

Chromaticity measurements

Solutions 1 :

$$q_1 - q_2 := \Delta q = Q' \Delta p/p$$

$$\delta \Delta q = \delta Q' / (\Delta p/p) = 10^{-3}$$

$$\rightarrow \delta q_1 = \delta q_2 = \sqrt{2} \cdot 10^{-3}$$

This precision is achievable with 1024 or 2048 sample points, provided the damping time of the oscillation is sufficiently long. Since chromaticity is one of the parameters, which determine the damping time, this precision can only be achieved for small chromaticities. The other limiting factor is that the initial oscillation amplitude has to be 10...100 times the BPM resolution.

Solutions 2:

- a) basic measurement:
point by point tune measurements for different beam momenta and linear regression.
- b) Damping time: This method is used in almost all machines in a qualitative way: The important parameter is the width of the tune peak. This is observed in 2 ways:
 - i) in time domain: After a kick stimulus the time envelope of the betatron oscillation is plotted over many turns (exponential decay).
 - ii) in frequency domain: as a direct observable in the measured beam transfer function.Unfortunately chromaticity is not the only parameter, which determines the width of the tune line: Space charge effects, radiation damping... also give rise to a width of the tune line and hence the method is only used for a qualitative observation.
- c) This method is not used in practice, it is just mentioned for completeness. The amplitude of synchrotron sidebands (of the betatron tune line) is proportional to chromaticity. Unfortunately many other parameters enter into the equation, such that this proportionality can not be exploited in a numerical way. But changes in chromaticity could be observed like that. It should also be pointed out that structural resonances of the lattice can change the amplitude of a sideband, if it coincides with this resonance.
- d) This method corresponds to method a), just by the fact of modulating periodically the beam momentum and by continuous tune tracking, many measurement points of tune versus beam momentum are collected.

Solutions 3:

- a) The beta squeeze is a jargon for the reduction of the beta* (the value of the beta function at the IP). In almost all colliders the beams are injected into an untuned

insertion in order to have more physical aperture. After accelerator the β^* is reduced (giving higher luminosity), but as consequence the beta values rise to large values in the insertion quadrupoles. During this process the tune and the chromaticity has be kept constant.

b)

$$\Delta Q = Q' \frac{\Delta p}{p} = \left(\frac{1}{\gamma^2} - \alpha \right) Q' \frac{\Delta f}{f}$$

$$Q' = \Delta q * p / \Delta p = \Delta q * \alpha * f / \Delta f$$

for a value of Δq of $3 \cdot 10^{-3}$ the above equation gives chromaticity values of about 5 units. Note that in the plotted curves in the horizontal plane the chromaticity changes sign during a short period.