

Excited QCD 2022

# MACHINE LEARNING FOR HADRON SPECTROSCOPY

Mount Etna

**César Fernández Ramírez**

#SOMOS2030

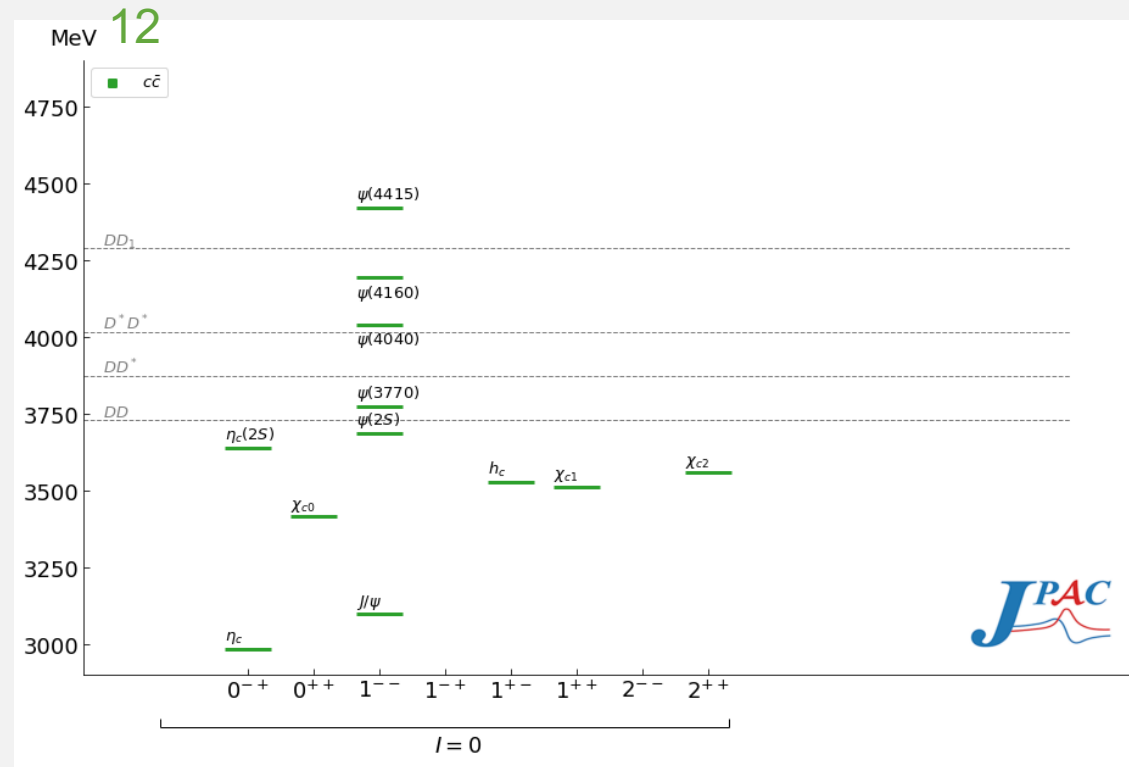


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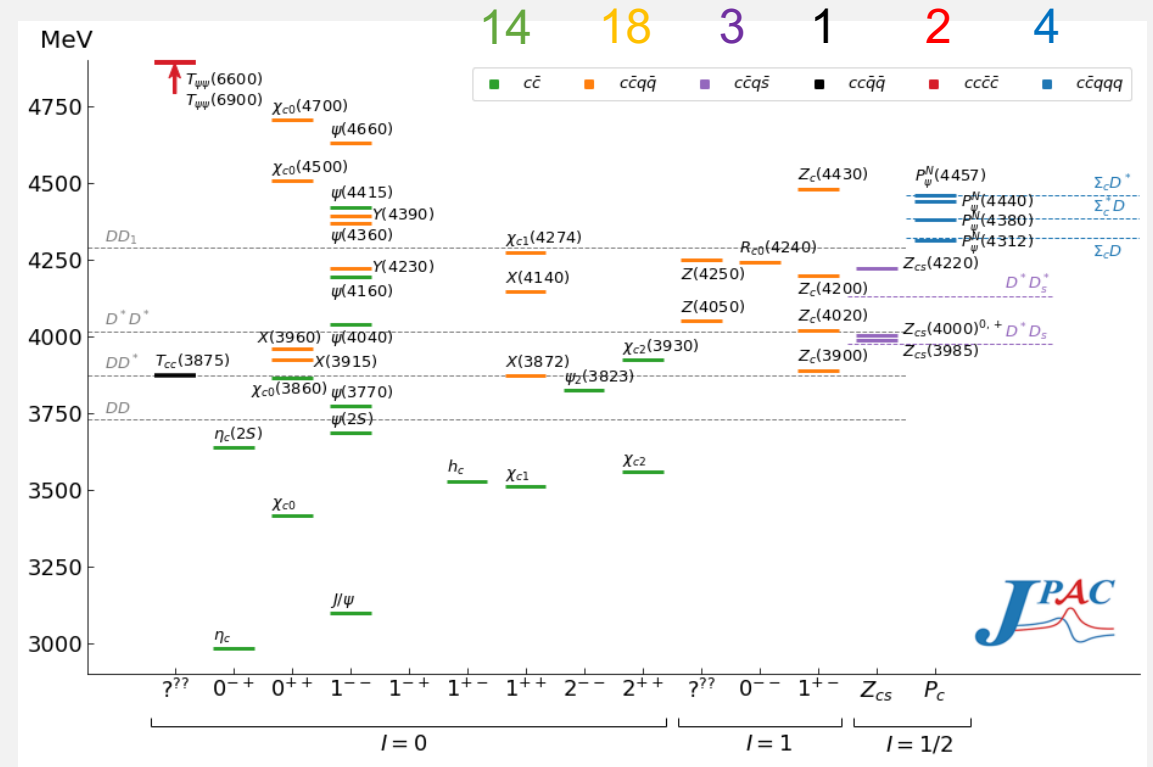


# Charmonia(-like)

2003

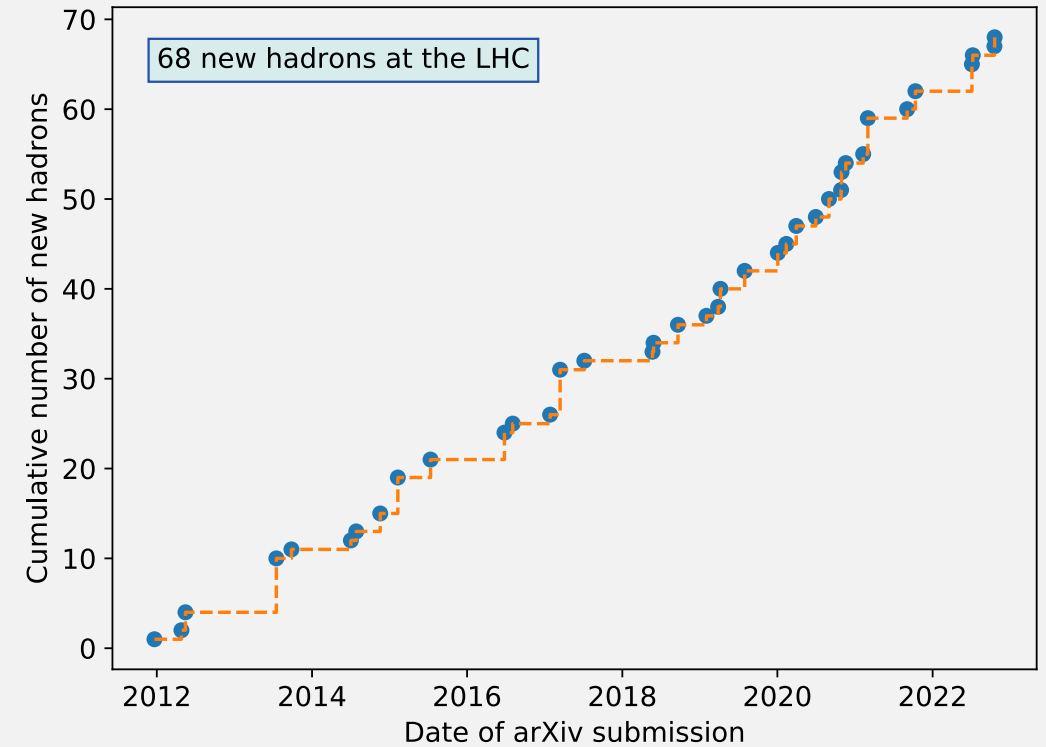
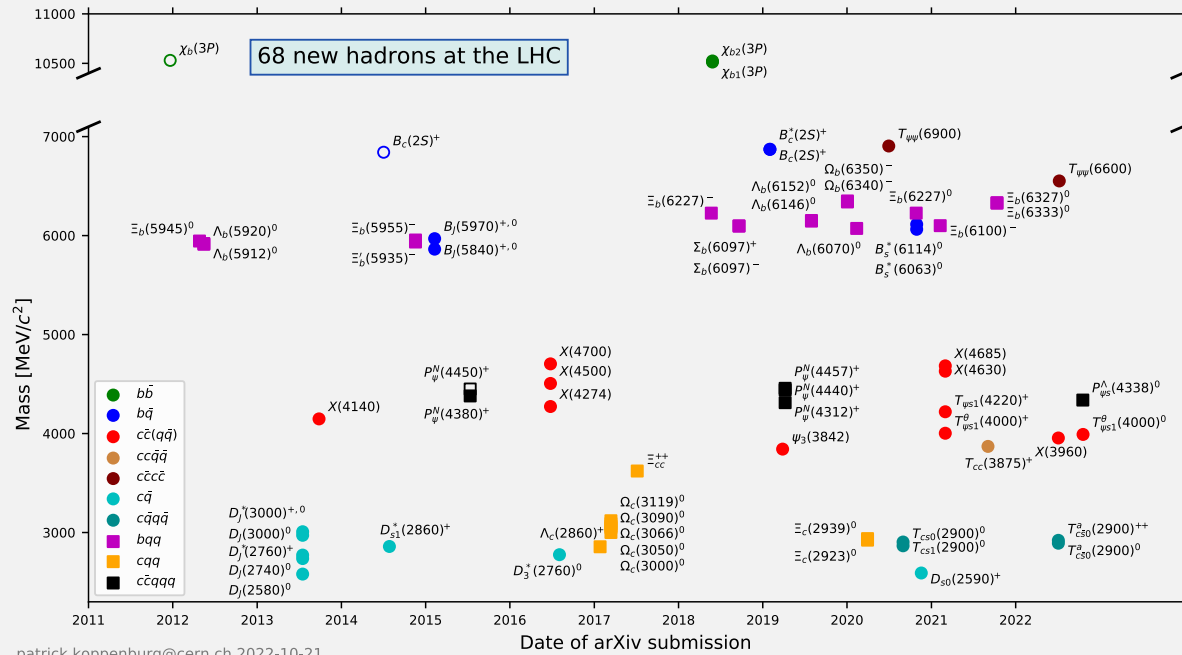


2022



And  $P_{\psi S}^{\Delta}(4338), T_{c\bar{s}0}^a(2900)^{0,++}$

# Just LHC





# Standard approach to lineshape analysis

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Build a model for the amplitude and assume it is true

Fit data using  $\chi^2$

Extract model parameters and get pole positions and compute uncertainties

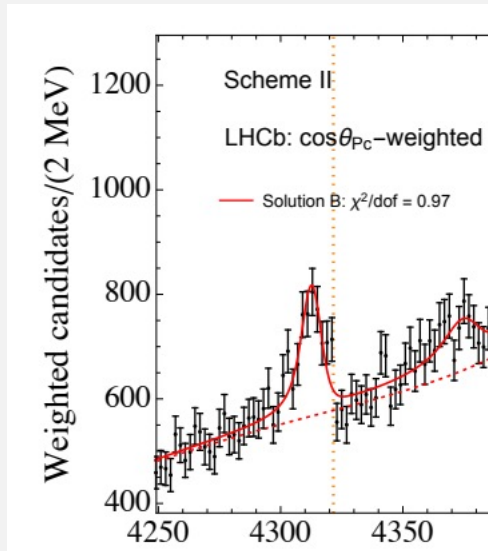
Asses the probability that those data were generated by your model

If everything is fine, you can claim that the interpretation embedded in the model is a possible explanation of the data

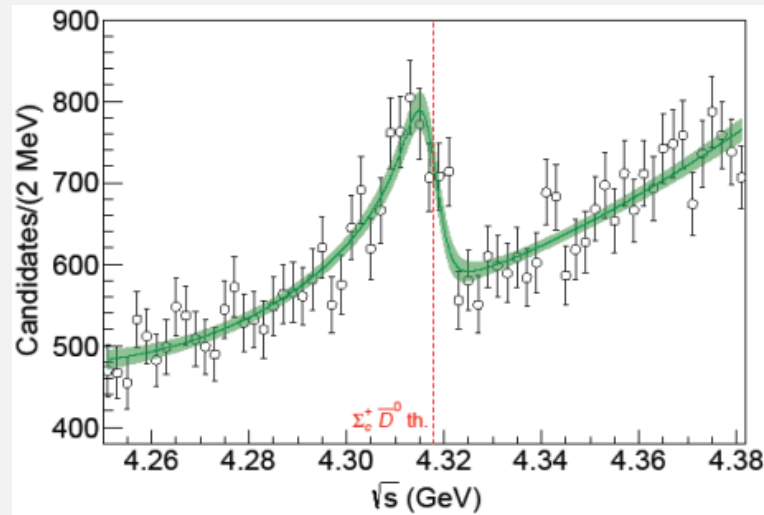
You can do this with different models with different underlying dynamics

Compare models? Compare dynamics?

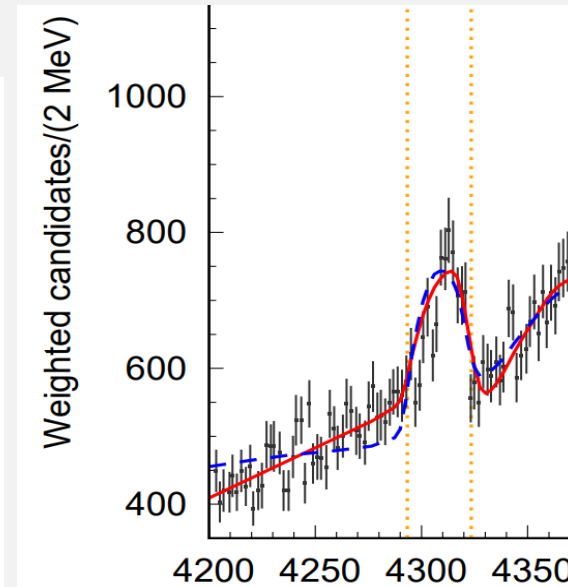
# Example $P_c(4312)$



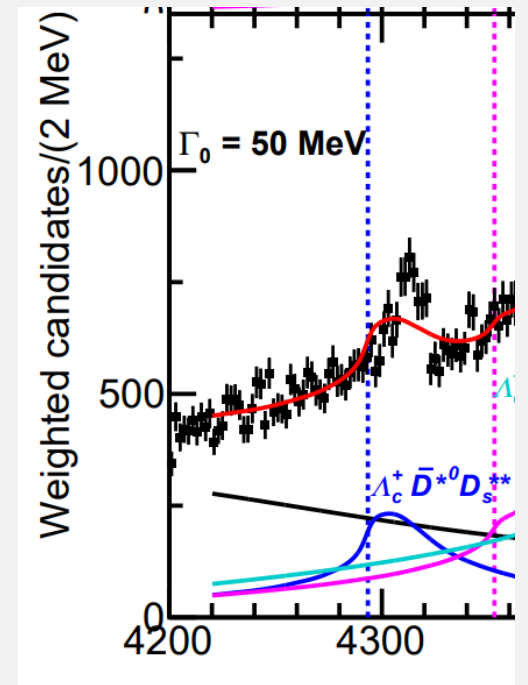
Molecule  
Du et al., 2102.07159



Virtual  
CFR et al. (JPAC), 1904.10021

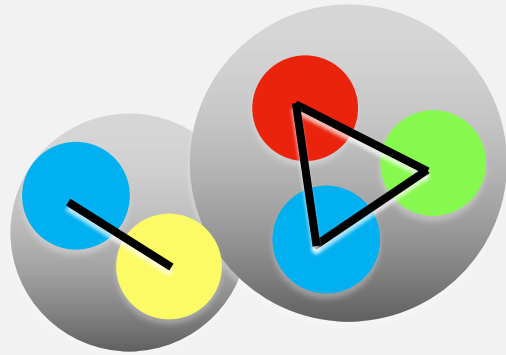


Double-triangle (w. complex  
coupl. in the Lagrangian)  
Nakamura, 2103.06817

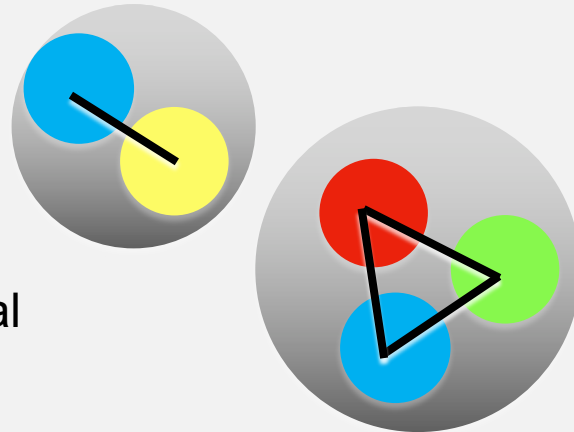


Single triangle  
(ruled out)  
LHCb, 1904.03947

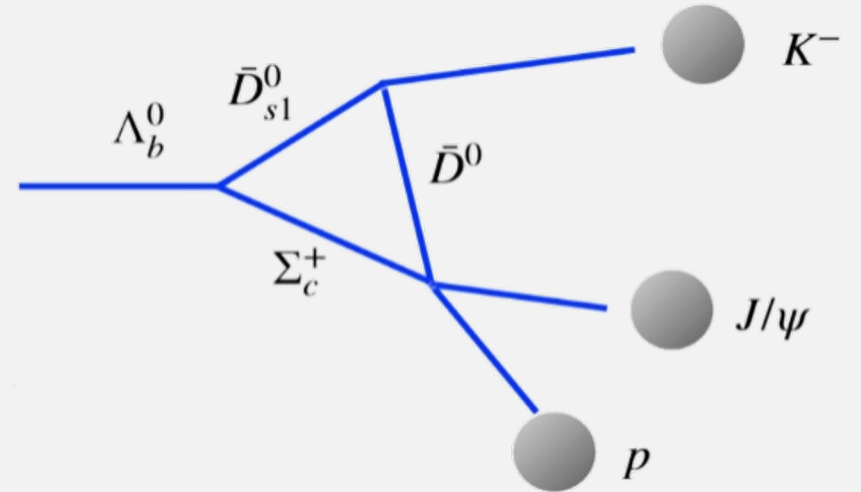
# Internal dynamics



Molecule



Virtual



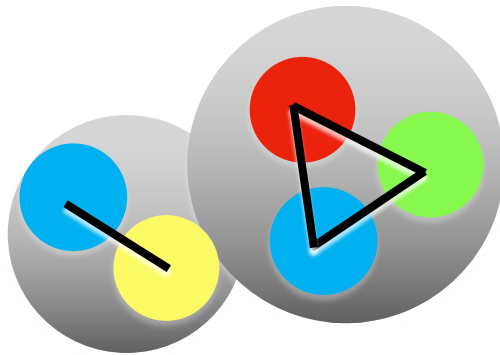
Single triangle  
(double triangle=same idea but more complicated)

# Threshold generated (residual interaction)

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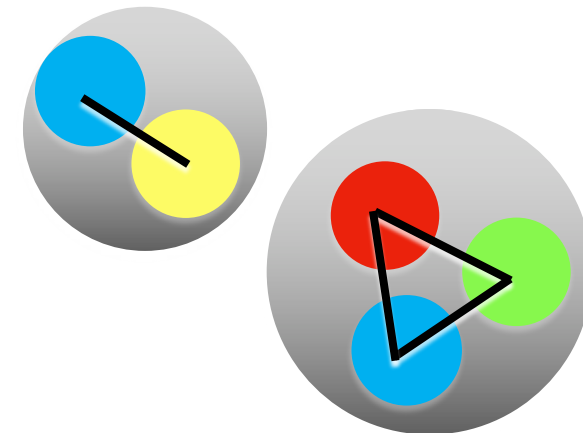
## Molecular state

- Generated through the opening of a new channel
- Residual part of the strong interaction bounds the system
- Finite radius



## Virtual state

- Generated through the opening of a new channel
- Residual part of the strong interaction generates the signal
- Infinite radius





# Can machine learning help us?

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First explorations of deep neural networks as classifiers for hadron spectroscopy:  
Sombillo et al., 2003.10770, 2104.141782, 2105.04898

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**Can we train a neural network to analyze a lineshape and get as a result what is the probability of each possible dynamical explanation?**

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If possible, what other information can we gain by using machine learning techniques?

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Still far away from answering those question but we are advancing

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Benchmark case:  $P_c(4312)$  lineshape

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# Outlook

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Amplitude analysis of  $P_c(4312)$  (benchmark case):  
CFR et al. (JPAC), 1904.10021

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Deep neural networks: classifiers

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Application to our benchmark case  
Ng et al. (JPAC), 2110.13742

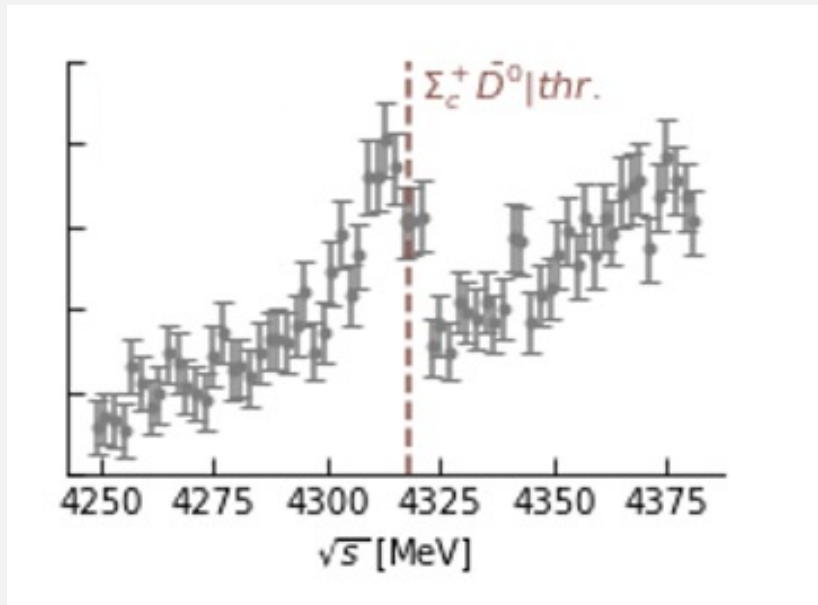
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Additional information: SHAP values

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Takeaways

# $J/\psi p$ projection data



Data from LHCb, 1904.03947

We focus on  $\Sigma_c^+ \bar{D}^0$  threshold region where the  $P_c(4312)$  signal appears

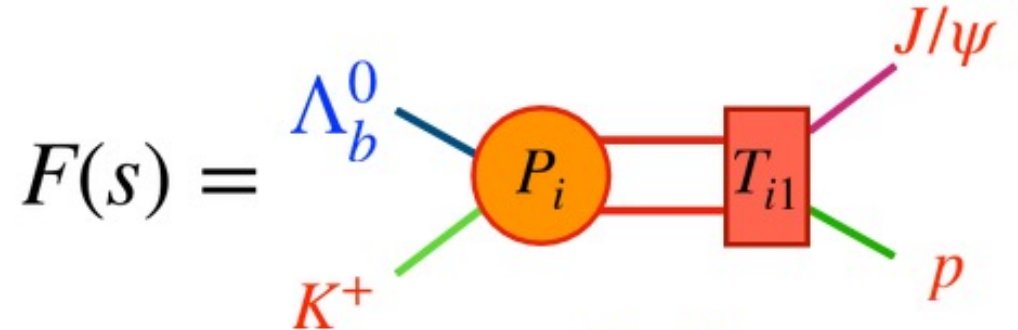
We assume only one partial wave contributes to the  $P_c(4312)$

The threshold is responsible for the dynamics (no compact state interpretation)

Other singularities are irrelevant

# Near- $\Sigma_c^+ \bar{D}_0$ threshold theory (two channels)

$$\frac{dN}{d\sqrt{s}} = \rho(s) \left[ |F(s)|^2 + B(s) \right]$$



$$F(s) = P_1(s)T_{11}(s) \quad (T^{-1})_{ij} = M_{ij} - ik_i\delta_{ij}$$

1 :  $J/\psi p$   
2 :  $\Sigma_c^+ \bar{D}_0$

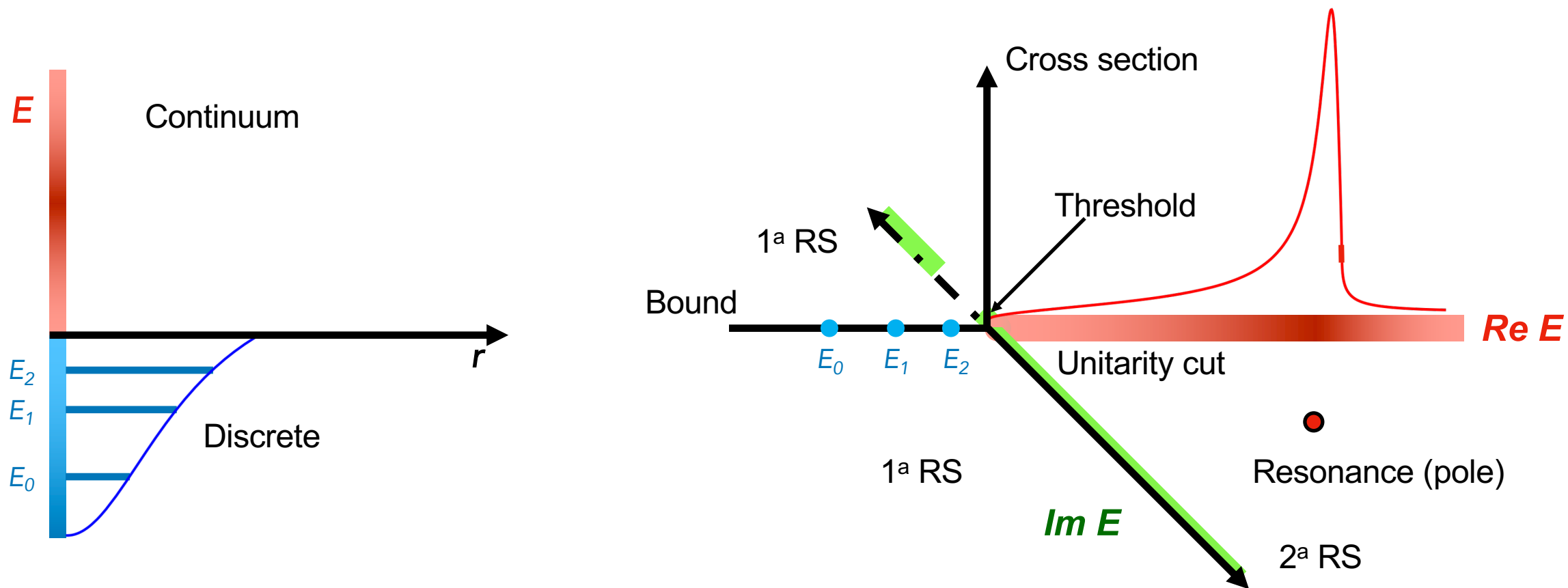
Scattering length      Effective range  
↓                              ↓

$$M_{ij}(s) = m_{ij} - c_{ij}s$$

Matrix elements  $M_{ij}$  are singularity free and can be Taylor expanded

Frazer, Hendry, PR134 (1964) B1307

# Resonances (aka poles in unphysical Riemann sheets)

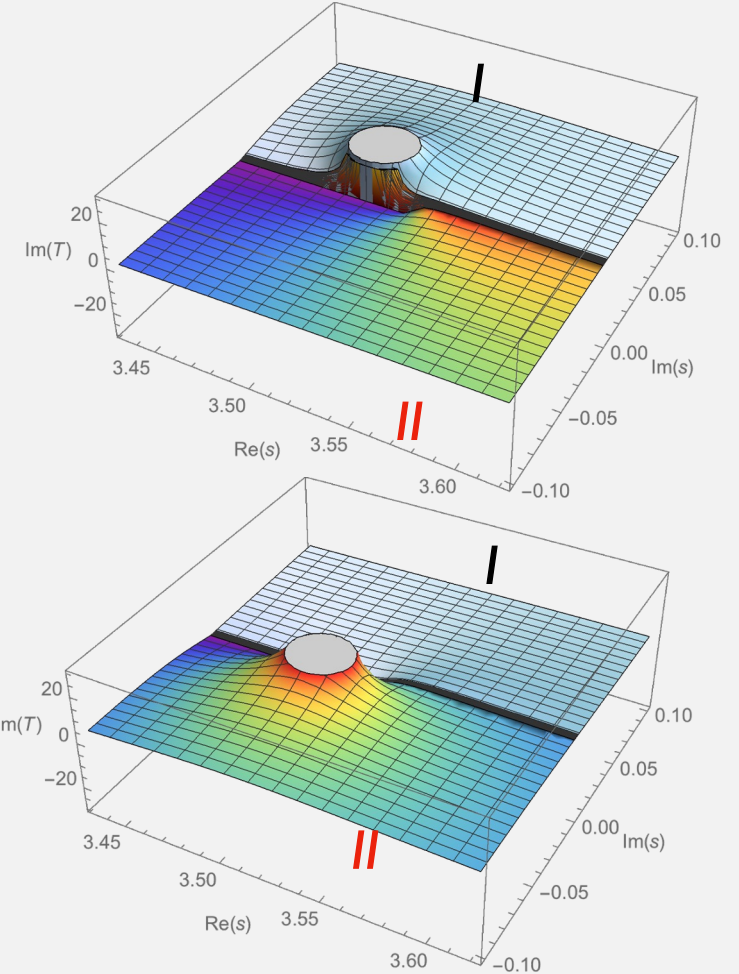
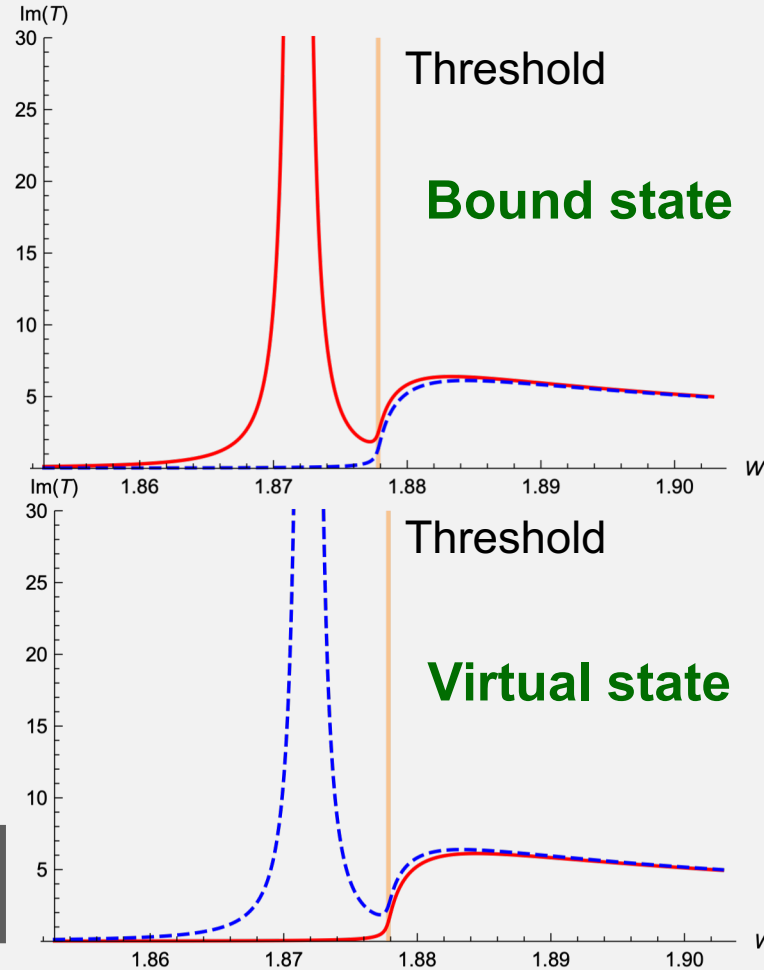


# Bound vs. virtual: pole position

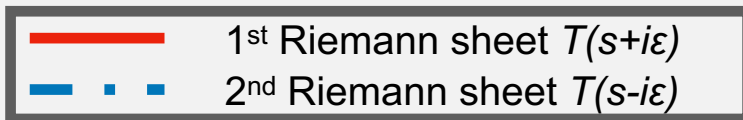
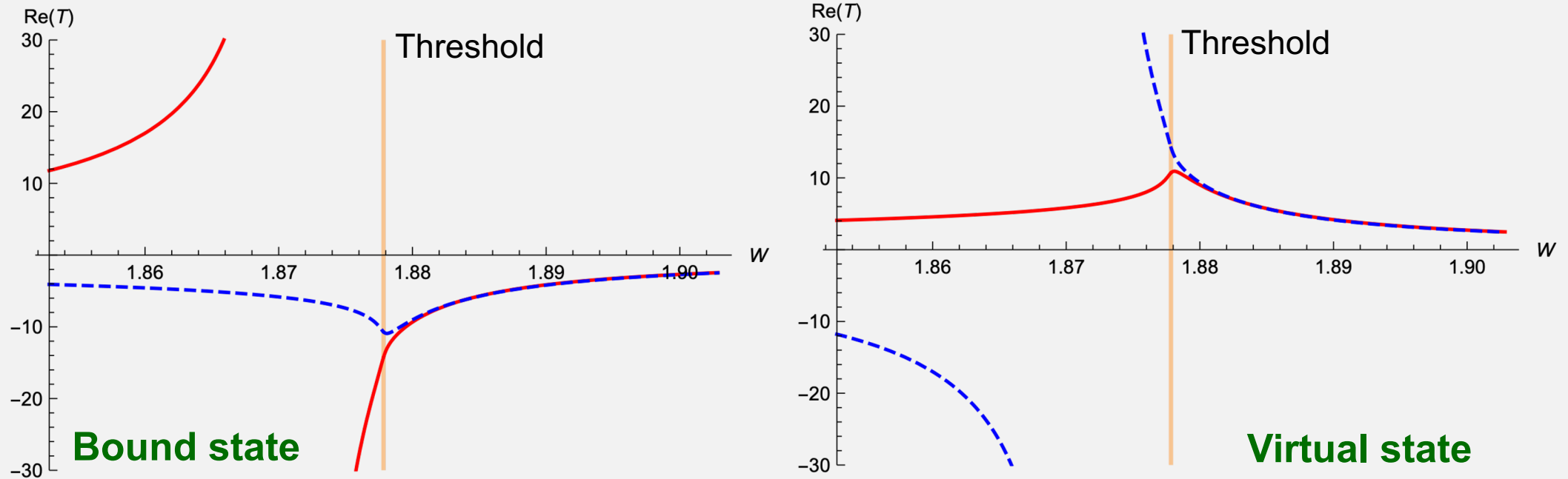
Amplitude close to the threshold  $\rho \simeq k$

$$T = \frac{1}{-\frac{1}{a} + \frac{r_e}{2}k^2 - ik}$$

— 1<sup>st</sup> Riemann sheet  $T(s+i\epsilon)$   
- - - 2<sup>nd</sup> Riemann sheet  $T(s-i\epsilon)$



# Bound vs. virtual: scattering length



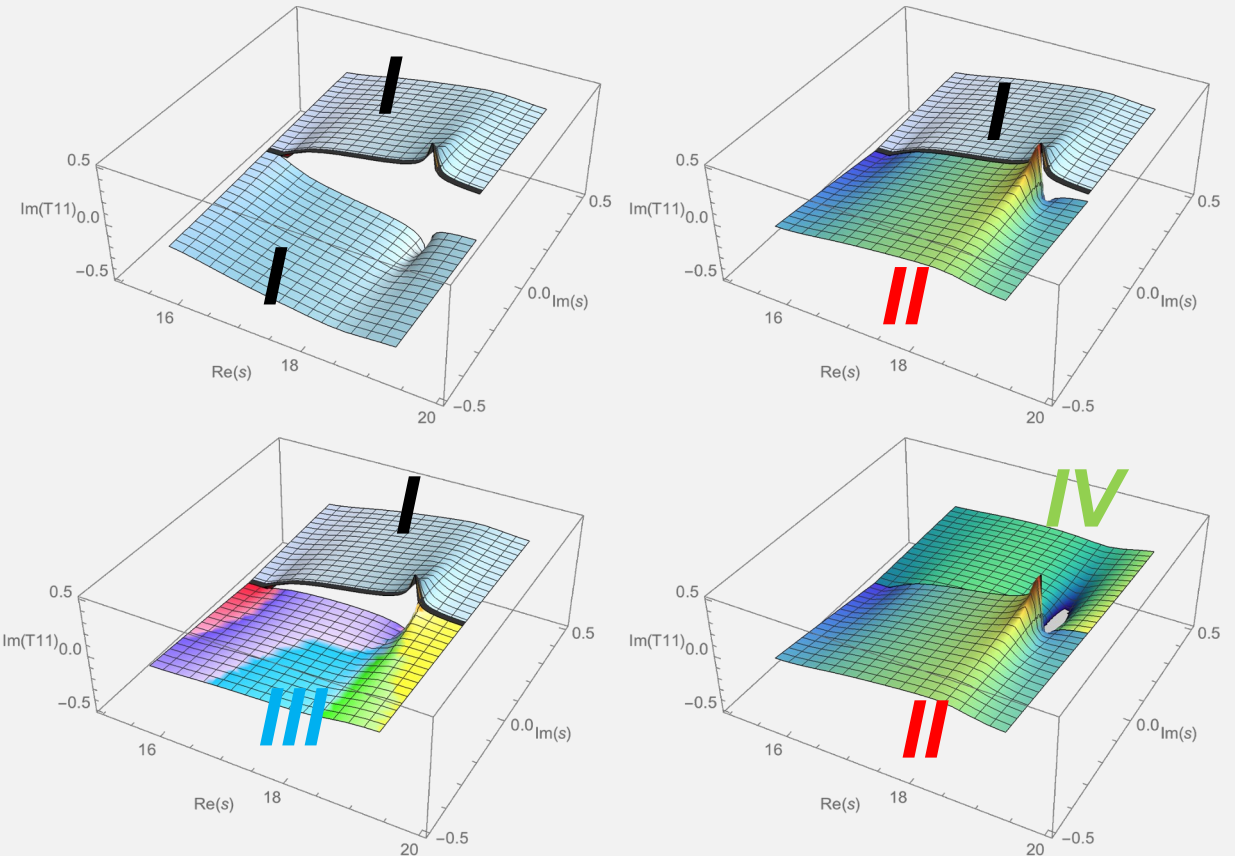
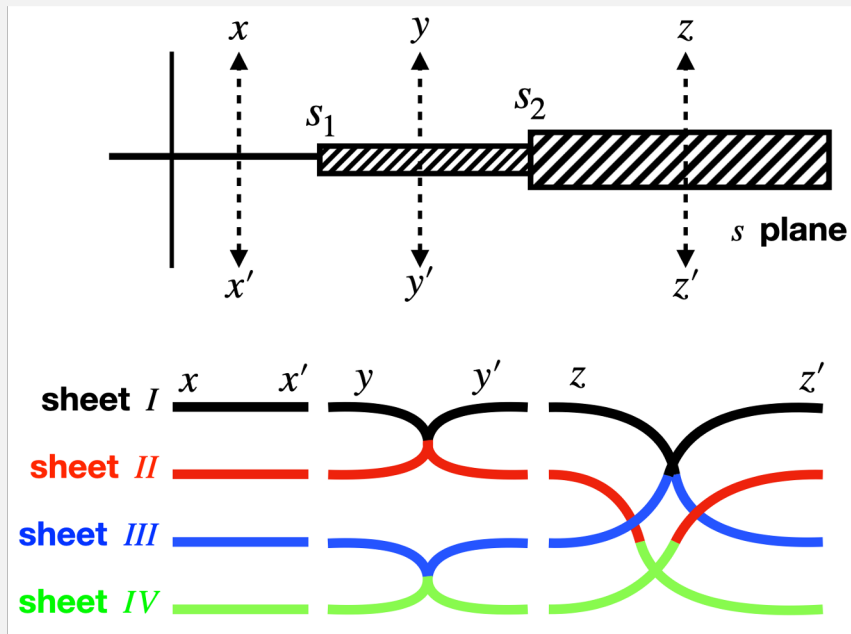
$$k \cot \delta(k) \simeq -\frac{1}{a} + \frac{r_e}{2} k^2$$

$$a_s(pn \rightarrow pn) \simeq 5.4 \text{ fm (deuteron, } ^3S_1)$$

$$a_t(pn \rightarrow pn) \simeq -24 \text{ fm (} ^1S_0)$$

# Riemann sheets structure (two channels)

$$\begin{bmatrix} J/\psi p \rightarrow J/\psi p & J/\psi p \rightarrow \Sigma_c \bar{D}^0 \\ \Sigma_c \bar{D}^0 \rightarrow J/\psi p & \Sigma_c \bar{D}^0 \rightarrow \Sigma_c \bar{D}^0 \end{bmatrix}$$

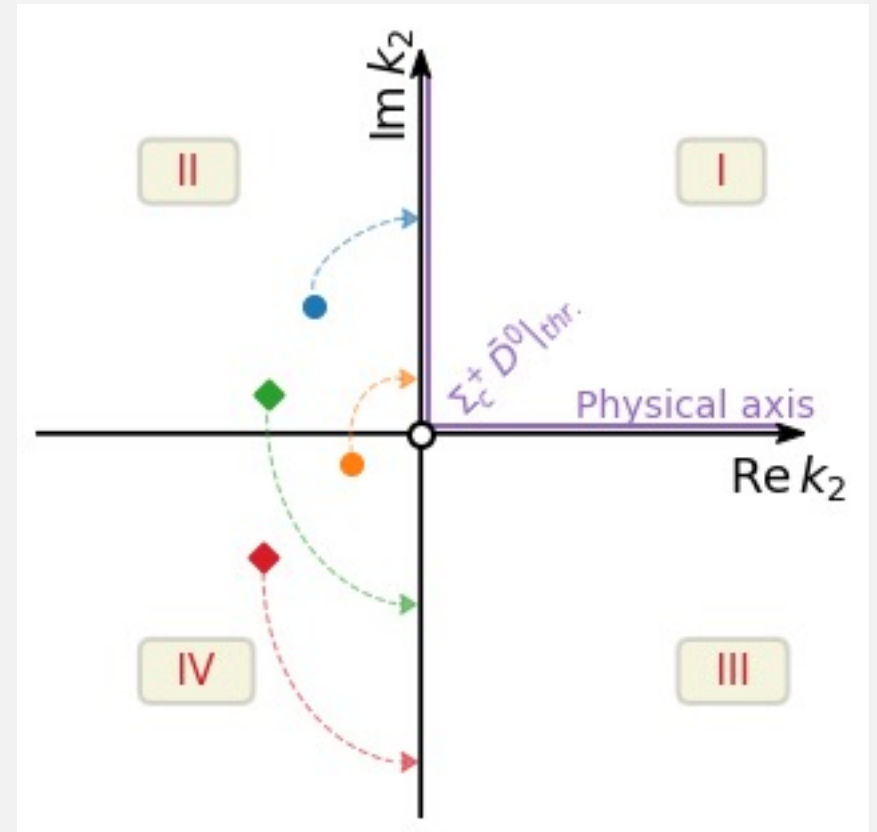




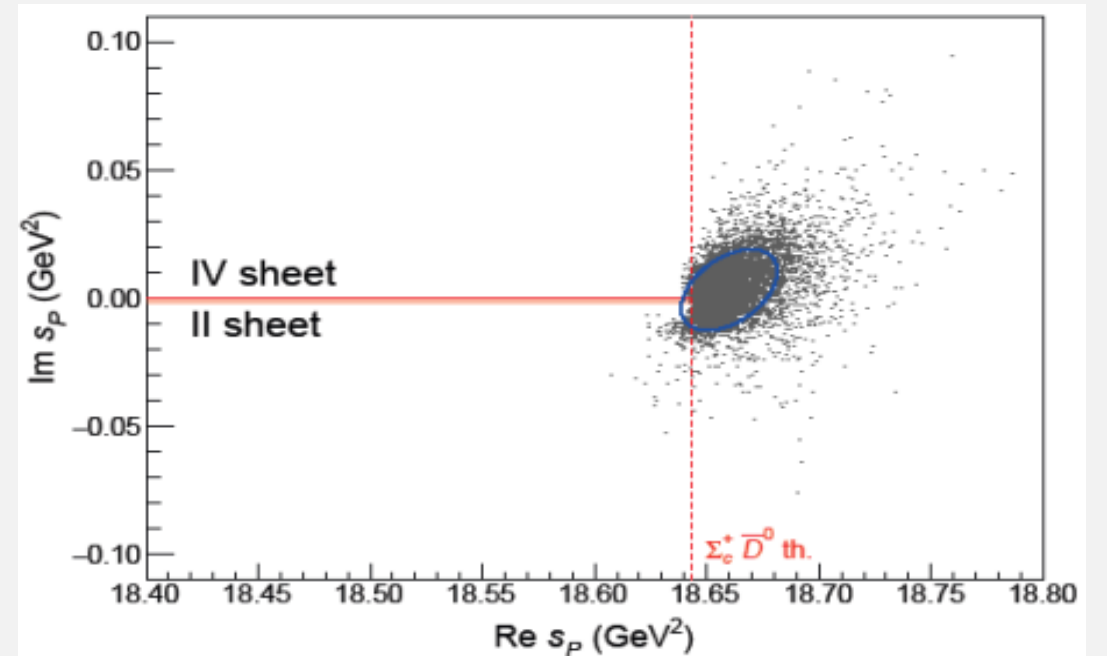
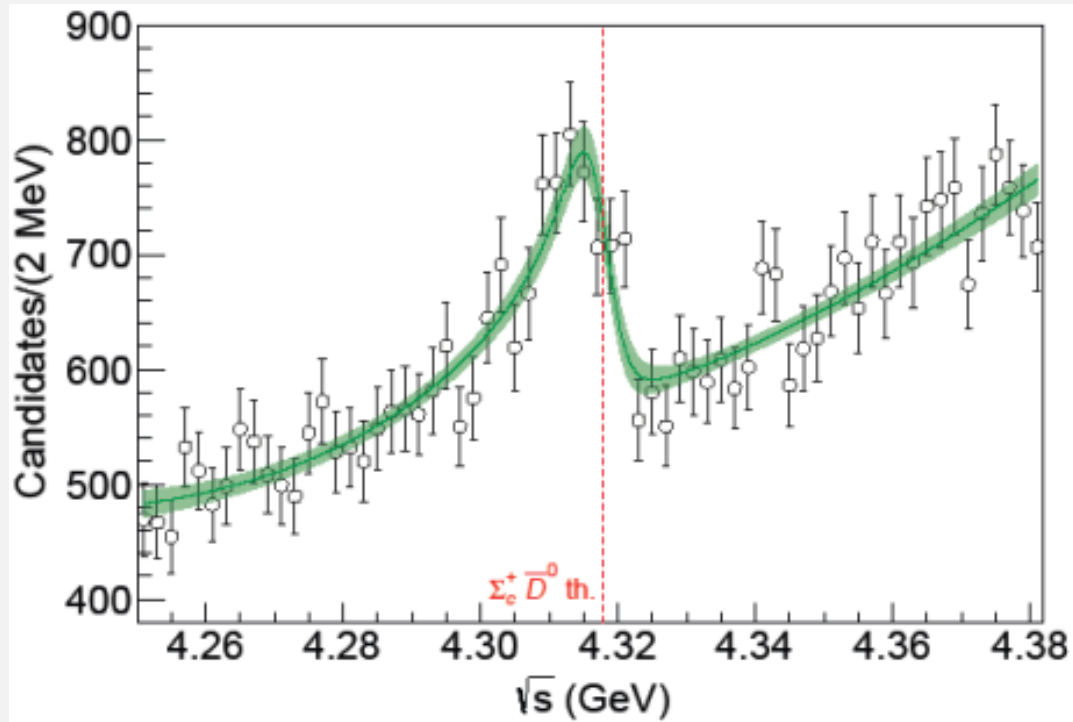
# Virtual and bound states

Under the the scattering length approximation the physical interpretation is given by the sign of the  $m_{22}$  parameter. Four options:

- Bound state on IV RS:  $b|4$
- Virtual state on IV RS:  $v|4$
- Bound state on II RS:  $b|2$
- Virtual state on II RS:  $v|2$



# Amplitude analysis result

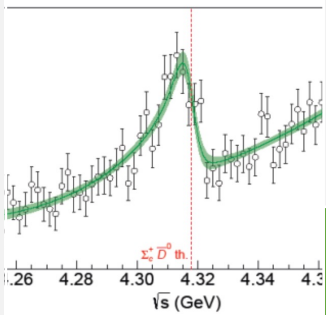


Interpretation obtained: Virtual state on IV RS (v|4)

$$M = 4319.7 \pm 1.6 \text{ MeV}$$

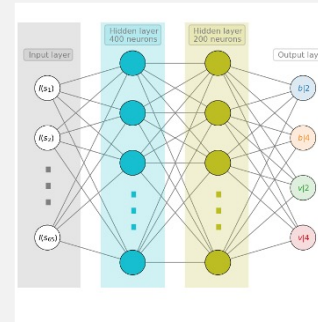
$$\Gamma = -0.8 \pm 2.4 \text{ MeV}$$

# Dictionary



## AMPLITUDE ANALYSIS

- Datapoints (lineshape)
- Model with random parameters convoluted with experimental resolution
- Experimental uncertainties
- Physical interpretation
- Objective (minimization) function

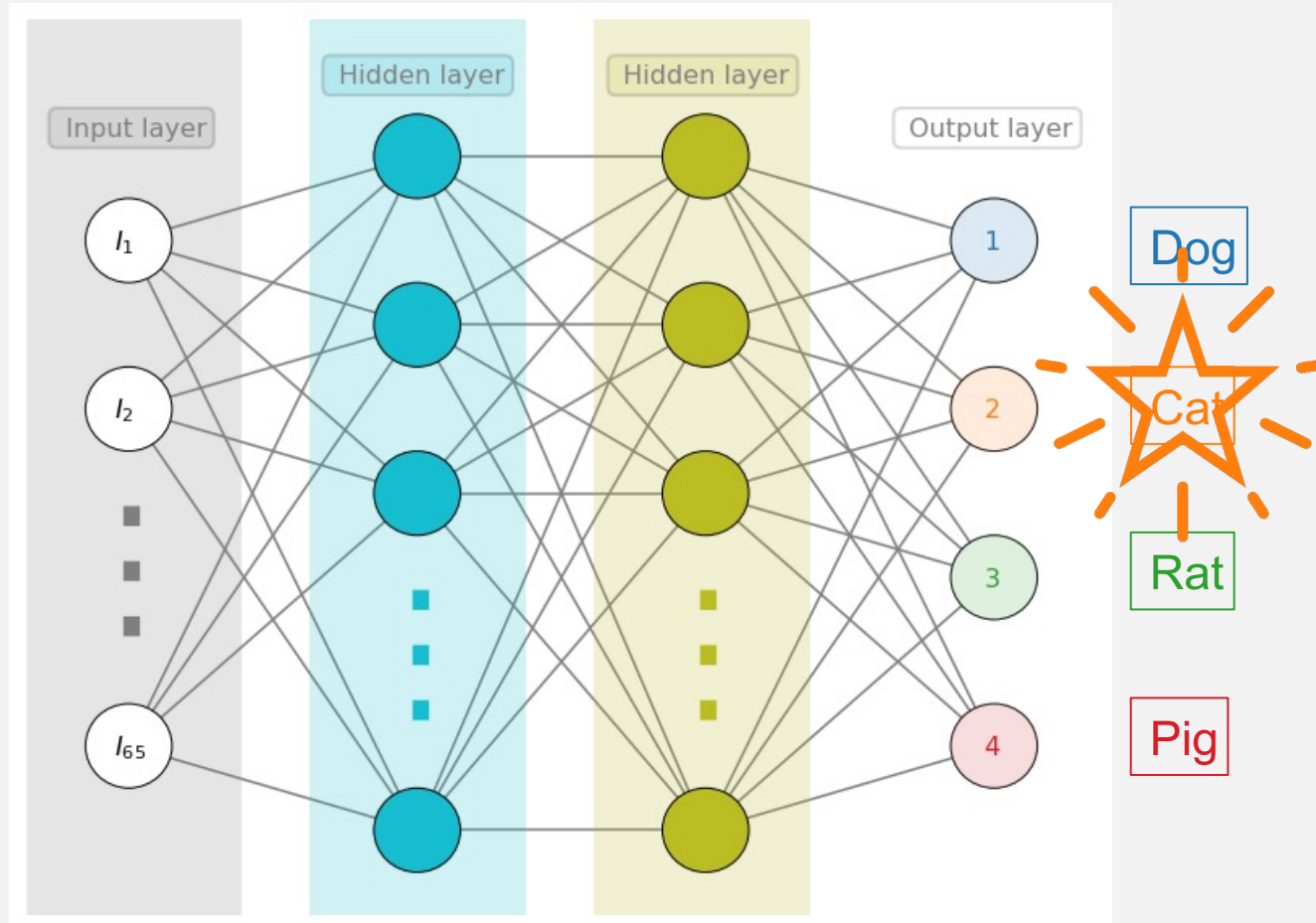
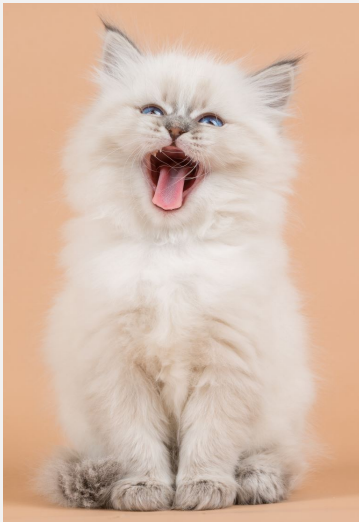


## MACHINE LEARNING

- Features
- Training set
- Noise
- Classes
- Minimize the DNN (Adam algorithm)

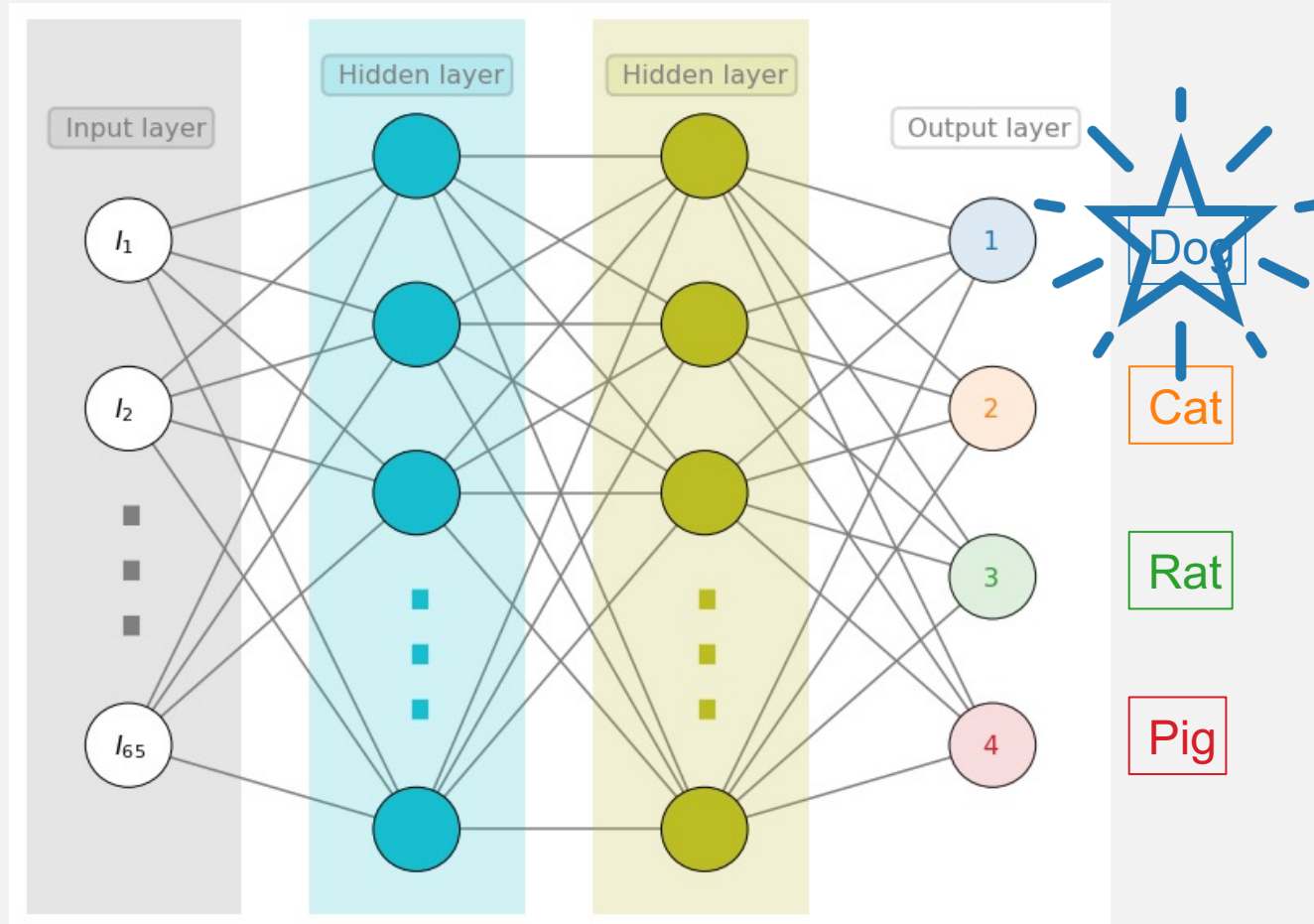
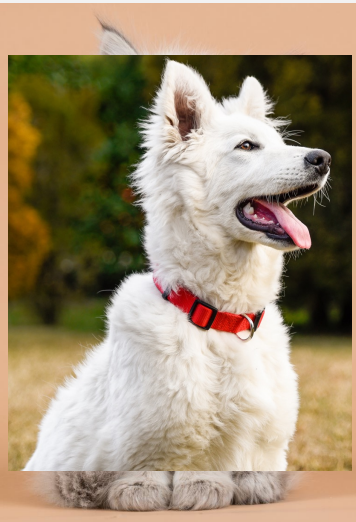
# Neural networks as classifiers

## Training



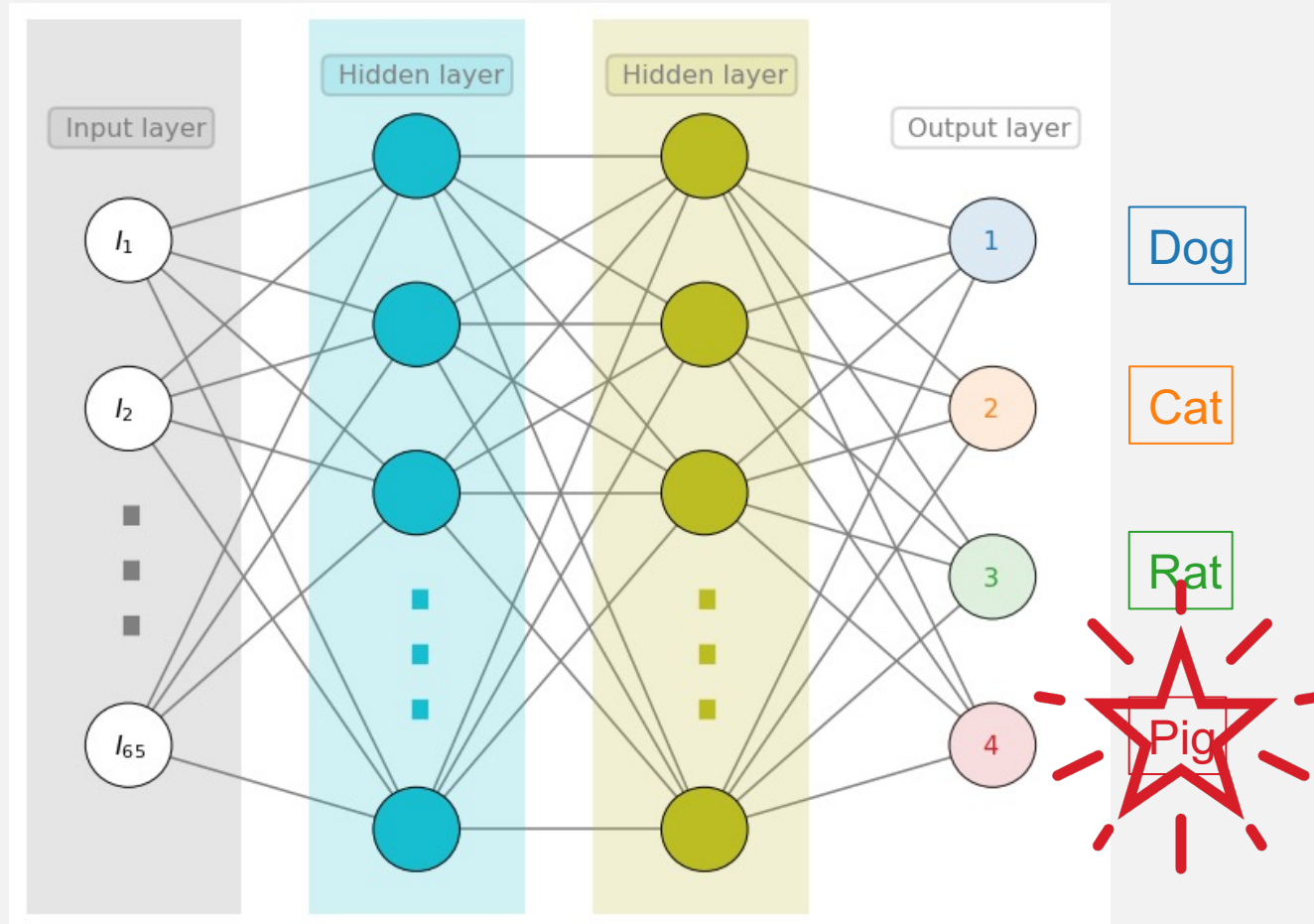
# Neural networks as classifiers

## Training



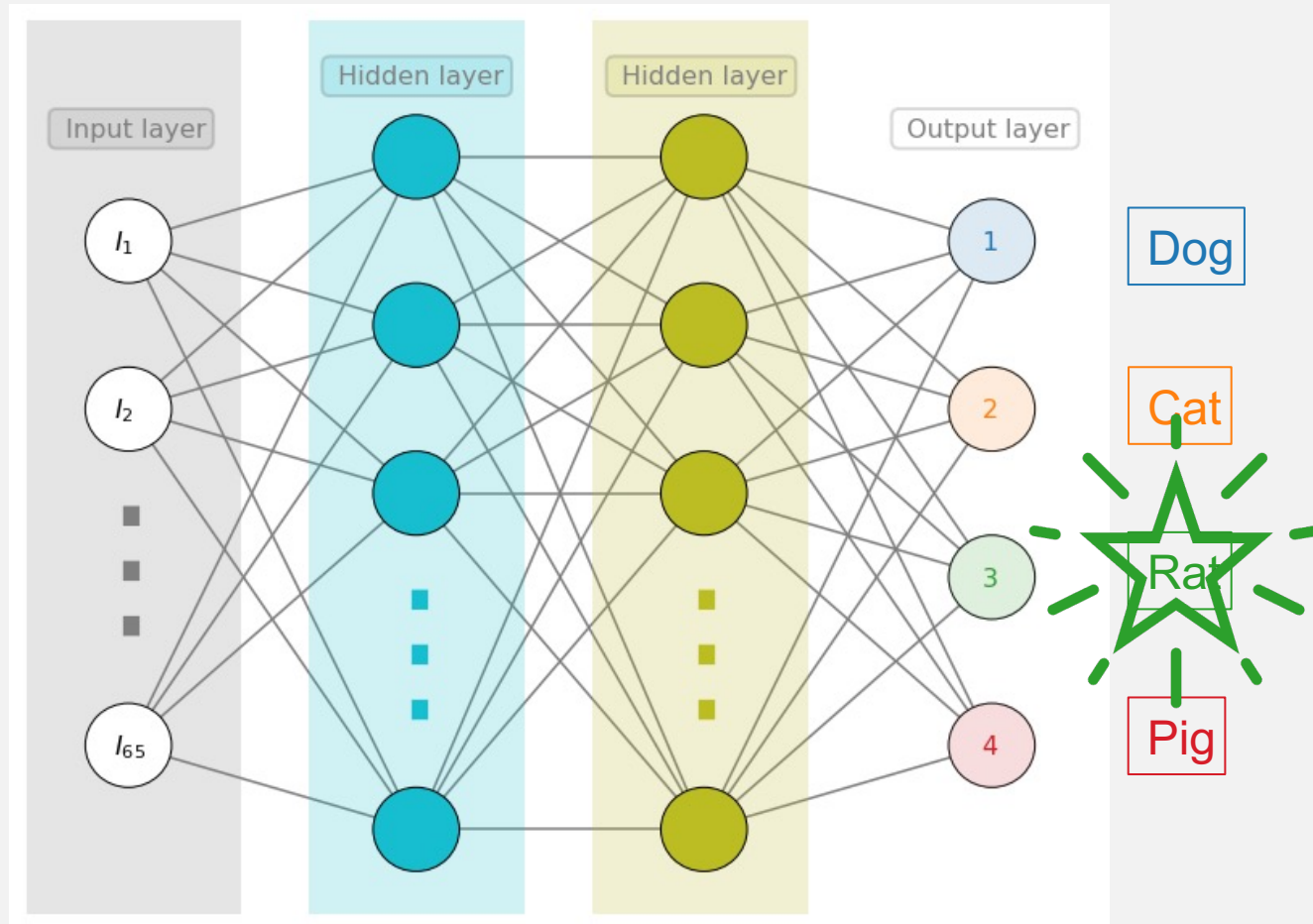
# Neural networks as classifiers

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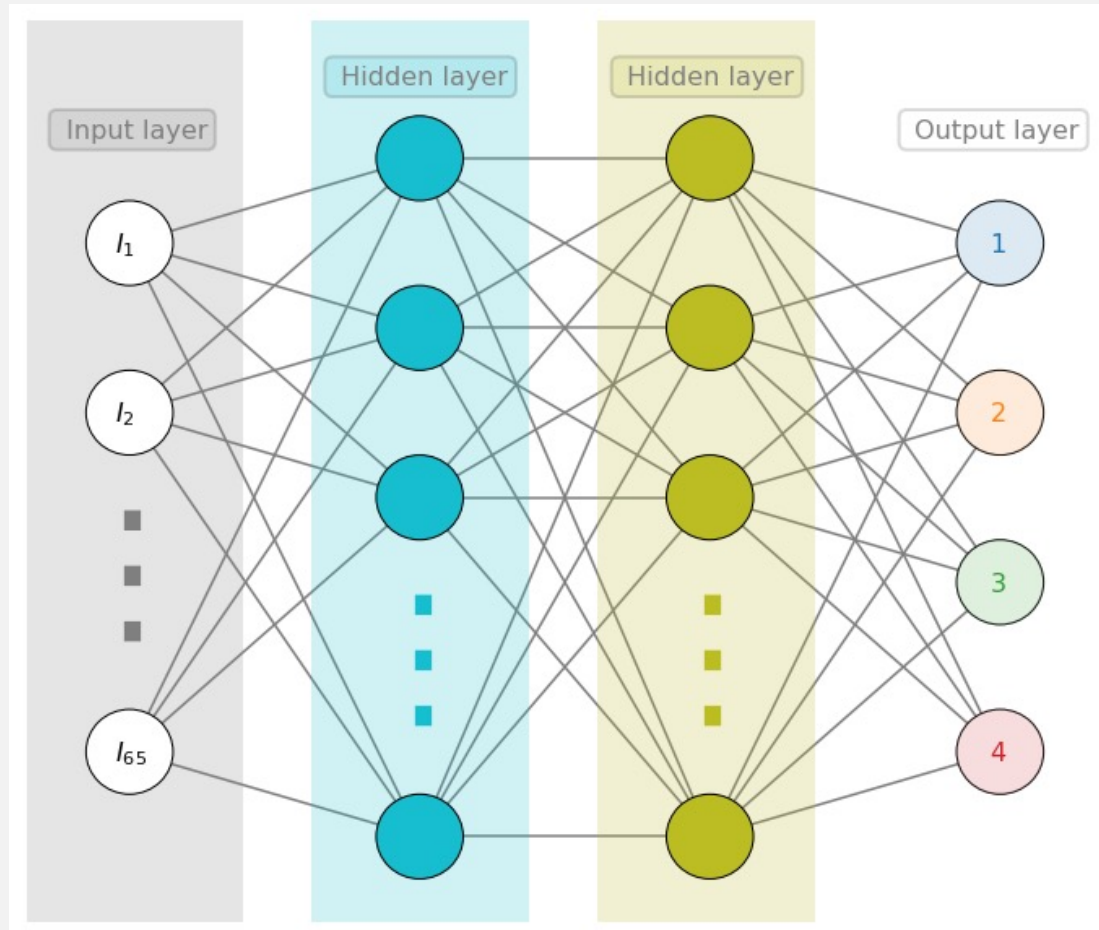
# Neural networks as classifiers

## Training



# Neural networks as classifiers

New picture



Dog

3%

Cat

96.8%

Rat

0.1%

Pig

0.1%



# The deep neural network

The input layer contains 65 experimental datapoints

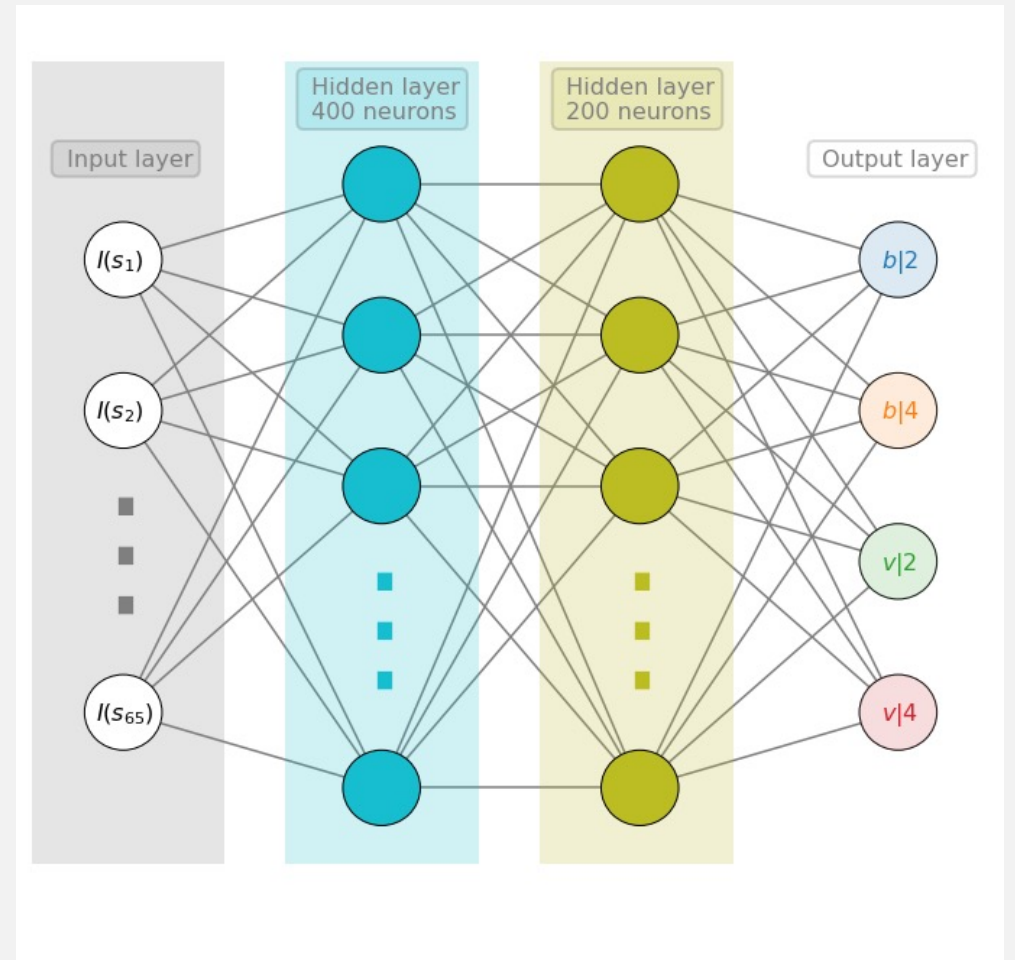
Two hidden layers with 400 and 200 neurons respectively

Output layer, four classes:  $v|4$ ,  $b|4$ ,  $b|2$ ,  $v|2$

Probabilities are obtained using the softmax function  $p(x_i) = \frac{e^{x_i}}{\sum e^{x_j}}$

Minimizing multi-class cross-entropy loss

Implemented with PyTorch (alt-version using Keras)



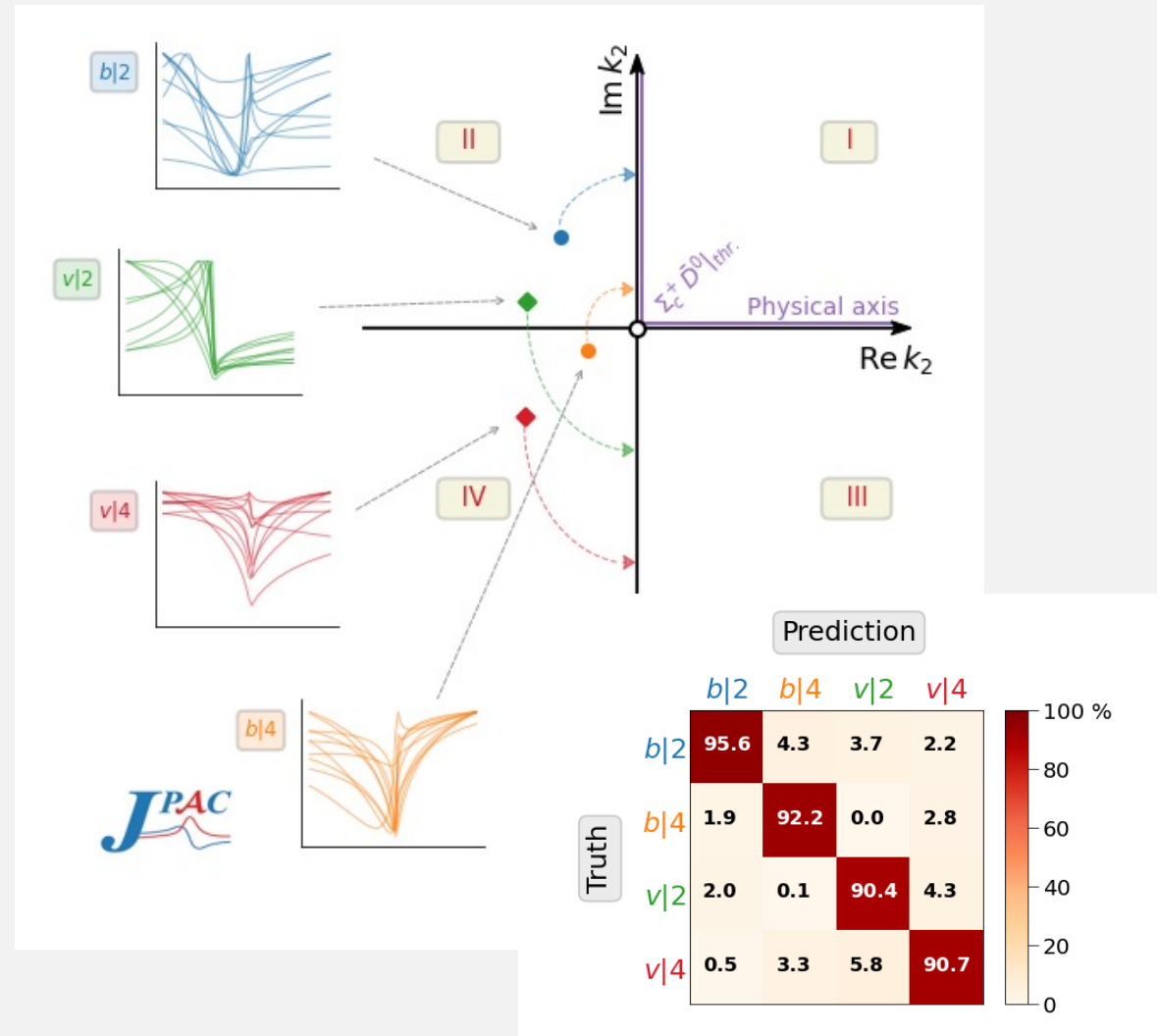
# Building training set. Training the DNN

$10^5$  training curves generated by randomly choosing parameter values

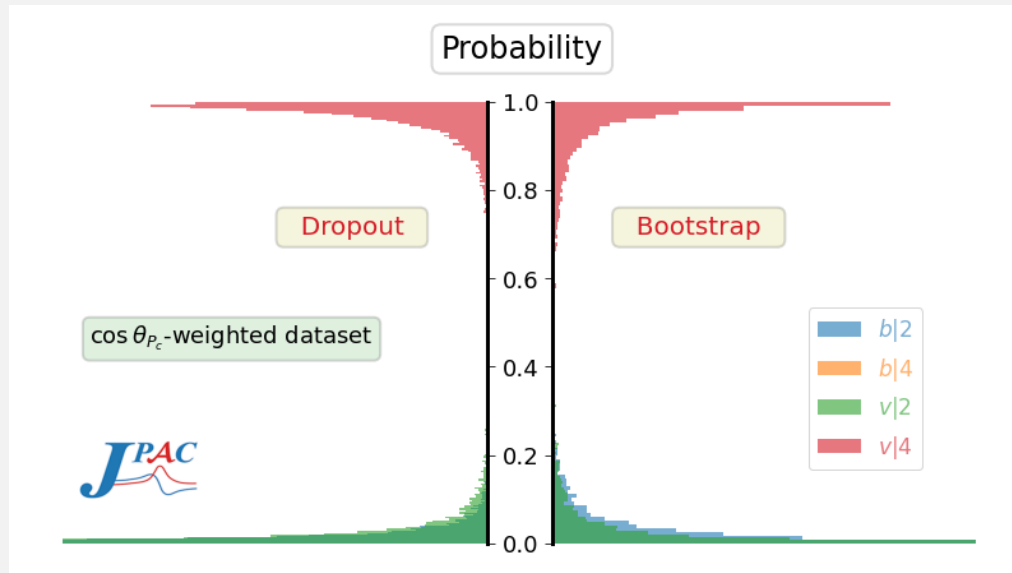
To mimic uncertainties a 5% noise is included in the training set

The lineshapes are convoluted with the experimental resolution

Experimental uncertainty limits the accuracy of the DNN



# Applying the DNN



We pass the three LHCb datasets (full, cut in  $m_{Kp}$  and weighted) through the DNN to obtain an answer. Note that we pass the three dataset through the same DNN

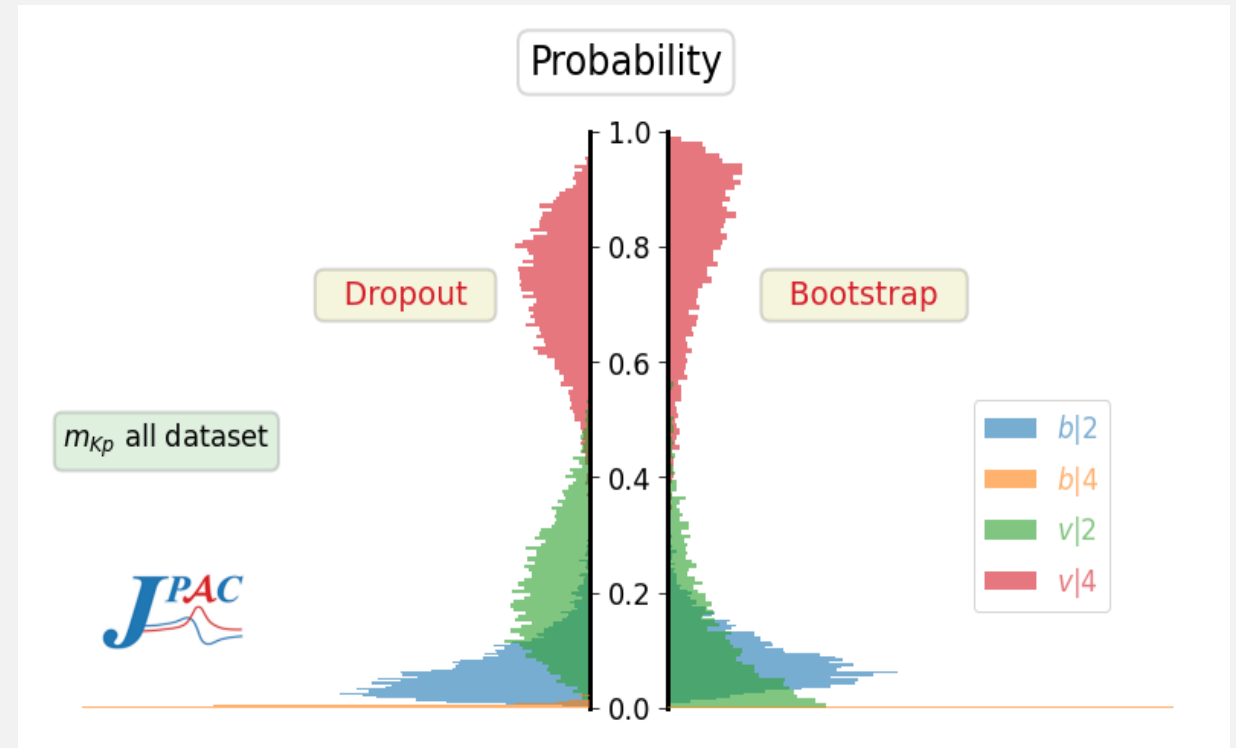
To account for uncertainties, we use two procedures: Bootstrap and Dropout

We (unsurprisingly) recover the same result as with the standard approach:  $v|4$

But we are learning new things...

# Three datasets analyzed with the same DNN

	$b 2$	$b 4$	$v 2$	$v 4$
$\cos \theta_{P_c}$ -weighted	0.6%	< 0.01%	1.1%	98.3%
$m_{Kp} > 1.9 \text{ GeV}$	1.4%	< 0.1%	1.6%	97.0%
$m_{Kp}$ all	5.4%	< 0.1%	21.0%	73.6%



# What we get from the DNN

The DNN targets specific regions of the parameter space (which yield stable solutions) that might be difficult to reach during optimization or might require high-resolution data

Standard  $\chi^2$  fit can be indeed unstable, and a small change in the input data can induce large changes in the parameter values and therefore in the physics interpretation

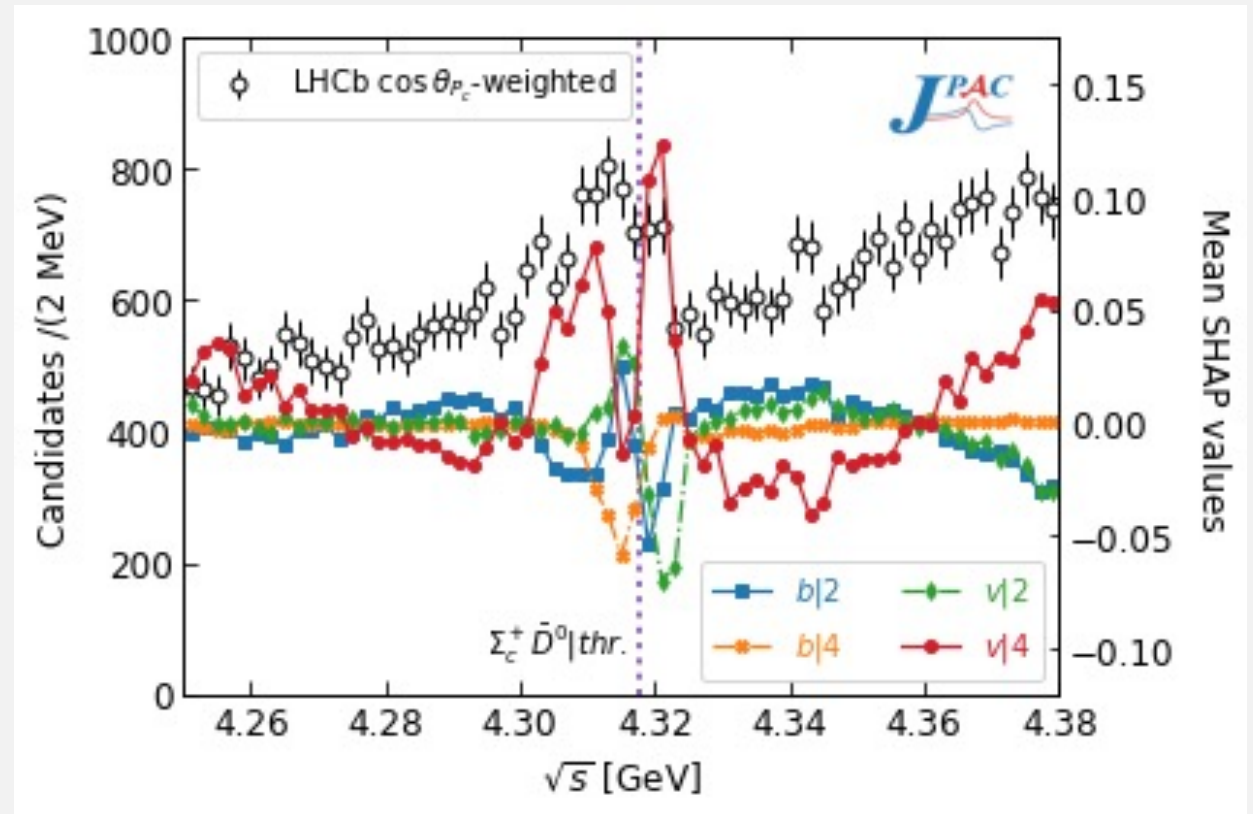
Rather than testing a single model hypothesis as a  $\chi^2$  fit would, the DNN determines the probability of each of the classes of interest, given the experimental uncertainties. The latter is possible, since the DNN learns the subtle classification boundary between the different classes

But, there's more...

# SHapley Additive exPlanations (SHAP) values

Technique inherited from game theory

Allows to determine how a given feature in the input layer (in our case an experimental datapoint) impacts the decision made by the DNN in the output layer (the classes)



# Takeaways

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Deep neural networks open new possibilities to answer the question on the underlying nature of a given resonance

Possibility to gain physics insight on how the data impact the obtained interpretation

DNN does NOT substitute the standard approach. They are complementary (we still want the amplitude and the pole position)

DNN allows a true comparison among interpretations

The BIG objective: Be able to train the DNN with every amplitude possibility we can devise with our twisted minds and throw the data to it, returning probabilities for each class

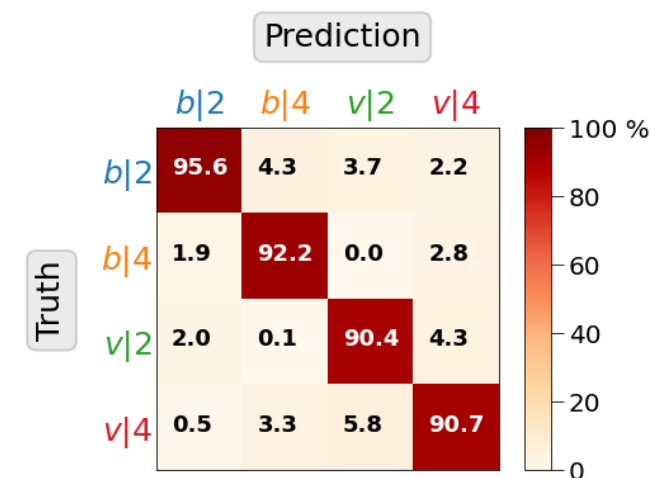
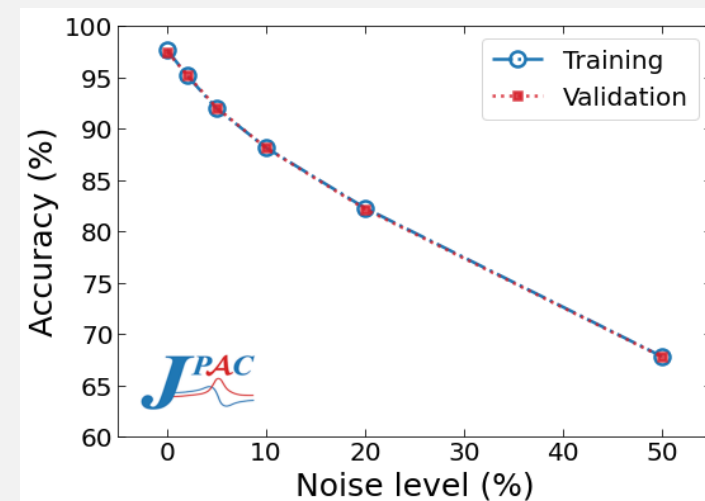
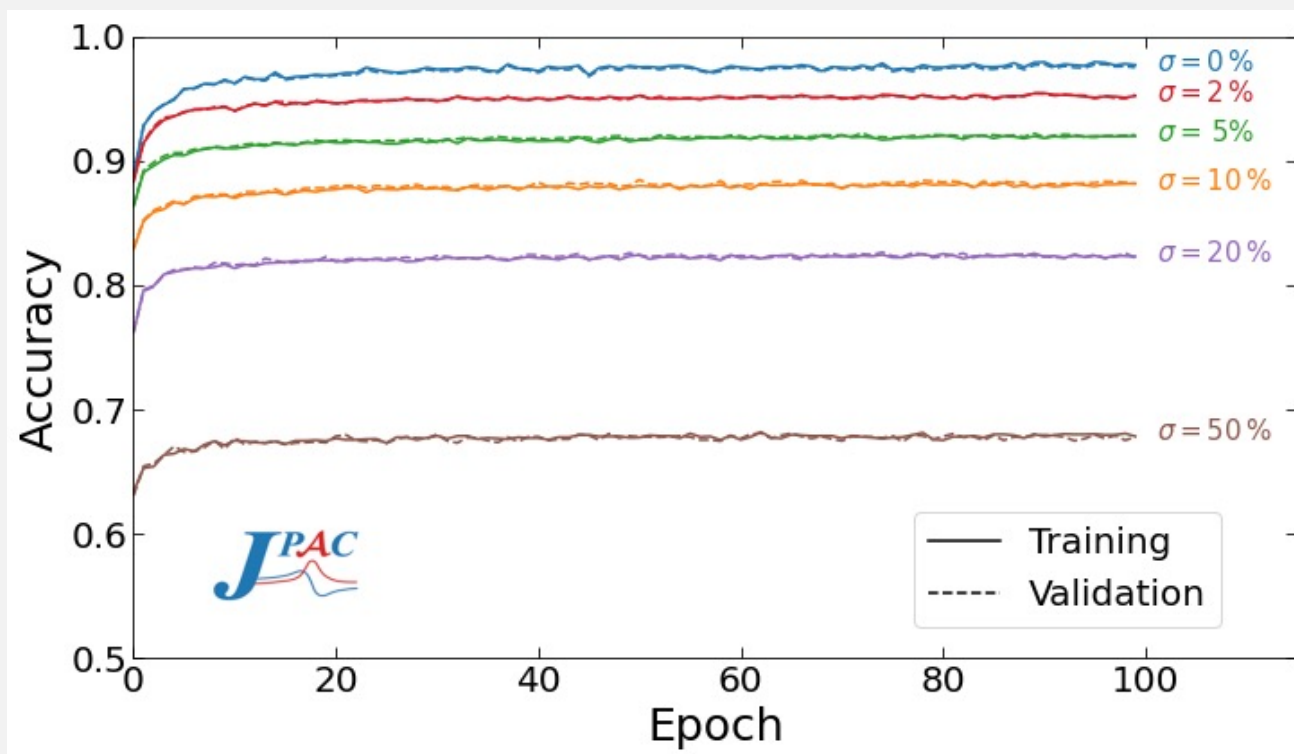
Uncharted territory so we are taking baby steps: Ng et al. (JPAC), 2110.13742

We are (hopefully) just in the beginning...

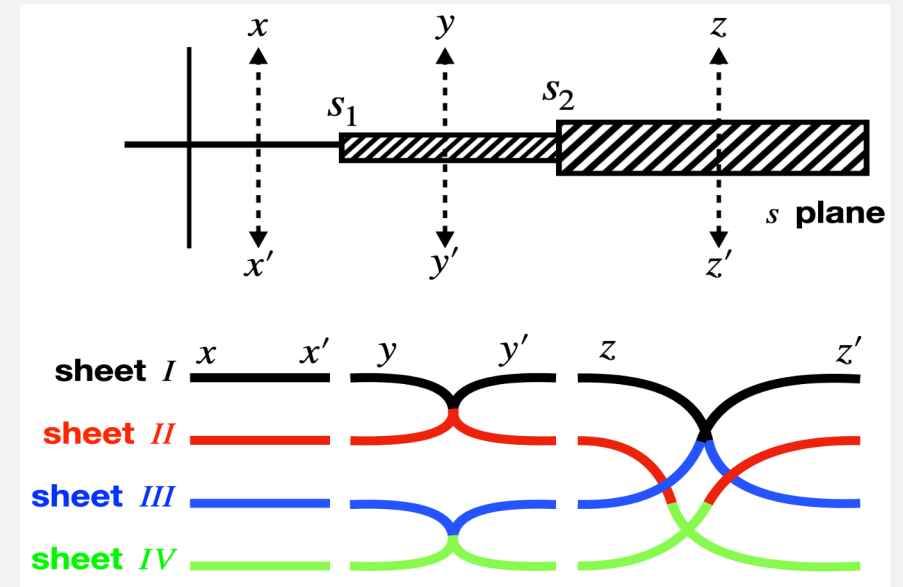
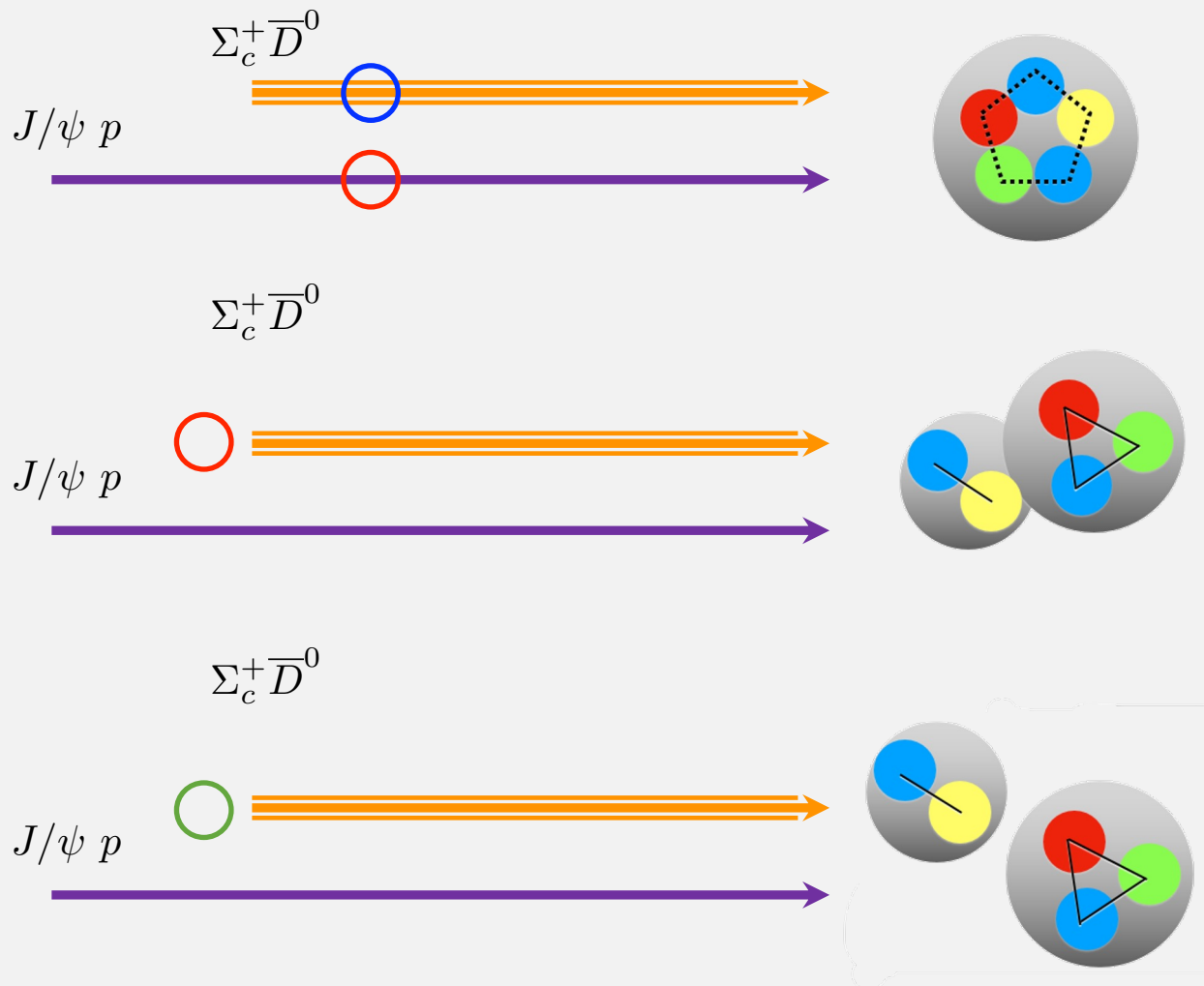
**Additional  
material**



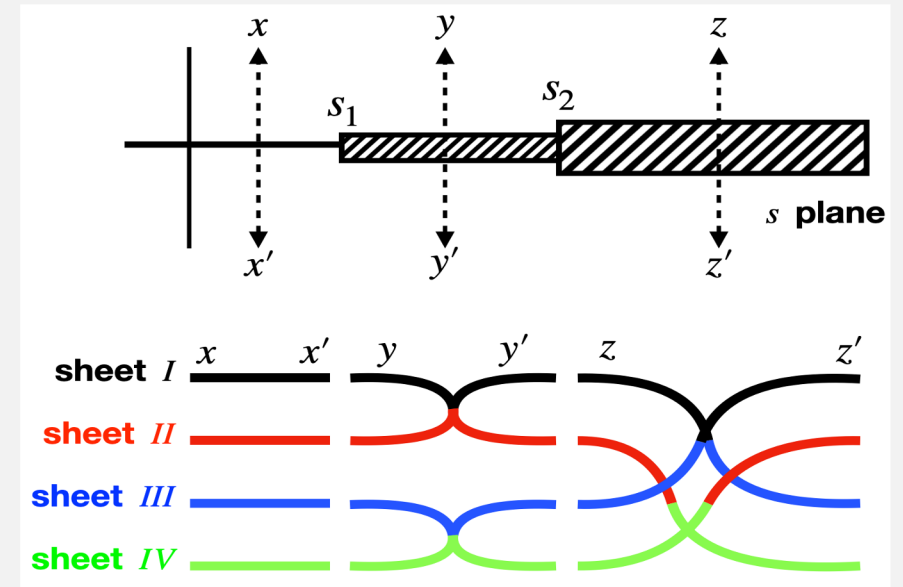
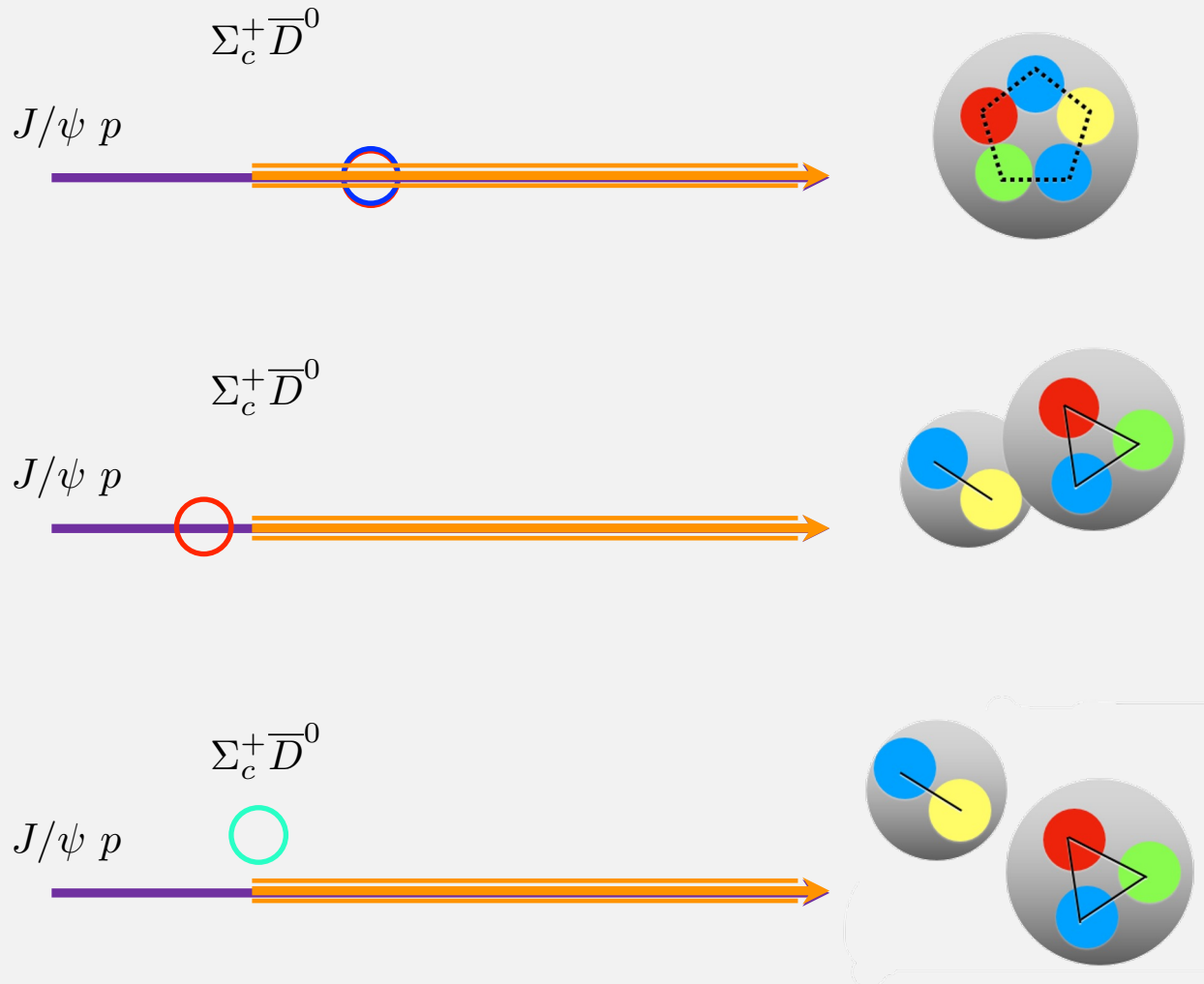
# Training



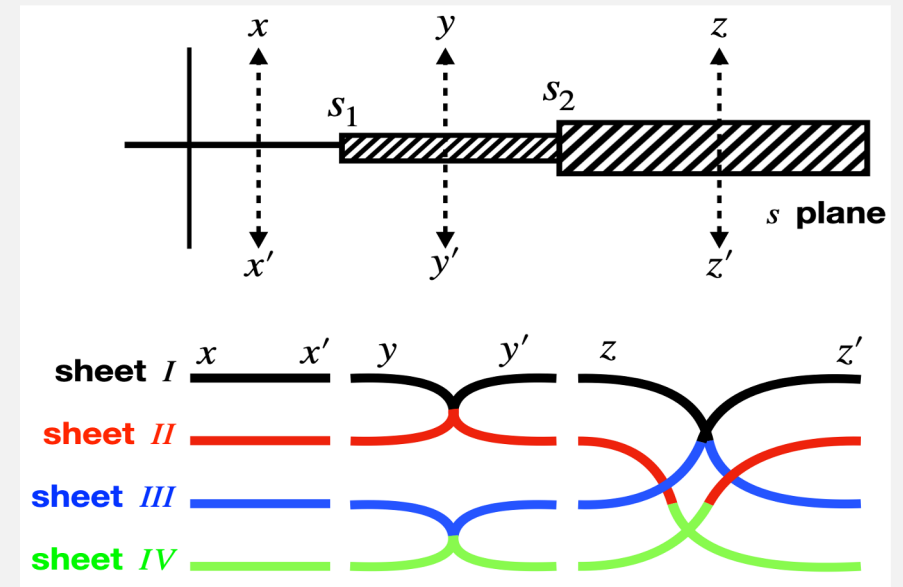
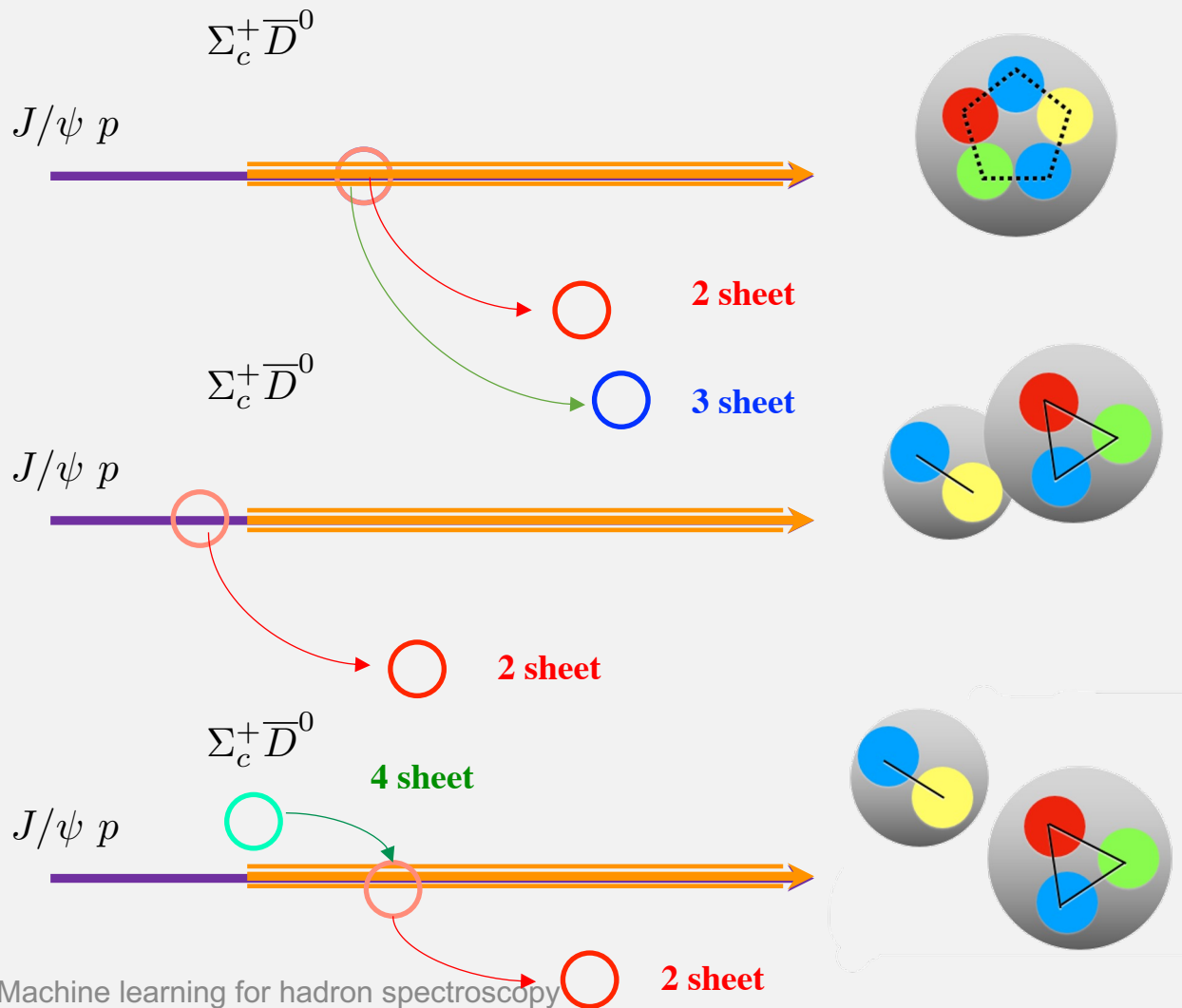
# Pole movement (model dependent)



# Pole movement (model dependent)

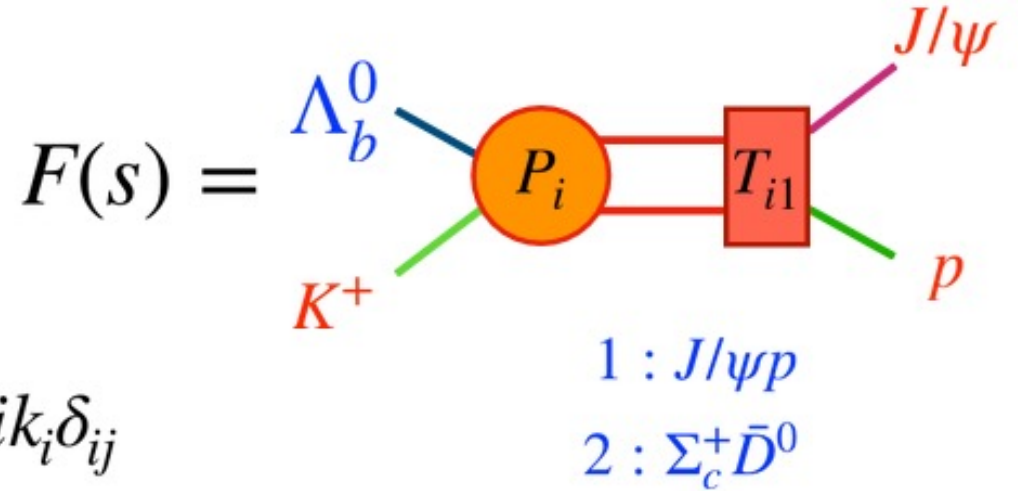


# Pole movement (model dependent)



# Near-threshold theory (two channels)

$$\frac{dN}{d\sqrt{s}} = \rho(s) \left[ |F(s)|^2 + B(s) \right]$$



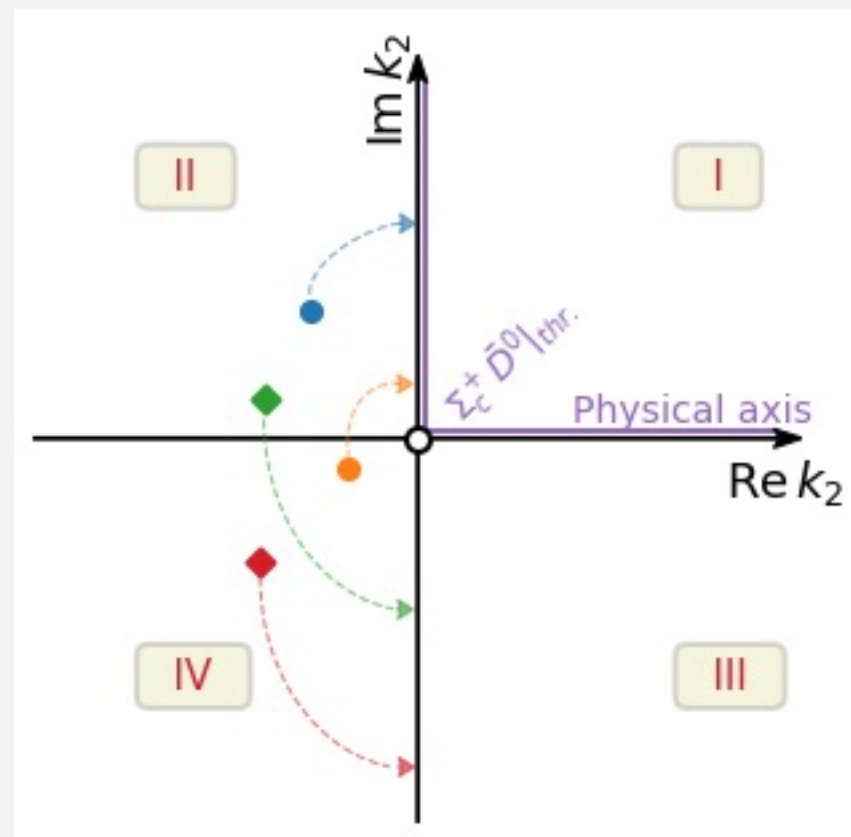
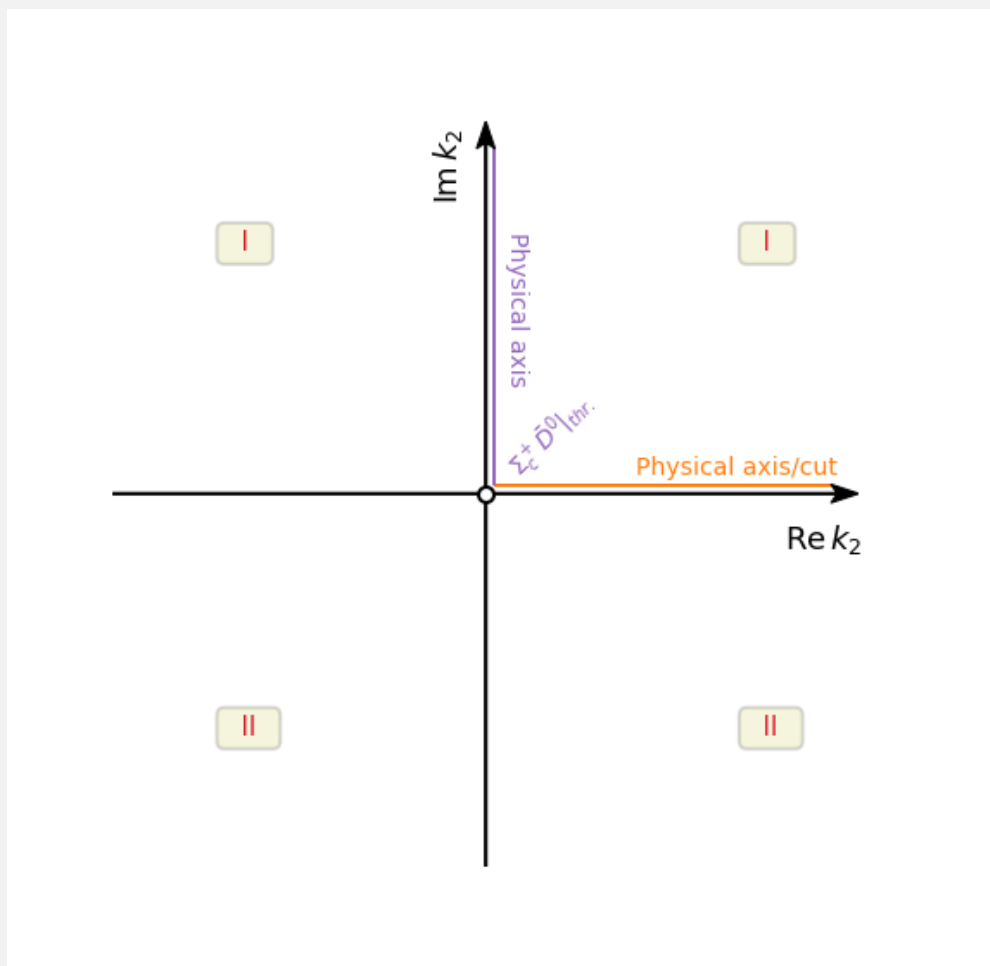
$$F(s) = P_1(s)T_{11}(s) \quad (T^{-1})_{ij} = M_{ij} - ik_i\delta_{ij}$$

Scattering length approximation for  $T$

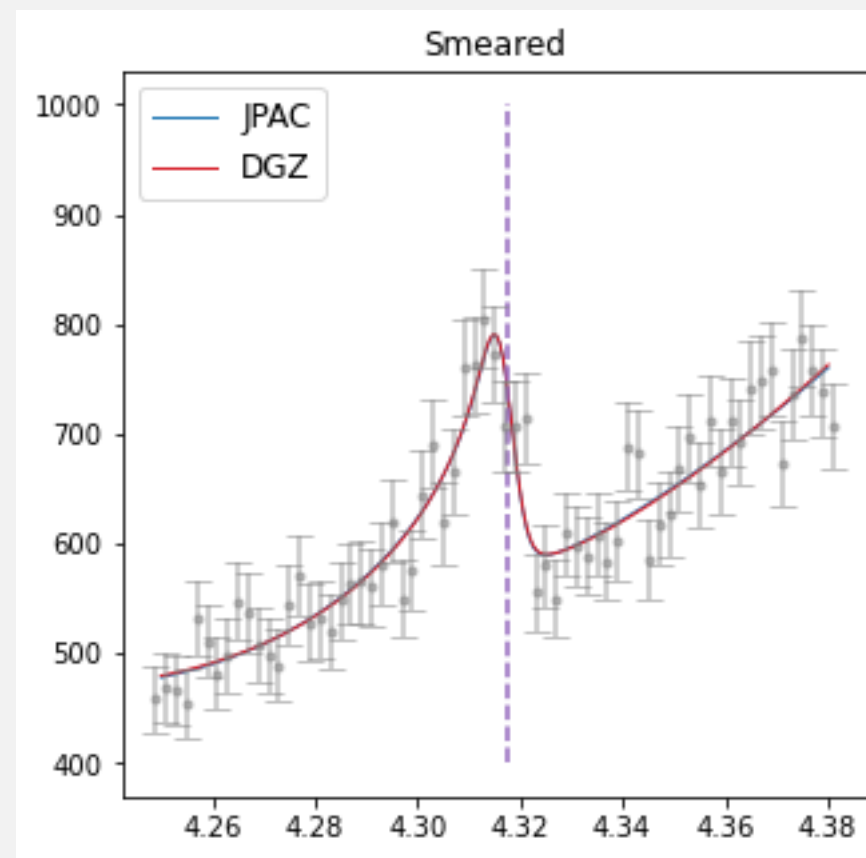
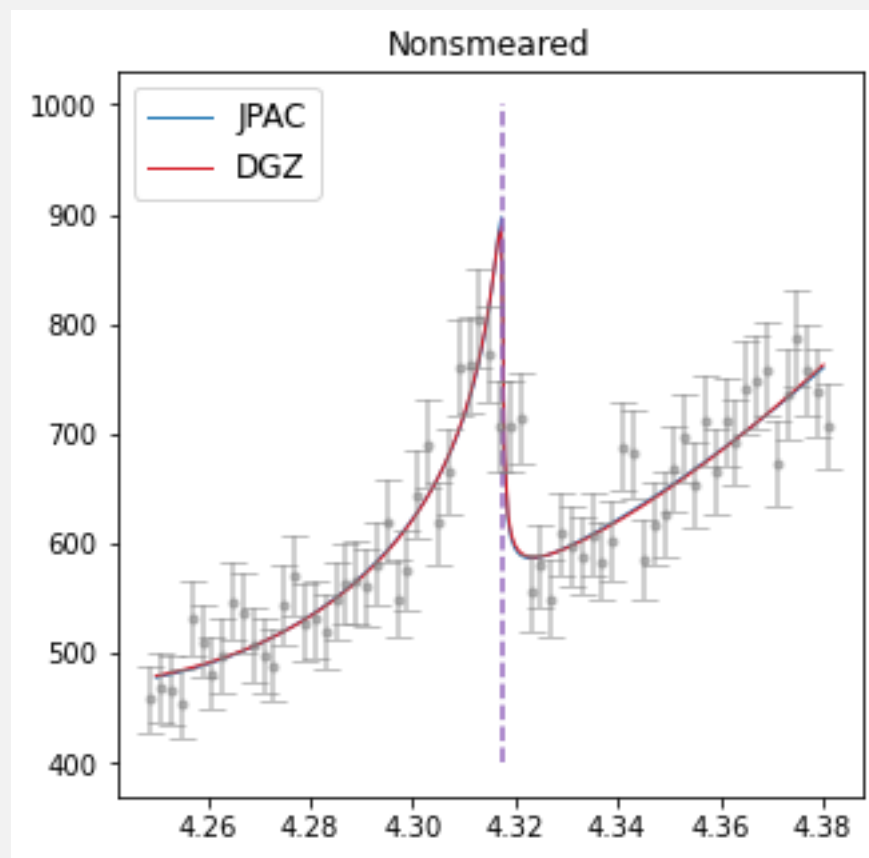
Dong, Guo & Zou (DGZ), 2011.14517  
CFR et al. (JPAC), 1904.10021

$$M_{ij}(s) = m_{ij}$$

# DGZ vs. JPAC (cut)



# DGZ vs. JPAC amplitudes (fit to data+resolution)



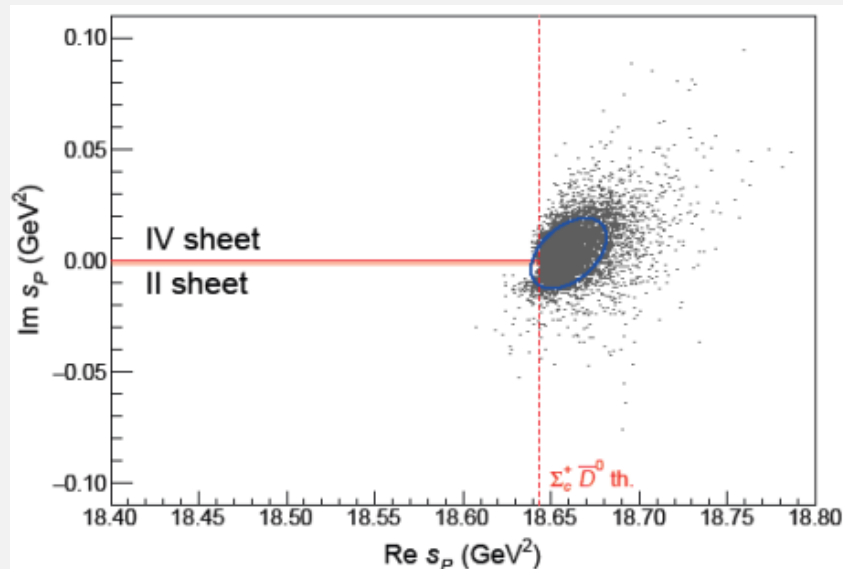
# DGZ vs. JPAC (pole position)

Using JPAC amplitude for  $T$

$$M_P = 4319 \pm 1.6 \text{ MeV}$$
$$\Gamma_P = -0.84 \pm 2.4 \text{ MeV}$$

Using DGZ amplitude for  $T$

$$M_P = 4319.45 \text{ MeV}$$
$$\Gamma_P = 0.95 \text{ MeV}$$



*Note: minus sign refers to 4th Riemann sheet*



# Difference in momenta (actually irrelevant)

