Critical Endpoint of QCD in a Finite Volume and Mesonic Contributions to the Columbia Plot

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Based on:
JB, Fischer, Isserstedt, Schaefer, PRD 104 (2021) 074035
and
JB, Fischer, Isserstedt (arXiv: 2208.01981)

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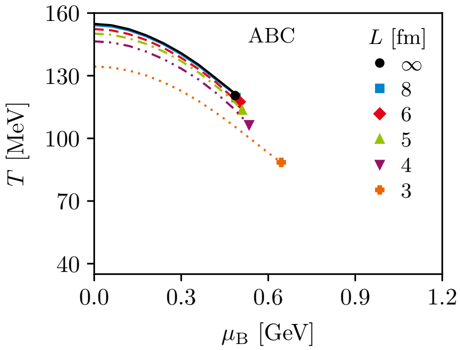
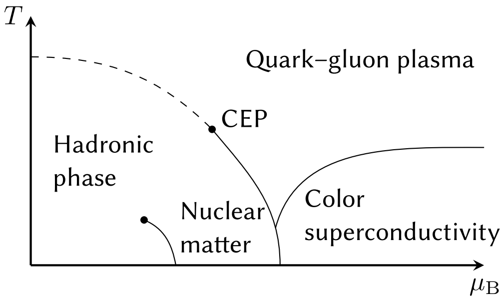
Giardini Naxos, Italy
2022-10-25

Outline

First Objective: QCD Phase Diagram in a Finite Volume



Motivation: Why Finite Volume?

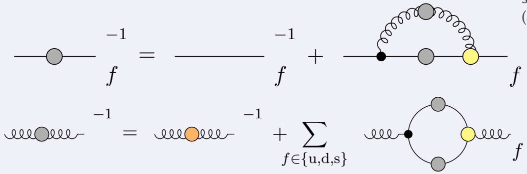


- Goal of many experiments is to locate critical endpoint in QCD phase diagram
- "Fireball" of heavy-ion collisions has finite spatial extent
- Impact of volume effects on CEP is important for comparison between theory and experiment
- Cross-check between different theoretical approaches: lattice QCD (by construction formulated in a finite volume) vs. functional methods

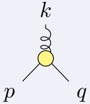
Functional Framework: Truncated Set of DSEs

Truncated Dyson–Schwinger Equations for Quark and Gluon Propagators

see Fischer, PPNP 105 (2019) 1
(and references therein)



Quark–Gluon Vertex Ansatz



$$\Gamma_\mu^f(k,p,q) = \Gamma(k,p,q)\Gamma_\mu^{f,\mathrm{BC}}(p,q)$$ (Information about quarks)

Quenched Gluon Propagator

$$D_{\mu\nu}^{\mathrm{que}}(k) = D_{\mu\nu}^{\mathrm{que}}(k;T)$$ (Temperature-dependent fit to lattice data)

reference for lattice data: Fischer, Maas, Müller, EPJC 68 (2010) 165-181
Maas, Pawlowski, von Smekal, Spielmann, PRD 85 (2012) 034037

Finite Volume Framework: Ansatz

- Feasible shape as ansatz: cube with edge length L:

$$\int_{\mathbb{R}^3} \mathrm{d}^3x\,\mathcal{L} \;\rightarrow\; \int_{[0,L]^3} \mathrm{d}^3x\,\mathcal{L}$$

- For quarks, free to choose between

$$\psi(\boldsymbol{x} + L\boldsymbol{e}_i) = +\psi(\boldsymbol{x}) \quad \text{periodic boundary conditions (PBC)}$$
$$\psi(\boldsymbol{x} + L\boldsymbol{e}_i) = -\psi(\boldsymbol{x}) \quad \text{antiperiodic boundary conditions (ABC)}$$

- For gluons, need PBC for kinematic reasons

$\rightarrow$ Only discrete values possible in momentum space

Finite Volume Framework: Implications

→ Possible discrete momentum values given by:

Spatial Matsubara Modes

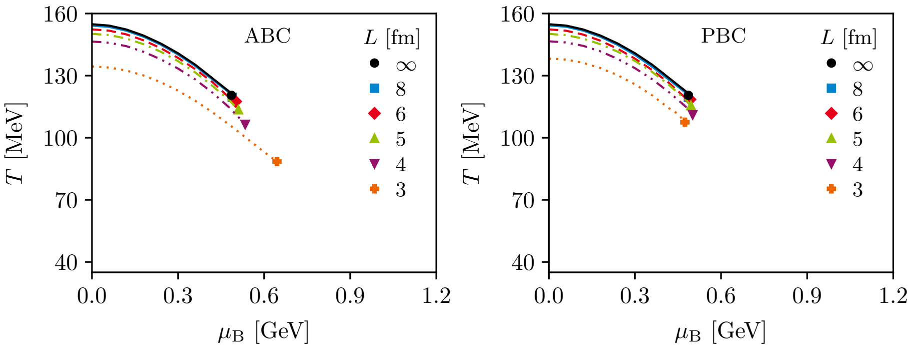
$$\omega_n^L = \begin{cases} 2n\pi/L & \text{for PBC}\,, \\ (2n+1)\pi/L & \text{for ABC}\,, \end{cases} \qquad n \in \mathbb{Z}$$

- Momentum integrals become sums

$$\int \frac{\mathrm{d}^3 q}{(2\pi)^3}\, K(\boldsymbol{q}) \to \frac{1}{L^3} \sum_{\boldsymbol{n}\in\mathbb{Z}^3} K(\boldsymbol{q_n})\,,$$

where $\boldsymbol{q_n} := \sum_{i=1}^{3} \omega_{n_i}^L \boldsymbol{e}_i$ are allowed momentum vectors

Results: QCD Phase Diagram in a Finite Volume



- Consistent infinite-volume limit
- For decreasing L, pseudocritical temperature decreases and CEP (mostly) moves to higher μ
- Visible volume effects for $L \leq 4$ fm
- Very similar results for ABC and PBC above $L \geq 4$ fm

JB, Fischer, Isserstedt, Schaefer, PRD 104 (2021) 074035

Baryon Number Fluctuations

Fluctuations of Conserved Charges from QCD's Grand Potential

$$\chi_{ijk}^{\mathrm{BQS}} = -T^{(i+j+k)-4} \frac{\partial^{i+j+k}}{\partial\mu_{\mathrm{B}}^{i}\partial\mu_{\mathrm{Q}}^{j}\partial\mu_{\mathrm{S}}^{k}} \Omega$$

B: baryon number
Q: electric charge
S: strangeness

- Relation to cumulants of baryon number distribution:

$$C_n^{\mathrm{B}} = V T^3 \chi_n^{\mathrm{B}} \quad \text{where} \quad \chi_n^{\mathrm{B}} = \chi_{n00}^{\mathrm{BQS}}$$

- Directly linked to moments of baryon number distribution:

$$\sigma_{\mathrm{B}}^2 = C_2^{\mathrm{B}}\,, \quad S_{\mathrm{B}} = C_3^{\mathrm{B}}(C_2^{\mathrm{B}})^{-3/2}\,, \quad \kappa_{\mathrm{B}} = C_4^{\mathrm{B}}(C_2^{\mathrm{B}})^{-2}\,, \quad \ldots$$

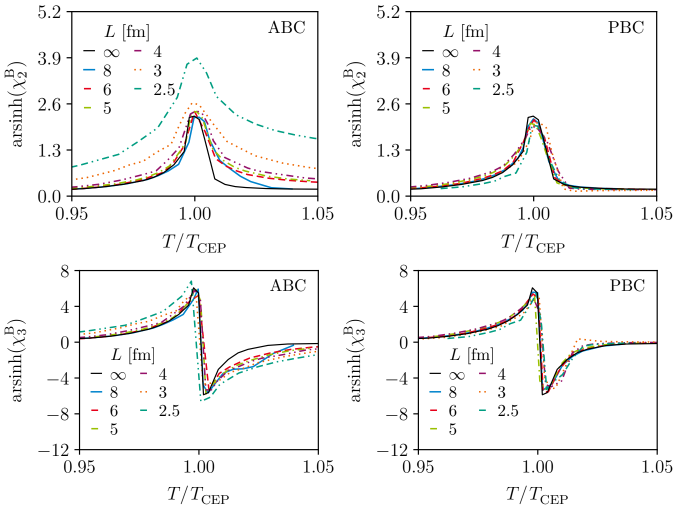
- Ratios relate theoretical and experimental quantities:

$$\chi_3^{\mathrm{B}}/\chi_2^{\mathrm{B}} = S_{\mathrm{B}}\sigma_{\mathrm{B}}\,, \quad \chi_4^{\mathrm{B}}/\chi_2^{\mathrm{B}} = \kappa_{\mathrm{B}}\sigma_{\mathrm{B}}^2\,, \quad \ldots$$

$\rightarrow$ Explicit volume dependence drops out!

Reviews: Luo, Xu, Nucl. Sci. Tech. 28 (2017) 112
Bzdak, Esumi, Koch, Liao, Stephanov, Xu, Phys. Rep. 853 (2020) 1

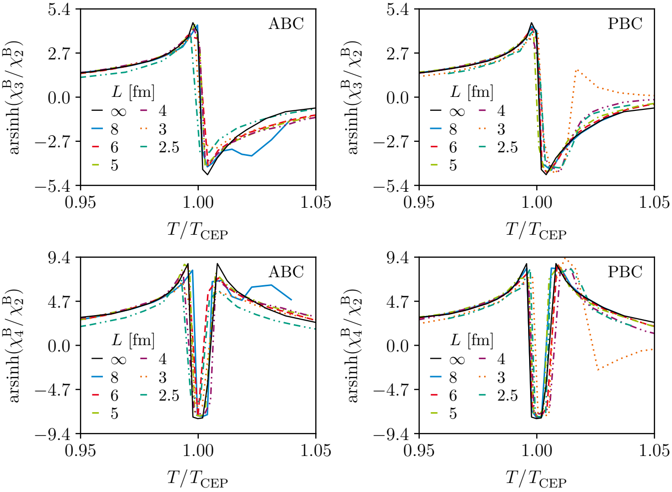
Results: Baryon Number Fluctuations at $\mu_B = \mu_B^{CEP}$



- Visible volume effects (especially for ABC)

JB, Fischer, Isserstedt (arXiv: 2208.01981)

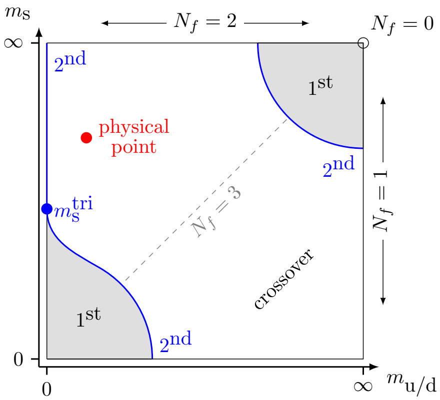
Results: Baryon Number Fluctuation Ratios at $\mu_B = \mu_B^{CEP}$



- (Essentially) independent of system size → no implicit volume dependence

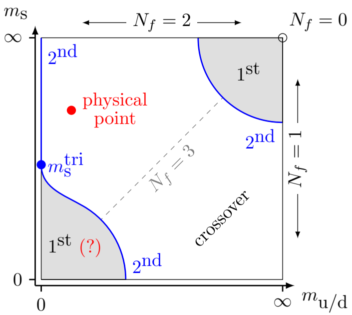
JB, Fischer, Isserstedt (arXiv: 2208.01981)

Second Objective: Columbia Plot and (Up Quark) Chiral Limit



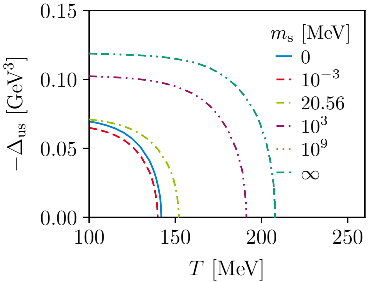
Motivation: Columbia Plot(s)

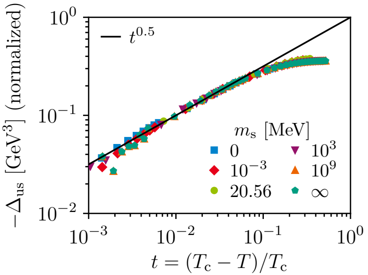
for reference on upper right corner in DSE framework,
see Fischer, Luecker, Pawlowski, PRD 91 (2015) 014024

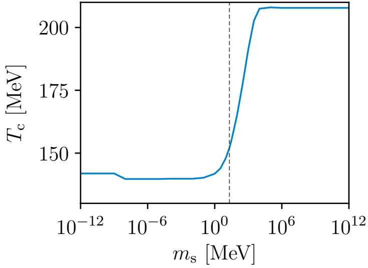


- Two different scenarios for Columbia Plot: anomalously broken (left) or restored (right) $U_A(1)$-symmetry
- Existence of first order region in lower left corner (of left scenario) is not yet clear see Cuteri, Philipsen, Sciarra, JHEP 11 (2021) 141
- Chiral limit is difficult for lattice QCD but no conceptual problem for our framework

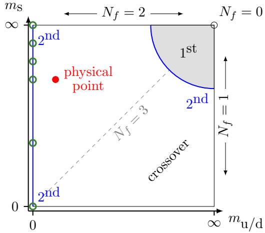
Results: Condensate and Critical Scaling in Chiral Limit







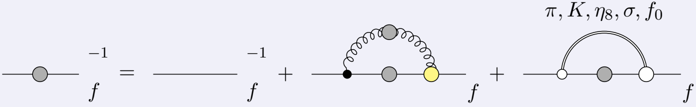
Resulting Columbia Plot:



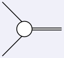
Meson Backcoupling Ansatz

- Improve truncation in chiral limit: long-range correlations in vertex become important → add meson backcoupling diagram to quark DSE

Modified Quark DSE



Bethe–Salpeter Amplitudes



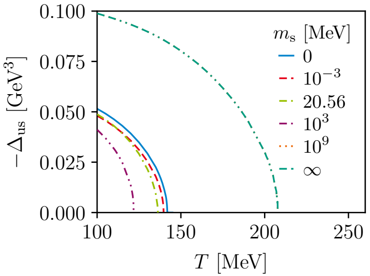
(Obtained from quarks: Goldberger–Treiman-like relations)

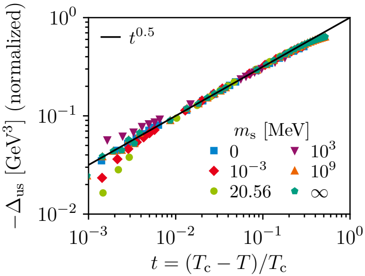
Free Meson Propagator

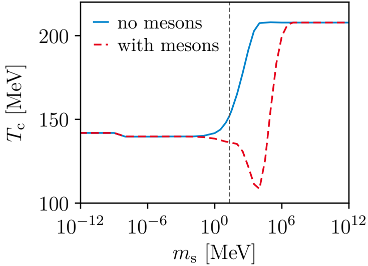
(Mass from Gell-Mann–Oakes–Renner fit)

details on meson backcoupling: Fischer, Müller, PRD 84 (2011) 054013

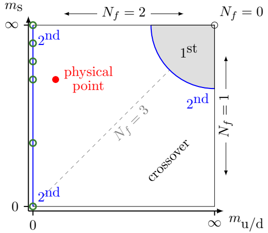
Results for Meson Backcoupling







Resulting Columbia Plot:



Conclusion and Outlook

Conclusion:

- Studied finite-volume effects on QCD phase diagram using DSEs beyond rainbow–ladder truncation for ABC and PBC
- Crossover line and CEP exhibit visible volume effects for $L \leq 4\,\mathrm{fm}$
- Baryon number fluctuations show volume dependence, ratios do not
- Second order phase transition across whole left edge of Columbia Plot (both with and without meson backcoupling)

Outlook:

- Implement proper QCD scaling to meson backcoupling
- Study finite-volume effects in chiral limit