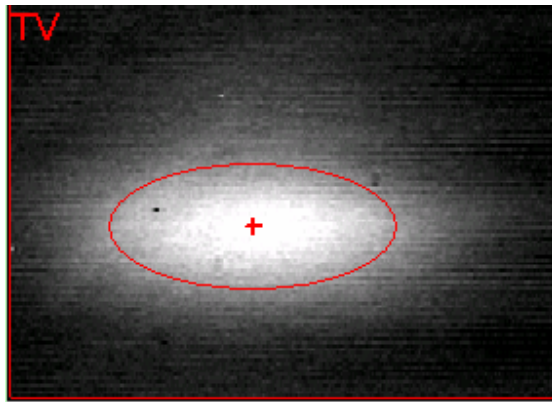


Solutions: Synchrotron Light Profile Monitor



by. K. Wittenburg, G. Kube

DESY

Solutions: Synchrotron light profile monitor

An Example: HERAe

$$R = \rho = 604814.3 \text{ mm}$$

$$G = O-L = 6485.5 \text{ mm}$$

$$B = L-Z = 1182.3 \text{ mm}$$

$$O-S1 = 6216 \text{ mm}$$

$$L = O_a-O_i = 1035 \text{ mm}$$

$$\text{opening angle (horizontal): } \tan\theta/2 = d/2/6216 \Rightarrow \theta/2 = \text{arc tan } d/2/6216 = \underline{0.85 \text{ mrad}}$$

$$\text{opening angle (vertikal): } \Psi(\lambda) = 1/\gamma (\lambda/\lambda_c)^{1/3}$$

with

$$\gamma = E/m_0c^2 = E [\text{MeV}]/0.511 = 23483 \text{ at } 12 \text{ GeV and } 52838 \text{ at } 27 \text{ GeV}$$

and

$$\lambda_c = \frac{4\pi\rho}{3\gamma^3} = 0.017 \text{ nm} < \lambda_c < 0.19 \text{ nm}$$

Exercise SR1: Which problems with the setup can be expected?:

Heating of mirror => total emitted Power per electron:

$$P = \frac{e^2 c \gamma^4}{6\pi\epsilon_0 \rho^2}$$

total Power of 46 mA circulating electrons at 27 GeV (Number of electrons $N_e = 6 \cdot 10^{12}$)

$$P_{\text{tot}} = 6 \cdot 10^6 \text{ W}$$

The mirror will get $P_{\text{tot}} \cdot \Theta / (2\pi) = 1600 \text{ W}$ (Integral over all wavelength!!!)

=> mirror is moveable, mirror has to be cooled

=> Material with low Z is nearly not visible for short wavelength => Beryllium

Exercise SR2: What limits the spatial resolution?

Diffraction, depth of field, arc, camera are physical reasons

Mirror and lenses, vibrations and alignment has to be made very precisely, => technical solutions

How to calibrate the optics?

Grid (yardstick) at point of emission, orbit bumps, ...

Diffraction:

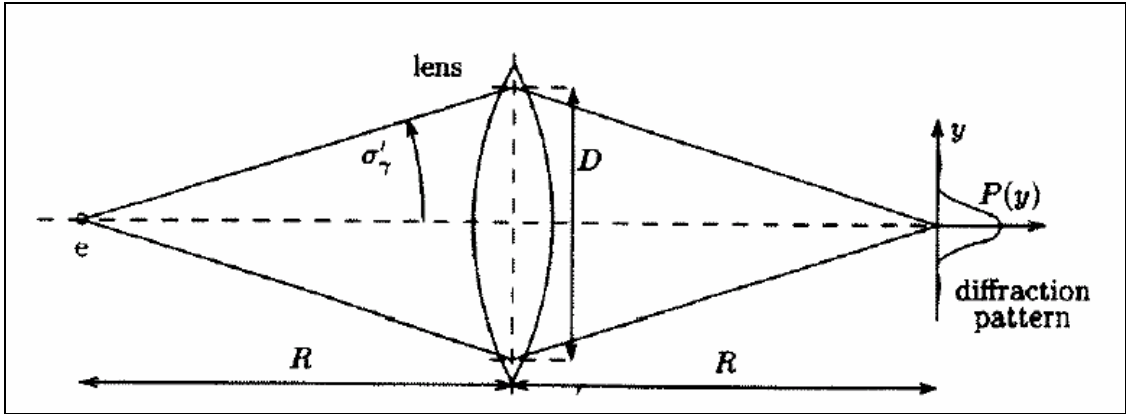
EQ 1:

Diffraction limit (for Object):

For normal slit:

$$\sigma_{\text{Diff}} = 0.47 * \lambda/\theta/2 \text{ (horizontal, mirror defines opening angle } \theta)$$

$$\sigma_{\text{Diff}} \approx 0.47 * \lambda/\Psi \text{ (vertikal)}$$



Depth of field:

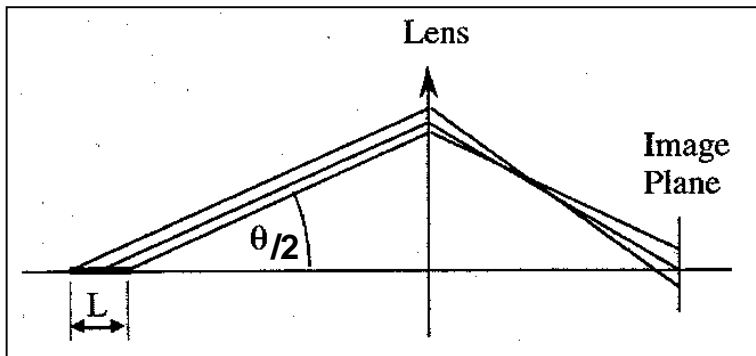
EQ 2:

depth of field:

Vertical: $\Delta_{\text{depth}} \approx L/2 * \Psi = \sigma_{\text{depth}}$

Horizontal: $\Delta_{\text{depth}} \approx L/2 * \theta/2 = \sigma_{\text{depth}}$ (mirror defines opening angle θ)

$L \approx \rho \tan\theta$ or $2\rho (\theta/2 + \Psi)$



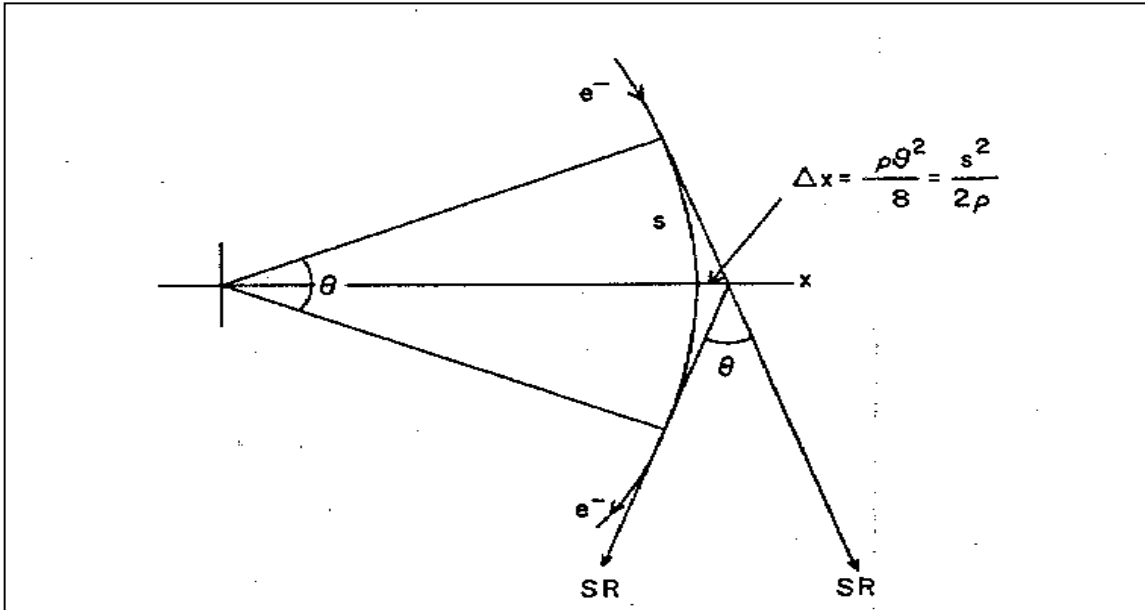
Arc:

EQ 3:

Arc (horizontal):

Observation of the beam in the horizontal plane is complicated by the fact that the light is emitted by all points along the arc. The horizontal width of the apparent source is related to the observation angle as:

$$\Delta x_{\text{arc}} = \rho \theta^2 / 8 = \sigma_{\text{arc}} \text{ (mirror defines opening angle } \theta \text{)}$$



Camera:

EQ 4:

Camera:

$$\text{image gain} = G/B = 5.485$$

$$\text{typical resolution of camera CCD chip: } \sigma_{\text{chip}} = 6.7 \mu\text{m}$$

$$\sigma_{\text{camera}} = \sigma_{\text{chip}} * G/B = 37 \mu\text{m}$$

λ not monochromatic !

$$\sigma_{\text{Diff}} = 0.47 * \lambda / \theta \text{ (horizontal)} = ???$$

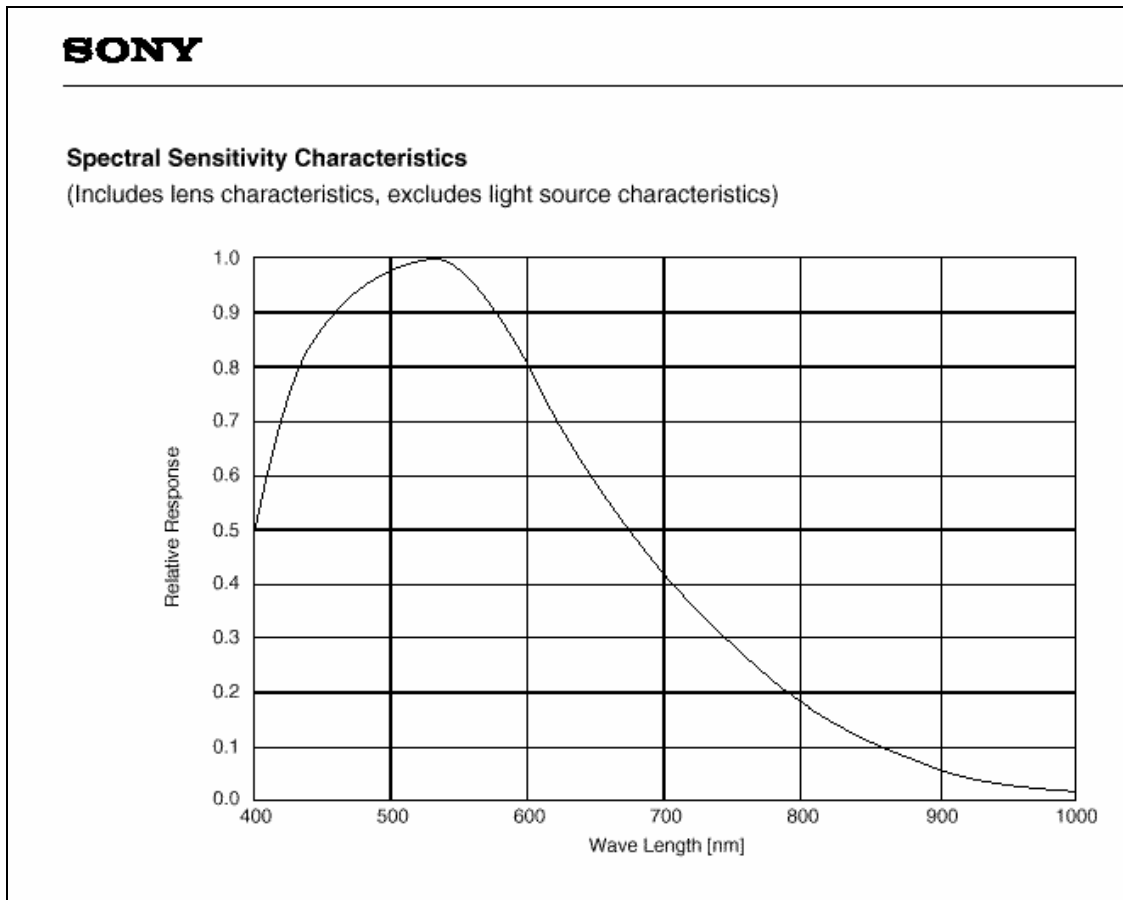
$$\sigma_{\text{Diff}} = 0.47 * \lambda / \Psi \text{ (vertikal)} = ???$$

$$\sigma_{\text{depth}} = L/2 * \theta/2 = 440 \mu\text{m}$$

$$\sigma_{\text{arc}} = \rho \theta^2 / 8 = 219 \mu\text{m (horizontal)}$$

$$\sigma_{\text{camera}} = \sigma_{\text{chip}} * G/B = 37 \mu\text{m}$$

typical spectral sensitivities from CCD Sensors:



Assume: $\lambda = 550 \text{ nm}$;

$$(\gamma = E/m_0c^2)$$

$$\gamma_{12} = 2.35 * 10^4 \text{ (E = 12 GeV)}$$

$$\gamma_{35} = 6.85 * 10^4 \text{ (E = 35 GeV)}$$

$$\lambda_{c,12} = (4\pi\rho)/(3\gamma^3) = 0.195 \text{ nm at 12 GeV}$$

$$\lambda_{c,35} = (4\pi\rho)/(3\gamma^3) = 0.008 \text{ nm at 35 GeV}$$

opening angle (horizontal): $\tan\theta/2 = d/2/6216 \Rightarrow \theta/2 = \text{arc tan } d/2/6216 = \underline{0.85 \text{ mrad}}$

opening angle (vertikal): $\Psi(\lambda) = 1/\gamma (\lambda/\lambda_c)^{1/3} = [(3\lambda)/(4\pi\rho)]^{1/3} = 0.6 \text{ mrad}$ (indep. on γ for $\lambda \gg \lambda_c$)

$$\sigma_{\text{diff}} = 0.47 * \lambda/\theta/2 = \underline{304 \mu\text{m}} \text{ (horizontal)}$$

$$\sigma_{\text{diff}} = 0.47 * \lambda/\Psi = \underline{431 \mu\text{m}} \text{ (vertical, mirror has to be larger than spot on mirror)}$$

$$\sigma_{\text{depth}} = L/2 * \theta/2 = \underline{440 \mu\text{m}}$$

$$\sigma_{\text{arc}} = \rho \theta^2/8 = \underline{219 \mu\text{m}} \text{ (horizontal)}$$

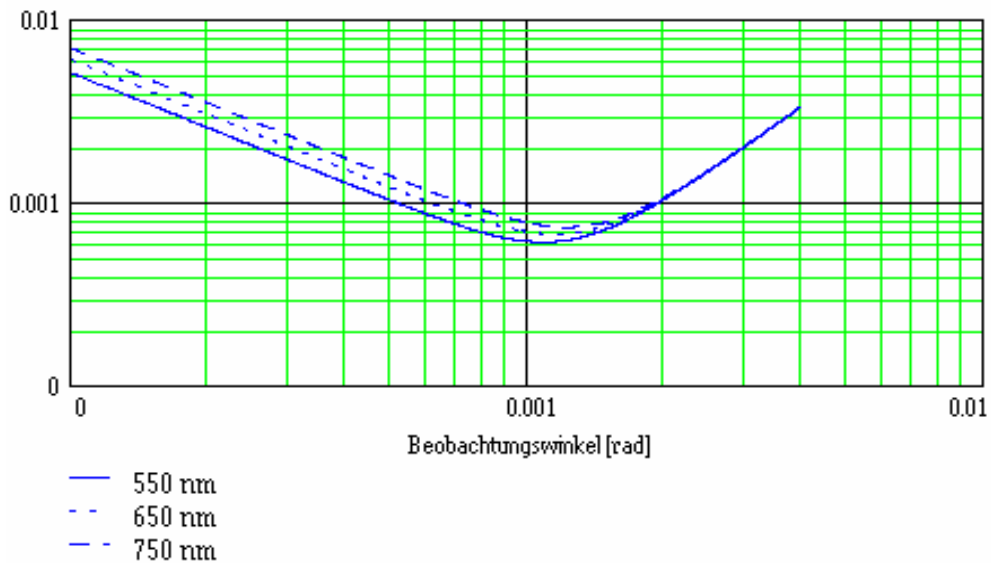
$$\sigma_{\text{camera}} = \sigma_{\text{chip}} * G/B = \underline{37 \mu\text{m}}$$

$$\sigma_{\text{cor}} = (\sigma_{\text{diff}}^2 + \sigma_{\text{depth}}^2 + \sigma_{\text{arc}}^2 + \sigma_{\text{camera}}^2)^{1/2} = 579 \mu\text{m} ; (\text{horizontal})$$

$$\sigma_{\text{cor}} = (\sigma_{\text{diff}}^2 + \sigma_{\text{depth}}^2 + \sigma_{\text{camera}}^2)^{1/2} = 617 \mu\text{m} ; (\text{vertical})$$

Horizontal:

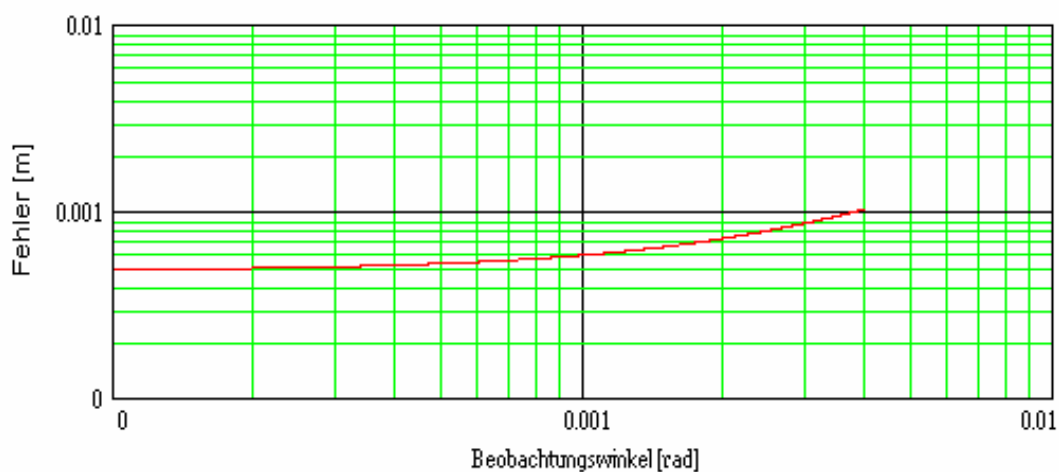
$$\text{resolution} = [(\rho \theta^2/8)^2 + (L/2 * \theta/2)^2 + (0.47 * \lambda/\theta/2)^2]^{1/2} \quad \text{with } L \approx \rho \tan \theta \approx \rho \theta$$



Minimum at $\theta = 1.05$ rad

Vertical:

$$\text{Resolution} = [(L/2 * \Psi)^2 + (0.47 * \lambda/\Psi)^2]^{1/2} \quad \text{with } L \approx \rho \tan \theta \approx \rho \theta$$



Not the whole truth:

1) Diffraction:

a) Ψ_{exact} is larger than the Gauss approximation (e.g. 0.79 \rightarrow 1.08 mrad at Tristan)

b) For a gaussian beam the diffraction width is $\sigma_{\text{diff}} \approx 1/\pi * \lambda/\Psi$

(Ref: **ON OPTICAL RESOLUTION OF BEAM SIZE MEASUREMENTS BY MEANS OF SYNCHROTRON RADIATION**. By A. Ogata (KEK, Tsukuba). 1991. Published in Nucl.Instrum.Meth.A301:596-598,1991)

$\Rightarrow \sigma_{\text{diff}} \approx 1/\pi * \lambda/\Psi_{\text{exact}} = \underline{218 \mu\text{m}}$ ($\Psi_{\text{exact}} = 0.8 \text{ mrad}$, $\lambda = 550 \text{ nm}$) vertical

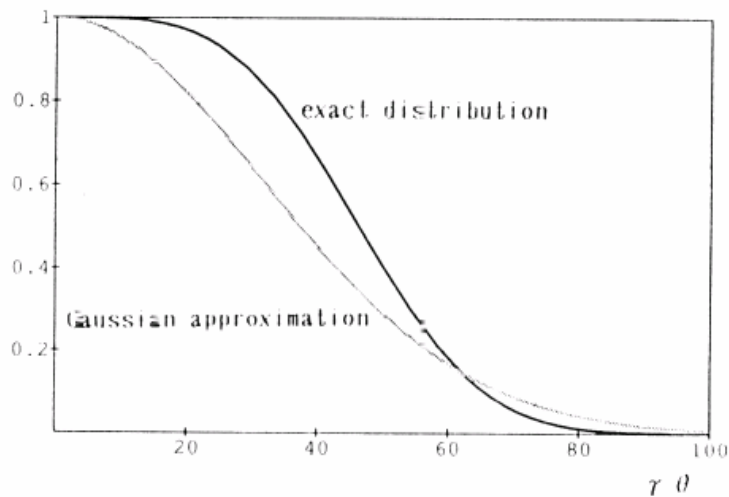
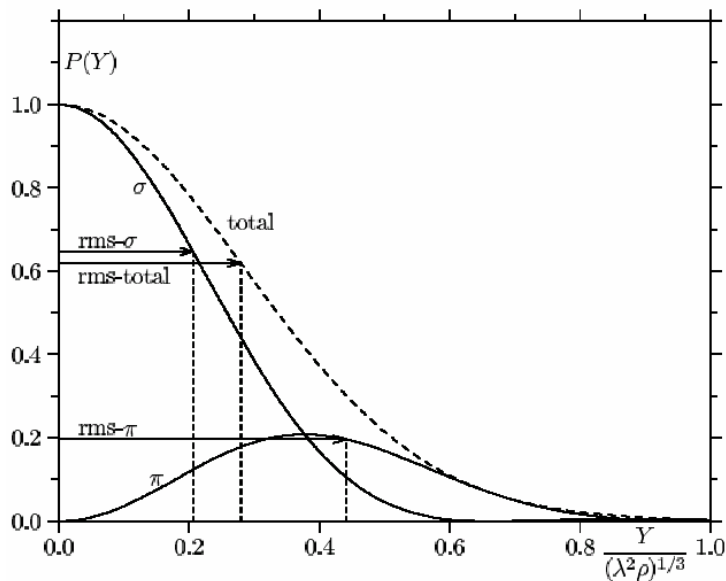


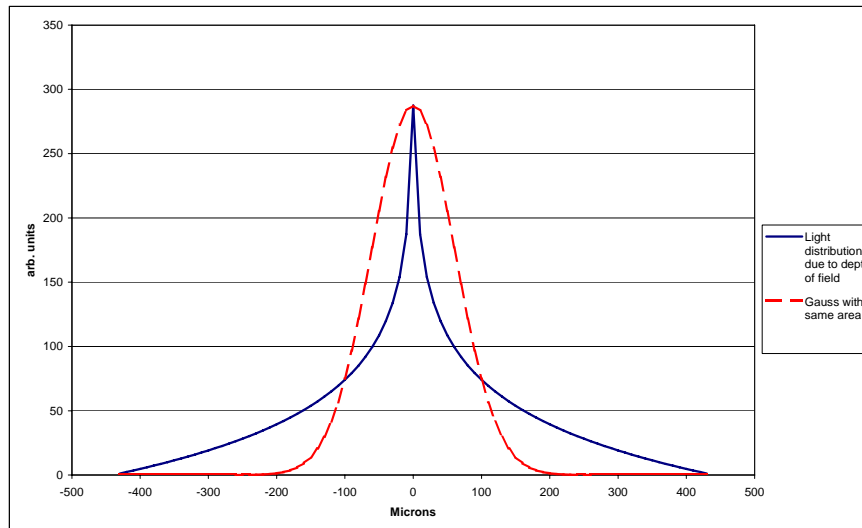
Fig. 1 Angular distribution of the 500 nm component of the synchrotron radiation from the TRISTAN MR bending magnet ($\approx 46 \text{ m}$ bending radius) operated at 30 GeV, and its Gaussian approximation.



8: Fraunhofer diffraction for synchrotron radiation from long magnets

2) Depth of field:

The formula $R_{\text{depth}} = L/2 * \theta/2$ describes the radius of the distribution due to the depth of field effect. It is not gaussian and has long tails. The resolution of an image is probably much better than the formula above. A gaussian approximation with the same integral is shown in the figure below resulting in a width of $\sigma_{\text{depth}} = 61 \mu\text{m}$.



$$\begin{aligned} \sigma_{\text{diff}} &= 0.47 * \lambda/\theta/2 &&= \underline{304 \mu\text{m}} \text{ (horizontal)} \\ \sigma_{\text{diff}} &= 1/\pi * \lambda/\Psi &&= \underline{218 \mu\text{m}} \text{ (vertical)} && \underline{(431 \mu\text{m})} \\ \sigma_{\text{depth}} &= L/2 * \theta/2 &&= \underline{61 \mu\text{m}} && \underline{(440 \mu\text{m})} \\ \sigma_{\text{arc}} &= \rho \theta^2/8 &&= \underline{219 \mu\text{m}} \text{ (horizontal)} \\ \sigma_{\text{camera}} &= \sigma_{\text{chip}} * G/B &&= \underline{37 \mu\text{m}} \end{aligned}$$

$$\begin{aligned} \sigma_{\text{cor}} &= (\sigma_{\text{diff}}^2 + \sigma_{\text{depth}}^2 + \sigma_{\text{arc}}^2 + \sigma_{\text{camera}}^2)^{1/2} &&= \underline{381 \mu\text{m}} ; \text{ (horizontal)} \\ \sigma_{\text{cor}} &= (\sigma_{\text{diff}}^2 + \sigma_{\text{depth}}^2 + \sigma_{\text{camera}}^2)^{1/2} &&= \underline{229 \mu\text{m}} ; \text{ (vertical)} \end{aligned}$$

$$\text{Beam width } \sigma_{\text{beam}} = (\sigma_{\text{fit_measured}}^2 - \sigma_{\text{cor}}^2)^{1/2}$$

Exercise SR3: Discuss possible improvements:

Monochromator at shorter wavelength

use optimum readout angle

Polarization - filter

use x-ray ($\lambda < 0.1 \text{ nm}$)

More:

Interferometer

The principle of measurement of the profile of an object by means of spatial coherency was first proposed by H.Fizeau and is now known as the Van Cittert-Zernike theorem. It

is well known that A.A. Michelson measured the angular dimension (extent) of a star with this method.

Referenzen

Characteristics of synchrotron radiation/ Hofmann, A ;

In: CAS - CERN Accelerator School : Synchrotron Radiation and Free Electron Lasers, Grenoble, France, 22 - 27 Apr 1996 - CERN, Geneva, 1998. [CERN-98-04]
<http://doc.cern.ch/archive/cernrep//1998/98-04/p303.pdf>

A Review of Optical Diagnostics Techniques for Beam Profile Measurements.

J A Clarke

http://www.astec.ac.uk/preprints/epac94/JAC-Profile_paper.pdf

Further readings:

Optical resolution of beam cross-section measurements by means of synchrotron radiation,

A. Hofmann, F. Meot, Nucl. Instr. Meth. 203 (1982) 483

Accurate and Efficient Computation of Synchrotron Radiation in the Near Field Region

O. Chubar, P. Elleaume, ESRF, Grenoble, France

Proc. EPAC 1998 Stockholm, Sweden

<http://accelconf.web.cern.ch/AccelConf/e98/PAPERS/THP01G.PDF>

Synchrotron Radiation Profile Monitor for HERA Positron Beam

G. Kube, Rainer Fischer, Kay Wittenburg - Deutsches Elektronen Synchrotron DESY

[11th Beam Instrumentation Workshop](#), May 3-6 2004, Knoxville, Tennessee, USA

Detailed Resolution Studies of the Synchrotron Radiation Profile Monitor for HERAe

Gero Kube, Rainer Fischer, Kay Wittenburg (DESY, Hamburg)

Proc. DIPAC 2005 Lyon

Interferometer:

SPATIAL COHERENCY OF THE SYNCHROTRON RADIATION AT THE VISIBLE LIGHT REGION AND ITS APPLICATION FOR THE ELECTRON BEAM PROFILE MEASUREMENT.

By T. Mitsuhashi (KEK, Tsukuba). KEK-PREPRINT-97-56, May 1997. 4pp. Talk given at 17th IEEE Particle Accelerator Conference (PAC 97): Accelerator Science, Technology and Applications, Vancouver, Canada, 12-16 May 1997.

Intensity Interferometer and its application to Beam Diagnostics, Elfim Gluskin, ANL, publ. PAC 1991 San Francisco

MEASUREMENT OF SMALL TRANSVERSE BEAM SIZE USING INTERFEROMETRY

T. Mitsuhashi

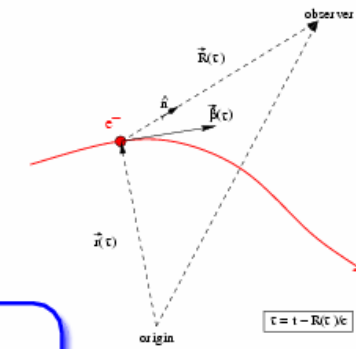
High Energy Accelerator Research Organisation, Oho, Tsukuba, Ibaraki, 305-0801 Japan

DIPAC 2001 Proceedings - ESRF, Grenoble

Still not the whole truth:

(Diffraction Limited) Resolution

Liénard –Wiechert potentials: $\varphi(t) = \left[\frac{-e}{R(1-\hat{n}\cdot\vec{\beta})} \right]_{ret}$, $\vec{A}(t) = \left[\frac{-e\vec{\beta}}{R(1-\hat{n}\cdot\vec{\beta})} \right]_{ret}$



Liénard –Wiechert fields:
 $\vec{E}(t) = -e \left(\frac{(1-\beta^2)(\hat{n}-\vec{\beta})}{R^2(1-\hat{n}\cdot\vec{\beta})^3} + \frac{\hat{n} \times [(\hat{n}-\vec{\beta}) \times \dot{\vec{\beta}}]}{cR(1-\hat{n}\cdot\vec{\beta})^3} \right)_{ret}$
 $\vec{H}(t) = \hat{n} \times \vec{E}(t)$

Fourier transform:
 $\varphi_\omega = -e \int \frac{1}{R(\tau)} e^{i\omega(\tau+R(\tau)/c)} d\tau$
 $\vec{A}_\omega = -e \int \frac{\vec{\beta}(\tau)}{R(\tau)} e^{i\omega(\tau+R(\tau)/c)} d\tau$

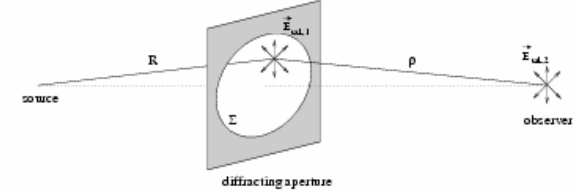
Fourier transform:
 $\vec{E}_\omega \approx -\frac{i\omega e}{cR} \int [\hat{n} \times [\hat{n} \times \vec{\beta}]] e^{i\omega(\tau+R(\tau)/c)} d\tau$

Electric field:
 $\vec{E}_\omega = -\frac{i\omega e}{c} \int \left[\frac{\vec{\beta}-\hat{n}}{R(\tau)} - \frac{ic}{\omega} \frac{\dot{\hat{n}}}{R^2(\tau)} \right] e^{i\omega(\tau+R(\tau)/c)} d\tau$

point charge on circular orbit
 ↔ analytical far field expressions for E_σ, E_π

numerical near field calculation depending on field geometry
 ↔ free codes available (SRW, SPECTRA, ...)

propagation in frame of scalar diffraction theory
 $\vec{E}_{\omega\perp,2} = -i \frac{\omega}{2\pi c} \int \int_\Sigma \frac{\vec{E}_{\omega\perp,1}}{\rho} \exp[i\omega/c\rho] d\Sigma$



spectral flux density
 $\frac{d^2 N_\omega}{dA d\omega/\omega} = \frac{\alpha c^2}{4\pi^2 e^2} |\vec{E}_{\omega\perp,2}|^2$

- Fraunhofer diffraction pattern for spherical wave
 ↔ depth of field, orbit curvature: additional contributions

- accounts for the fact that SR is not actually a spherical wave
 ↔ depth of field and orbit curvature included

A. Hofmann and F. Méot, Nucl. Instr. Meth. **203** (1982) 483

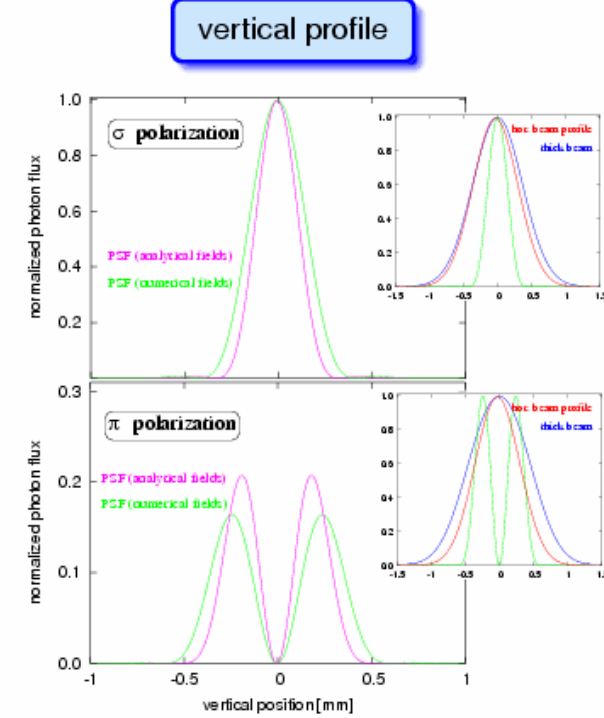
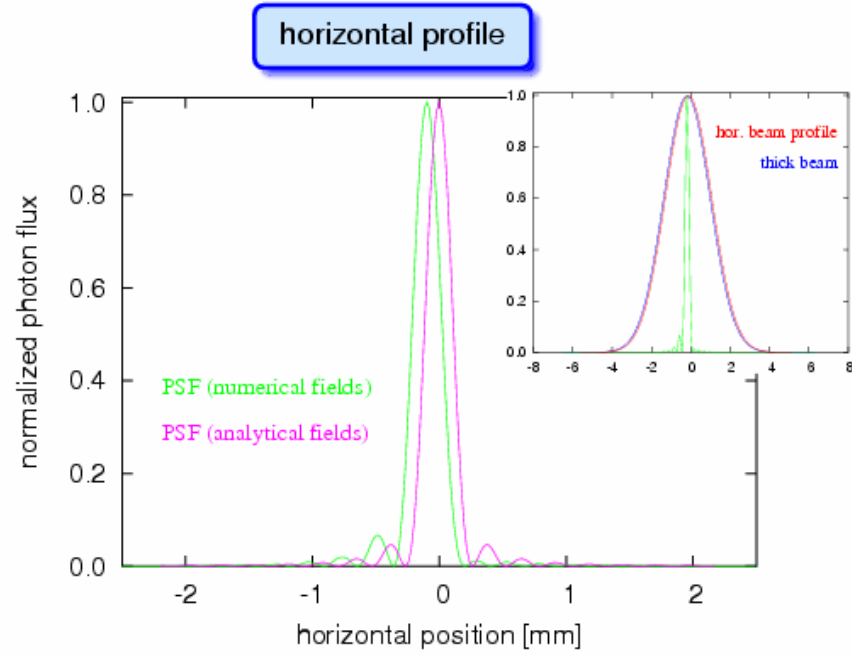
O. Chubar and P. Elleaume, Proceedings of the EPAC98, Stockholm, Sweden (1998) 1177

Resolution Broadening Effects for the HERAe Emittance Monitor

determination:

- calculation of spatial SR intensity distribution including beam emittance
- quadratical subtraction of beam size

For ∞ mirror size



● broadening contribution: $\sigma_{res} = 203 \mu\text{m}$

numerical

● broadening contribution: $\sigma_{res} = 138 \mu\text{m}$

● $\sigma_{diffract} = 188 \mu\text{m}$
 $\sigma_{dof+curv} = 275 \mu\text{m}$

\leftrightarrow $\sigma_{res} = 333 \mu\text{m}$

analytical

● $\sigma_{diffract} = 106 \mu\text{m}$
 $\sigma_{dof} = 134 \mu\text{m}$

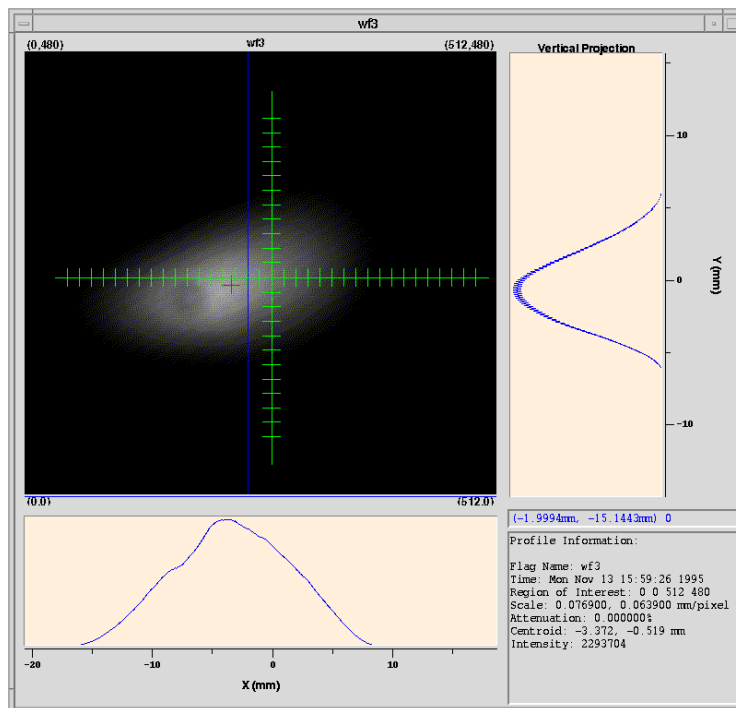
\leftrightarrow $\sigma_{res} = 171 \mu\text{m}$

381 mm

229 mm

- G. Kube, R. Fischer, K. Wittenburg, in *Proceedings of BIW2004*, AIP Conf. Proc. **732** (2004), p.350–357
- G. Kube, R. Fischer, Ch. Wiebers, K. Wittenburg, in *Proceedings of DIPAC2005* (in press)

Solutions: LINACS/Transport Lines Emittance Measurement



by Kay Wittenburg, -DESY-

Solutions: LINACS/Transport Lines Emittance Measurement

1) **Explain ways of measuring the emittance of a charged particle beam in a Linear accelerator or a transport line without knowing the beam optic parameters α , β , γ .**

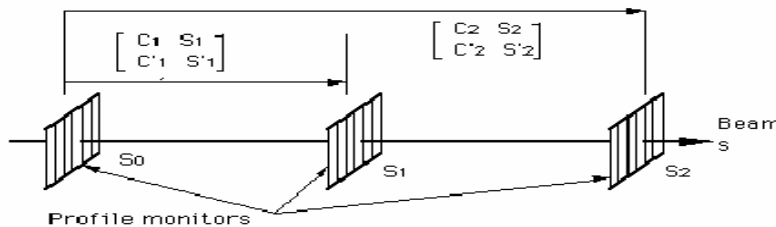
a) Exercise L1: Which one is the preferable method for a high energy proton transport line ($p > 5 \text{ GeV}/c$)?

Solution: 3 (thin) screens/SEM grids or varying quadrupole which measure the different beam widths σ . For pepper pot or slits one needs a full absorbing aperture.

b) Exercise L2: Assuming that the geometry between the measurement stations and the transport matrices M of the transport line are well defined (including magnetic elements), describe a way to get the emittance using 3 screens and the σ -matrix.

$$\sigma = \begin{pmatrix} \sigma_{11} & \sigma_{12} \\ \sigma_{21} & \sigma_{22} \end{pmatrix} = \begin{pmatrix} \sigma_y^2 & \sigma_{yy'} \\ \sigma_{yy'} & \sigma_{y'}^2 \end{pmatrix} = \varepsilon_{rms} \begin{pmatrix} \beta & -\alpha \\ -\alpha & \gamma \end{pmatrix} = \sigma \text{ matrix}$$

If β is known unambiguously as in a circular machine, then a single profile measurement determines ε by $\sigma_y^2 = \varepsilon\beta$. But it is not easy to be sure in a transfer line which β to use, or rather, whether the beam that has been measured is matched to the β -values used for the line. This problem can be resolved by using three monitors (see Fig. 1), i.e. the three width measurements determine the three unknowns α , β and ε of the incoming beam.



σ elements at first Screen or Quadrupole (Ref. 1).

$$\sigma = \begin{pmatrix} \sigma_{11} & \sigma_{12} \\ \sigma_{21} & \sigma_{22} \end{pmatrix} = \begin{pmatrix} \sigma_y^2 & \sigma_{yy'} \\ \sigma_{yy'} & \sigma_{y'}^2 \end{pmatrix} = \varepsilon_{rms} \begin{pmatrix} \beta & -\alpha \\ -\alpha & \gamma \end{pmatrix} = \sigma \text{ matrix}$$

Beam width_{rms} of measured profile = $\sigma_{11} = \sqrt{\beta(s) \cdot \varepsilon}$,

L_1, L_2 = distances between screens or from Quadrupole to screen and Quadrupole field strength are given, therefore the transport matrix M is known.

Employing transfer matrix gives: $M \cdot \sigma \cdot M^t$

$$\begin{pmatrix} M_{11} & M_{12} \\ M_{21} & M_{22} \end{pmatrix} \cdot \begin{pmatrix} \sigma_{11} & \sigma_{12} \\ \sigma_{21} & \sigma_{22} \end{pmatrix} \cdot \begin{pmatrix} M_{11} & M_{21} \\ M_{12} & M_{22} \end{pmatrix} = \sigma^{measured} = \begin{pmatrix} \sigma_y^2 & \sigma_{yy'} \\ \sigma_{y'y} & \sigma_{y'}^2 \end{pmatrix}^{measured} = \varepsilon_{rms} \begin{pmatrix} \beta & -\alpha \\ -\alpha & \gamma \end{pmatrix}$$

$$\begin{aligned}
&= \begin{pmatrix} M_{11} & M_{12} \\ M_{21} & M_{22} \end{pmatrix} \cdot \begin{pmatrix} \sigma_{11}M_{11} + \sigma_{12}M_{12} & \sigma_{11}M_{21} + \sigma_{12}M_{22} \\ \sigma_{21}M_{11} + \sigma_{22}M_{12} & \sigma_{21}M_{21} + \sigma_{22}M_{22} \end{pmatrix} \\
&= \begin{pmatrix} M_{11}(\sigma_{11}M_{11} + \sigma_{12}M_{12}) + M_{12}(\sigma_{21}M_{11} + \sigma_{22}M_{12}) & \dots \\ \dots & \dots \end{pmatrix}
\end{aligned}$$

$$\sigma_{11}^{\text{measured}} = M_{11}^2 \sigma_{11} + 2M_{11} M_{12} \sigma_{12} + M_{12}^2 \sigma_{22} \quad (\sigma_{12} = \sigma_{21}) \quad (1)$$

Solving σ_{11} σ_{12} and σ_{22} while Matrix elements are known: Needs minimum of three different measurements, either three screens or three different Quadrupole settings with different field strength.

$$\varepsilon_{rms} = \sqrt{\det \sigma} = \sqrt{\sigma_{11}\sigma_{22} - \sigma_{12}^2} \quad (\text{from } \beta\gamma - \alpha^2 = 1) \quad (2)$$

- c) Exercise L3: In a transport line for $p = 7.5$ GeV/c protons are two measurement stations. The first is located exactly in the waist of the beam and shows a beam width of $\sigma_y = 3$ mm, the second at a distance of $s = 10$ m shows a width of $\sigma_y = 9$ mm. Assuming no optical elements in this part, calculate the emittance and the normalized emittance of the beam.

$$\text{No optical elements} \Rightarrow M = \begin{pmatrix} 1 & s \\ 0 & 1 \end{pmatrix} \quad (3)$$

$$\text{Waist} \Rightarrow \alpha = \sigma_{12} = \sigma_{21} = 0 \quad \Rightarrow \varepsilon_{rms} = \sqrt{\sigma_{11}\sigma_{22}} \quad (4)$$

Momentum $p = 7.5$ GeV/c \Rightarrow relativistic $\gamma\beta \approx 7.5$

$$\text{Measured width at } s = 0 \Rightarrow (3 \text{ mm})^2 = \sigma_y^2 (s=0) = \sigma_{11} \quad (5)$$

$$\text{Calculate } \sigma_{22} \text{ with width measured at } s = 10 \text{ m and with (1, 4)} \Rightarrow (9 \text{ mm})^2 = \sigma_y^2 (s=10) = M_{11}^2 \cdot \sigma_{11} + M_{12}^2 \cdot \sigma_{22} = \sigma_{11} + s^2 \cdot \sigma_{22} \quad (\sigma_{11}, \sigma_{22} \text{ at } s=0) \quad (6)$$

$$\text{with (5)} \Rightarrow \sigma_{22} = \frac{\sigma_y^2(10) - \sigma_y^2(0)}{s^2} \quad (7)$$

With (4) and (7) \Rightarrow

$$\begin{aligned}
\varepsilon_{rms} &= \sqrt{\sigma_{11}\sigma_{22}} = \sqrt{\sigma_y^2(0) \cdot \frac{\sigma_y^2(10) - \sigma_y^2(0)}{s^2}} = \frac{\sigma_y(0)}{s} \sqrt{\sigma_y^2(10) - \sigma_y^2(0)} \\
&= \underline{2.5 \cdot 10^{-6} \text{ m rad}}
\end{aligned}$$

$$\varepsilon^{\text{normalized}} = \varepsilon_{rms} \gamma \beta = 19 \cdot 10^{-6} \text{ m rad} = \underline{19 \text{ mm mrad}}$$

Additional exercise: Calculate $\beta(s=0$ and $s=10\text{m})$

$$\text{Beam width } \sigma_{\text{rms}} = \sqrt{\beta(s) \cdot \varepsilon}$$

$$\text{At } s=10 \text{ m: } \sigma^2 = \beta\varepsilon \Rightarrow \beta = 32.4 \text{ m}$$

$$\text{At } s=0 \text{ m : } \beta = 3.6 \text{ m}$$

What is the influence on the emittance ε assuming at $s = 10\text{m}$ this β , a dispersion of $D = 1 \text{ m}$ and a momentum spread of $\Delta p/p = 10^{-3}$?

$$\varepsilon = \frac{\sigma^2 - \left(D \cdot \frac{\Delta p}{p}\right)^2}{\beta} = \frac{81 \cdot 10^{-6} - 1 \cdot 10^{-6}}{32.4} = 2.469 \pi \text{ mm mrad}$$

or $\approx 1\%$ which is less than the typical accuracy of a profile measurement

References

Ref. 1: S.Y. Lee, Accelerator Physics, World Scientific, pp 54-55 is attached

Ref. 2: Emittance Measurements at the Bates Linac

K.D. Jacobs, J.B. Flanz, T. Russ

http://accelconf.web.cern.ch/accelconf/p89/PDF/PAC1989_1529.PDF

attached

Ref. 3: Basic accelerator course, like Schmüser and Rossbach in previous CAS CERN 94-01 v 1

<http://doc.cern.ch/cgi-bin/tiff2pdf?/archive/cernrep/1994/94-01/p17.tif>

Ref. 4: Beam-line instruments, [Raich, U](#) ; *In: Joint US-CERN-Japan-Russia Particle Accelerators School on Beam Measurement, Montreux, Switzerland, 11-20 May 1998 - World Sci., Singapore, 1999. - pp.263-276*

For solutions attached: Criegee, PLIN note 88-04

Emittance Measurements for Linac III

Principle

The measurement of the *Courant-Snyder* beam parameters α , β , γ , ϵ is based on the evolution of the beam matrix σ . If the transport is described by the 2×2 matrix

$$R = \begin{pmatrix} R_{11} & R_{12} \\ R_{21} & R_{22} \end{pmatrix},$$

with $R_{11}R_{22} - R_{12}R_{21} = 1$, the beam evolution is given by¹

$$\sigma = \begin{pmatrix} \epsilon\beta & -\epsilon\alpha \\ -\epsilon\alpha & \epsilon\gamma \end{pmatrix} = R \begin{pmatrix} \epsilon\beta_0 & -\epsilon\alpha_0 \\ -\epsilon\alpha_0 & \epsilon\gamma_0 \end{pmatrix} R^T.$$

$\epsilon\beta$ is the measurable squared beam envelope \hat{x}^2 at the end of the transport. It depends linearly on the initial beam matrix elements:

$$\hat{x}^2 = \epsilon\beta = R_{11}^2 \cdot \epsilon\beta_0 - 2R_{11}R_{12} \cdot \epsilon\alpha_0 + R_{12}^2 \cdot \epsilon\gamma_0$$

To obtain $\epsilon\alpha_0$, $\epsilon\beta_0$, $\epsilon\gamma_0$, and $\epsilon^2 = \epsilon\beta_0 \cdot \epsilon\gamma_0 - \epsilon\alpha_0 \cdot \epsilon\alpha_0$, one has set R to three (sufficiently different) values, measure the resulting \hat{x}^2 , and solve three linear equations.

3-Position Method

One simple example is the simultaneous measurement with three profile harps as foreseen for the HEBT. Here the transport matrix

$$R(S) = \begin{pmatrix} 1 & S \\ 0 & 1 \end{pmatrix}$$

describes just a variable drift space. The three settings $S_1 = -L$, $S_2 = 0$, $S_3 = +L$ give the beam transport from a reference position S_2 to two symmetric ones up- and downstream. They lead to the equations

$$\left. \begin{aligned} \hat{x}_1^2 &= \epsilon\beta_2 - 2L \cdot \epsilon\alpha_2 + L^2 \cdot \epsilon\gamma_2 \\ \hat{x}_2^2 &= \epsilon\beta_2 \\ \hat{x}_3^2 &= \epsilon\beta_2 + 2L \cdot \epsilon\alpha_2 + L^2 \cdot \epsilon\gamma_2 \end{aligned} \right\} \text{with the solution } \begin{cases} \epsilon\beta_2 &= \hat{x}_2^2 \\ \epsilon\alpha_2 &= (\hat{x}_3^2 - \hat{x}_1^2)/(4L) \\ \epsilon\gamma_2 &= (\hat{x}_1^2 - 2\hat{x}_2^2 + \hat{x}_3^2)/(2L^2) \\ \epsilon^2 &= \epsilon\beta_2 \cdot \epsilon\gamma_2 - (\epsilon\alpha_2)^2 \end{cases}$$

The measurements should be done in the vicinity of a beam waist where the curvature due to $\epsilon\gamma$ is most noticeable. If the beam waist is exactly at S_2 , we have $\hat{x}_3 = \hat{x}_1$, $\alpha_2 = 0$. The emittance formula is then reduced to

$$\epsilon = \hat{x}_2 \sqrt{\hat{x}_3^2 - \hat{x}_2^2} / L \quad (\text{for } \hat{x}_2 = \text{minimum})$$

¹see K.L.Brown et al., CERN 80-04 (TRANSPORT manual)

$$p \cdot j - \alpha^2 = 1$$

3-Gradient Method

If the 3-Position method cannot be used, the beam parameters can be measured by varying the gradient of a quadrupole. The system of the quadrupole and a subsequent transport section with constant elements C , S , C' and S' ($CS' - C'S = 1$) is described by

$$R(k) = \begin{pmatrix} C & S \\ C' & S' \end{pmatrix} \begin{pmatrix} \cos kl & k^{-1} \sin kl \\ -k \sin kl & \cos kl \end{pmatrix} \text{ with } \begin{cases} R_{11}(k) = C \cos kl - Sk \sin kl \\ R_{12}(k) = Ck^{-1} \sin kl + S \cos kl \end{cases}$$

The measurement of \hat{x}^2 , as defined above, for three different quadrupole settings k yields the linear equations to determine all beam parameters.

Simplified 3-Gradient Method

The function $\hat{x}^2(k)$ is greatly simplified² if the quadrupole can be described as a thin lense ($kl \rightarrow 0$, $k^2l \rightarrow q = f^{-1}$):

$$\begin{aligned} R_{11}(k) &\rightarrow R_{11}(q) = C - q \cdot S \\ R_{12}(k) &\rightarrow R_{12}(q) = S \end{aligned}$$

This leads to

$$\begin{aligned} \hat{x}^2(q) &= (C - q \cdot S)^2 \cdot \epsilon \beta_0 - 2S(C - qS) \cdot \epsilon \alpha_0 + S^2 \cdot \epsilon \gamma_0 \\ &= S^2 \epsilon^2 / \hat{x}_0^2 + \hat{x}_0^2 \cdot (q - q_{min})^2 . \end{aligned}$$

with $q_{min} = C/S - \alpha_0/\beta_0$ being the setting for minimal \hat{x}^2 , and $\hat{x}_0^2 = \epsilon \beta_0$ the squared envelope before the quadrupole. The emittance is then given by

$$\epsilon = \frac{\hat{x}(q_{min}) \sqrt{\hat{x}^2(q) - \hat{x}^2(q_{min})}}{S^2 |q - q_{min}|} .$$

The other beam parameters (at the entrance of the quadrupole) are

$$\begin{aligned} \hat{x}_0 &= \sqrt{\hat{x}^2(q) - \hat{x}^2(q_{min})} / (S |q - q_{min}|) \\ \beta_0 &= \hat{x}_0^2 / \epsilon \\ \alpha_0 &= \beta_0 (C/S - q_{min}) . \end{aligned}$$

Some comments may be in order:

1. The formula for the emittance is simple enough to be coded in POCAL.
2. The emittance measurement after linac tank I forseees the variation of quad No 52, while No 53 is turned off³. The above formalism allows also measurements with No 53 powered (fixed $C' \neq 0$) and also to account for the defocussing by (linear) space charge and by RF acceleration.
3. Suitable quadrupole settings and the quality of the approximations can be investigated by beam transport calculations.

²G.Jacobs, private communication

³S.H. Wang, PLIN - Note 88-01 (June 24, 1988)

An other method: (P.J. Bryant, 5th CAS, Finland)

By definition, Eq. (4),

$$\varepsilon = \pi \frac{\sigma_0^2}{\beta_0} = \pi \frac{\sigma_1^2}{\beta_1} = \pi \frac{\sigma_2^2}{\beta_2} \quad (5)$$

where β_0 , β_1 and β_2 are the β -values corresponding to the beam and are therefore uncertain. Although we may not know β and α , we do know the transfer matrices and how β and α propagate through the structure (see lectures by K. Steffen in these proceedings).

$$\begin{pmatrix} \beta \\ \alpha \\ \gamma \end{pmatrix}_1 = \begin{pmatrix} C^2 & -2CS & S^2 \\ -CC' & CS' + SC' & -SS' \\ C'^2 & -2C'S' & S'^2 \end{pmatrix} \begin{pmatrix} \beta \\ \alpha \\ \gamma \end{pmatrix}_0 \quad (6)$$

where $\gamma = (1 + \alpha^2)/\beta$. Thus, from Eq. (6)

$$\beta_1 = C_1^2 \beta_0 - 2C_1 S_1 \alpha_0 + \frac{S_1^2}{\beta_0} (1 + \alpha_0^2) \quad (7)$$

$$\beta_2 = C_2^2 \beta_0 - 2C_2 S_2 \alpha_0 + \frac{S_2^2}{\beta_0} (1 + \alpha_0^2) \quad (8)$$

and from Eq. (5),

$$\beta_0 = \pi \frac{\sigma_0^2}{\varepsilon} \quad (9)$$

$$\beta_1 = \left(\frac{\sigma_1}{\sigma_0} \right)^2 \beta_0 \quad (10)$$

$$\beta_2 = \left(\frac{\sigma_2}{\sigma_0} \right)^2 \beta_0 \quad (11)$$

From Eqs. (7) and (8), we can find α_0 and using Eqs. (10) and (11), we can express α_0 as,

$$\alpha_0 = \frac{1}{2} \beta_0 \Gamma \quad (12)$$

where

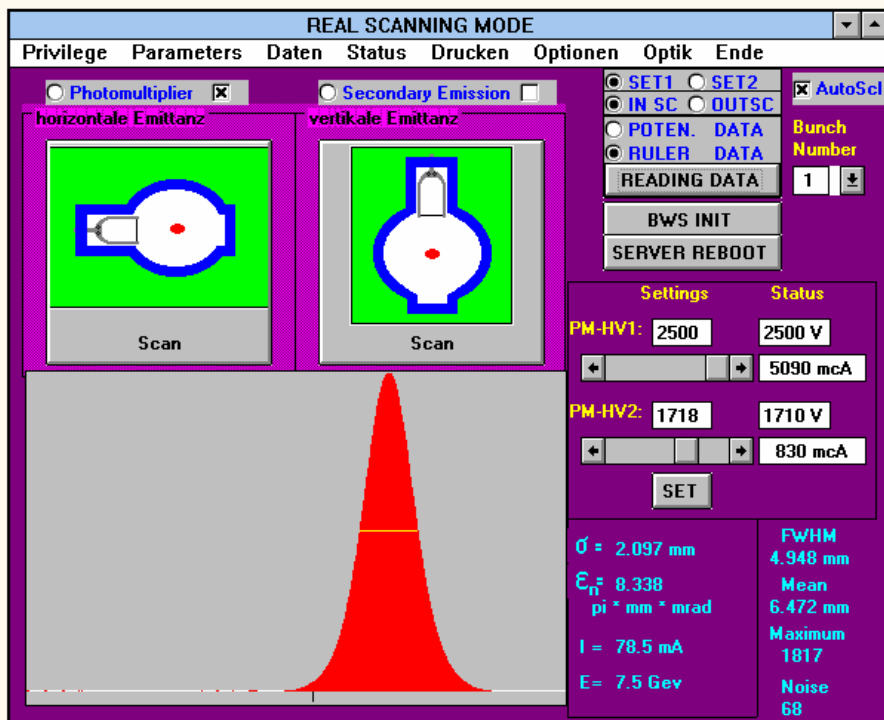
$$\Gamma = \frac{(\sigma_2 / \sigma_0)^2 / S_2^2 - (\sigma_1 / \sigma_0)^2 / S_1^2 - (C_2 / S_2)^2 + (C_1 / S_1)^2}{(C_1 / S_1) - (C_2 / S_2)}. \quad (13)$$

Since Γ is fully determined, direct substitution back into Eq. (7) or Eq. (8), using Eq. (10) or Eq. (11) to re-express β_1 or β_2 , yields β_0 which via Eq. (9) gives the emittance,

$$\beta_0 = 1 / \sqrt{\left(\frac{\sigma_1}{\sigma_0} \right)^2 / S_1^2 - \left(\frac{C_1}{S_1} \right)^2 + \left(\frac{C_1}{S_1} \right) \Gamma - \Gamma^2 / 4} \quad (14A)$$

$$\varepsilon = (\pi \sigma_0^2) \sqrt{\left(\frac{\sigma_2}{\sigma_0} \right)^2 / S_2^2 - \left(\frac{C_2}{S_2} \right)^2 + \left(\frac{C_2}{S_2} \right) \Gamma - \Gamma^2 / 4}. \quad (14B)$$

Solutions: Wire Scanners



By Kay Wittenburg, -DESY-

Solutions: Wire Scanner

Exercise WIRE: Discuss where one should locate the Scintillator in case of a proton and an electron accelerator? Do you expect any difference?

Projected angular distribution could be approximated by Gaussian with a width given by:

$$\Theta_{mean} = \frac{0.014 \text{ GeV}}{pc} \cdot \sqrt{\frac{d'}{L_{rad}}} \cdot \left(1 + 1/9 \cdot \log_{10} \frac{d'}{L_{rad}} \right)$$

$d' = 1.5 \times 10^{-3}$ cm – the thickness of the target, $X_0 = 12.3$ cm – quartz-wire radiation length, $x/X_0 = 1.22 \times 10^{-4}$

Note that the angle depends on the momentum of the particle!

It is corresponding to:

$$\Theta_{mean} \approx 3.0 \times 10^{-6} \text{ rad}$$

for electron momentum of 30 GeV/c.

Scattered particles will arrive a vacuum chamber of radius $R = 2$ cm at:

$$z \approx \frac{R}{\Theta_{mean} \sqrt{2}} \approx 4.9 \text{ km!!!!}$$

What to do?

Do Monte Carlo simulations of best location for scintillators. Simulation should include all magnetic fields as well as all elastic and inelastic scattering cross sections. For protons the inelastic cross section is very high, therefore one can locate the detector (scintillator) close to the scanner, while for electrons one has to calculate (simulate) the best location.

Higher Order Modes

Exercise WIRE1: Discuss methods of proving this behavior. What are possible solutions against the RF coupling?

Methods:

- a) Measurement of wire resistivity
- b) Measurement of thermo-ionic emission
- c) Optical observation of glowing wire
- d) Measurement of RF coupling in Laboratory with spectrum analyzer

a) Measurement of wire resistivity

The wire resistivity will change depending on the temperature of the wire, even without scanning.

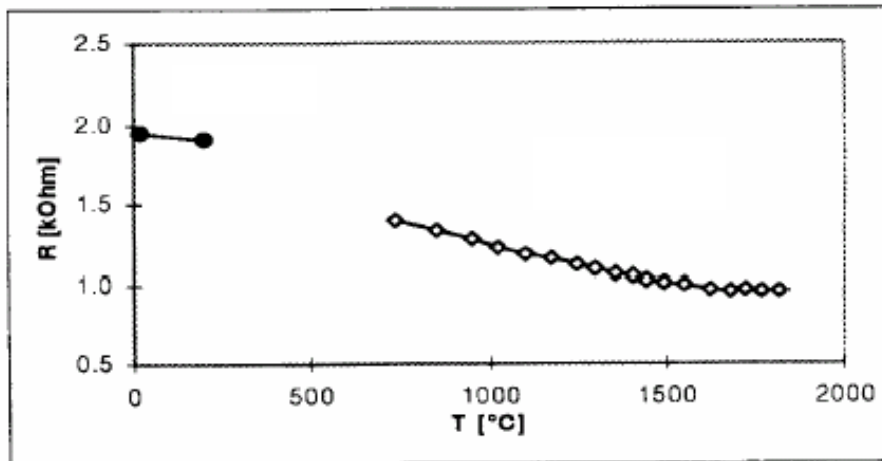


Fig 4: Measured wire resistance variations with temperature.

Here: 8 μm Carbon wire

(from [OBSERVATION OF THERMAL EFFECTS ON THE LEP WIRE SCANNERS](#). By J. Camas, C. Fischer, J.J. Gras, R. Jung, J. Koopman (CERN). CERN-SL-95-20-BI, May 1995. 4pp. Presented at the 16th Particle Accelerator Conference - PAC 95, Dallas, TX, USA, 1 - 5 May 1995. Published in IEEE PAC 1995:2649-2651)

b) Measurement of thermoionic emission

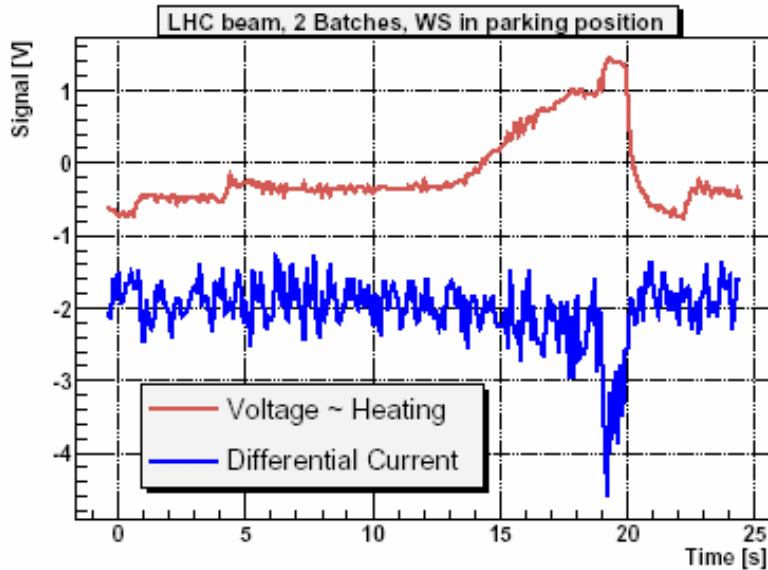


Figure WIRE5: Wire heating due to the LHC beam injection in the SPS (No scan, wire in parking position). The beam energy ramp/bunch length decreasing begin t=11 s.

A constant current was supplied to the wire and the voltage drop across it was fed to a digital scope together with the difference between the input and output currents. The differential current ($I_{out}-I_{in}$) grow up is due to the wire heating and consequent emission of electrons for thermionic effect. Fig. WIRE5 shows such voltage and differential current evolutions during the SPS cycle with LHC type beam. No scans were performed along this cycle. It is thus evident that the wire heating does not depend on the direct wire-beam interaction only.

(From [CAVITY MODE RELATED WIRE BREAKING OF THE SPS WIRE SCANNERS AND LOSS MEASUREMENTS OF WIRE MATERIALS](#)

F. Caspers, B. Dehning, E.Jensen, J. Koopman, J.F. Malo, CERN, Geneva, Switzerland
F. Roncarolo, CERN/University of Lausanne, Switzerland; DIPAC03)

c) Optical observation of glowing wire

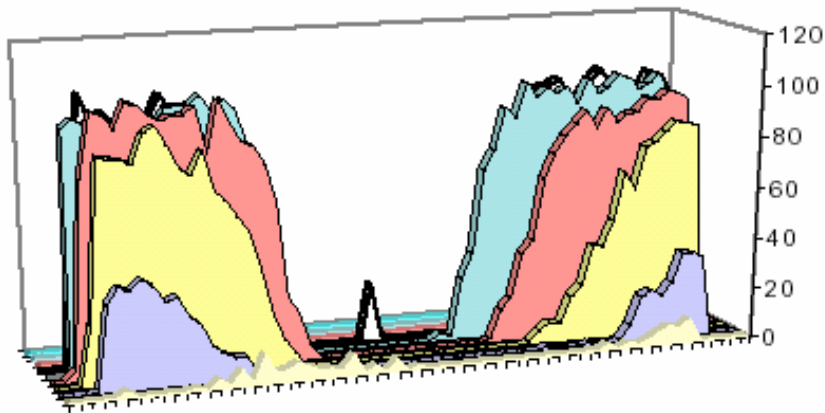


Figure WIRE6: Digitized video recording of an 8 μm carbon wire scanning a 0.8 mA beam. The wire is parallel to the horizontal axis, and the light intensity is plotted along the vertical axis (arbitrary units). Successive profiles are separated by 20 ms. The central spot corresponds to the passage of the wire through the beam. Thus, RF heating led to (huge) thermal glowing before the beam interacts with the wire.

(from: QUARTZ WIRES VERSUS CARBON FIBERS FOR IMPROVED BEAM HANDLING CAPACITY OF THE LEP WIRE SCANNERS.

By C. Fischer, R. Jung, J. Koopman (CERN). CERN-SL-96-09-BI, May 1996. 8pp. Talk given at 7th Beam Instrumentation Workshop (BIW 96), Argonne, IL, 6-9 May 1996.

d) Measurement of RF coupling with spectrum analyzer

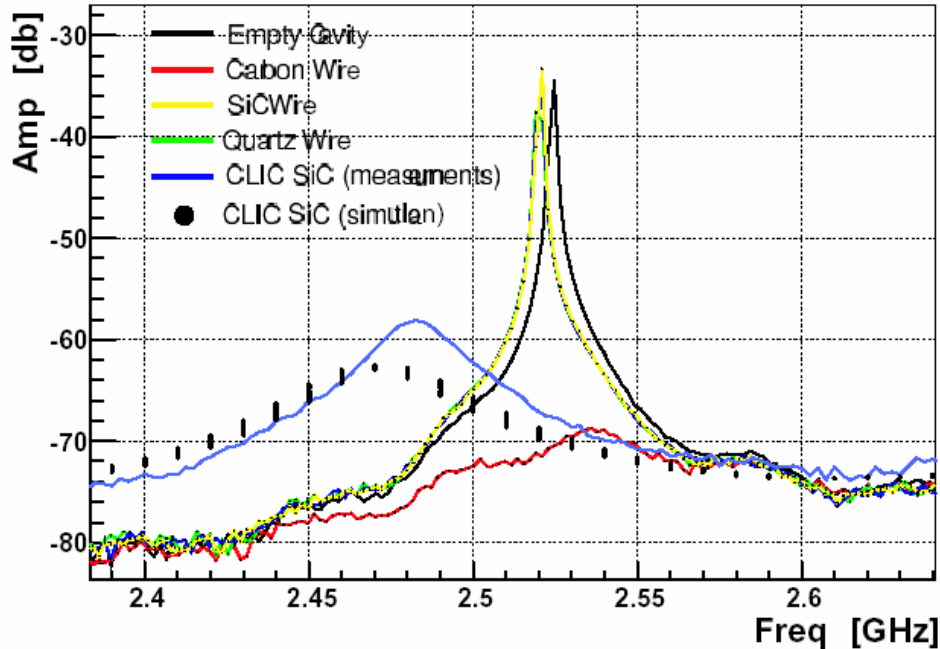


Figure WIRE7: Resonant cavity signal in presence of Carbon (36 μm), Silicon Carbide and Quartz wires

The plot qualitatively proves the RF power absorption of Carbon, and the non-absorption of Silicon Carbide and Quartz. Absorbed energy is mainly converted into heat.

Solutions:

- ❖ Damping of Higher Order Modes with Ferrites etc.
- ❖ Non conducting wires

Wire heat load

Exercise WIRE2: Which kind of wire Material you will prefer for a wire scanner in this accelerator? Estimate the wire temperature after one scan with a speed v (assume no cooling mechanisms).

Solving G:

G [g] is the mass of the part of the wire interacting with the beam. The mass G is defined by the beam dimension in the direction of the wire (perpendicular to the measuring direction) and by the wire diameter d' :

$$G = \text{wire volume} \cdot \rho = 2 \cdot \sigma_v \cdot d'^2 \cdot \rho \quad [g].$$

Solving N:

The number of particles N hitting the wire during one scan depends on the speed of the scan ($\sim 1/v$), the revolution frequency ($\sim f_{rev}$), the wire diameter ($\sim d'$) and the beam current ($\sim NB \cdot n_{bunch}$):

$$N = \frac{d' \cdot f_{rev}}{v} \cdot (NB \cdot n_{bunch}).$$

Fig. WIRE8 shows the a graphical representation of the parameters. The quotient $d \cdot f / v$ is the ratio of the scanned beam area or, in other words, like a grid seen by one bunch, assuming that all bunches are equal. However, the ratio can exceed the value 1 (a foil) if the scanning distance between two bunches is smaller than the wire diameter. Note that N does not depend on the beam widths σ .

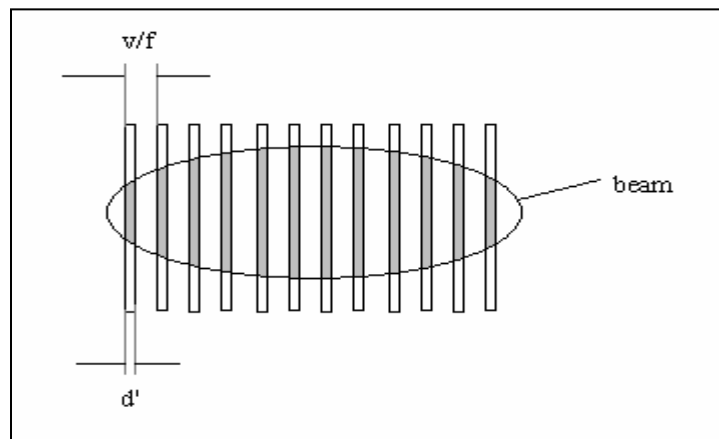


Figure WIRE8: Geometrical meaning of the parameters v/f and d'

Therefore, the temperature increase of the wire after one scan becomes:

$$T_h = C \cdot dE / dx_m \cdot d' \cdot \frac{d' \cdot f_{rev}}{v} \cdot (NB \cdot n_{bunch}) \cdot \frac{1}{c_p \cdot 2 \cdot \sigma_v \cdot d'^2 \cdot \rho} \cdot \alpha \quad [^{\circ}C]$$

with dE/dx_m [MeV/cm].

$$\text{with } \frac{dE / dx_m}{\rho} = dE / dx \left[\frac{\text{MeV} \cdot \text{cm}^2}{\text{g}} \right] \quad \text{and} \quad f_{rev} \cdot NB = f_{bunch}$$

T becomes:

$$T_h = C \cdot dE / dx \cdot n_{bunch} \cdot \frac{f_{bunch}}{v} \cdot \frac{1}{c_p \cdot 2 \cdot \sigma_v} \cdot \alpha \quad [^{\circ}C]$$

Where h , denotes the horizontal (h) scanning direction. The cooling factor ' α ' is described in the next section. Note that the temperature does not depend on the wire diameter and that it depends on the beam dimension perpendicular to the measuring direction. The temperature increase is inverse proportional to the scanning speed, therefore a faster scanner has a correspondingly smaller temperature increase.

The wire parameters $dE/dx / c_p$ and the Quotient T_h/T_m should be minimal for a choice of the material ($\alpha = 1$):

Material	$dE/dx / c_p$	$T_h [^{\circ}C]$	T_h/T_m
AL	7.7	$1.1 \cdot 10^4$	16.9
W	50.6	$7.1 \cdot 10^4$	20.9
C	5.4	$0.77 \cdot 10^4$	2.2
Be	4.1	$0.58 \cdot 10^4$	4.8
SiO2	12.9	$1.8 \cdot 10^4$	10.6

Table WIRE3: Temperature

From Table WIRE3 follows, that even the best material (Carbon) will be a Factor 2.2 above its melting temperature.

Exercise WIRE2a: Discuss cooling mechanisms which will cool the wire.

- 1) Secondary particles emitted from the wire
- 2) Heat transport along the wire
- 3) Black body radiation
- 4) Change of c_p with temperature

1) Secondaries: Some energy is lost from the wire by secondary particles. In the work in (J. Bossert et al.; The micron wire scanner at the SPS, CERN SPS/86-26 (MS) (1986)) about 70% is assumed. In DESY III (example above) no carbon wire was broken during more than 10 years of operation. At HERA, the theoretical temperature of the carbon wire (without secondaries) exceeds the melting temperature after a scan by far ($T = 12\,800\text{ }^{\circ}C$). Considering the loss by secondaries of 70%, the temperature reaches nearly the melting point. In practice, the wire breaks about once in 6 months. The observation is that the wire becomes thinner at the beam center. This may indicate, that during a scan some material of the wire is emitted because of nuclear interactions or is vaporized because it is very close to the melting temperature. This supports the estimate of the 70% loss and one has to multiply the factor $\alpha = 0.3$ in the equation above.

2) Heat transport: The transport of heat along the wire does not contribute to short time cooling of the wire (P. Lefevre; Measure tres peu destructive des distribution transversales dans le PS de 800 MeV a 26 GeV, CERN PS/DL/Note 78-8). However, frequent use of the scanner heats up also the ends of the wire and its connection to the wire holders (fork).

3) Black body radiation: The temperature T_{bb} at which the radiated power is equal to the deposited power in the wire during one scan P_{dep} [MeV/s] can be calculated from the Stefan-Boltzmann-law:

$$T_{bb} = \sqrt{\sqrt{\frac{P_{dep}}{s \cdot A}}}$$

where $s = 35.4 \text{ MeV} / (\text{s}^1 \text{ cm}^2 \text{ } ^0\text{K}^4)$ is the Stefan-Boltzmann-constant and A is the area of radiating surface. The surface of the heated wire portion A is $2 \cdot \sigma_v \cdot d \cdot \pi [\text{cm}^2]$. The power can be calculated by:

$$P_{dep\ h,v} = \alpha \cdot dE / dx \cdot d' \cdot n_{bunch} \cdot \frac{f_{bunch} \cdot d'}{v} \cdot \frac{1}{t_{scan}} \quad [MeV / s]$$

where $t_{scan} = 2 \cdot \sigma_{h,v} / v$ is the time for a scan (in the assumption of 2σ it is neglected that only about 70% of the power is concentrated within 2σ). α is the expected loss from secondaries.

For the example above $T_{bb} = 3900 \text{ } ^0\text{C}$. Therefore the black body radiation is a fraction of cooling for fast scans.

4) $c_p(T)$: The heat capacitance is a function of the temperature. Fig. 2 shows the increase of c_p for Carbon with T . The expected temperature after a scan is inversely proportional to c_p . Therefore one can expect a slightly smaller resulting temperature because of this dependence.

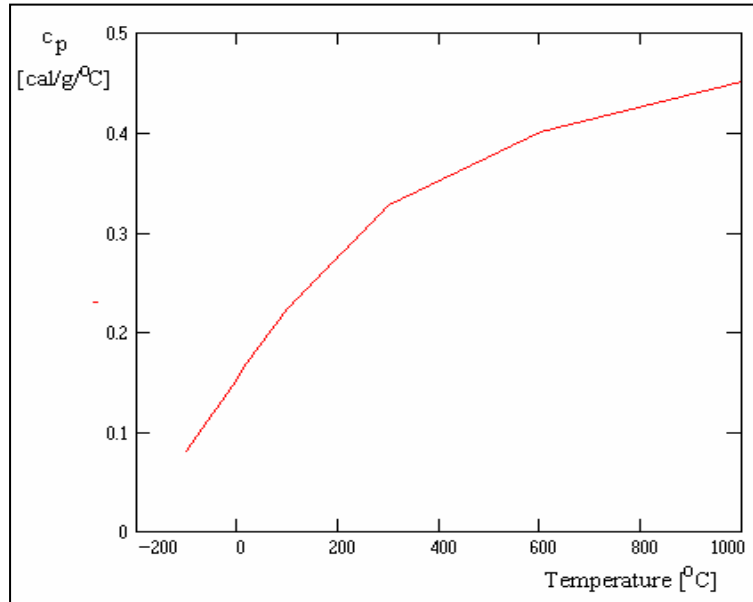


Figure WIRE9: The heat capacitance versus the temperature of Carbon.

Emittance blow up

Exercise WIRE3: Calculate the emittance blowup of the proton beam after one scan at a position with $\beta = 11.8 \text{ m}$ for $p = 0.3$ and 7 GeV/c (Carbon wire):

Assume a measurement position close to a Quadrupole ($\alpha=0$)

For small deflection angles a good approximation for average root mean square scattering angle is given by:

$$\delta \Theta = \frac{0.014 \text{ GeV}}{pc} \cdot \sqrt{\frac{d'}{L_{rad}}} \cdot \left(1 + 1/9 \cdot \log_{10} \frac{d'}{L_{rad}} \right)$$

A fraction Ψ of the circulating beam particles will hit the wire:

$$\Psi = \frac{d \cdot f_{rev}}{v} \quad (\text{see exercise WIRE2}),$$

The resulting mean deflection angle is than:

$$\Theta = \delta \Theta \cdot \Psi$$

and the emittance blowup: from: $\gamma(s)y^2 + 2\alpha(s)yy' + \beta(s)y'^2 = \varepsilon$

with $\alpha = 0$ and $y'^2 = \sqrt{2\pi} \cdot \delta \Theta^2 \cdot \Psi^2$ (this angle adds to the angular spread of the beam)

$$\delta \varepsilon_{rms} = \sqrt{2\pi} \cdot \delta \Theta^2 \cdot \Psi^2 \cdot \beta = 5.1 \cdot 10^{-8} \pi \text{ rad} = 5.1 \cdot 10^{-2} \pi \text{ mm mrad}$$

$$\Rightarrow \varepsilon_{rms} = 15.05 \pi \text{ mm mrad}$$

$\sqrt{2\pi}$ from ???; $\pi/2$ from D. Möhl's paper

$$\delta \varepsilon = \pi \frac{1}{4} \cdot \Theta_{rms}^2 \cdot \beta \quad \text{from M. Giovannozzi (CAS 2005)}$$

$\delta \varepsilon / \varepsilon$ [%/scan]

