

Exercises for tutorial on "Non-linear Dynamics" at the CAS 2011 on Chios

W. Herr, BE Department, CERN, 1211-Geneva 23

1st September 2011

1 Exercise 1

1.1 Problem:

a) Compute the map:

$$X(L) = ?$$

$$P(L) = X'(L) = ?$$

for a thick sextupole (1D) (length L , strength k) with the equation of motion:

$$x'' = k \cdot x^2$$

up to order $\mathcal{O}(L^2)$, using the symplectic integration method.

b) Compute the map:

$$X(L) = ?$$

for a thick sextupole (2D) with the Hamiltonian (to give the equation of motion above):

$$H = \frac{1}{3}k(x^3 - 3xy^2) + \frac{1}{2}(p_x^2 + p_y^2)$$

using the Lie transformation method, compare with the solution from a).

2 Exercise 2

2.1 Problem:

Starting from the transfer matrix, derive the Lie operators representing:

- a) a thick, focusing quadrupole
- b) a thick, defocusing quadrupole

3 Exercise 3

3.1 Problem:

Assume a matrix M of the type:

$$M = \begin{pmatrix} m_{11} & m_{12} \\ m_{21} & m_{22} \end{pmatrix}$$

described by a generator f . Use the properties of Lie transforms to evaluate the effect of this matrix on the moments x^2, xp, p^2 :

$$e^{:f:} x^2 = ?$$

$$e^{:f:} p^2 = ?$$

$$e^{:f:} xp = ?$$