy loss:

$$-\frac{dE}{dx} = 2\pi \int_{b_{min}}^{b_{max}} b n_e \Delta E\, db = \frac{4\pi Q^2 e^4}{(4\pi\epsilon_0)^2 m_e v^2} n_e \ln\frac{b_{max}}{b_{min}}$$



Coulomb logarithm $L_C = \ln(b_{max}/b_{min}) \approx 10$ (typical value)

Characteristics of Electron Cooling Force

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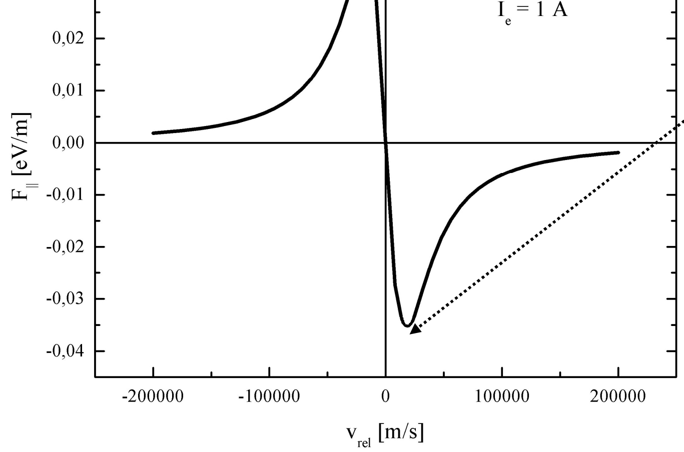
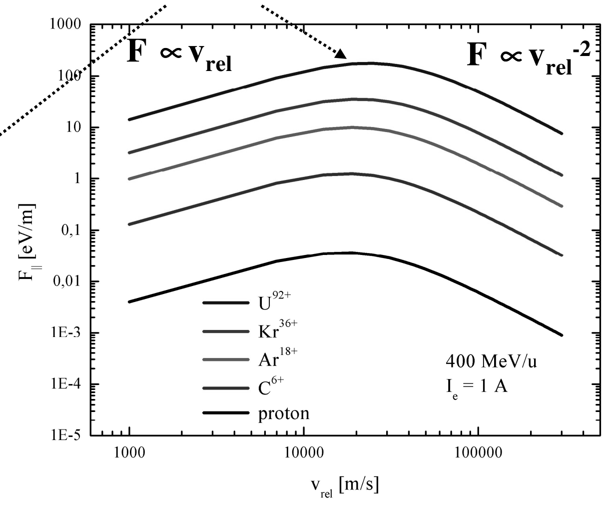
$$\boxed{\vec{F}(\vec{v_i}) = -\frac{4\pi Q^2 e^4 n_e}{(4\pi\epsilon_0)^2 m_e}\int L_C(\vec{v}_{rel}) f(\vec{v_e}) \frac{\vec{v}_{rel}}{v_{rel}^3} d^3\vec{v_e}}$$

$$\vec{v}_{rel} = \vec{v_i} - \vec{v_e}$$

cooling force F
for small relative velocity: $\propto v_{rel}$
for large relative velocity: $\propto v_{rel}^{-2}$
increases with charge: $\propto Q^2$
maximum of cooling force
at effective electron temperature





Electron Cooling Time

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first estimate:
(Budker 1967) $$\tau = \frac{3}{8\sqrt{2\pi} n_e Q^2 r_e r_i c L_C} \left(\frac{k_B T_e}{m_e c^2} + \frac{k_B T_i}{m_i c^2}\right)^{3/2}$$

for large relative velocities

cooling time $$\tau_z \propto \frac{A}{Q^2} \frac{1}{n_e \eta} \beta^3 \gamma^5 \theta_z^3 \quad \begin{cases} \theta_{x,y} = \dfrac{v_{x,y}}{\gamma\beta c} \\ \theta_{\parallel} = \dfrac{v_{\parallel}}{\gamma\beta c} \end{cases}$$

<u>cooling rate:</u>

- slow for hot beams $\propto \theta^3$
- decreases with energy $\propto \gamma^{-2}$ ($\beta\gamma\theta$ is conserved)
- linear dependence on electron beam intensity n_e and cooler length $\eta = L_{ec}/C$
- favorable for highly charged ions Q^2/A
- independent of hadron beam intensity

for small relative velocities

cooling rate is constant and maximum at small relative velocity

$F \propto v_{rel} \Rightarrow \tau = \Delta t = p_{rel}/F =$ constant

Models of the Electron Cooling Force

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binary collision model

description of the cooling process by successive collisions of two particles and integration over all interactions

analytic expressions become very involved, various regimes (multitude of Coulomb logarithms)

dielectric model

interaction of the ion with a continuous electron plasma (scattering off of plasma waves)

fails for small relative velocities and high ion charge

a simple empiric formula (Parkhomchuk):

$$\vec{F} = -4 \frac{n_e}{m_e} \frac{(Qe^2)^2}{(4\pi\epsilon_0)^2} \ln\left(\frac{b_{max} + b_{min} + r_c}{b_{min} + r_c}\right) \frac{\vec{v}_{ion}}{(v_{ion}^2 + v_{eff}^2)^{3/2}}$$

$$b_{min} = \frac{Qe^2/4\pi\epsilon_0}{m_e v_{ion}^2}; \quad b_{max} = \frac{v_{ion}}{max(\omega_{pe}, 1/T_{cool})}, \quad v_{eff}^2 = v_{e,\parallel}^2 + v_{e,\perp}^2$$

Electron Beam Properties

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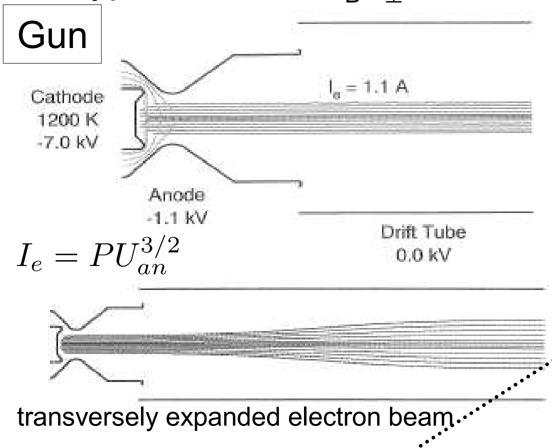
electron beam temperature

transverse $k_BT_\perp = k_BT_{cat}$, with transverse expansion ($\propto B_c/B_{gun}$)

longitudinal $k_BT_{//} = (k_BT_{cat})^2/4E_0 \ll k_BT_\perp$ lower limit : $k_BT_\parallel \geq 2e\frac{n_e^{1/3}}{4\pi\epsilon_0}$

typical values: $k_BT_\perp \approx$ 0.1 eV (1100 K), $k_BT_{//} \approx$ 0.1 - 1 meV

Gun



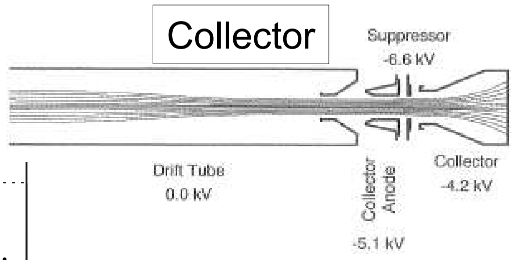
$I_e = PU_{an}^{3/2}$

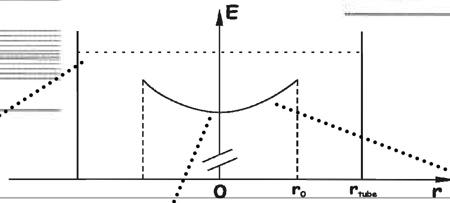
transversely expanded electron beam

electron beam confined by longitudinal magnetic field (from gun to collector)

Drift Section (Cooling)

Collector





radial variation of electron energy due to space charge:

$$E(r) = eU_{cat} - n_e\pi r_0^2 r_e m_e c^2[1 + 2\ln(r_{tube}/r_0)] + n_e\pi r_e m_e c^2 r^2$$

Electron Motion in Longitudinal Magnetic Field

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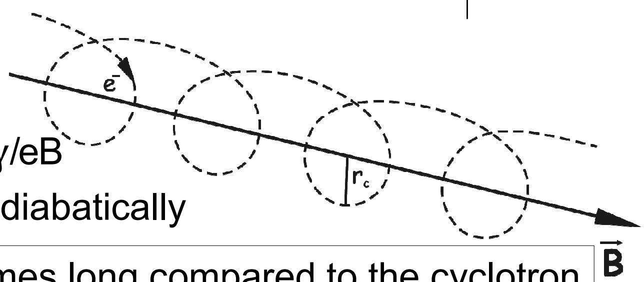
single particle cyclotron motion

cyclotron frequency $\omega_c = eB/\gamma m_e$

cyclotron radius $r_c = v_\perp/\omega_c = (k_BT_\perp m_e)^{1/2}\,\gamma/eB$

electrons follow the magnetic field line adiabatically



important consequence: for interaction times long compared to the cyclotron period the ion does not sense the transverse electron temperature
magnetized cooling ($T_{eff} \approx T_{//} \ll T_\perp$)

electron beam space charge:

transverse electric field + B-field $\Rightarrow$azimuthal drift $v_{azi} = r\omega_{azi} = r\frac{2\pi r_e n_e c^2}{\gamma\omega_c}$

$\Rightarrow$ electron and ion beam should be centered

Favorable for optimum cooling (small transverse relative velocity):

- high parallelism of magnetic field lines $\Delta B_\perp/B_0$
- large beta function (small divergence) in cooling section

Imperfections and Limiting Effects in Electron Cooling

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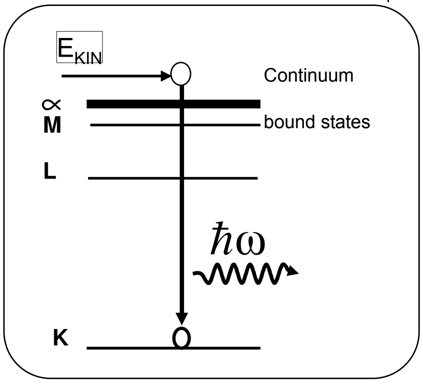
technical and physical issues:

ripple of accelerating voltage

magnetic field imperfections

beam misalignment

space charge of electron beam and compensation



losses by recombination (REC)

loss rate $\tau^{-1} = \gamma^{-2}\alpha_{REC} n_e \eta$

$$\alpha_{REC} = \frac{1.92 \times 10^{-13} Q^2}{\sqrt{k_B T}} \left(\ln \frac{5.66 Q}{\sqrt{k_B T}} + 0.196 \left(\frac{k_B T}{Q^2}\right)^{1/3} \right) [cm^3 s^{-1}]$$

Examples of Electron Cooling

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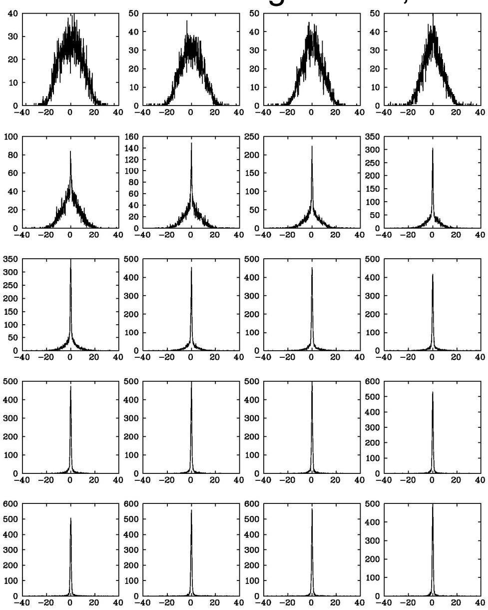
fast transverse cooling at TSR, Heidelberg



profile every 0.1 s. x [mm]

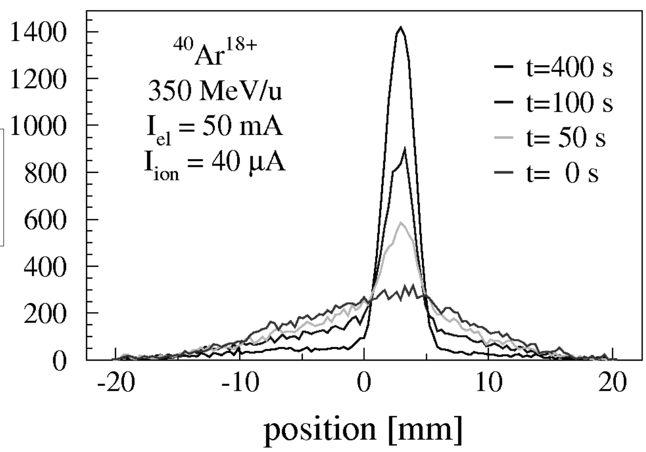
cooling of 6.1 MeV/u C^{6+} ions

0.24 A, 3.4 keV electron beam

n_e = 1.56 ×10^7 cm^{-3}

measured with residual gas ionization beam profile monitor

transverse cooling at ESR, Darmstadt



cooling of 350 MeV/u Ar^{18+} ions

0.05 A, 192 keV electron beam

n_e = 0.8 ×10^6 cm^{-3}

Accumulation of Heavy Ions by Electron Cooling

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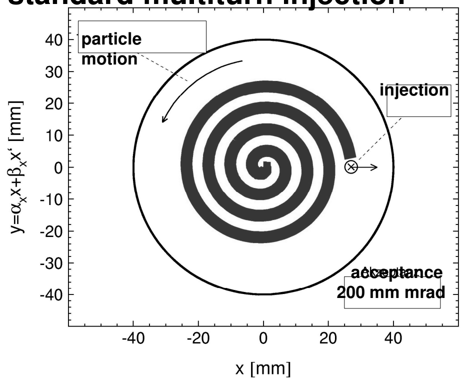
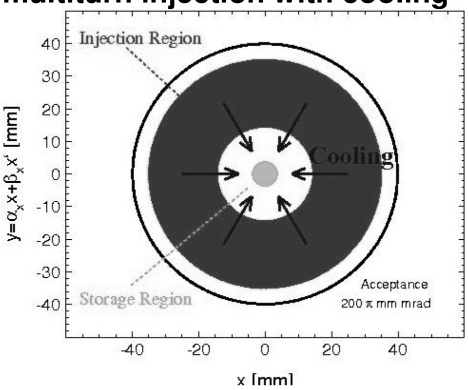
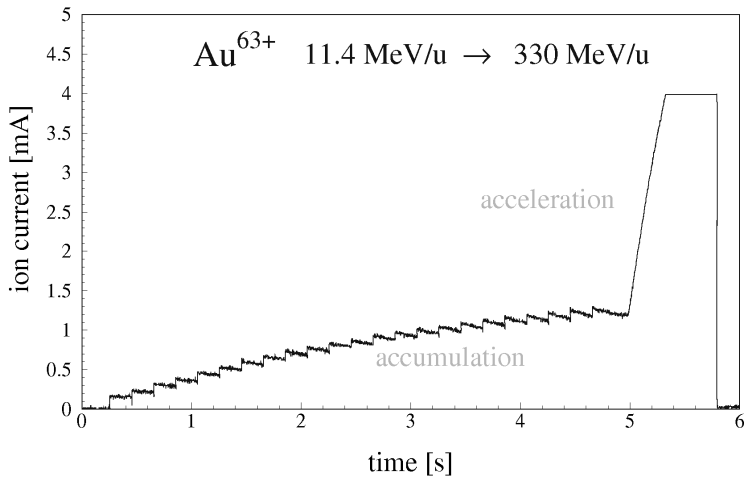
standard multiturn injection



multiturn injection with cooling



fast electron cooling of highly charged ions (Au^{63+}) at injection energy allows accumulation with a repetition rate of 5 Hz



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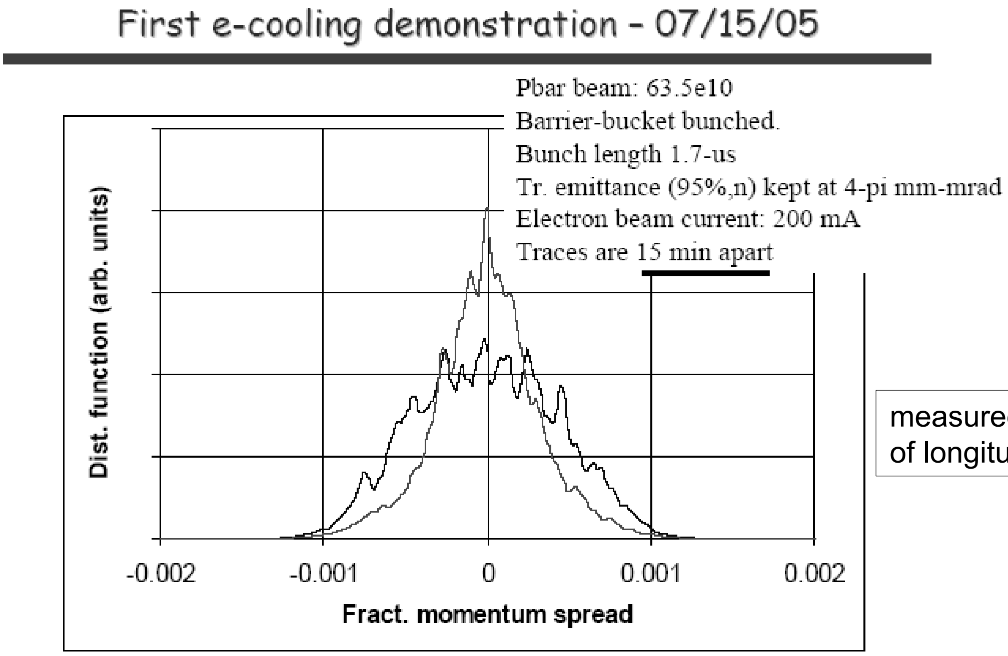
first high energy electron cooling of 8 GeV antiprotons
longitudinal cooling with 0.2 A, 4.4 MeV electron beam



measured by detection
of longitudinal Schottky noise

Electron Cooling Systems

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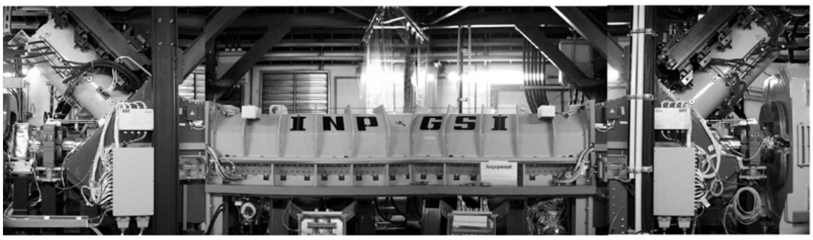
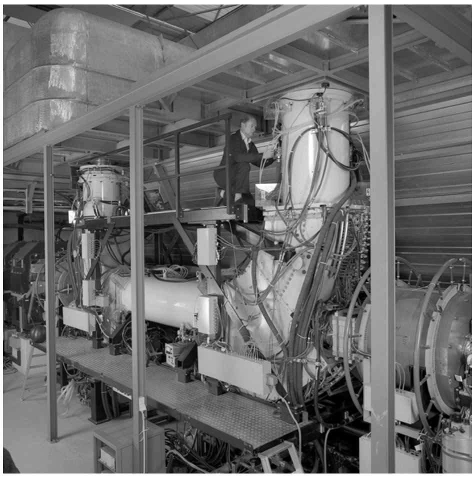
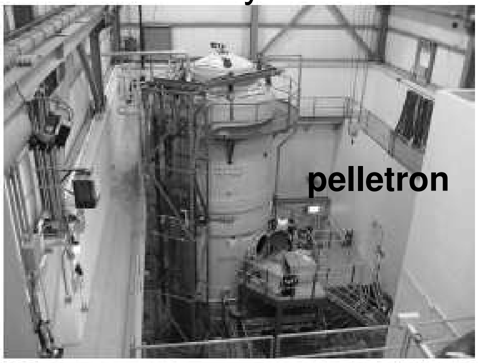
Low Energy: 35 keV SIS/GSI



Medium Energy:
300 keV ESR/GSI



High Energy:
4.3 MeV Recycler/FNAL



2. Ionization Cooling

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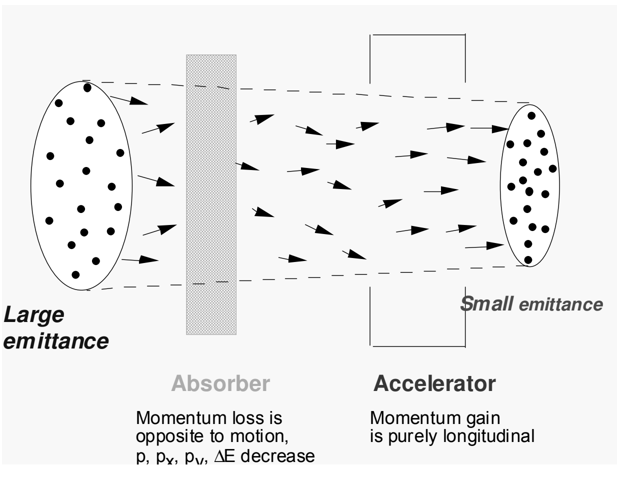
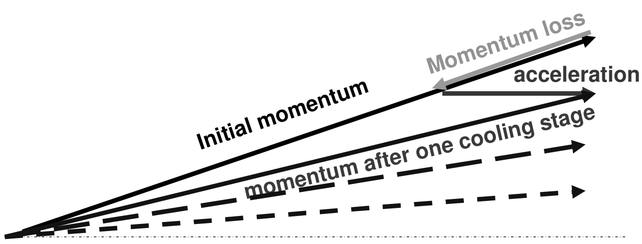
makes use of energy loss in matter



proposed for muon cooling



transverse cooling

$$\frac{d\epsilon_N}{ds} = -\frac{1}{\beta^2 E}\frac{dE}{ds}\epsilon_N + \frac{\beta\gamma\beta_\perp}{2}\frac{\langle\theta_{rms}^2\rangle}{ds}$$

$$= -\frac{1}{\beta^2 E}\frac{dE}{ds}\epsilon_N + \frac{\beta_\perp E_s^2}{2\beta^3 m_\mu c^2 L_R E}$$

small $\beta_\perp$ at absorber in order to minimize multiple scattering

large $L_R/(dE/ds)$ $\Rightarrow$ light absorbers (H_2)

Ionization Cooling

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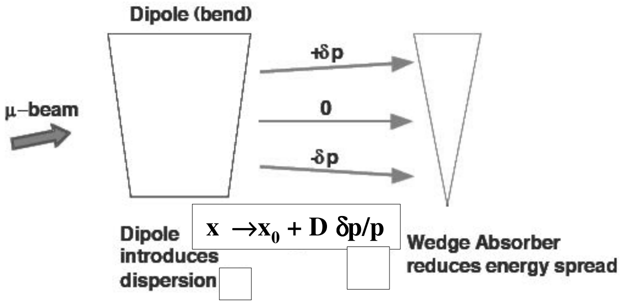
increased longitudinal cooling
by longitudinal-transverse emittance exchange



$$\frac{d\sigma_E^2}{ds} = -2\frac{\partial(dE/ds)}{\partial E}\sigma_E^2 + \frac{d\langle\Delta E_{rms}^2\rangle}{ds}$$

cooling term heating term

cooling, if $\frac{\partial(dE/ds)}{\partial E} > 0$

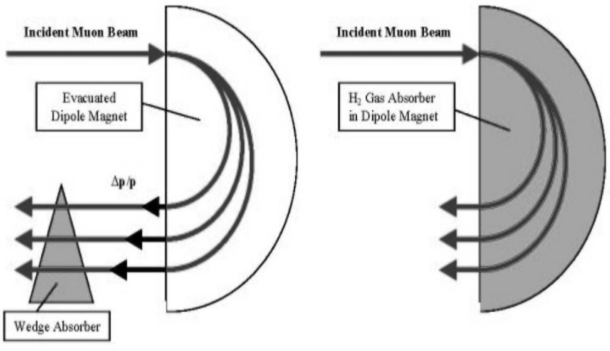


Figure 1. Use of a Wedge Absorber for Emittance Exchange

Figure 2. Use of Continuous Gaseous Absorber for Emittance Exchange

emittance exchange

increased longitudinal cooling

$$\frac{\partial \frac{dE}{ds}}{\partial E} \Rightarrow \frac{\partial \frac{dE}{ds}}{\partial E}\Big|_0 + \frac{dE}{ds}\frac{D\rho'}{\beta c p \rho_0}$$

reduced transverse cooling

$$\frac{d\epsilon_N}{ds} = -\frac{1}{\beta^2 E}\frac{dE}{ds}\left(1 - \frac{D\rho'}{\rho_0}\right)\epsilon_N$$

Applications of Ionization Cooling

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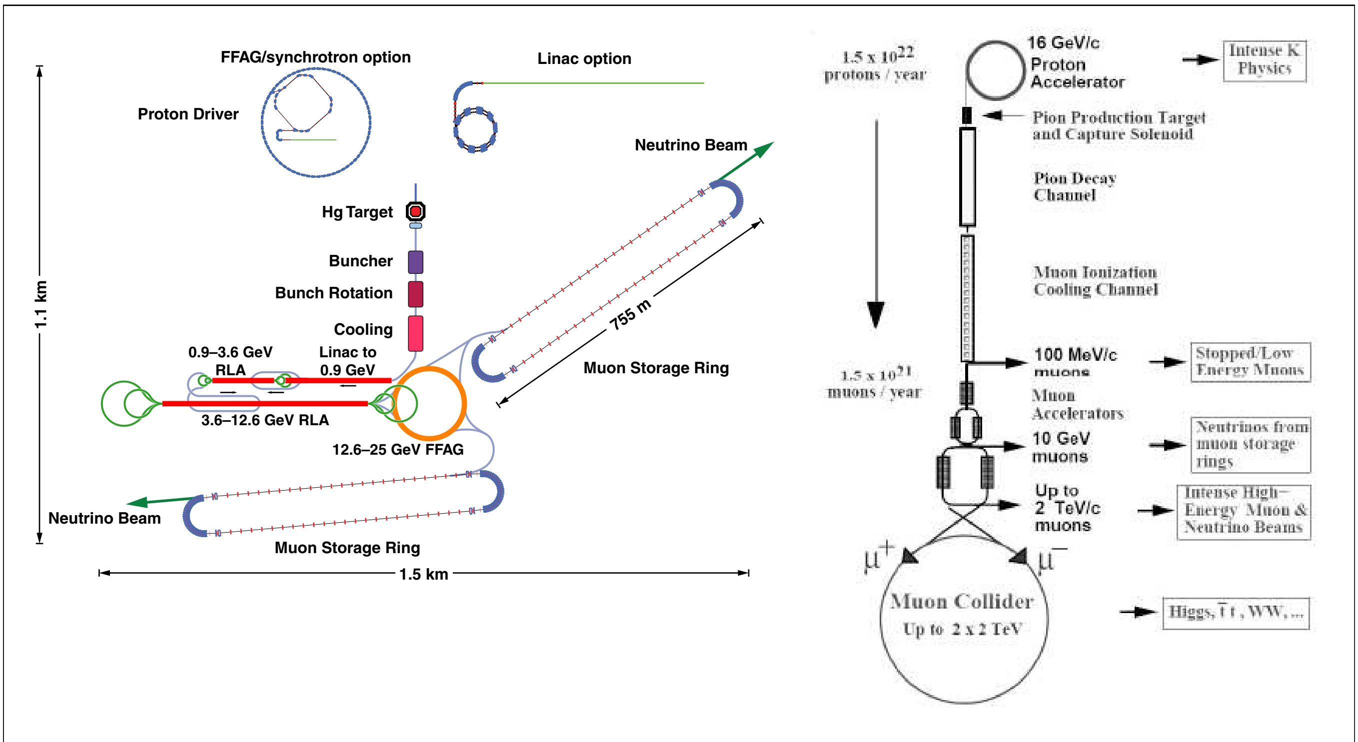
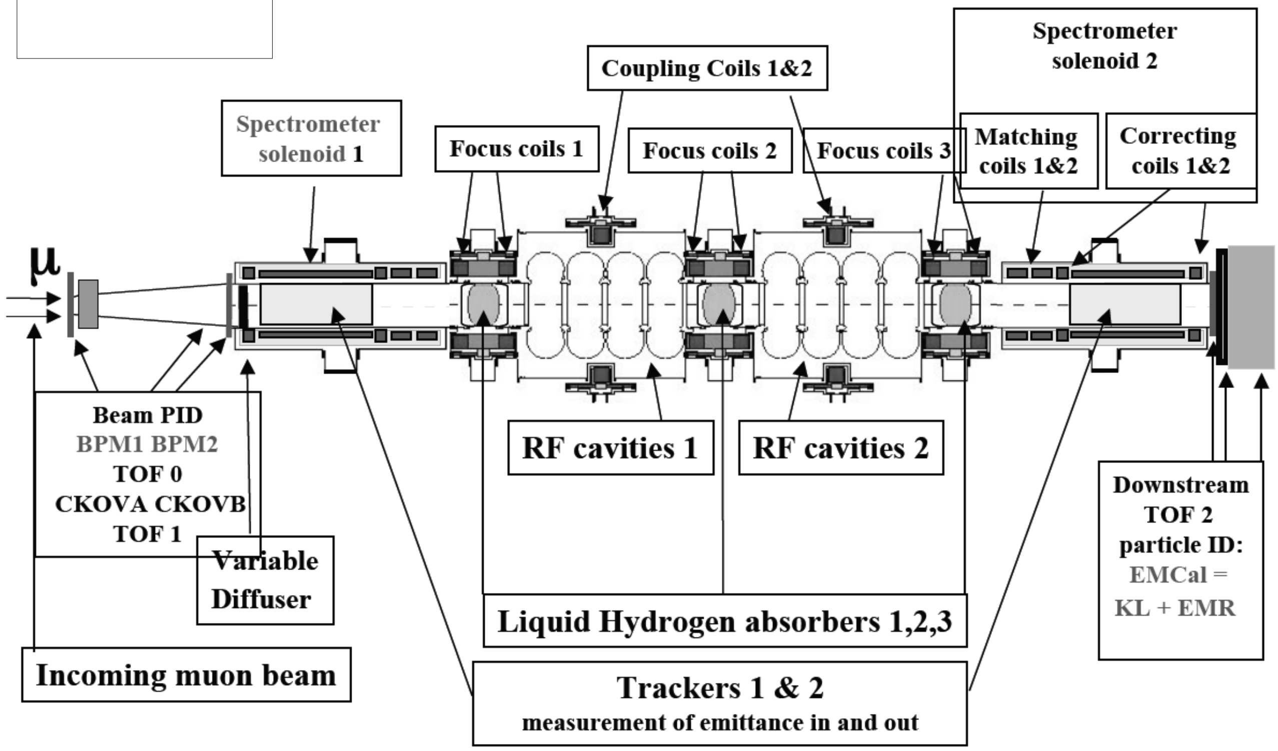
Neutrino Factory

Muon Collider



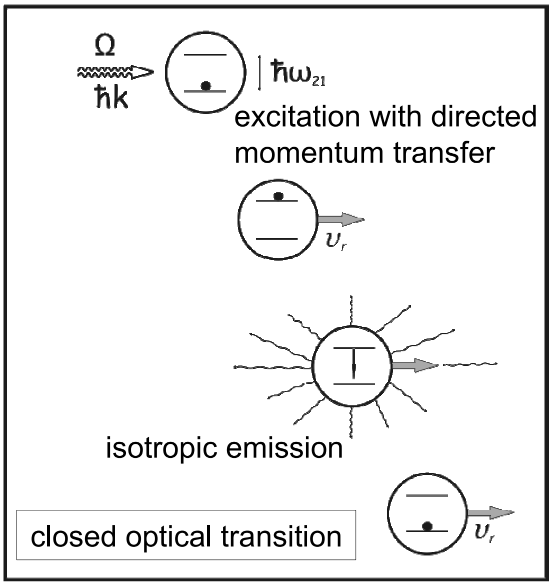
MICE

Muon Ionization Cooling Experiment at RAL

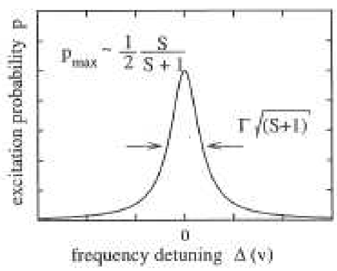


3. Laser Cooling

$$\Omega = \gamma\omega_{21}(1 \pm \beta\cos\theta)$$



the directed excitation and isotropic emission result in a transfer of velocity v_r



cooling force

$$\boxed{\vec{F}(\vec{v},\vec{k}) = \frac{\hbar\vec{k}}{2}S\Gamma\frac{(\Gamma/2)^2}{(\omega-\omega_{21}-\vec{v}\vec{k})+(\Gamma/2)^2(1+S)}}$$

Lorentzian with width Γ/k ~ 10 m/s

minimum temperature $T_D = \dfrac{\hbar\Gamma}{2k_B}$ (Doppler limit)
typical 10^{-5} □ 10^{-4} K

typical cooling time ~ 10 μs

only longitudinal cooling

Laser Cooling

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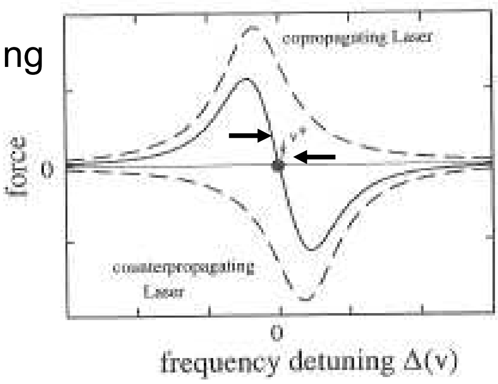
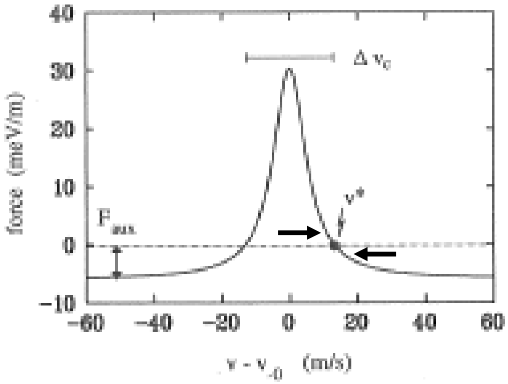
a single laser does not provide cooling (only acceleration or deceleration)

schemes for cooling





two counter-propagating lasers (matched to beam velocity)

auxiliary force (betatron core, rf)

capture range of laser is limited ⇒frequency sweep (snowplow)

ions studies so far: ^{7}**Li**$^{1+}$, ^{9}**Be**$^{1+}$, ^{24}**Mg**$^{1+}$, ^{12}**C**$^{3+}$ in future: Li-like heavy ions

Laser Cooling at ESR

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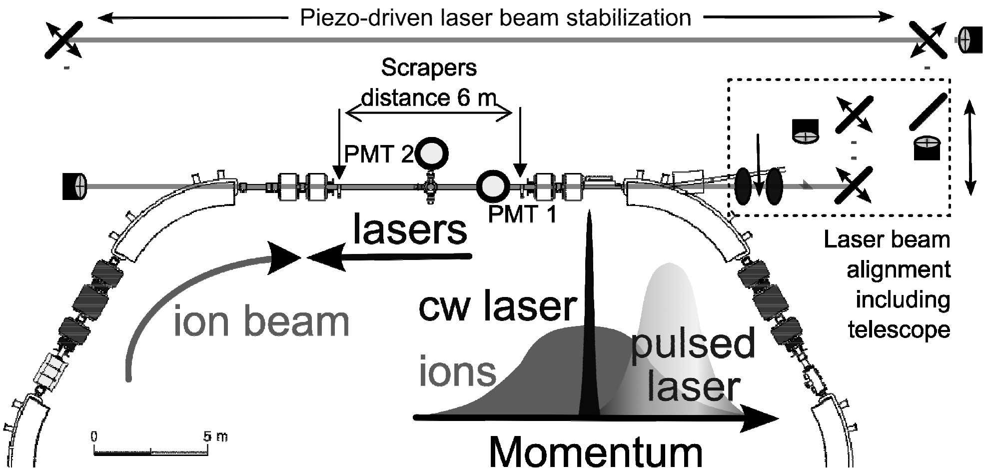
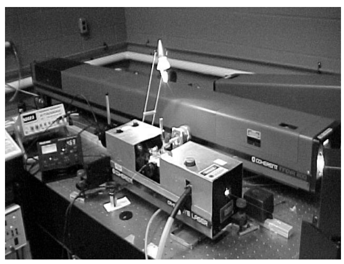
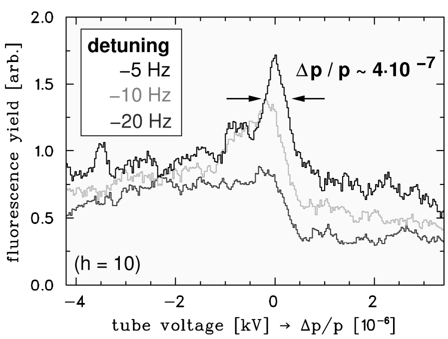
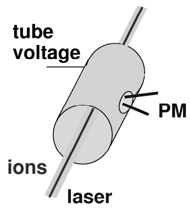




Argon ion laser (257.3 nm) frequency doubled



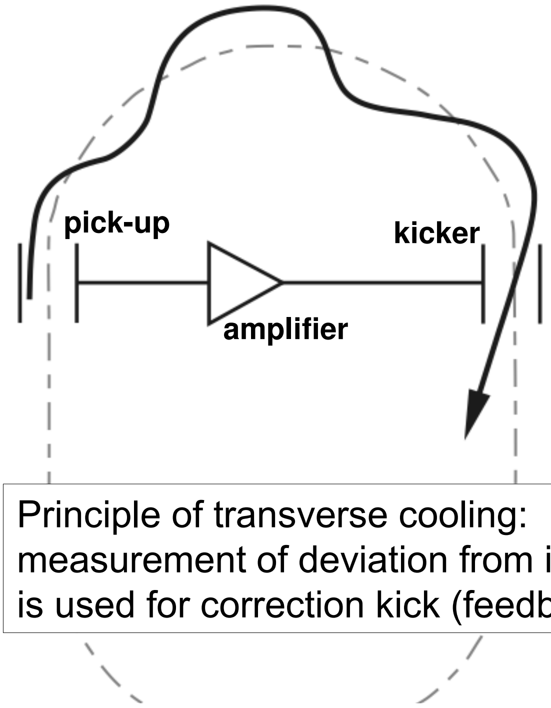


diagnostics by

fluorescence light detection

4. Stochastic Cooling

First cooling method which was successfully used for beam preparation



Principle of transverse cooling:
measurement of deviation from ideal orbit
is used for correction kick (feedback)

S. van der Meer, D. Möhl, L. Thorndahl et al.

Conditions:

Betatron phase advance
(pick-up to kicker): (n + ½) π

Signal travel time = time of flight of particle
(between pick-up and kicker)

Sampling of sub-ensemble of total beam

Stochastic Cooling

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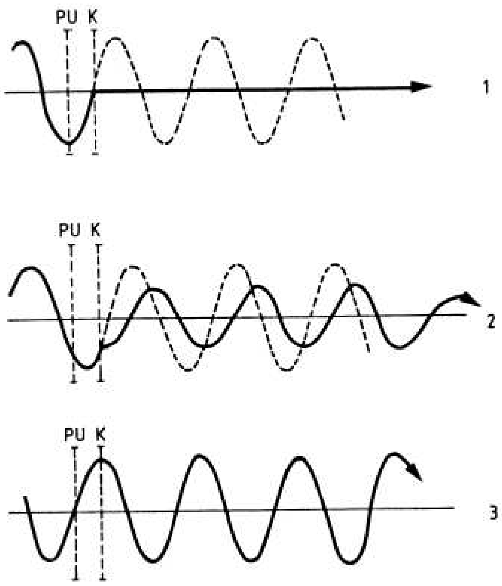
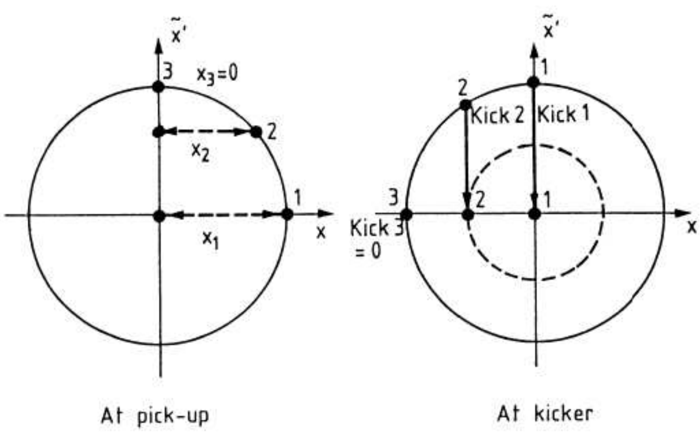
single particle betatron motion
along storage ring
without and with correction kick



projection to two-dimensional
horizontal phase area



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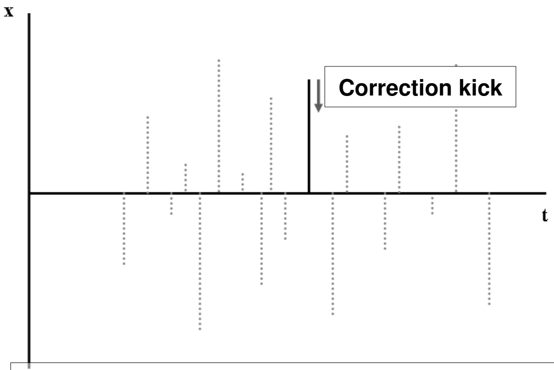
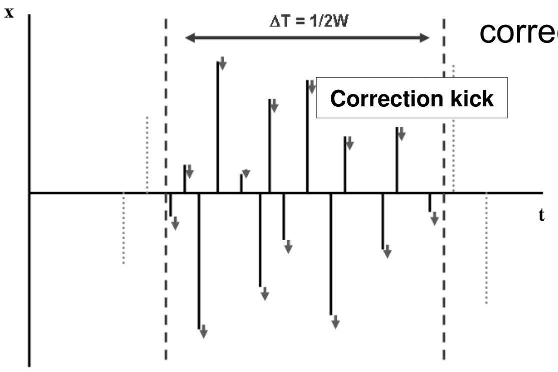


correction kick
(unlimited resolution)

$$\Delta x = g \times x$$

Nyquist theorem: a system with a band-width Δf = W in frequency domain can resolve a minimum time duration ΔT=1/(2W)



correction kick $\Delta x = \frac{g}{N_s} \times \sum_{i=1..N_s} x_i \ , \ \ N_s = N\frac{\Delta T}{T_0} = \frac{N}{2WT_0}$

For exponential damping ($x(t)=x(t_0) \exp(-(t-t_0)/\tau)$):

$$\tau^{-1} = T_0^{-1} \times \frac{\Delta x}{x} = \frac{g2W}{N}, \ if \sum_{i=1..N_s} x_i = x$$

$$\underline{\tau^{-1} \leq \frac{2W}{N}}, \ if \ g \leq 1$$

Stochastic Cooling

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some refinements of cooling rate formula

noise: thermal or electronic noise adds to beam signal

mixing: change of relative longitudinal position of particles due to momentum spread

cooling rate $\lambda = \tau^{-1} = \frac{2W}{N}(\underbrace{2g}_{\text{cooling}} - \underbrace{g^2(M+U)}_{\text{heating}})$

M mixing factor
U noise to signal ratio

maximum of cooling rate

$$\boxed{\lambda_{max} = \frac{2W}{N}\frac{1}{M+U}}$$

$$\frac{d\lambda}{dg} = 0 \Rightarrow g = \frac{1}{M+U}$$

further refinement (wanted ↔ unwanted mixing):

with wanted mixing M (kicker to pick-up)
and unwanted mixing $\tilde{M}$ (pick-up to kicker)

$$\lambda = \tau^{-1} = \frac{2W}{N}(2g(1-\tilde{M}^2) - g^2(M+U))$$

Stochastic Cooling Circuit

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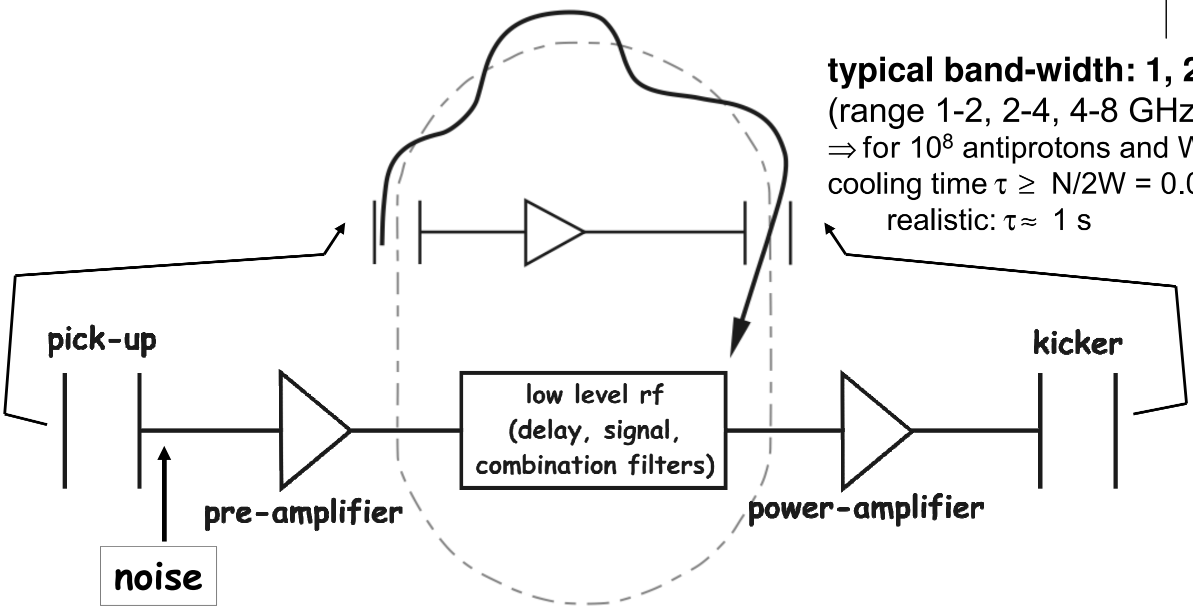
typical band-width: 1, 2 , 4 GHz
(range 1-2, 2-4, 4-8 GHz)
⇒ for 10^8 antiprotons and W = 1 GHz
cooling time $\tau \geq$ N/2W = 0.05 s
realistic: $\tau \approx$ 1 s



Transfer Function:

$$Z_{pick-up} \cdot G_{pick-up}(E) \cdot H(t_{delay}) \cdot F(E) \cdot G \cdot G_{kicker}(E) \cdot Z_{kicker}$$

Longitudinal Stochastic Cooling

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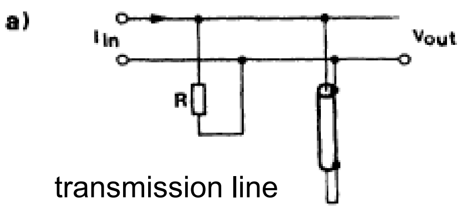
1) **Palmer cooling**

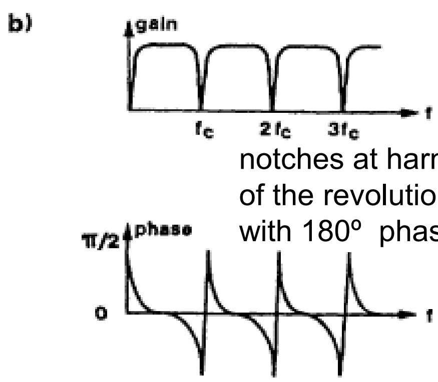
pick-up in dispersive section detects horizontal position
⇒ correcting acceleration/deceleration kick

2) **Notch filter cooling**

filter creates notches at the harmonics of nominal
revolution frequency
⇒ particles are forced to circulate at the nominal frequency



transmission line
short circuit at all harmonics
of the revolution frequency



notches at harmonics
of the revolution frequency
with 180° phase jump

Antiproton Accumulation by Stochastic Cooling

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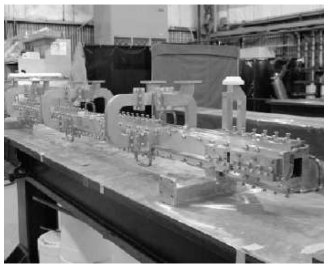
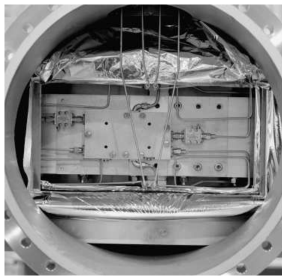
accumulation of 8 GeV antiprotons at FNAL

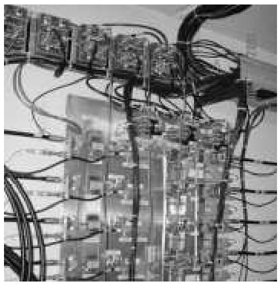
momentum distribution of accumulated antiproton beam



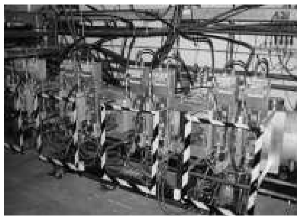
kicker array



cryogenic microwave amplifier



microwave electronics



power amplifiers (TWTs)

Stochastic Cooling at GSI

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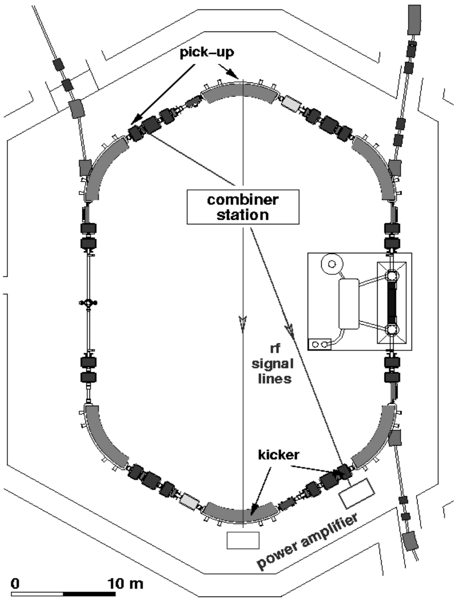
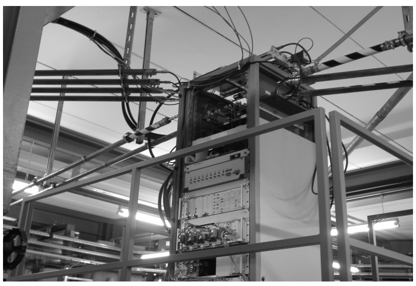
fast pre-cooling of hot fragment beams

energy 400 (-550) MeV/u
bandwidth 0.8 GHz (range 0.9-1.7 GHz)
$\delta p/p = \pm 0.35\ \% \rightarrow \delta p/p = \pm 0.01\ \%$
$\varepsilon = 10 \times 10^{-6}$ m $\rightarrow \varepsilon = 2 \times 10^{-6}$ m



electrodes installed inside magnets



combination of signals from electrodes

power amplifiers for generation of correction kicks

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Laser Cooling:

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Biannual Workshops on Beam Cooling: e. g. COOL'11, Alushta, Ukraine

Trends in Beam Cooling

Stochastic cooling was mainly developed for the production of high intensity antiproton beams for colliders (CERN, FNAL, 1970 □2010).

Electron cooling was and still is used in low energy storage rings (protons, ions, secondary beams (antiprotons, rare isotopes).

First demonstration of bunched beam stochastic cooling (2008) with ions (BNL) made it also attractive for ion colliders.

Electron Cooling still is interesting for low energy storage rings, but also application at higher energies (MeV electron energies) is envisaged after the successful demonstration of the 4 MeV electron cooler at FNAL.

Other cooling methods, like muon (ionization) cooling or coherent electron cooling are under investigation, but still far from implementation in a full scale machine.