Relativistic spin-(magneto)hydrodynamics

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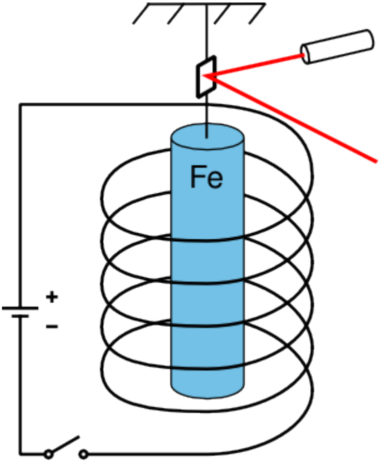
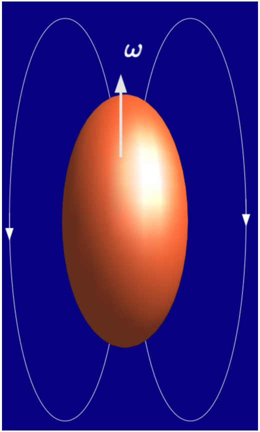
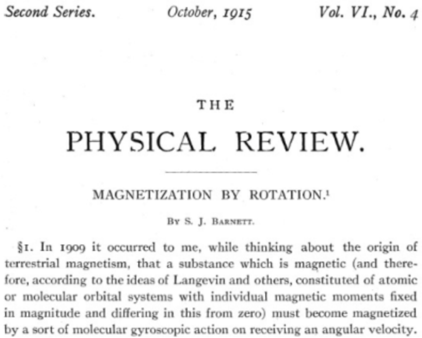
Einstein-de Haas effect



Electron spins get aligned in external magnetic field which is compensated by rotation of the ferromagnetic material.

Converse: Barnett effect





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THE

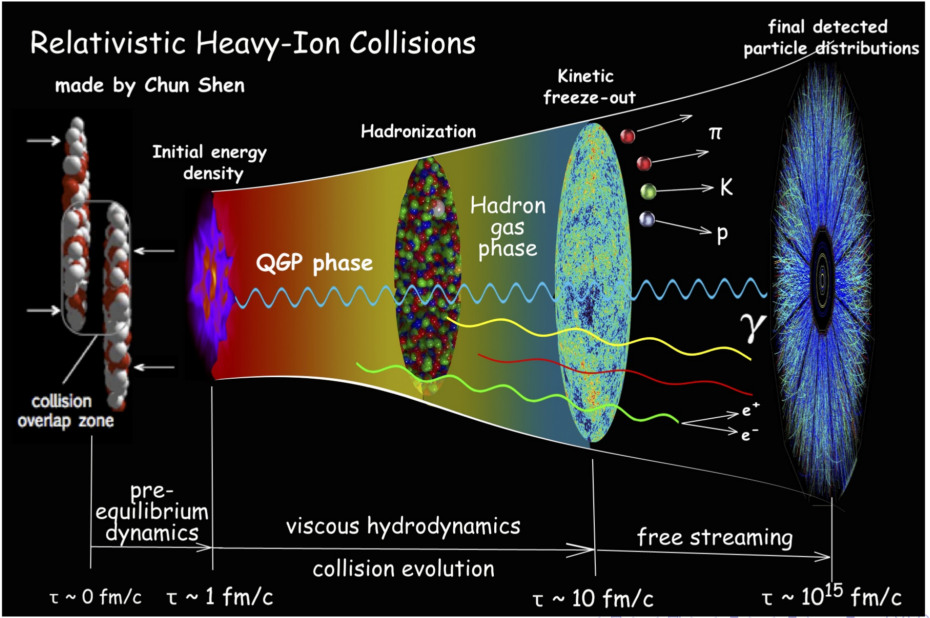
PHYSICAL REVIEW.

MAGNETIZATION BY ROTATION.[1]

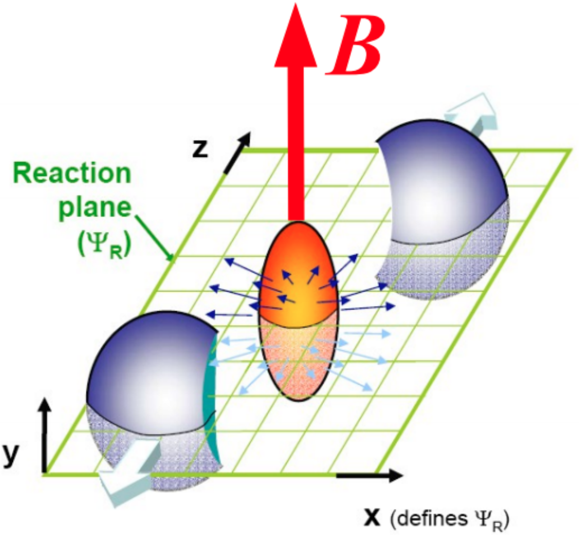
BY S. J. BARNETT.

§1. In 1909 it occurred to me, while thinking about the origin of terrestrial magnetism, that a substance which is magnetic (and therefore, according to the ideas of Langevin and others, constituted of atomic or molecular orbital systems with individual magnetic moments fixed in magnitude and differing in this from zero) must become magnetized by a sort of molecular gyroscopic action on receiving an angular velocity.

Spontaneous magnetization when spun around. Transformation of orbital angular momentum into spin alignment. Angular velocity decreases with appearance of magnetic field. Explanation appeals to spin-orbit coupling.

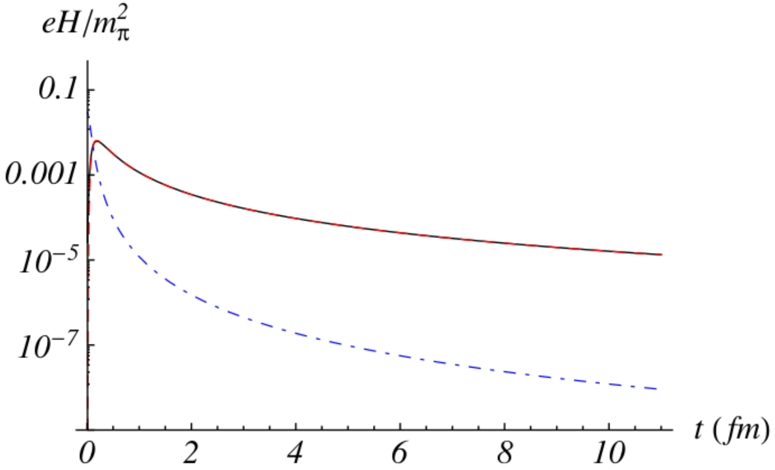

Relativistic Heavy-Ion Collisions
made by Chun Shen
final detected particle distributions
Kinetic freeze-out
Hadronization
Initial energy density
Hadron gas phase
QGP phase
π
K
p
γ
e+
e−
collision overlap zone
pre-equilibrium dynamics
viscous hydrodynamics
free streaming
collision evolution
τ ~ 0 fm/c
τ ~ 1 fm/c
τ ~ 10 fm/c
τ ~ 10^15 fm/c

Generation of magnetic field in heavy ion collisions



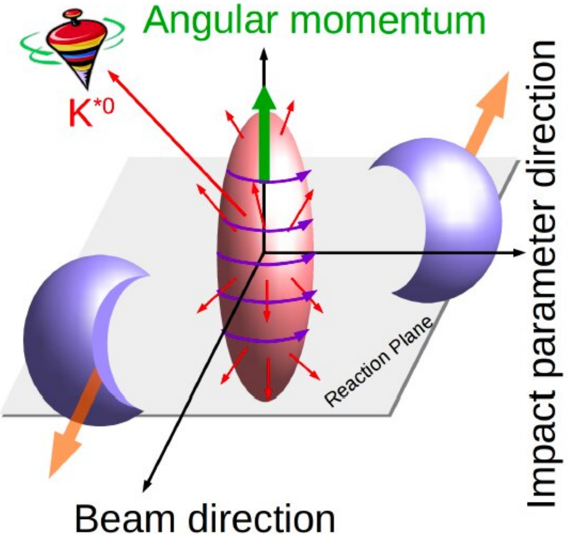
[Adapted from D. Kharzeev @ CPOD 2013.]

Magnetic field time evolution



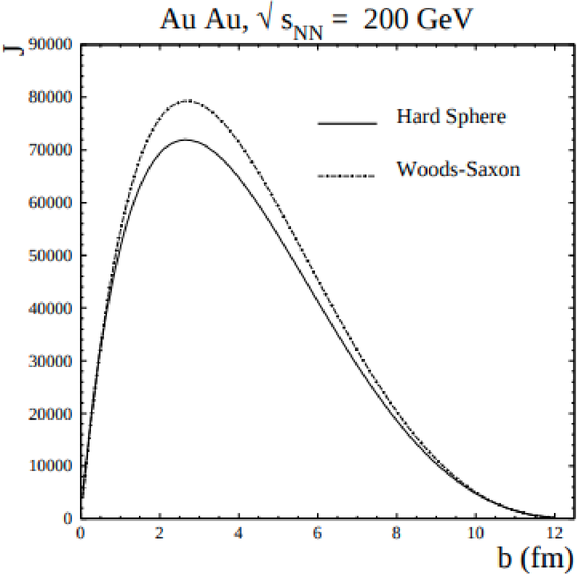
[K. Tuchin, Int. J. Mod. Phys. E23, 1430001 (2014).]

Global angular momentum in heavy ion collisions



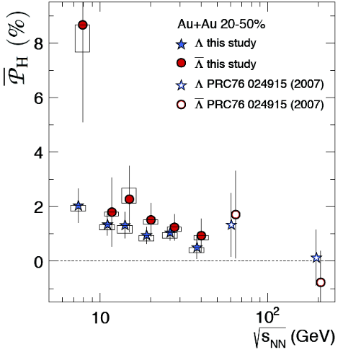
[B. Mohanty, ICTS News 6, 18-20 (2020).]

Angular momentum generation in non-central collisions



[F. Becattini, et al., Phys. Rev. C77, 024906 (2008).]

STAR Collaboration, *Global Lambda hyperon polarization in nuclear collisions,* Nature 548 62-65, 2017



First evidence of a quantum effect in (relativistic) hydrodynamics

Adapted from F. Becattini 'Subatomic Vortices'

Hydrodynamics with angular momentum conservation

- Apply relativistic (magneto)-hydrodynamics to understand particle polarization.
- Hydrodynamic evolution should ensure total angular momentum conservation.
- Formulation of relativistic magneto-hydrodynamics with angular momentum conservation necessary.
- Polarization processes are generally dissipative in nature.
- Dissipation should also be incorporated in the formulation.
- Relativistic kinetic theory framework.
- Geometrical approach with torsionful curved background.

Angular momentum conservation: particles

- Angular momentum of a particle with momentum $\vec{p}$:

$$\vec{L} = \vec{x} \times \vec{p} \quad \Rightarrow \quad L_i = \varepsilon_{ijk}\, x_i\, p_j$$

- One can obtain the dual tensor:

$$L_{ij} \equiv \varepsilon_{ijk}\, L_k \quad \Rightarrow \quad L_{ij} = x_i\, p_j - x_j\, p_i$$

- We know that both definitions are equivalent.
- In absence of external torque, $\dfrac{d\vec{L}}{dt} = 0$, we also have: $\partial_i L_{ij} = 0$.
- Relativistic generalization: $L^{\mu\nu} = x^\mu p^\nu - x^\nu p^\mu$ and $\partial_\mu L^{\mu\nu} = 0$.
- This treatment valid for point particles.
- For fluids, particle momenta → "generalized fluid momenta"

The energy-momentum tensor

Angular momentum conservation: fluid

- The orbital angular momentum for relativistic fluids is defined as

$$L^{\lambda,\mu\nu} = x^\mu T^{\lambda\nu} - x^\nu T^{\lambda\mu}$$

- Keeping in mind the energy-momentum conservation, $\partial_\mu T^{\mu\nu} = 0$:

$$\partial_\lambda L^{\lambda,\mu\nu} = T^{\mu\nu} - T^{\nu\mu}$$

- Obviously, for symmetric $T^{\mu\nu}$, orbital angular momentum is automatically conserved. Classically $T^{\mu\nu}$ symmetric.

- For medium constituent with intrinsic spin, different story

$$J^{\lambda,\mu\nu} = L^{\lambda,\mu\nu} + S^{\lambda,\mu\nu} + L\text{-}S \text{ couplings}$$

- Ensure total angular momentum conservation: $\partial_\lambda J^{\lambda,\mu\nu} = 0$.

- Basis for formulation of spin Hydrodynamics.
  [Florkowski et. al., Prog.Part.Nucl.Phys. 108 (2019) 103709; Bhadury et. al., Eur.Phys.J.ST 230 (2021) 3, 655-672]

Pseudo-gauge transformations

- Ignoring the L-S coupling terms,

$$J^{\lambda,\mu\nu} = L^{\lambda,\mu\nu} + S^{\lambda,\mu\nu}$$

- With $\partial_\mu T^{\mu\nu} = 0$, and $\partial_\lambda L^{\lambda,\mu\nu} = T^{\mu\nu} - T^{\nu\mu}$, $\partial_\lambda J^{\lambda,\mu\nu} = 0$ leads to,

$$\partial_\lambda S^{\lambda,\mu\nu} = T^{\nu\mu} - T^{\mu\nu}$$

- Hence the final hydrodynamic equations can be written as

$$\partial_\mu T^{\mu\nu} = 0, \qquad \partial_\lambda S^{\lambda,\mu\nu} = T^{\nu\mu} - T^{\mu\nu}$$

- Also holds with the following redefinition

$$\tilde{T}^{\mu\nu} = T^{\mu\nu} + \frac{1}{2}\partial_\lambda \left(\Phi^{\lambda,\mu\nu} - \Phi^{\mu,\lambda\nu} - \Phi^{\nu,\lambda\mu}\right)$$

$$\tilde{S}^{\lambda,\mu\nu} = S^{\lambda,\mu\nu} - \Phi^{\lambda,\mu\nu}$$

- Freedom due to space-time symmetry; including torsion fixes this.

[Gallegos et. al., SciPost Phys. 11, 041 (2021); Hongo et. al., JHEP 11 (2021) 150]

Non-dissipative spin-hydrodynamics from kinetic theory

- The phase-space for single particle distribution function gets extended $f(x,p,s)$.
- The equilibrium distribution for Fermions is given by

$$f_{eq}(x,p,s) = \frac{1}{\exp\left[\beta\cdot p - \xi - \frac{1}{2}\,\omega : s\right] + 1} \qquad \begin{cases}\beta\cdot p \equiv \beta_\mu p^\mu \\ \omega : s \equiv \omega_{\mu\nu} s^{\mu\nu}\end{cases}$$

- Quantities $\beta^\mu = u^\mu/T,\ \xi = \mu/T,\ \omega_{\mu\nu}$ are functions of x.
- $\xi,\ \beta^\mu,\ \omega^{\mu\nu}$: Lagrange multipliers for conserved quantities.
- $s^{\mu\nu}$: Particle spin, similar to particle momenta p^μ.
- Hydrodynamics: average over particle momenta and spin.
- Classical treatment of spin.

Dissipative spin-hydrodynamics Bhadury et. al., PLB 814, 136096 (2021)

- Introduce out-of-equilibrium distribution function $f(x,p,s)$.
- Use Boltzmann equation for evolution of $f(x,p,s)$.
- Express hydrodynamic quantities in terms of $f(x,p,s)$.

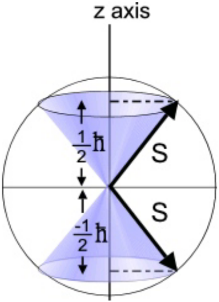
$$T^{\mu\nu}(x) = \int dPdS\; p^\mu p^\nu \left[f(x,p,s) + \bar{f}(x,p,s)\right]$$

$$N^{\mu}(x) = \int dPdS\; p^\mu \left[f(x,p,s) - \bar{f}(x,p,s)\right]$$

$$S^{\lambda,\mu\nu}(x) = \int dPdS\; p^\lambda s^{\mu\nu} \left[f(x,p,s) + \bar{f}(x,p,s)\right]$$

$$dP \equiv \frac{d^3p}{E_p(2\pi)^3}, \quad dS \equiv m\frac{d^4s}{\pi\,\mathfrak{s}}\,\delta(s\cdot s + \mathfrak{s}^2)\,\delta(p\cdot s)$$

$$\mathfrak{s}^2 = \frac{1}{2}\left(\frac{1}{2}+1\right) = \frac{3}{4}: \quad s^\mu \equiv \frac{1}{2m}\epsilon^{\mu\nu\alpha\beta}p_\nu s_{\alpha\beta}$$



- Classical treatment of spin: internal angular momentum.

Relativistic spin-magnetohydrodynamics [arXiv:2204.01357]

- The particle four-current and its conservation is given by

$$N^\mu = nu^\mu + n^\mu, \qquad \partial_\mu N^\mu = 0$$

- Total stress-energy tensor of the system: $T^{\mu\nu} = T^{\mu\nu}_{\rm f} + T^{\mu\nu}_{\rm int} + T^{\mu\nu}_{\rm em}$

$$T^{\mu\nu}_{\rm f} = \epsilon u^\mu u^\nu - (P + \Pi)\,\Delta^{\mu\nu} + \pi^{\mu\nu},$$

$$T^{\mu\nu}_{\rm int} = -\Pi^\mu u^\nu - F^\mu{}_\alpha M^{\nu\alpha}$$

$$T^{\mu\nu}_{\rm em} = -F^{\mu\alpha} F^\nu{}_\alpha + \frac{1}{4} g^{\mu\nu} F^{\alpha\beta} F_{\alpha\beta}$$

- Maxwell's equation: $\partial_\mu H^{\mu\nu} = J^\nu$ and $H^{\mu\nu} = F^{\mu\nu} + M^{\mu\nu}$,

$$\partial_\mu T^{\mu\nu}_{\rm em} = F^\nu{}_\alpha J^\alpha$$

- Current generating external field, $J^\mu = J^\mu_{\rm f} + J^\mu_{\rm ext}$ where $J^\mu_{\rm f} = \mathfrak{q} N^\mu$,

$$\partial_\mu T^{\mu\nu} = -f^\nu_{\rm ext}, \qquad f^\nu_{\rm ext} = F^\nu{}_\alpha J^\alpha_{\rm ext}$$

Equations of motion

- Divergence of matter part of energy-momentum tensor,

$$\partial_\nu T_{\mathrm{f}}^{\mu\nu} = F^{\mu}{}_{\alpha} J_{\mathrm{f}}^{\alpha} + \frac{1}{2}\left(\partial^\mu F^{\nu\alpha}\right) M_{\nu\alpha}$$

- Next, consider total angular momentum conservation:

$$J^{\lambda,\mu\nu} = L^{\lambda,\mu\mu} + S^{\lambda,\mu\nu}$$

- In presence of external torque its divergence leads to,

$$\partial_\lambda J^{\lambda,\mu\nu} = -\tau_{\mathrm{ext}}^{\mu\nu}, \qquad \tau_{\mathrm{ext}}^{\mu\nu} = x^\mu f_{\mathrm{ext}}^{\nu} - x^\nu f_{\mathrm{ext}}^{\mu}$$

- Torque due to moment of external force; "pure" torque ignored.
- The orbital part of angular momentum and its divergence is

$$L^{\lambda,\mu\nu} = x^\mu T^{\lambda\nu} - x^\nu T^{\lambda\mu}, \qquad \partial_\lambda L^{\lambda,\mu\nu} = -\tau_{\mathrm{ext}}^{\mu\nu}$$

- Spin part of the total angular momentum is conserved

$$\partial_\lambda S^{\lambda,\mu\nu} = 0$$

- Along with particle four-current conservation, $\partial_\mu N^\mu = 0$.

Boltzmann equation

- One can also define the polarization-magnetization tensor as

$$M^{\mu\nu} = m \int dP dS \, m^{\mu\nu} \left(f + \bar{f}\right)$$

- Boltzmann equation (BE) in relaxation-time approximation (RTA)

$$\left(p^\alpha \frac{\partial}{\partial x^\alpha} + m\,\mathcal{F}^\alpha \frac{\partial}{\partial p^\alpha} + m\,\mathcal{S}^{\alpha\beta} \frac{\partial}{\partial s^{\alpha\beta}}\right) f = C[f] = -\left(u \cdot p\right) \frac{f - f_{\rm eq}}{\tau_{\rm eq}}$$

- The force term is:

$$\mathcal{F}^\alpha = \frac{\mathfrak{q}}{m} F^{\alpha\beta} p_\beta + \frac{1}{2}\left(\partial^\alpha F^{\beta\gamma}\right) m_{\beta\gamma}, \qquad m^{\alpha\beta} = \chi s^{\alpha\beta}$$

- There is a "pure" torque term:

$$\mathcal{S}^{\alpha\beta} = 2\, F^{\gamma[\alpha}\, m^{\beta]}{}_{\gamma} - \frac{2}{m^2}\left(\chi - \frac{\mathfrak{q}}{m}\right) F_{\phi\gamma}\, s^{\phi[\alpha}\, p^{\beta]} p^\gamma$$

- We ignore this "pure" torque term for now.

Einstein-de Haas and Barnett effects

- The equilibrium polarization-magnetization tensor is

$$M^{\mu\nu}_{eq} = m \int dPdS \, m^{\mu\nu} \left(f_{eq} + \bar{f}_{eq} \right)$$

- Magnetic dipole moment $m^{\mu\nu} = \chi s^{\mu\nu}$.
- χ: resembles the gyromagnetic ratio.
- Integrating over the momentum and spin degrees of freedom,

$$M^{\mu\nu}_{eq} = a_1 \, \omega^{\mu\nu} + a_2 \, u^{[\mu} u_\gamma \omega^{\nu]\gamma}$$

- In global equilibrium, $\omega^{\mu\nu}$ corresponds to rotation of the fluid.
- Rotation produces magnetization (Barnett effect) and vice versa (Einstein-de Hass effect).

Hydrodynamic equations from kinetic theory

- Impose Landau frame and extended matching conditions

$$u_\mu T^{\mu\nu} = \epsilon u^\nu, \quad \epsilon = \epsilon_{\rm eq}, \quad n = n_{\rm eq}, \quad u_\lambda \delta s^{\lambda,\mu\nu} = 0$$

- Zeroth, first and "spin" moment of the RTA collision vanishes

$$\int dP dS\, C[f] = \int dP dS\, p^\mu\, C[f] = \int dP dS\, s^{\mu\nu} C[f] = 0$$

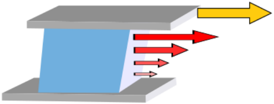
- Using definitions of hydro quantities, these moments of BE gives

$$\partial_\mu N^\mu = 0, \quad \partial_\nu T^{\mu\nu}_{\rm f} = F^\mu{}_\alpha J^\alpha_{\rm f} + \frac{1}{2} \left(\partial^\mu F^{\nu\alpha}\right) M_{\nu\alpha}, \quad \partial_\lambda S^{\lambda,\mu\nu} = 0$$

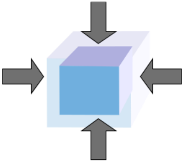
- Same equations as obtained from macroscopic arguments.
- Polarization/magnetization effects are dissipative in nature.
- Boltzmann equation $\rightarrow$ dissipative spin-magnetohydodynamics.

Dissipative effects

- In simple terms: $\pi^{yx} = 2\,\eta\,\partial^{\langle y}u^{x\rangle}, \quad \Pi = -\zeta\,\partial\cdot u, \quad n^x = \kappa\,\partial^{\langle x\rangle}a$
  - Shear viscosity: fluid's resistance to shear forces



  - Bulk viscosity: fluid's resistance to compression



- Charge/heat conductivity: fluid's resistance to flow of charge/heat.
- Dissipation to spin current: development of Kubo formalism.

Our work in this direction within kinetic theory

- Ideal spin-hydrodynamics:
  - W. Flokowski, B. Friman, A. Jaiswal and E. Speranza, Physical Review C 97, 041901 (2018).
  - W. Flokowski, B. Friman, A. Jaiswal, R. Ryblewski and E. Speranza, Physical Review D 97, 116017 (2018).
- Dissipative spin-hydrodynamics:
  - S. Bhadury, W. Flokowski, A. Jaiswal, A. Kumar, and R. Ryblewski, Physics Letters B 814 (2021) 136096.
  - S. Bhadury, W. Flokowski, A. Jaiswal, A. Kumar, and R. Ryblewski, Physical Review D 103, 014030 (2021).

- Relativistic spin-magnetohydrodynamics [arXiv:2204.01357]; to be published in Physical review Letters.

Ongoing work from geometrical approach

- Starting from the symmetries of the Lagrangian of a given theory, one can construct conserved currents using Noether's theorem.
- Energy-momentum tensor-variation of Lagrangian with metric $g^{\mu\nu}$: conservation is a consequence of diffeomorphism invariance.
- Conserved charge current-variation with gauge field A^{μ}: consequence of local gauge symmetry.
- Spin-current can be constructed similarly.
- Price to pay: introduce torsion in metric, non-Riemannian geometry.
- Spin current: variation w.r.t torsion.
- Angular momentum conservation: consequence of local Lorentz invariance.

Other relevant works

- Other parallel approaches from Wigner function [N. Weickgenannt, X.-l. Sheng, E. Speranza, Q. Wang and D. Rischke, PRD 100 (2019) 056018].
- Appraoch based on chiral kinetic theory [S. Shi, C. Gale and S. Jeon, PRC 103 (2021) 044906].
- Appraoch based on Lagrangian method [D. Montenegro and G. Torrieri, PRD 100 (2019) 056011].
- Formulation with torsion in metric [A. D. Gallegos, U. Gürsoy and A. Yarom, SciPost Phys. 11, 041 (2021); M. Hongo, X.-G. Huang, M. Kaminski, M. Stephanov, H.-U. Yee, JHEP 11 (2021) 150].
- Useful reviews on spin hydro: [W. Florkowski, R. Ryblewski and A. Kumar, Prog.Part.Nucl.Phys. 108 (2019) 103709; S. Bhadury, J. Bhatt, A. Jaiswal and A. Kumar, Eur.Phys.J.ST 230 (2021) 3, 655-672].
- Relativistic spin-magnetohydrodynamics: unexplored area.
- Much work needed in this direction.

Thank you!