

## 1 Answers to Discussion on General Features of BPMs

### Definition of phrases for 1<sup>st</sup> discussion topic:

For the characterization of a position measurement system several phrases are frequently used, which is compiled in the following (modified version based on R. Shafer paper):

**Position sensitivity:** It is the proportional constant between the beam displacement and the signal strength. It is defined by the derivative

$$S_x(x) = d/dx (\Delta U_x / \Sigma U_x) = [\%/mm] \text{ (linear scheme)}$$

$$S_x(x) = d/dx (\log U_{right} / U_{left}) = [dB/mm] \text{ (logarithmic scheme)}$$

for the horizontal direction and for the vertical direction, correspondingly. The unit is  $S = [\%/mm]$  in case of linear processing or  $S = [dB/mm]$  in case of logarithmic processing. For small displacements a constant value is expected. For larger displacements the horizontal sensitivity  $S_x$  might depend on beam position in horizontal direction, called non-linearity, and additionally in vertical direction, called horizontal-vertical coupling. Moreover, it might depend on the evaluation frequency. In the most general case the position sensitivity  $S(x, y, f)$  is a function of horizontal and vertical displacement as well as frequency. Often the inverse of  $S$  is used having the abbreviation  $k = 1/S$  and is given in the unit of  $k = [mm]$ .

**Accuracy:** It refers to the ability of position reading relative to a mechanical fix-point or to any other absolutely known axis e.g. the symmetry axis of a quadrupole magnet. The accuracy is mainly influenced by the BPM's mechanical tolerances as well as the long-term stability of the mechanical alignment. Beside these mechanical properties, the accuracy is influenced by electronics properties like amplifier drifts, noise and pickup of electro-magnetic interference. By calibrating the electronics in regular time intervals long-term drifts can be compensated. The digitalization leads to a granularity of values, which might limit the reachable accuracy; for modern installations this limitation is compensated by improved ADC technologies.

**Resolution:** It refers to the ability for measuring small displacement variations. In contrast to the accuracy relative values are compared here. In most cases, the resolution is much better than the accuracy. It depends strongly on the measurement time because averaging procedures can exceed the accuracy by a factor of 100. A typical value for the resolution for a broadband, single bunch reading is  $10^{-3}$  of the beam pipe radius or roughly 100  $\mu m$ . For averaged readings on typical time scale of 10 to 1000 ms, a resolution of  $10^{-5}$  of the beam pipe radius or roughly 1  $\mu m$  can be reached. As for the accuracy, it depends on the electronics noise contribution as well as short-term and long-term drifts.

**Analog bandwidth:** The lower and upper cut-off frequencies of the analog electronics have to be matched to the frequency spectrum delivered by the bunched beam. For noise reduction, the bandwidth can be limited by analog filters.

**Acquisition bandwidth:** It refers to the frequency range over which the beam position is recorded and should be matched to the analog bandwidth. For monitoring fast changes of beam parameters a much larger bandwidth is required, resulting in a lower position resolution. The same is valid for short beam deliveries, e.g. in transport lines, preventing from averaging. The bandwidth can be restricted to achieve a high resolution in case of slow varying beam parameters.

**Real-time bandwidth:** The data rate of producing an analog or digital position signal with predictable latency to be used e.g. by an orbit feedback system is characterized by this quantity.

**Dynamic range:** It refers to the range of beam current for which the system has to respond to. In most cases the signal adoption is done by a variable gain amplifier at the input stage of the electronics processing chain. Within the dynamic range, the position reading should have a negligible dependence with respect to the input level.

**Signal-to-noise ratio:** It refers to the ratio of wanted signal to unwanted noise. An unavoidable contribution is given by thermal noise. Cooling of the first stage amplifier reduces this thermal noise. Other sources, like electro-magnetic pickup or ground-loops can contribute significantly to an unwanted signal disturbance and are incorporated in this phrase even though they are not caused by noise. Careful shielding and grounding is required to suppress these disturbances.

**Detection threshold:** It refers to the minimal beam current for which the system delivers usable information. It is limited by noise contributions. Sometimes this quantity is called signal sensitivity.

**Types of BPMs for 2<sup>nd</sup> discussion topic:**

Most frequent types of BPMs and their main advantages and disadvantages are summaries in the table below (the related simplifications have to be discussed):

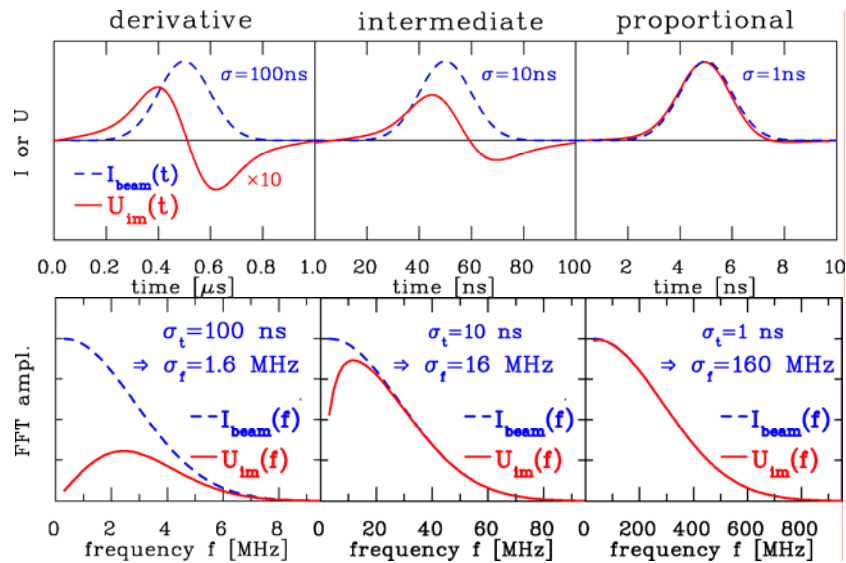
Type	Usage	Precaution	Advantage	Disadvantage
Shoe-box	p-Synch.	Long bunches $f_{rf} < 10$ MHz	Very linear No x-y coupling Sensitive For broad beams	Complex mechanics Capacitive coupling between plates
Button	p-Linacs, all e- acc.	$f_{rf} > 10$ MHz	Simple mechanics	Non-linear, x-y coupling Possible signal deformation
Stipline	colliders p-Linacs all e- acc.	best for $\beta \approx 1$ , short bunches	Directivity 'Clean' signals Large Signal	Complex 50 $\Omega$ matching Complex mechanics
Ind. WCM	all	non	Broadband	Complex, long insertion
Cavity	e- Linacs (e.g. FEL)	Short bunches Special appl.	Very sensitive	Very complex, high frequency

**2 Answer to Numerical Examples for Signal Estimation of capacitive BPMs**

**2.1 Estimation of Beam Spectrum**

**2.1.1 Answer**

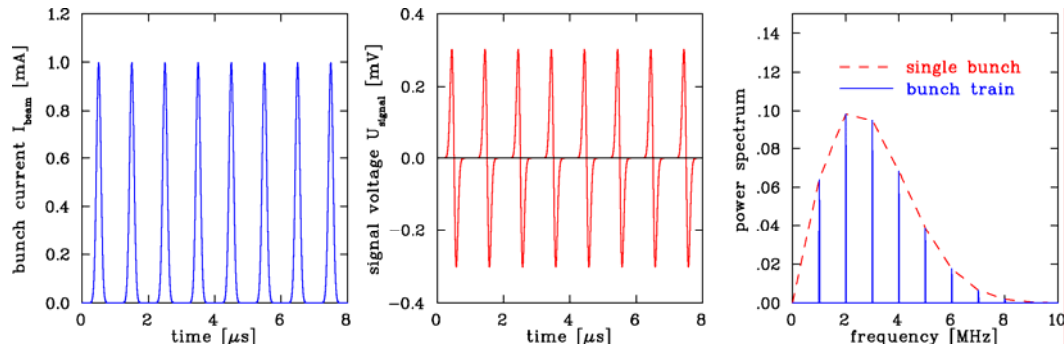
The top plots show the time domain functions of a single bunch  $I_{beam}(t)$  and the resulting signal voltage  $U_{signal}(t)$  and the for the given example. The bottom plots are the bunch spectra  $I_{beam}(f)$  and the signal voltage  $U_{signal}(f)$  in frequency domain. For Gaussian shaped bunches the frequency spectrum extends to a frequency of about  $\approx 1/\sigma_t$ .



**2.1.2 Answer**

The figure below shows the signal simulation of 8 Gaussian bunches of width  $\sigma_t = 100$  ns accelerated with  $f_{acc} = 1$  MHz circulating in a synchrotron with a revolution frequency of  $f_{rev} = 125$

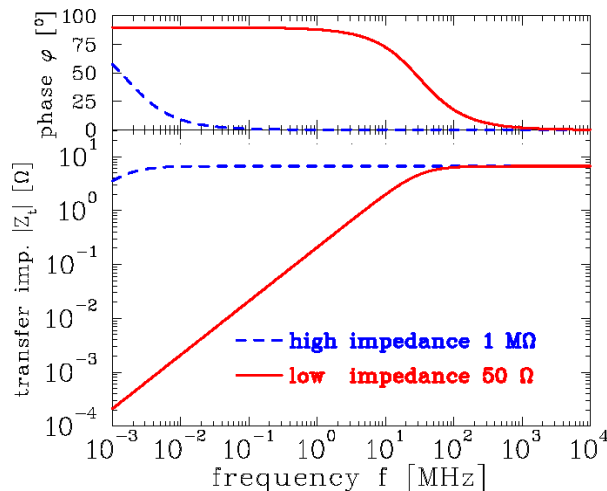
kHz. The beam current is shown left, the signal voltage in the middle for a BPM with a transfer impedance of  $Z_t(f > f_{cut}) \approx 7 \Omega$  and a  $R = 50 \Omega$  termination. The resulting signal voltage spectrum is shown right. This spectrum is now composed of lines with lines separated by the  $f_{acc}$  with an envelope function given by the single bunch spectrum. This estimation is important for the specification of the applied electronics. For typical values at proton synchrotrons, a broadband electronics up to the 10<sup>th</sup> harmonics of the acceleration frequency is well suited.



## 2.2 Estimation of Signal Voltage for a capacitive Shoe-box BPM

### 2.2.1 Answer

The cut-off is  $f_{cut} = 1/(2\pi RC) = 32$  MHz.  $Z_t$  shows the characteristic of a first order high pass filter. The amplitude damping for low frequencies are accompanied by the phase change of  $90^\circ$ , corresponding to the derivative of the signal.



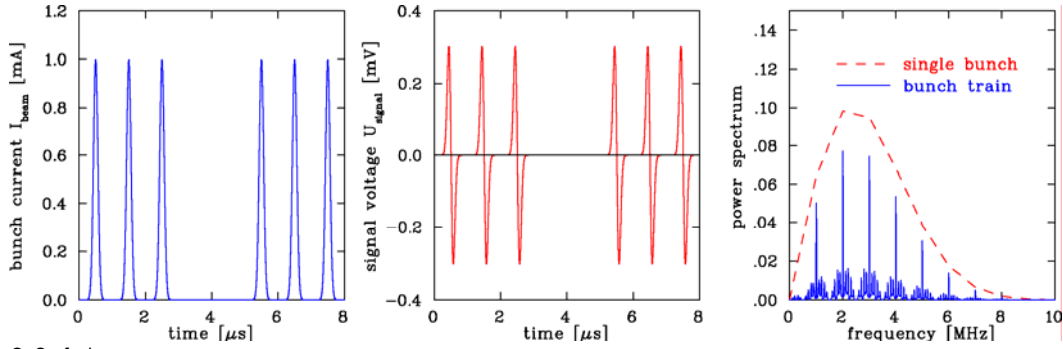
### 2.2.2 Answer

The plots are shown at answer 2.1.1. The signal calculation can be performed by the following steps: Given  $I_{beam}(t) \rightarrow$  FFT yields  $I_{beam}(f) \rightarrow$  multiplying by  $Z_t(f)$  yields  $U_{signal}(f) = Z_t(f) \cdot I_{beam}(f) \rightarrow$  inverse FFT yields  $U_{signal}(t)$ . This numerically extensive calculation is useful because the electronics for the signal chain is normally described in frequency domain as well. But in principle the properties can be also calculated in time domain using the response function  $H(t)$ .

### 2.2.3 Answer

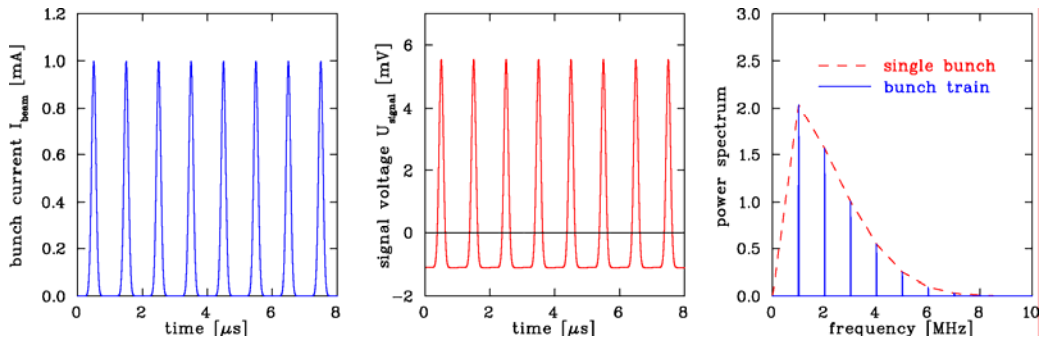
The spectrum is presented and discussed for Answer 2.1.2.

If only some of the buckets are filled the main frequency lines within the Fourier-Spectrum are lower as the single bunch envelope value and side bands in the Fourier-Spectrum appear.



### 2.2.4 Answer

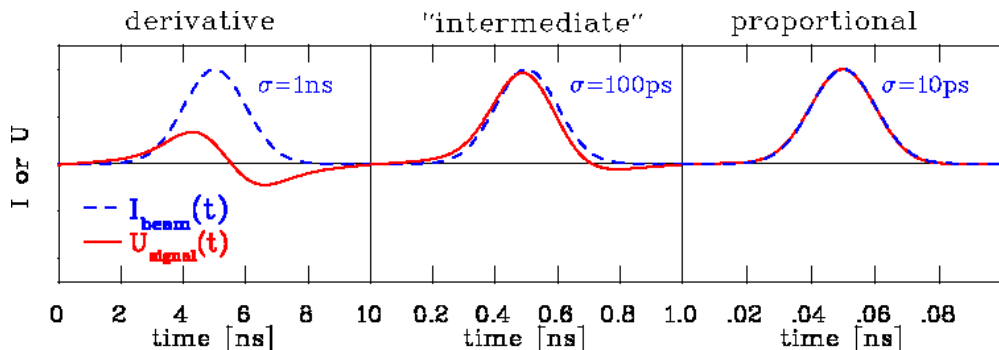
The same type calculation as above is shown for a high impedance termination below. The single bunch spectrum is extended to lower frequencies due to the cut-off  $f_{cut}=1.6$  kHz. Because most frequency components are now above the cut-off frequency, the signal voltage is of proportional shape. The value of the signal voltage corresponds to a transfer impedance of  $Z_t(f > f_{cut}) \approx 7 \Omega$ . Even though, the cut-off frequency is only  $\approx 1$  kHz, no dc-component can be transmitted resulting in a baseline shift. (The baseline is shifted in such a way, that positive and negative contributions add up to zero.) The signal is now larger by nearly a factor of 30 compared to the  $50 \Omega$  termination, which is a big improvement in case of low current operation. But for position determination thermal noise is an important factor, which is increased by a factor 141 of due to the scaling  $U_{eff} \propto \sqrt{R}$ .



## 2.3 Estimation of Signal Voltage for capacitive Button BPM

### 2.3.1 Answer

Due to the 20-fold lower capacitance, the cut-off frequency is larger by a factor 20, with a  $50 \Omega$  termination it results in  $f_{cut}=640$  MHz. The general properties are the same, but due to the short bunches the derivative behavior is suppressed.



### 2.3.2 Answer

**Numerical example:** For the case of a  $N=10^{10}$  electrons within boxcar-like bunch shape with width  $6\sigma_t$  the beam current is:  $I_{beam} \approx eN/6\sigma_t$  :

$\Rightarrow$  Bunch current  $I_{beam}=2.67$  A for  $\sigma_t=100$  ps.

⇒ The main frequency components are above  $f_{cut} \Rightarrow U_{signal} \approx 2.67 \text{ V}$

If one assumes a more realistic Gaussian bunch shape, the maximum is about a factor of 2.5 larger than the average value

⇒ the peak value for  $U_{signal} = 6.65 \text{ V}$ .

If the bunch length is comparable to the button size, signal propagation starts to contribute, leading to a signal deformation not covered by the electro-static model used for the signal estimation. Moreover, the mechanical transition for the button to the cable is difficult to manufacture obeying  $50 \Omega$  signal impedance up to high frequencies. A typical value of the button bandwidth is limited to several GHz.

### 2.3.3 Answer

For a beam offset of 0.1 mm, the relative difference voltage is only 0.1 %. For the example above the difference voltage is only  $\Delta U = 10^{-3} \cdot \Sigma U = 600 \mu\text{V}$ . For a bandwidth of  $\Delta f = 1 \text{ GHz}$  the thermal noise is  $U_{eff} = 30 \mu\text{V}$ . The signal-to-noise ratio for this case is  $\approx 20$ . For a position evaluation at least a signal-to-noise ratio of 2 is required; the related minimum beam current is about 27 mA or  $10^9$  electrons per bunch. A realistic noise contribution from the used amplifier is at least a factor of 2 or 3 larger than the given thermal noise.

By a bandwidth limitation the thermal noise is decreased and therefore the position resolution, see discussion on BPM electronics. (With band-limited filters the signal shape is modified, with the help of the related frequency response functions  $H(f)$  the signal strength and shape can be calculated via  $Z_{tot}(f) = H(f) \cdot Z_t(f)$ .)

## 3 Answer to Signal Estimation of different BPM types

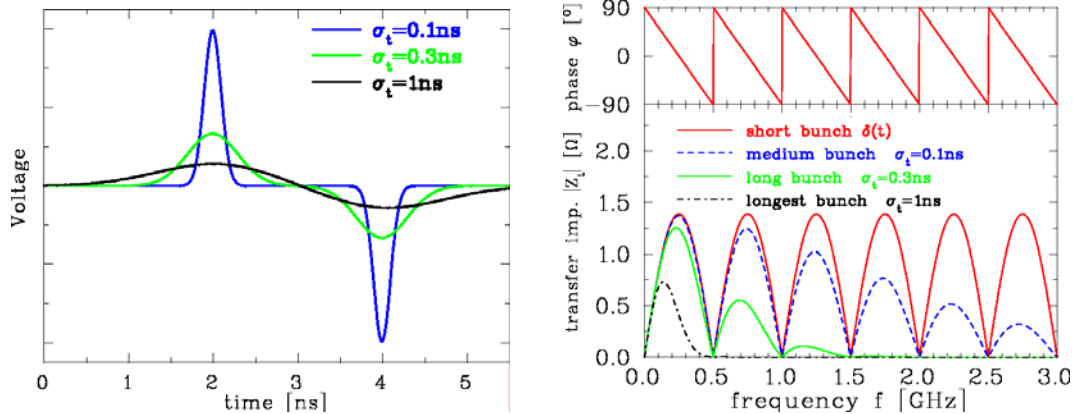
### 3.1 Answer

The signal shape was discussed for item 2.3.1.

### 3.2 Answer

The signals from the stripline are shown below together with the transfer impedance:

For the  $\sigma_t = 0.3 \text{ ns}$  bunch length overlapping of the direct and reflected pulse is just avoid, the  $\sigma_t = 1 \text{ ns}$  bunch length overlapping leads to a significant damping. This is equivalent to the spectrum of  $Z_t(f)$ , where the function of the short  $\delta$ -bunch has to be folded with the single bunch Fourier transformation. For longer bunches even the first maximum is lowered.

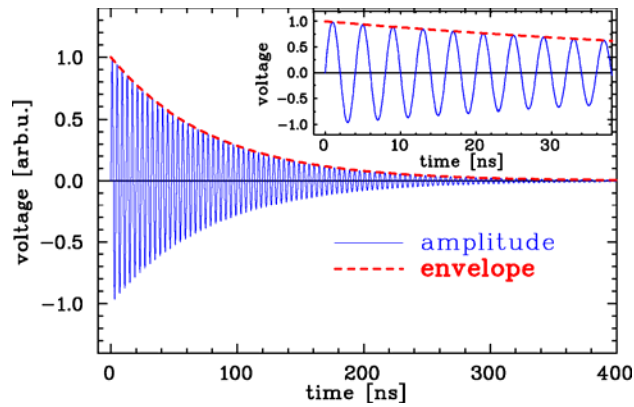


### 3.3 Answer

The passing bunch excites an oscillation as described by  $U_{signal}(t) = U_0 \cdot e^{-2\pi f/2Q \cdot t} \cdot \sin(2\pi f \cdot t)$  and shown below. For the given quality factor  $Q$ , the damping time is  $\tau = 2Q/2\pi f \approx 80 \text{ ns}$  (this time concerns the voltage, concerning power  $P \propto U^2$  it is a factor two shorter).

A larger value of  $Q$  results in a longer oscillation and therefore to higher available signal strength for the monopole and dipoles mode. Moreover, a small beam displacement leads to higher dipole mode power, which is equivalent to much larger position sensitivity as for button BPMs. However, a large  $Q$  reduces the time resolution and by that the possibility to resolve consecutive

pulses. Cavity BPMs are well suited for short beam pulses below 1  $\mu\text{s}$  in connection with a required resolution below 1  $\mu\text{m}$ .



### 3.4 Answer

Button BPMs are easier to produce and have simpler installation scheme. Stripline BPMs have lower signal deformation for short bunches and offer directivity, which is important only for collider operation with to counter-propagating beams within one beam pipe. Cavity BPMs have much higher position resolution for single bunches.