4-quark states from functional methods

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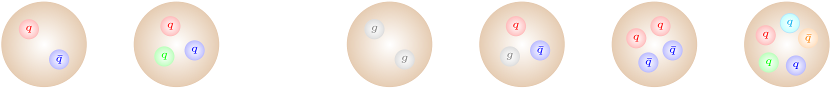
Motivation

Conventional Hadrons: Exotic Hadrons:

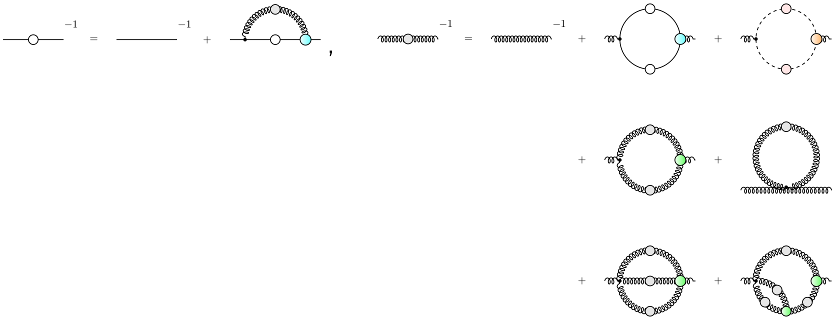


Mesons | Baryons | Glueballs | Hybrids | 4-quark states | Pentaquarks

- Powerful toolkit to classify conventional hadrons: Quark Model (QM).
- Lot of particles measured that do not fit into the QM picture.
- A few examples:
  - Light scalar mesons: σ, κ, a_0, f_0
  - Exotic XYZ-states: $X(3872)$, $X(3915)$, $Z_c(3900)$, $Z_c(4430)$
- More naturally explained as **4-quark states**.

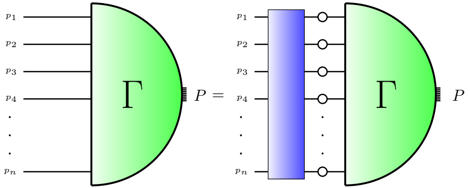
Functional Framework

- Non-perturbative, fully relativistic framework.
- To compute the properties of bound states, use combination of:
  - DSEs: The QCD quantum equations of motion,



Functional Framework

- Non-perturbative, fully relativistic framework.
- To compute the properties of bound states, use combination of:
  - DSEs: The QCD quantum equations of motion,
  - Hadronic bound state equations: BSEs, Faddeev eqs. .

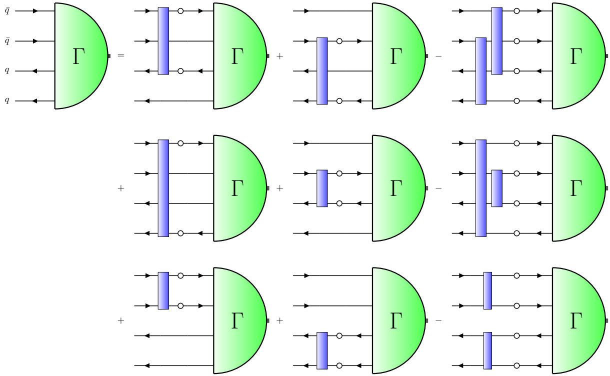


Eigenvalue equation

$$\lambda(P^2)\,\Gamma^{(n)} = K^{(n)}G^{(n)}\,\Gamma^{(n)}$$

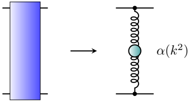
with $\lambda(P^2 = -M^2) = 1$

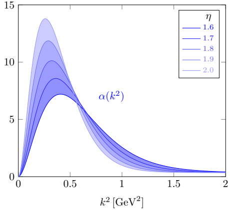
The 4-quark BSE:



$$\Gamma(k,q,p,P) = \sum_i f_i \underbrace{(\ldots)}_{\text{Lorentz-invariants}} \tau_i(k,q,p,P) \otimes \Gamma_C \otimes \Gamma_F$$

Calculations are done in the *Rainbow-Ladder truncation*:



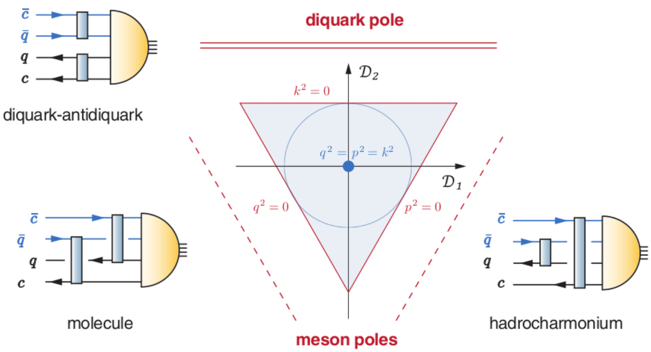


Maris, Tandy, PRC 60 (1999)
Qin et al., PRC 84 (2011)

- Can cast the Lorentz-invariants into multiplets of S_4:

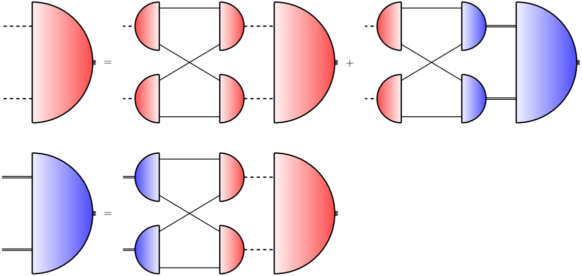
Eichmann, Fischer, Heupel, Phys.Lett.B 753 (2016) 282-287

  - One singlet: S_0
  - One doublet: $D = \begin{pmatrix} D_1 \\ D_2 \end{pmatrix}$
  - Two triples: T_0, T_1 $\rightarrow$ subleading
- Dressing functions: $f_i(S_0, D)$
- Poles dynamically generated in D
- "Physical basis": put poles in externally: $f_i(S_0, D) \rightarrow f_i(S_0) \cdot P_{ab} \cdot P_{cd}$



2-body approach

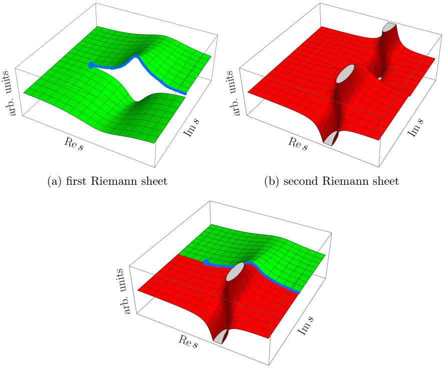
- Assume dominant 2-body forces → simplify the 4-body BSE to get the **2-body approach**.
  Santowsky, Fischer, Eur.Phys.J.C 82 (2022) 4, 313
  Santowsky, Eichmann, Fischer, Wallbott, Williams, Phys.Rev.D 102 (2020) 5, 056014



- It is a coupled system of meson-meson and diquark-antidiquark components which interact via quark exchange.
- This approach is close in spirit to an effective field theory description.

Results in 2-body formalism

Resonances



(a) first Riemann sheet (b) second Riemann sheet

(c) transition from first to second Riemann sheet

P.A. Zyla et al. (Particle Data Group), Prog. Theor. Exp. Phys. 2020, 083C01 (2020) and 2021 update

- Poles in scattering amplitude:
  - Bound state: on physical sheet
  - Resonance: on unphysical (2nd) sheet

Complex plane & Branch cuts in BSE

$$\lambda(P^2)\,\Gamma^{(n)} = K^{(n)} G^{(n)}\,\Gamma^{(n)}$$

- It is possible to solve the BSE for complex eigenvalues.
- Usually carry out computations in rest frame of the bound state: $P^\mu = i\mathrm{M}\,\hat{e}_4^\mu$. (Eucl. space-time)
- Add term: $P^\mu = i\mathrm{M}\,\hat{e}_4^\mu \rightarrow \left(i\mathrm{M} + \frac{\Gamma}{2}\right)\,\hat{e}_4^\mu$.
- Pole is identified if Re$(\lambda(P^2)) = 1$ & Im$(\lambda(P^2)) = 0$.
- Caveat: Only possible to calculate in the first Riemann sheet $\rightarrow$ need analytic continuation.

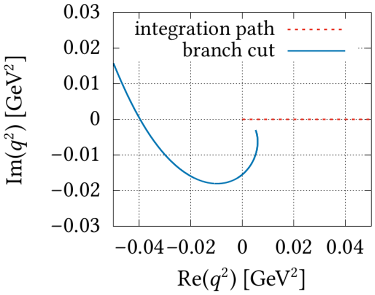
Branch cut in BSE:

$$\Gamma^{(n)}(p^2) = \int \mathrm{d}q^2 \int \mathrm{d}z \int \mathrm{d}y \int \mathrm{d}\phi \, K^{(n)}(p^2, q^2) G^{(n)} \, \Gamma^{(n)}(q^2)$$

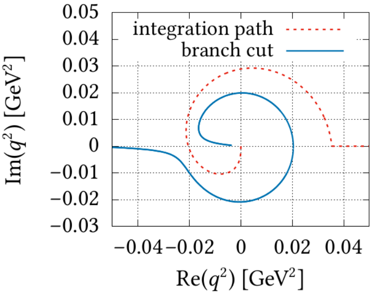
- Pole structures

$$\frac{1}{(q \pm P)^2 + m^2} = \frac{1}{q^2 \pm 2\sqrt{q^2}\sqrt{P^2} z + P^2 + m^2}$$

in the interaction topologies → restrict the range of P^2.



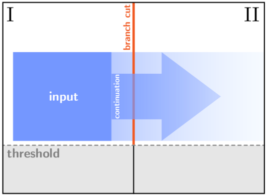
Branch cut for $\mathrm{P} < \mathrm{P}_{\text{crit.}}$

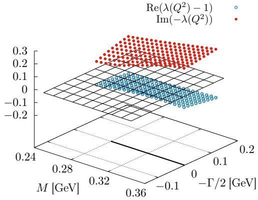


Branch cut for $\mathrm{P} \geq \mathrm{P}_{\text{crit.}}$

Santowsky, Fischer, Phys.Rev.D 105 (2022) 3, 034025

- Used clever combination of path deformation and analytic continuation in the 2-body approach.
- Extract the **mass + decay width** of the σ, a_0, f_0.





PDG (M$-i\Gamma/2$): σ: $(400-550)-i(200-350)$ MeV
a$_0$: $(960-1030)-i(20-70)$ MeV
f$_0$: $(980-1010)-i(20-35)$ MeV

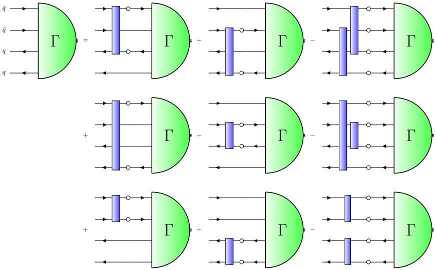
σ	state	M	$\Gamma/2$
	$\pi\pi + 0^+0^+ + q\bar{q}$	291(5)	121(22)
	$\pi\pi + 0^+0^+$	302(7)	148(31)
	$\pi\pi$	301(7)	158(29)
	$q\bar{q}$	661(8)	0

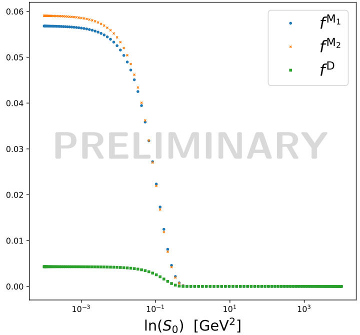
f_0	a_0	setup	M	$\Gamma/2$
•	•	$K\bar{K} + 0^+0^+$	1001(4)	24(16)
•		$s\bar{s}$	1073(10)	0
•		$s\bar{s} + K\bar{K}$	927(18)	1(3)
•		$s\bar{s} + K\bar{K} + 0^+0^+$	915(20)	2(3)

Santowsky, Fischer, Phys.Rev.D 105 (2022) 3, 034025

Results in 4-body formalism

Dominant substructure





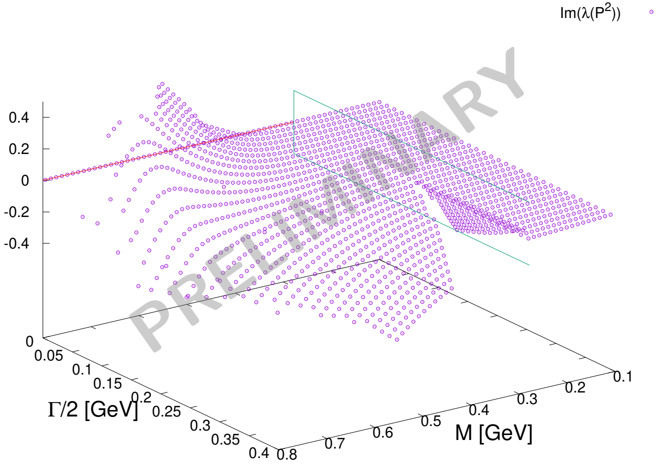
- $\Gamma = f^{\mathrm{M}_1} \cdot \tau^{\mathrm{M}_1} + f^{\mathrm{M}_2} \cdot \tau^{\mathrm{M}_2} + f^{\mathrm{D}} \cdot \tau^{\mathrm{D}}$
- Channels for a_0 (quark content $\bar{s}\bar{q}qs$):
  - M$_1$: $K\bar{K}$
  - M$_2$: $\eta\pi$
  - D : $0_s^+ 0_s^+$

$q = u/d$

$m_\pi \approx 140$ MeV
$m_K \approx 500$ MeV
$m_\eta \approx 550$ MeV
$m_{0_s^+} \approx 1100$ MeV

Complex plane in the 4-body approach

Computation with path deformation candidate:



Summary:

- Motivated why 4-quark states are a highly interesting area of research.
- Functional methods are a great toolkit to make qualitative predictions about masses and internal structure of 4-quark candidates.
- Showed results for **masses + decay widths** obtained in the 2-body approach.
- Showed preliminary results for computation in the complex plane in the 4-body approach.

Outlook:

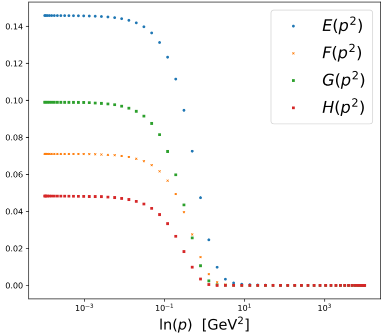
- Further investigation of the BSE in the complex plane.
- Check if masses + decay widths can be extracted in the 4-body framework using the technique developed in the 2-body approach.

Backup slides

Pion BSE

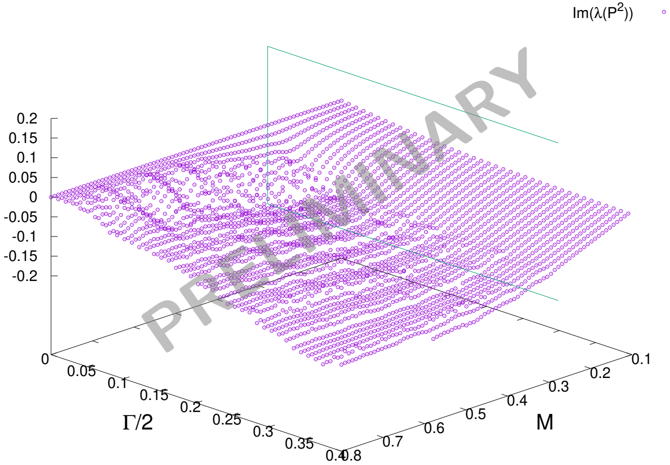
Pion Bethe-Salpeter amplitude is given by:

$$\Gamma_{\text{pion}}(p^2) = E(p^2) \cdot \tau_1(p,P) + F(p^2) \cdot \tau_2(p,P) + G(p^2) \cdot \tau_3(p,P) + H(p^2) \cdot \tau_4(p,P)$$

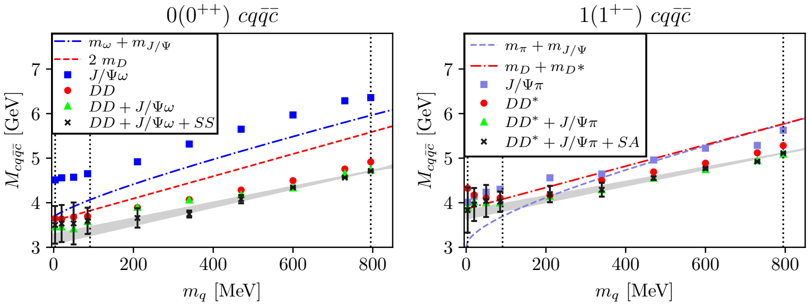


Complex plane in the 4-body approach

Computation without path deformation:



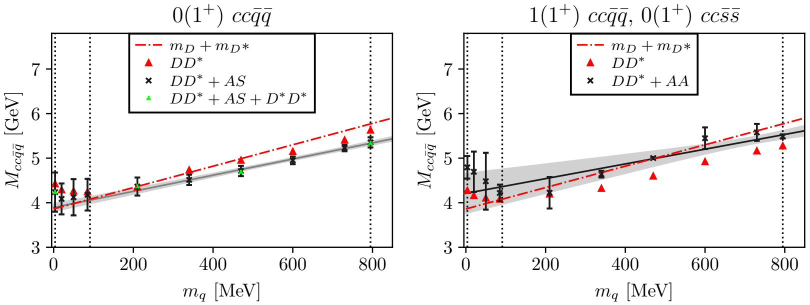
Results (Heavy-light mesons $c\bar{c}q\bar{q}$ (hidden charm))



$I(J^{PC})$	exp. candidate	mass 2-body BSE [GeV]	mass 4-body BSE [GeV]
$0(0^{++})$	$X(3915)$	3.49(25)	3.50(42)
$0(1^{++})$	$\chi_{c1}(3872)$	3.85(18)	3.92(7)
$1(1^{+-})$	$Z_c(3900)$	3.79(31)	3.74(9)

Wallbott, Eichmann, Fischer, Phys.Rev.D 102 (2020) 5, 051501
Santowsky, Fischer, Eur.Phys.J.C 82 (2022) 4, 313

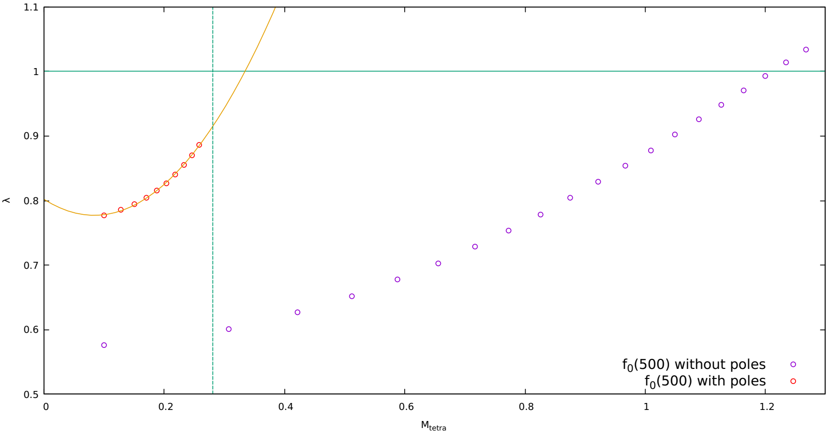
Results (Heavy-light mesons $cc\bar{q}\bar{q}$ (open charm))



$I(J^P)$	exp. candidate	mass 2-body BSE [GeV]	mass 4-body BSE [GeV]
$1(0^+)$	–	3.21(2)	3.80(10)
$0(1^+)$	$T_{cc}^+(\sim 3875)$	3.49(48)	3.90(8)
$1(1^+)$	–	3.47(24)	4.22(44)

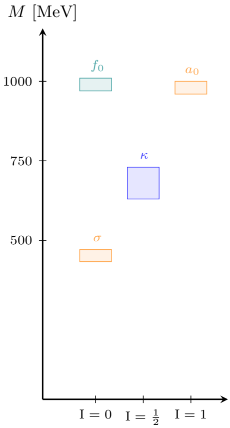
Wallbott, Eichmann, Fischer, Phys.Rev.D 102 (2020) 5, 051501
Santowsky, Fischer, Eur.Phys.J.C 82 (2022) 4, 313

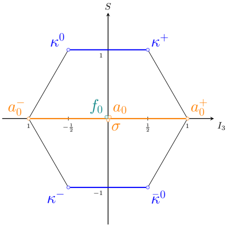
Eigenvalue curve



Light scalar mesons

The light scalar (0^{++}) mesons is an example where the Quark Model yields wrong predictions:





$f_0(980)$ $s\bar{s}$

$\kappa(700)$ $u\bar{s},\, d\bar{s}$

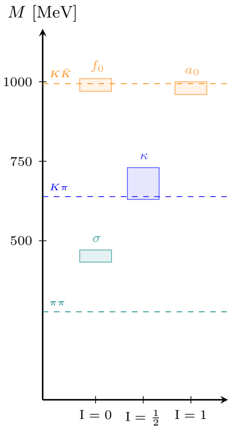
$\left.\begin{matrix} a_0(980) \\ \sigma(500) \end{matrix}\right\} u\bar{u},\, d\bar{d},\, u\bar{d}$

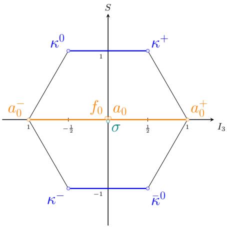
- Why are a_0, f_0 almost mass degenerate?
- Why are the decay widths so different?

  $\Gamma(\sigma, \kappa) \approx 550\,\mathrm{MeV}$
  $\Gamma(a_0, f_0) \approx 50 - 100\,\mathrm{MeV}$
- Why are they so light?

Light scalar mesons

Suppose they were tetraquarks: Jaffe 1977; Close, Tornqvist 2002; Maiani, Polosa, Riquer 2004





$f_0(980)$, $a_0(980)$ $\big\}$ $us\bar{u}\bar{s}, \ldots$

$\kappa(700)$ $us\bar{u}\bar{d}, \ldots$

$\sigma(500)$ $ud\bar{u}\bar{d}$

- Explains mass ordering and decay widths:
  a_0, f_0 couple to $K\bar{K}$,
  σ, κ large decay widths
- Non-$q\bar{q}$ nature of σ is supported by dispersive analyses, unitarized ChPT, large N_C, extended linear σ models, quark models

Pelaez, Phys.Rept. 658 (2016)

Branch cut in 4-body approach