The Jet/CGC Correspondence: A Perspective through Conformal Transformations.

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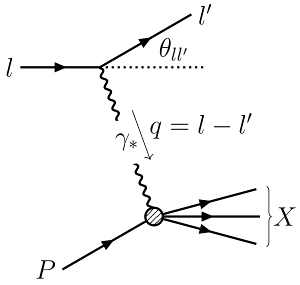
Deep Inelastic Scattering

Consider a leptonic projectile which is imposed upon a hadronic target at a large centre-of-mass energy,



- Hard scale, $-Q^2 := q^2$
- Final state mass, $X^2 = (P + q)^2$
- Momentum fraction, $x_{\mathrm{BJ}} := \frac{Q^2}{2P.q}$;

Bjorken x

$$Q^2 > 0 \text{ and } X^2 > P^2 \implies x_{\mathrm{BJ}} = \frac{Q^2}{Q^2 + (X^2 - P^2)} \in [0, 1) \tag{1}$$

Bjorken Limit of DIS

Hard scale dominates, $Q^2 \to \infty$ - at fixed x_{BJ}

- The off-shell photon momentum is large and spacelike - $0 > q^2 = -Q^2$
- Transverse resolution of hadronic constituents $\delta A_\perp \approx 1/Q^2$
- Hadron consituents like quarks and gluons are called **partons**.

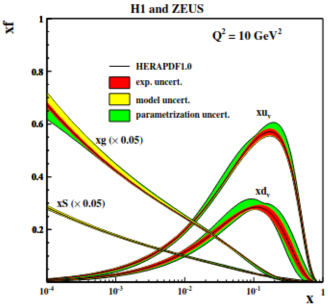


Figure: the x_{BJ}-evolution of the gluon, sea quark, and valence quark distributions for $Q^2 = 10\text{GeV}^2$ measured at HERA [arXiv: 1111.5452]

QCD Phase Space

Regge-Gribov Limit of DIS

Momentum transfer dominates, $x_{\mathrm{BJ}} \to 0$ - at fixed hard scale Q^2.

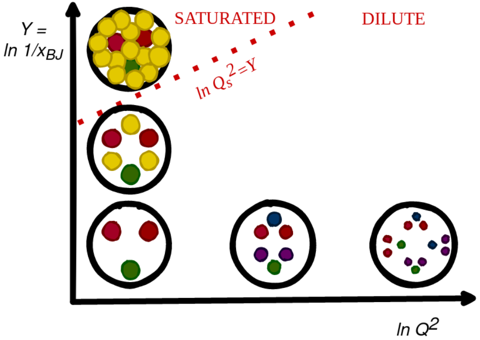


Figure: the "phase–diagram" for QCD evolution; each colored dot represents a parton with transverse area $\delta A_\perp \approx 1/Q^2$.

- Increasing Q^2 at fixed Y adds more partons at finer resolutions. Hadron remains dilute.
- Increasing Y at fixed Q^2 adds more partons (mainly gluons) at fixed resolution. Hadron becomes condensed beyond the **saturation scale** for $Y > \ln Q_s^2$

The Color Glass Condensate

The CGC is an effective description of the hadron at the point of gluon saturation (at high rapidity or small-x_{BJ})

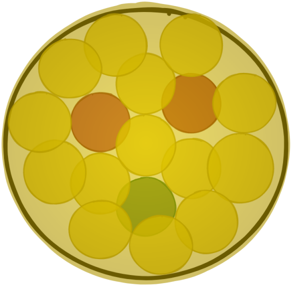


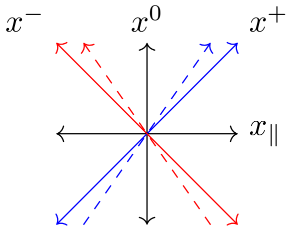
Figure: Artistic rendition of the CGC

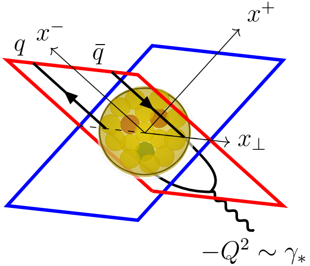
COLOR	charge carried by quarks and gluons
GLASS	the CGC appears, in experiments, frozen due to time dilation effects
CONDENSATE	allusion to innate gluon saturation effects

- Gluons behave in a coherent manner due to saturation.
- CGC is modelled as a **large background gauge field**, $b(x) := \delta(x^-)\beta(x_\perp)$.

The Spacetime Picture of DIS

The simplest QCD interaction with the CGC ($b(x)$) involves the spacelike virtual photon γ_* splitting into a quark-antiquark pair ($q\bar{q}$)





Interaction with the CGC involves a rotation in the color orientation by the appropriate representation of a Wilson line,

$$\overset{\text{rep}}{U}_x := P\exp\left[-ig\int_{-\infty}^{+\infty} dx^- b(x)\right]; \quad b(x) = \delta(x^-)\beta^a(x_\perp)t^a_{\text{rep}} \qquad (2)$$

where $\overset{\text{rep}}{U}_x$ is a representation of $SU(N_c)$.

CGC Evolution

The color orientations of quarks, antiquarks, and gluons rotate according to the fundamental, antifundamental, and adjoint representations of a Wilson line,

$$\psi(x) \mapsto U_x \psi(x), \quad \bar{\psi}(y) \mapsto \bar{\psi}(y) U_y^\dagger, \quad \alpha^{\mu b}(z) \mapsto \tilde{U}_z^{ab} \alpha^{\mu b}(z). \tag{3}$$

JIMWLK Equation

$$\begin{aligned} &\frac{d}{d \ln(1/x_{\mathrm{BJ}})} \left\langle \mathrm{tr}(U_x U_y^\dagger) \right\rangle \\ &\qquad = \frac{2\alpha_s}{\pi} \int \frac{d^2 \mathbf{z}_\perp}{2\pi} K_{xzy} \Big\{ \left\langle \tilde{U}_z^{ab} \mathrm{tr}(t^a U_x t^b U_y^\dagger) \right\rangle \\ &\qquad\qquad - C_f \left\langle \mathrm{tr}(U_x U_y^\dagger) \right\rangle \Big\} \end{aligned} \tag{4}$$

Refer to [arXiv: 0501087]

- The general stucture of CGC evolution is found in Jet evolution.

The BMS equation (finite N_c generalization thereof) is,

$$\begin{aligned}\frac{d}{d\ln(E/E_{\text{out}})}&\langle\text{tr}(V_p V_q^\dagger)\rangle \\ &= \frac{2\alpha_s}{\pi}\int\frac{d^2\hat{\boldsymbol{k}}}{4\pi}W_{pkq}\Big\{u(k)\big\langle\tilde{V}_k^{ab}\text{tr}(t^a V_p t^b V_q^\dagger)\big\rangle \\ &\qquad - 1C_f\big\langle\text{tr}(V_p V_q^\dagger)\big\rangle\Big\},\end{aligned} \tag{5}$$

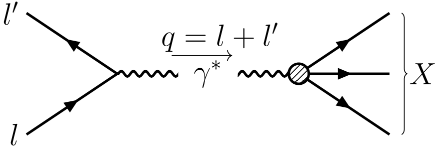
where $d^2\hat{\boldsymbol{k}} := \sin\theta_k d\theta_k d\phi_k$ is an angular measure and $u(k)$ is a geometric constraint on the real emission of gluons. Refer to [arXiv: 0312050].

Annihilation

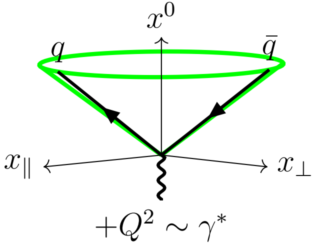
Consider a collision between a lepton and an antilepton at a large centre-of-mass energy,

$$+Q^2 = q^2 = s.$$

Q is once again the hard scale of the experiment.



The simplest QCD final state involves the timelike virtual photon γ^* splitting into a quark-antiquark pair ($q\bar{q}$).



Wilson Lines

The Wilson lines featured in Jet evolution are,

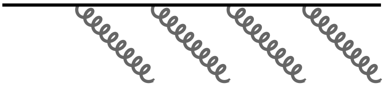
$$V_k := P \exp\left[-ig\int_0^{+\infty} d\tau \eta_\mu A^\mu(\eta\tau)\right], \text{ such that } \eta.\eta = 0. \tag{6}$$

The subscript k alludes to the Fourier transform of the gauge field,

$$A^\mu(\eta\tau) = \int \frac{d^4k}{(2\pi)^4} e^{-i\eta\tau.k} A^\mu(k), \tag{7}$$

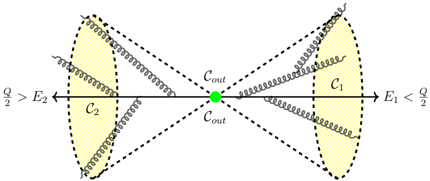
Wilson line describes a parton (with momentum k) radiating/absorbing soft gluons $(\omega_k \gg \omega_1, \omega_2, \ldots, \omega_i)$,

$$\overset{\text{rep}}{V}_k = \sum_{\text{gluons}}$$



$$. \tag{8}$$

Non-global Observables



$$E = \omega_{p_1} + \omega_{p_2} + \sum_{\text{all } k} \omega_k$$

$$E_{\text{out}} = \sum_{k \in \mathcal{C}_{\text{out}}} \omega_k$$

$$E = Q \text{ and } E_{out} \ll Q$$

Soft gluon real emissions outside the jets are suppressed,

$$u(k) = \Theta_{\text{in}}(k) + e^{-\nu\omega}\Theta_{\text{out}}(k). \quad (9)$$

Non-global observables are important for understanding the mechanism of color neutralization. [arXiv: 0206076]

Jet Evolution

BMS Equation (Finite N_c Generalization)

$$
\begin{aligned}
\frac{d}{d\ln(E/E_{\text{out}})}\langle \mathrm{tr}(V_p V_q^\dagger)\rangle \\
= \frac{2\alpha_s}{\pi}\int \frac{d^2\hat{\boldsymbol{k}}}{4\pi} W_{pkq}\Big\{ u(k)\big\langle \tilde{V}_k^{ab}\mathrm{tr}(t^a V_p t^b V_q^\dagger)\big\rangle \\
- 1 C_f \big\langle \mathrm{tr}(V_p V_q^\dagger)\big\rangle \Big\}
\end{aligned}
\tag{10}
$$

Relating Kernel and Measure Geometry

- Both Jet and CGC evolution equations track changes to the color rotation of a quark-antiquark pair.
- At leading order, changes are tracked w.r.t color orientations induced by the *hardest* soft gluon.

$$\text{Jet Kernel:} \qquad W_{pkq} := \frac{(1-\hat{\boldsymbol{p}}\cdot\hat{\boldsymbol{q}})^2}{(1-\hat{\boldsymbol{p}}\cdot\hat{\boldsymbol{k}})^2(1-\hat{\boldsymbol{k}}\cdot\hat{\boldsymbol{q}})^2} \tag{11}$$

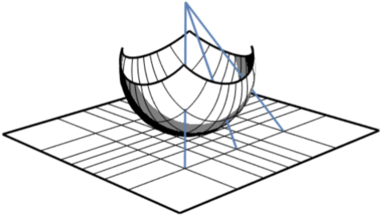
$$\text{CGC Kernel:} \qquad K_{xzy} := \frac{(\boldsymbol{x}_\perp-\boldsymbol{y}_\perp)^2}{(\boldsymbol{x}_\perp-\boldsymbol{z}_\perp)^2(\boldsymbol{z}_\perp-\boldsymbol{y}_\perp)^2} \tag{12}$$

Is there a way to relate the relevant measures and kernels?

$$\int \frac{d^2\hat{\boldsymbol{k}}}{4\pi} W_{pkq} \leftrightarrow \int \frac{d^2\boldsymbol{z}_\perp}{4\pi} K_{xzy} \tag{13}$$

Stereographic Projection

Hatta used a steregraphic projection to relate Jet and CGC evolution [arXiv: 0810.0889]



$$d^2\hat{\boldsymbol{k}}_c = \left(\frac{2}{1+\boldsymbol{z}_{c\perp}^2}\right)^2 d^2\boldsymbol{z}_{c\perp} \tag{14}$$

$$1-\hat{\boldsymbol{k}}_a\cdot\hat{\boldsymbol{k}}_b = \frac{2(\boldsymbol{z}_{a\perp}-\boldsymbol{z}_{b\perp})^2}{(1+\boldsymbol{z}_{a\perp}^2)(1+\boldsymbol{z}_{b\perp}^2)} \tag{15}$$

The measures and kernels, together, are related by a stereographic projection,

$$\begin{aligned}\int\frac{d^2\hat{\boldsymbol{k}}_c}{4\pi}&\frac{(1-\hat{\boldsymbol{k}}_a\cdot\hat{\boldsymbol{k}}_b)^2}{(1-\hat{\boldsymbol{k}}_a\cdot\hat{\boldsymbol{k}}_c)^2(1-\hat{\boldsymbol{k}}_c\cdot\hat{\boldsymbol{k}}_b)^2}\\ =\int\frac{d\boldsymbol{z}_\perp}{2\pi}&\frac{(\boldsymbol{z}_{a\perp}-\boldsymbol{z}_{b\perp})^2}{(\boldsymbol{z}_{a\perp}-\boldsymbol{z}_{c\perp})^2(\boldsymbol{z}_{c\perp}-\boldsymbol{z}_{b\perp})^2}\end{aligned} \tag{16}$$

Relating Wilson Line Geometry

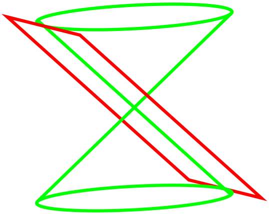


Figure: Geometries of Wilson lines contributing to Jet evolution (green) and CGC evolution (red).

Properties of the stereographic projection may be extended to Minkowski spacetime provided that:

- Euclidean stereographic projection is preserved in the spatial coordinates.
- Minkowski mapping is conformal.

Vladimirov presented a mechanism to generate the conformal map [arXiv: 1701.07606],

$$\mathrm{c}^{\mu}(x^{+}\bar{n} + x_{\perp} + x^{-}n) = \Omega_{\mathrm{c}} n_{\mathrm{c}}^{\mu} + x^{+}\bar{n}^{\mu}; \tag{17}$$

where

$$\Omega_{\mathrm{c}} = -\frac{1}{x^{-}}, \quad n_{\mathrm{c}}^{\mu} = \frac{-x_{\perp}^{2}}{2}\bar{n}^{\mu} + x_{\perp}^{\mu} + n^{\mu} \tag{18}$$

Impacts on Causality

$$c^{\mu}(x) = \Omega_c n_c^{\mu} + \cancelto{0}{x^+} \bar{n}^{\mu}; \quad \Omega_c = -\frac{1}{x^-} \tag{19}$$

(a) $(x^+ = 0, x_\perp, x^-)$-plane (b) $(x^+ = 0, x_\perp, \Omega_c)$-plane

Figure: Graphical depiction of the inversion of the x^- performed by the conformal factor $\Omega_c = -1/x^-$.

Mapping of the Large Background Field

$$c^{\mu}(x) = \Omega_c n_c^{\mu} + \cancelto{0}{x^{+}}\bar{n}^{\mu}; \quad \Omega_c = -\frac{1}{x^{-}}, \quad n_c^{\mu} = \frac{-x_{\perp}^{2}}{2}\bar{n}^{\mu} + x_{\perp}^{\mu} + n^{\mu} \quad (20)$$

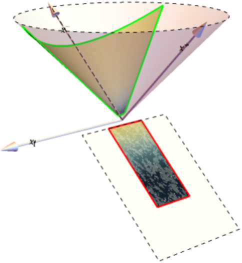


Figure: Colour plot of the conformal map $c^{\mu}(x)$.

CGC is modelled a localized background field

$$b(x) = \delta(x^{-})\beta(x_{\perp}). \qquad (21)$$

The localized field is stretched under the conformal map.

Conclusions and Outlook

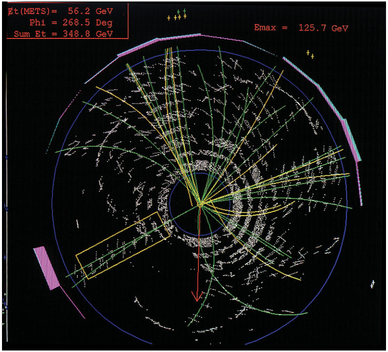
Conclusions

- Kernels and measures show equivalence under stereographic projection.
- Large Background Field interpretation of jet evolution is facilitated.
- Leading order CGC and Jet evolution mapped.

Outlook

- Collinear singularities at NLO CGC evolution (JIMWLK equation) need to be removed/regularized.

Jets



- As the (anti)quark moves out from the collision point it radiates gluons.
- Emission of gluons rotates the color orientation of the (anti)quark.
- Gluons split into other gluons and quark–antiquark pairs.
- When the collection has spread out over length scales of order Λ_{QCD}^{-1}, the quarks and gluons hadronize into color-neutral objects.

Inversion of Lightcone Coordinate x^-

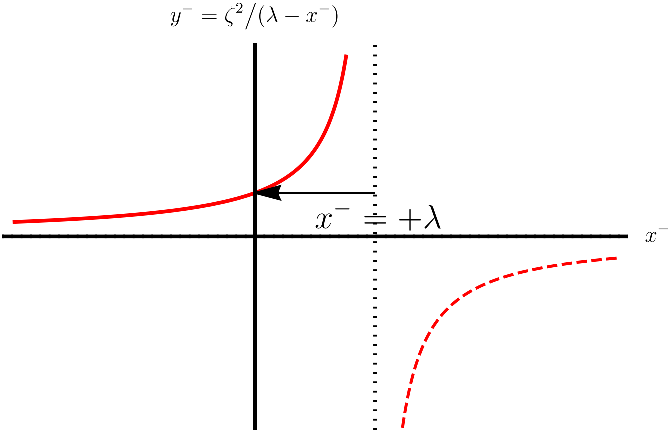


Figure: Plot of conformal factor $\Omega_c(x)$. Dotted black line denotes the singularity $x^- = +\lambda$. Dashed red line denotes strictly negative part of $\Omega_c(x)$. Solid red line denotes the strictly positive part of $\Omega_c(x)$.