Higgs boson decay into gluons: IR cancellation in the decay rate at NLO using Implicit Regularization

Ana Isabel Pereira

ana.pereira98@icloud.com

Excited QCD 2022

1 2 9 0

UNIVERSIDADE D COIMBRA

Ana Pereira, Adriano Cherchiglia, Marcos Sampaio, Brigitte Hiller, arXiv:2207.05187

Divergences in perturbative QCD

Interactions in QCD

$\downarrow$

series expansion around a small coupling constant (PQCD)

$\downarrow$

Feynman diagrams

$\downarrow$

diagrams with loops $\longrightarrow \int_k \dots$

$\downarrow$

divergences in the integrals $\longrightarrow \int_k \dots \to \infty$

$\downarrow$

Regularization (UV div + UV fin(IR div+ IR fin))

$+$

Renormalization

$\downarrow$

Observables

- **Tradicional dimensional schemes (DS)**: Conventional Dimensional Regularization (**CDR**), t'Hooft Veltman (**HV**), Four dimensional Helicity (**FDH**), Dimensional Reduction (**DRED**).
- **Non-dimensional schemes**: Implicit Regularization (**IREG**), Four-dimensional Regularization (**FDR**), Four-Dimensional Unsubtraction (**FDU**), Differential Renormalization (**DREN**).

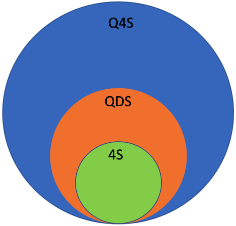
Regularization frameworks

[Gnendiger et al., 2017]

- The basic idea of all DS is to regularize divergent integrals by formally changing the dimensionality of space-time d or $d_s = 4 - 2\epsilon$.
- UV and IR divergent integrals lead to poles of the form $\dfrac{1}{\epsilon^n}$

	CDR	HV	FDH	DRED
singular VF	$g^{\mu\nu}_{[d]}$	$g^{\mu\nu}_{[d]}$	$g^{\mu\nu}_{[d_s]}$	$g^{\mu\nu}_{[d_s]}$
regular VF	$g^{\mu\nu}_{[d]}$	$g^{\mu\nu}_{[4]}$	$g^{\mu\nu}_{[4]}$	$g^{\mu\nu}_{[d_s]}$

Regularization frameworks



$$\int \frac{d^4 k_{[4]}}{(2\pi)^4} \longrightarrow \mu_{DS}^{4-d} \int \frac{d^d k_{[d]}}{(2\pi)^d} \quad (1)$$

- Regularized covariant derivative in QCD:

$$\mathcal{D}^{\mu}_{[d_s]}\psi_i = \partial^{\mu}_{[d]}\psi_i + i(g_s A^{\mu,a}_{[d]} + g_e A^{\mu,a}_{N_\epsilon})T^a_{ij}\psi_j \quad (2)$$

- Changes at the Lagrangian level lead to changes in Feynman rules, which may lead to complex computation at higher orders

IReg - framework that allows for a simpler computation of precision observables

Using DS in Quantum field theoretical models which are well-defined only in their physical dimension

- γ^5 Dirac algebra clashes with dimensional continuation in space-time dimensions
- Supersymmetric theories $\longrightarrow$ breaking of the supersymmetric relations

KLN theorem in a nutshell

IR divergences may occur in the expansion of the action when doing perturbative expansions, the IR divergences coming from loop integrals are cancelled by divergences coming from phase space integrals and the total result must be IR finite. [Kinoshita, 1962, Lee and Nauenberg, 1964]

- Match between virtual and real contributions to NLO to find a finite and regularization independent result (**compliance of IReg with KLN theorem**)
- Understand how IReg is applied in **renormalization of effective theories**
- Verify that in IReg **no modifications to the Lagrangian** are needed (comparison with dimensional regularization schemes)
- **More friendly calculations** since we do not change the Feynman rules

IReg to NLO + Renormalization

$$\downarrow$$

$$H \longrightarrow gg$$

$$\Gamma_v + \Gamma_r = \text{IR finite?}$$

$$\Gamma_v \longrightarrow \text{IReg + Ren}$$
$$\Gamma_r \longrightarrow \text{Spinor-helicity}$$

[Cherchiglia et al., 2011, Gnendiger et al., 2017, Torres Bobadilla et al., 2021]

- Operates on the **momentum space** and implemented to ***n*-loop order**
- Respects unitarity, locality and Lorentz invariance
- **Non-dimensional** (operates on the specific physical dimension of the theory) $\longrightarrow$ no changes in the Lagrangian
- Recursively algebraic **identity with IR regulator** μ to completely separate the UV divergent from the UV finite content

$$\int_k \frac{1}{k^2 - p^2} \longrightarrow \int_k \frac{1}{(k^2 - p^2) - \mu^2} \tag{3}$$

$$\int_k \frac{1}{(k^2 - p^2) - \mu^2} = \int_k \frac{1}{k^2 - \mu^2} + \int_k \frac{2k.p - p^2}{(k^2 - \mu^2)((k-p)^2 - \mu^2)} \tag{4}$$

$$\downarrow$$

- UV divergences in terms of BDI's - not depend on physical parameters (mass, external momenta)
- Every time we apply integrals become more IR divergent

A theory may be initially IR safe and because of the use of this identity IR divergences may appear

UV divergent and UV finite integrals

UV divergent integrals $\longrightarrow$ BDI's classified in all orders of perturbation

- Do not need to be evaluated to compute physical observables (do not contain any physics)
- Subtracted via renormalization

$$
\begin{aligned}
I_{quad} &= \int_k \frac{1}{(k^2-\mu^2)} \\
I_{log} &= \int_k \frac{1}{(k^2-\mu^2)^2}
\end{aligned}
\tag{5}
$$

UV finite integrals $\longrightarrow$ **(IR divergent or IR finite)**

- Information about physics
- Evaluated with *Package-X* of software *Mathematica*

Renormalization scale

Scale relation

$$\underbrace{I_{log}(\mu^2)}_{\infty\text{in UV and IR}} = \underbrace{I_{log}(\lambda^2)}_{\infty\text{in UV}} + \underbrace{\frac{i}{(4\pi)^2}\ln\frac{\lambda^2}{\mu^2}}_{\infty\text{in IR}} \tag{6}$$

- μ^2 parameterizes IR divergences
- $\lambda^2 \neq 0$ plays the role of renormalization scale
- $I_{log}(\lambda^2)$ is subtracted via renormalization
- $\frac{i}{(4\pi)^2}\ln\frac{\lambda^2}{\mu^2}$ will cancel with terms coming from the UV finite integrals

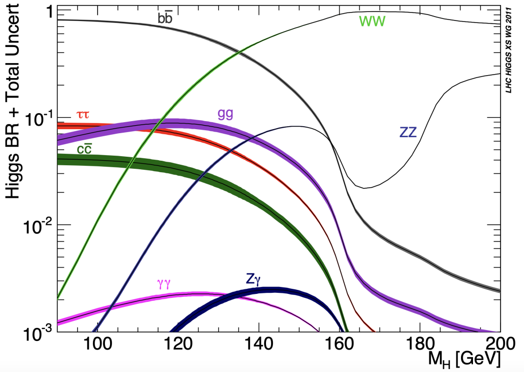


Figure 1: Higgs branching ratios and total uncert at low mass range. Source: [Denner et al., 2011]

An effective non-abelian field theory for the Higgs decay into gluons

Higgs does not couple with gluons at one-loop order

$\downarrow$

Effective non-abelian field theory $\longrightarrow$ top quark (large mass) is integrated out

$$L_{eff} = -\frac{1}{4} A H G^{a}_{\mu\nu} G^{a,\mu\nu} \tag{7}$$

$$A = \frac{\alpha_s}{3\pi v}\left(1 + \frac{11}{4}\frac{\alpha_s}{\pi}\right), \quad \longrightarrow \text{effective coupling} \tag{8}$$

$$G^{a}_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu + g f^{abc} A^b_\mu A^c_\nu \tag{9}$$

[Djouadi et al., 1991, Kauffman et al., 1997]

- $G^{\mu\nu}$, field strength of the $SU(3)$ gluon field
- f^{abc}, anti-symmetric $SU(3)$ structure constants
- $\alpha_s = \dfrac{g^2}{4\pi}$, strong coupling constant
- *H*, Higgs boson field

Virtual decay rate

$$(L_{eff})_{ren} = -\frac{1}{4} Z_{\alpha_s} Z_A A H G_{\mu\nu} G^{\mu\nu} \tag{10}$$

$$A^0_\mu = Z_A A_\mu, \qquad \alpha^0_s = Z_{\alpha_s} \alpha_s \tag{11}$$

Z_A and Z_{α_s} are given in first order in α_s by, [Sampaio et al., 2006]

$$Z_A = 1 + \delta_A \alpha_s \quad \longrightarrow \text{gluon-field renormalization constant} \tag{12}$$

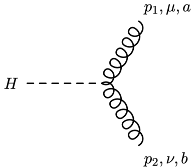
$$Z_{\alpha_s} = 1 + \delta_{\alpha_s} \alpha_s \quad \longrightarrow \text{color charge renormalization constant} \tag{13}$$

Counterterm

$$V_{count} = \alpha_s (\delta_{\alpha_s} + \delta_A) V_0 \tag{14}$$

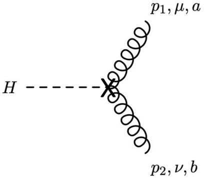
$$V_{count} = \frac{\alpha_s}{b\pi} \left[C_A \left(\frac{5}{12} I_{log}(\mu^2) - \frac{11}{12} I_{log}(\lambda^2) \right) - \frac{1}{3} T_F N_F \left(I_{log}(\lambda^2) - I_{log}(\mu^2) \right) \right] V_0. \tag{15}$$

V_0



$iA\delta^{ab}H^{\mu\nu}(p_1, p_2)$

V_{count}



where the tensor is $H^{\mu\nu}(p_1, p_2) = -p_1^\nu p_2^\mu + g^{\mu\nu} p_1 \cdot p_2$.

Virtual Feynman diagrams

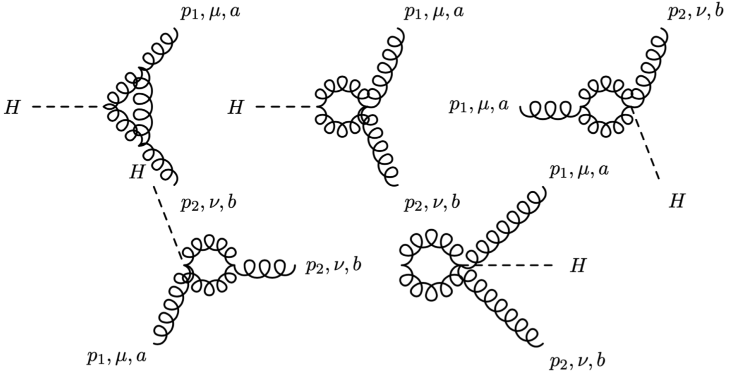
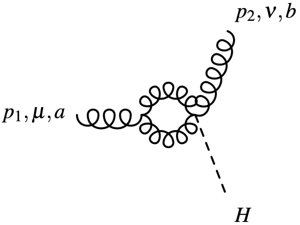


Figure 2: Virtual diagrams V_1 to V_5 contributing to the decay $H \longrightarrow gg$ obtained from the package *FeynArts* of order one loop



$$V_3 = \frac{1}{2} A g^2 C_A \delta^{ab} \int_k \frac{1}{k^2(k-p_2)^2} \Big(2k^2 g_{\mu\nu} - 2k.p_2 g_{\mu\nu} - 3p_1.p_2 g_{\mu\nu} + 2p_2^2 g_{\mu\nu} + 10 k_\mu k_\nu - 5k_\nu p_{2\mu} - 5k_\mu p_{2\nu} + 3p_{1\nu} p_{2\mu} + p_{2\mu} p_{2\nu} \Big) \tag{16}$$

First term

$$\int_k \frac{2k^2}{k^2(k-p_2)^2} = \int_k \frac{2}{(k-p_2)^2 - \mu^2}$$
$$= \int_k \frac{2}{k^2-\mu^2} + \int_k \frac{4k.p_2}{(k^2-\mu^2)^2} + \int_k \frac{2(2k.p_2)^2}{(k^2-\mu^2)^3} + \int_k \frac{2(2k.p_2)^3}{(k^2-\mu^2)^3((k-p_2)^2-\mu^2)}$$
$$= 2I_{quad}(\mu^2) + 2p_2^2 I_{log}(\mu^2) + \frac{2(2k.p_2)^3}{(k^2-\mu^2)^3((k-p_2)^2-\mu^2)} \tag{17}$$

$$V_1 + V_2 + V_3 + V_4 + V_5 = V_{div} + V_{rest} \tag{18}$$

Joining the divergent content of all diagrams

$$V_{div} = \frac{\alpha_s}{\pi} C_A \frac{I_{log}(\mu^2)}{2b} V_0 \tag{19}$$

Evaluating all the UV finite integrals in *package-X*

$$V_{rest} = V_1 + V_2 = \frac{\alpha_s}{\pi} C_A \left[-\frac{\ln(\mu_0)^2}{4} - \frac{i\pi \ln(\mu_0)}{2} + \frac{\pi^2}{4} \right] V_0 \tag{20}$$

$$\mu_0 = \frac{\mu^2}{q^2}, \quad q^2 = m_H^2$$

Renormalized amplitude

$$V_{ren} = V_{div} + V_{count} = \frac{\alpha_s}{b\pi}\left[\Big(I_{log}(\lambda^2) - I_{log}(\mu^2)\Big)\left(\frac{11}{12}C_A - \frac{1}{3}T_f N_F\right)\right] V_0 \tag{21}$$

Using the scale relation, eq. 6, $I_{log}(\mu^2) = I_{log}(\lambda^2) + b \ln \frac{\lambda^2}{\mu^2}$ we obtain

$$V_{ren} = \frac{\alpha_s}{\pi}\left[\left(\frac{11}{12}C_A - \frac{1}{3}T_f N_F\right)\ln\left(\frac{\lambda^2}{\mu^2}\right)\right] V_0, \tag{22}$$

rendering an UV finite result.

The amplitude is

$$V = V_0 + V_{ren} + V_{rest} \tag{23}$$

and the virtual decay rate is given by

$$\begin{aligned}\Gamma_v &= \frac{|V|^2}{32\pi m_H} \\ &= \Gamma_0 \left[1 + \frac{\alpha_s}{\pi}\left(-\Big(\frac{11}{6}C_A - \frac{1}{3}N_F\Big)\ln(\mu_0) + \frac{C_A}{2}\Big(-\ln(\mu_0)^2 + \pi^2\Big)\right)\right]\end{aligned} \tag{24}$$

- At the level of the virtual decay rate, IR divergences are still present

Real decay rate

Real diagrams contribution at the **same order** as the virtual ones (α_s^2)

$$H(q) \longrightarrow g(p_1) + g(p_2) + g(p_3), \quad \text{massless gluons}$$
$$H(q) \longrightarrow g(p_1) + q(p_2) + \bar{q}(p_3), \quad \text{light quarks}$$

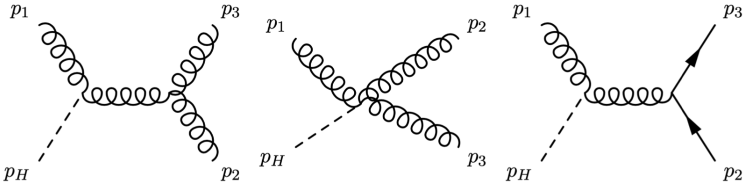


Figure 3: Real diagrams R_1, R_2 and R_3 contributing to the decay $H \longrightarrow gg(g)$.

Massless particles $\longrightarrow$ conservation of helicity $\longrightarrow$ helicity basis

$$p^{\alpha\dot{\alpha}} = p\rangle[p \tag{25}$$

$$p_{\alpha\dot{\alpha}} = p]\langle p \tag{26}$$

The polarization vector for a massless gauge boson is given by

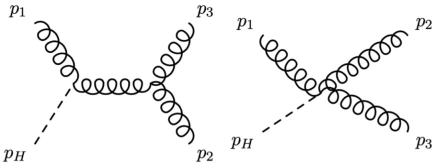
$$[\epsilon_p^-(r)]^{\alpha\dot{\alpha}} = \sqrt{2}\frac{p\rangle[r}{[pr]} \tag{27}$$

$$[\epsilon_p^+(r)]^{\alpha\dot{\alpha}} = \sqrt{2}\frac{r\rangle[p}{\langle rp\rangle} \tag{28}$$

where we defined a reference momentum r that is arbitrarily chosen.

$$\langle ij\rangle[ji] = 2p_ip_j = (p_i + p_j)^2 = s_{ij} \tag{29}$$

[Dixon, 2013]



Summing for all helicities

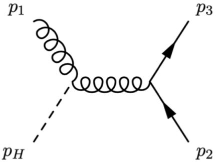
$$\begin{aligned}|M_g|^2 =& |M_g^{+++}|^2 + |M_g^{+--}|^2 + |M_g^{-+-}|^2 + |M_g^{--+}|^2 \\ &+ |M_g^{---}|^2 + |M_g^{-++}|^2 + |M_g^{+-+}|^2 + |M_g^{++-}|^2\end{aligned} \tag{30}$$

and the unpolarized amplitude is

$$|\overline{M}_g|^2 = \frac{1}{4}\sum_{col,polr}|M_g|^2 = A^2 192\pi\alpha_s \frac{1}{s_{12}s_{13}s_{23}}(s_{12}^4 + s_{13}^4 + s_{23}^4 + m_H^8) \tag{31}$$

$$s_{ij} = (p_i + p_j)^2$$
$$q^2 = m_H^2$$

Diagram with light quarks



$$M_q = iA\alpha_s t_b \epsilon_\nu(p_3) H^{\rho\nu}((-p_1 - p_2), p_3) \bar{u}^s(p_1)\gamma_\rho v^{s'}(p_2) \frac{1}{(p_1+p_2)^2 + i\epsilon}. \quad (32)$$

using the completeness relation for the sum over spin, one must retain the massive contribution,

$$|\overline{M_q}|^2 = A^2\alpha_s 16\pi\left(\frac{(s_{13}^2 + s_{12}^2)}{s_{23}} + \frac{4\mu^2}{s_{23}^2}\frac{(s_{13}+s_{12})^2}{2}\right) \quad (33)$$

Real decay rate

Phase space

$$\rho = \int \frac{d^3p_1}{(2\pi)^3 2\omega_1} \frac{d^3p_2}{(2\pi)^3 2\omega_2} \frac{d^3p_3}{(2\pi)^3 2\omega_3} (2\pi)^4 \delta^4(q - p_1 - p_2 - p_3) \qquad (34)$$

- consider massive phase space, $p_i^2 = \mu^2$ to parameterize the IR divergences
- define dimensionless variables, [Gnendiger et al., 2017]

$$\chi_i = \frac{(p_1 - q)^2}{q^2} - \mu_0, \quad \mu_0 = \frac{\mu^2}{q^2} \qquad (35)$$

In terms of these variables the decay rate is given by

$$\begin{aligned}
&\Gamma_r(H \longrightarrow gg(g), gq\bar{q}) = \\
&\Gamma_0 \frac{\alpha_s}{\pi} \int \Big[3\Big(2 + 3\chi_2 - \frac{4}{(\chi_2 + \mu_0)} + \frac{5\chi_1}{(\chi_2 + \mu_0)} - \frac{\chi_1^2}{(\chi_2 + \mu_0)} + \frac{1}{(\chi_1 + \mu_0)(\chi_2 + \mu_0)}\Big) \\
&+ N_F \Big(\frac{2\mu_0}{(\chi_2 + \mu_0)^2} + \frac{1}{(\chi_2 + \mu_0)} - \frac{2\chi_1}{(\chi_2 + \mu_0)} + \frac{2\chi_1^2}{(\chi_2 + \mu_0)} - 2 + 3\chi_2 \Big) \Big] d\chi_1 d\chi_2
\end{aligned} \qquad (36)$$

The tree-level decay rate in terms of real diagrams using spinor helicity is

$$\Gamma_0 = \frac{|M_{Hgg}|^2}{32\pi m_H} = \frac{A^2 m_H^3}{8\pi}. \tag{37}$$

and integrating over a massive phase-space

$$\Gamma_r(H \longrightarrow gg(g), gq\bar{q}) = \\ \Gamma_0 \frac{\alpha_s}{\pi}\left[3\left(\frac{73}{12} + \frac{11}{6}\ln(\mu_0) + \frac{\ln^2(\mu_0)}{2} - \frac{\pi^2}{2}\right) + N_F\left(\frac{-\ln(\mu_0)}{3} - \frac{7}{6}\right)\right]. \tag{38}$$

- IR divergences are still present

Total decay rate

$$\Gamma_T((H \longrightarrow gg(g), gq\bar{q})) = \Gamma_0 \left[1 + \frac{\alpha_s}{\pi}\left(\frac{95}{4} - \frac{7}{6}N_F\right)\right], \qquad (39)$$

- Result is a **correction to the tree-level decay**;
- all **IR divergences cancelled**, as well as the π^2;
- the **dependence on the regulator** μ **vanished**;
- $N_F = 5 \longrightarrow$ light quarks

Comparison with dimensional schemes

$$\Gamma_v^{\text{CDR}} = \Gamma_0 \left\{ 1 + \frac{\alpha_s}{\pi} \left[C_A \left(-\frac{1}{\epsilon^2} - \frac{11}{6\epsilon} + \frac{\pi^2}{12} \right) + \frac{N_F}{3\epsilon} \right] \right\} + \mathcal{O}(\epsilon)$$

$$\Gamma_v^{\text{FDH}} = \Gamma_0 \left\{ 1 + \frac{\alpha_s}{\pi} \left[C_A \left(-\frac{1}{\epsilon^2} - \frac{11}{6\epsilon} + \frac{\pi^2}{12} + \frac{1}{6} \right) + \frac{N_F}{3\epsilon} \right] \right\} + \mathcal{O}(\epsilon)$$

$$\Gamma_v^{\text{DRED}} = \Gamma_0 \left\{ 1 + \frac{\alpha_s}{\pi} \left[C_A \left(-\frac{1}{\epsilon^2} - \frac{11}{6\epsilon} + \frac{\pi^2}{12} \right) + \frac{N_F}{3\epsilon} + \frac{N_F}{6} \right] \right\} + \mathcal{O}(\epsilon)$$

$$\Gamma_v^{\text{IReg}} = \Gamma_0 \left\{ 1 + \frac{\alpha_s}{\pi} \left(-\left(\frac{11}{6} C_A - \frac{1}{3} N_F \right) \ln(\mu_0) + \frac{C_A}{2} \left(-\ln(\mu_0)^2 + \pi^2 \right) \right) \right\}$$

- $\frac{1}{\epsilon^n}$ refer to IR divergences
- We can compare these results to IReg with the correspondence $\epsilon^{-1} \to \ln \mu_0$, $\epsilon^{-2} \to \ln^2 \mu_0 / 2$

Comparison with dimensional schemes

Once the unpolarized amplitude is known in all schemes, one can obtain the part proportional to N_F of the real contribution to the decay rate

$$\Gamma_{q,r}^{\text{CDR/FDH}} = \Gamma_0 \frac{\alpha_s}{\pi} \left[-\frac{1}{3\epsilon} - \frac{7}{6} \right] N_F + \mathcal{O}(\epsilon) \tag{40}$$

$$\Gamma_{q,r}^{\text{DRED}} = \Gamma_0 \frac{\alpha_s}{\pi} \left[-\frac{1}{3\epsilon} - \frac{4}{3} \right] N_F + \mathcal{O}(\epsilon) \tag{41}$$

$$\epsilon^{-1} \to \log \mu_0$$

$$\Gamma_r^{\text{IReg}} = \Gamma_0 \frac{\alpha_s}{\pi} \left(\frac{33-2}{6} \ln \left(\frac{\lambda^2}{m_H^2} \right) - \frac{7}{6} \right) N_F \tag{42}$$

- At higher order the complexity of calculations in dimensional schemes blows up
- IReg framework allows for computations without changing the Feynman rules and shows to be a promising scheme to make simpler computations at higher orders

Conclusions

- We compute the decay rate $\Gamma((H \longrightarrow gg(g), gq\bar{q}))$ to α_s^3 in large top quark mass limit using an effective Lagrangian
- We achieve a full separation of BDI's from the UV finite integrals
- We single out the IR content, and the final decay rate is compliant with KLN theorem
- IReg does not require the use of evanescent fields at one loop level
- Additional degrees of freedom associated to ϵ scalars in some dimensional schemes have a counterpart in IReg

Thank you!

We acknowledge support from Fundação para a Ciência e Tecnologia (FCT) through the projects CERN/FIS-PAR/0040/2019 ('High precision effective modelling of the QCD phase diagram') and CERN/FIS-COM/0035/2019 ('Toolkit for precision calculations in the standard model and beyond, composite Higgs model and hadronic parity violation').

Backup slides

DS

2.1 Integration in d dimensions and dimensional schemes

Dimensional regularization [1, 2] and variants are the most common regularization schemes for practical calculations in gauge theories of elementary particle physics. In the following we summarize the basic definitions common to all dimensional schemes (DS) discussed in Secs. 2 and 3 and then provide specific definitions for four variants of DS which differ by the rules for the numerator algebra in analytical expressions.

The basic idea of all DS is to regularize divergent integrals by formally changing the dimensionality of space-time and of momentum space. In the present report we always denote the modified space-time dimension by d, and we set

$$d \equiv 4 - 2\epsilon\,. \tag{2.1}$$

Correspondingly, a four-dimensional loop integration is replaced by a d-dimensional one[1],

$$\int \frac{d^4 k_{[4]}}{(2\pi)^4} \quad \to \quad \mu_{\text{DS}}^{4-d} \int \frac{d^d k_{[d]}}{(2\pi)^d}\,, \tag{2.2}$$

including the scale of dimensional regularization, μ_{DS}. After this replacement, UV and IR divergent integrals lead to poles of the form $1/\epsilon^n$. In Refs. [3, 4], it is shown that such an operation can indeed be defined in a mathematical consistent way and that this operation has the expected properties such as linearity and invariance under shifts of the integration momentum.

To define a complete regularization scheme for realistic quantum field theories, it must be specified how to deal with γ matrices, metric tensors, and other objects appearing in analytical expressions. Likewise, it should be specified how to deal with vector fields in the regularized Lagrangian. On a basic level, two decisions need to be made,

- regularize only those parts of diagrams which can lead to divergences, or regularize everything;
- regularize algebraic objects like metric tensors, γ matrices, and momenta in d dimensions, or in a different dimensionality.

It turns out that there is an elegant way to unify essentially all common variants of DS in a single framework, where all definitions can be easily formulated and where the differences and relations between the schemes become transparent. This framework is based on distinguishing strictly four-dimensional objects, formally d-dimensional objects, and formally d_s-dimensional objects[2]. These objects can be mathematically realized [3–5] by introducing a strictly four-dimensional Minkowski space $\text{S}_{[4]}$ and *infinite*-dimensional vector spaces $\text{QS}_{[d_s]}$, $\text{QS}_{[d]}$, $\text{QS}_{[n_\epsilon]}$, which satisfy the relations

$$\text{QS}_{[d_s]} = \text{QS}_{[d]} \oplus \text{QS}_{[n_\epsilon]}\,, \qquad \text{S}_{[4]} \subset \text{QS}_{[d]}\,. \tag{2.3}$$

	CDR	HV	FDH	DRED
singular VF	$g^{\mu\nu}_{[d]}$	$g^{\mu\nu}_{[d]}$	$g^{\mu\nu}_{[d_s]}$	$g^{\mu\nu}_{[d_s]}$
regular VF	$g^{\mu\nu}_{[d]}$	$g^{\mu\nu}_{[4]}$	$g^{\mu\nu}_{[4]}$	$g^{\mu\nu}_{[d_s]}$

Table 1. Treatment of vector fields in the four different regularization schemes, i. e. prescription which metric tensor has to be used in propagator numerators and polarization sums. The quantity d_s is usually taken to be 4. This table is taken from Ref. [6].

The space $\text{QS}_{[d]}$ is the natural domain of CDR and of momentum integration in all considered schemes. Using

$$d_s \equiv d + n_\epsilon = 4 - 2\epsilon + n_\epsilon\,, \tag{2.4}$$

it is enlarged to $\text{QS}_{[d_s]}$ via a direct (orthogonal) sum with $\text{QS}_{[n_\epsilon]}$[3].

The structure of the vector spaces in Eq. (2.3) gives rise to the following decomposition of metric tensors and γ matrices

$$g^{\mu\nu}_{[d_s]} = g^{\mu\nu}_{[d]} + g^{\mu\nu}_{[n_\epsilon]}\,, \qquad \gamma^{\mu}_{[d_s]} = \gamma^{\mu}_{[d]} + \gamma^{\mu}_{[n_\epsilon]}\,. \tag{2.5}$$

Since the quantities in Eq. (2.5) do not have a finite-dimensional representation, in most of the practical calculations only their algebraic properties are relevant,

$$(g_{[dim]})^{\mu}{}_{\mu} \;=\; dim\,, \qquad (g_{[d]}\, g_{[n_\epsilon]})^{\mu}{}_{\nu} \;=\; 0\,, \tag{2.6a}$$

$$\{\gamma^{\mu}_{[dim]}, \gamma^{\nu}_{[dim]}\} \;=\; 2\, g^{\mu\nu}_{[dim]}\,, \qquad \{\gamma^{\mu}_{[d]}, \gamma^{\nu}_{[n_\epsilon]}\} \;=\; 0\,, \tag{2.6b}$$

with $dim \in \{4,\, d_s,\, d,\, n_\epsilon\}$.

Furthermore, a complete definition of the various dimensional schemes requires to distinguish two classes of vector fields (VF)[4]:

- Vector fields associated with particles in 1PI diagrams or with soft and collinear particles in the initial/final state are in the following called *singular* VF.
- All other vector fields are called *regular* VF.

Since UV and IR divergences are only related to *singular* VF there is some freedom in the treatment of the regular ones. In this report, we distinguish the following four DS:

- CDR and HV are two flavours of what is commonly called 'dimensional regularization'. They regularize algebraic objects in d dimensions, n_ϵ-dimensional objects are not used. In CDR, all VF are regularized, in HV only singular ones.
- FDH and DRED are two flavours of what is commonly called 'dimensional reduction'. They regularize algebraic objects in d_s dimensions. Sometimes d_s is identified as $d_s \equiv 4$ from the beginning, but it is possible to keep it as a free parameter, which is set to 4 only at the end of a calculation. In DRED, all VF are regularized, in FDH only singular ones.

EFT

EFT

where

$$\tilde{\mathscr{L}}_{\rm QCD}=-\tfrac{1}{4}\tilde{G}^a_{\mu\nu}\tilde{G}^{\mu\nu}_a+\sum_{h,l}\bar{\psi}(i\gamma^\mu\partial_\mu+\tilde{g}_0\gamma^\mu A^a_\mu T^a)\psi -\sum_h\bar{\psi}\tilde{M}_{j0}\psi-\frac{1}{2\tilde{\alpha}_0}(\partial^\mu A^a_\mu)^2+\bar{c}^a\partial^\mu\tilde{D}^{ab}_\mu c^b, \quad (2.2)$$

$$\tilde{\mathscr{L}}_{\rm int}=-(\sqrt{2}G_F)^{\frac{1}{2}}\phi J_H. \quad (2.3)$$

Use has been made of the notation $\tilde{G}^a_{\mu\nu}=\partial_\mu A^a_\nu-\partial_\nu A^a_\mu+\tilde{g}_0 f^{abc}A^b_\mu A^c_\nu$ and $\tilde{D}^{ab}_\mu=\delta^{ab}\partial_\mu-\tilde{g}_0 f^{abc}A^c_\mu$. The kinetic part of the Higgs Lagrangian $\tilde{\mathscr{L}}_{\rm Higgs}$ need not be specified. We shall use tilded quantities to denote those in the full theory. The summation in (2.2) is taken over light (l) and/or heavy (h) quarks, and the suffix "0" in $\tilde{g}_0$, $\tilde{M}_{j0}$ and $\tilde{\alpha}_0$ indicates that they are bare quantities. The scalar current J_H in the Yukawa interaction (2.3) is defined as usual by

$$J_H=\sum_h\bar{\psi}\tilde{M}_{j0}\psi. \quad (2.4)$$

Other notations in (2.2) and (2.3) are obvious. All the light quark masses are put equal to zero for simplicity, which, however, does not spoil the generality of our approach. For later purposes it is convenient to introduce here the renormalized parameters

$$\tilde{g},\tilde{\alpha},\tilde{M}_j. \quad (2.5)$$

To obtain the total hadronic decay width of the Higgs boson, we will consider effects to all orders in the QCD coupling, but restrict ourselves to the lowest order in the Weinberg-Salam coupling. This approximation is justified if the strength of the Yukawa coupling $(\sqrt{2}G_F)^{\frac{1}{2}}\tilde{M}_j$ is not so large as to invalidate the perturbative expansion, say, $\tilde{M}_j\lesssim$ a few hundred GeV.

The hadronic decay width of the Higgs boson can be expressed as

$$\Gamma=\Gamma\ (\text{Higgs}\to\text{heavy quarks}\to\text{hadrons}) =\frac{\sqrt{2}G_F}{m_H}\,{\rm Im}\,\Pi(q^2=m_H^2), \quad (2.6)$$

where

$$\Pi(q^2)=i\int d^Dx\, e^{iqx}\langle 0|T(J_H(x)J_H(0))|0\rangle. \quad (2.7)^\star$$

Thus our problem is reduced to investigating the behaviour of the spectral function (2.7) in the case of the mass hierarchy (1.1).

* To lowest order in the Weinberg-Salam coupling, the scalar current is not subject to renormalization. We therefore use the same notation for the renormalized and unrenormalized scalar current operators

3. Effective Lagrangian for the Higgs Boson Interaction

We are now in a position to give the low energy effective Lagrangian which consists of the light particle fields only,

$$\mathscr{L}=\mathscr{L}_{\rm QCD}+\mathscr{L}_{\rm int}+\mathscr{L}_{\rm Higgs}, \quad (3.1)$$

where

$$\mathscr{L}_{\rm QCD}=-\tfrac{1}{4}G^a_{\mu\nu}G^{\mu\nu}_a+\sum_l\bar{\psi}(i\gamma^\mu\partial_\mu+g_0\gamma^\mu A^a_\mu T^a)\psi -\frac{1}{2\alpha_0}(\partial^\mu A^a_\mu)^2+\bar{c}^a\partial^\mu D^{ab}_\mu c^b, \quad (3.2)$$

with $D^{ab}_\mu=\delta^{ab}\partial^\mu-g_0 f^{abc}A^c_\mu$. The kinetic part of the Higgs Lagrangian $\mathscr{L}_{\rm Higgs}$ is the same as that in (2.1). As will be shown later, the effective Higgs boson interaction with light particles is described in the lowest order of weak interactions, to which we restrict ourselves, by local operators, O_i,

$$\mathscr{L}_{\rm int}=-(\sqrt{2}G_F)^{\frac{1}{2}}\phi\sum_i C^i_0 O_i. \quad (3.3)$$

Only operators of dimension four will be considered. In contrast to Sect. 2 the quantities in the effective theory are untilded, and the renormalized coupling and renormalized gauge parameter in the MS scheme will be denoted hereafter by

$$g \quad\text{and}\quad \alpha, \quad (3.4)$$

respectively.

Renormalization of the fields and coupling parameters as well as the operators will be performed by employing the MS scheme in both the full theory and the effective light theory. The perturbative calculation of the coefficients C^i_0 and their renormalization is far easier in this scheme than in other subtraction schemes. At the same time it is possible to derive a consistent effective theory of light particles in the MS scheme.

Let us begin with the effective QCD Lagrangian (3.2), in which the heavy quarks' contributions are simply removed. To build a bridge between the full and effective QCD Lagrangians, (2.2) and (3.2), it is necessary to connect a (renormalized) Green's function with an arbitrary number of external light-particle legs

$$\tilde{\Gamma}^{(n,p,f_k)}(\tilde{g},\tilde{\alpha},\tilde{M}_j,\mu) \quad (3.5)$$

in the full theory with the corresponding quantity

$$\Gamma^{(n,p,f_k)}(g,\alpha,\mu) \quad (3.6)$$

in the effective theory, where μ is the mass scale of the dimensional regularization. The Green's function

(3.5) and (3.6) have n external gluon legs, p pairs of ghost-antighost external legs, and f_k pairs of external light fermions of flavour $k(k=u,d,\ldots)$.

The relation between (3.5) and (3.6) has been studied in perturbation theory by the authors of [10–12], who in fact confirmed by explicit calculation a relation of the form

$$\tilde{\Gamma}^{(n,p,f_k)}(\tilde{g},\tilde{\alpha},\tilde{M}_j,\mu) =(z_A)^{n/2}(z_G)^p(z_F)^f\,\Gamma^{(n,p,f_k)}(g,\alpha,\mu), \quad (3.7)$$

where $f=\sum_k f_k$ and power corrections (like $1/\tilde{M}_j^2$) are neglected. The finite renormalization factors z_A, z_G and z_F are to be attributed to the external gluon, ghost and fermion lines, respectively. Stating the relation (3.7) in another way, the parameters g and α together with the finite renormalization factors have been determined consistently in terms of the quantities in the full theory, namely, as

$$g=g(\tilde{g},\tilde{\alpha},\tilde{M}_j,\mu), \quad (3.8)$$

$$\alpha=\alpha(\tilde{g},\tilde{\alpha},\tilde{M}_j,\mu), \quad (3.9)$$

$$z_A=z_A(\tilde{g},\tilde{\alpha},\tilde{M}_j,\mu), \quad (3.10)$$

and similarly for z_G and z_F. By virtue of these "matching equations" the QCD Lagrangian (3.2) has been shown to be a consistent effective theory below the heavy quark threshold.

Incidently, it is worth noticing that the RGEs to be satisfied by the Green's functions in (3.7)

$$\left[\mu\frac{d}{d\mu}-n\tilde{\gamma}_A(\tilde{g},\tilde{\alpha})-2p\,\tilde{\gamma}_G(\tilde{g},\tilde{\alpha}) -2f\,\tilde{\gamma}_F(\tilde{g},\tilde{\alpha})\right]\tilde{\Gamma}^{(n,p,f_k)}=0, \quad (3.11)$$

$$\left[\mu\frac{d}{d\mu}-n\,\gamma_A(g,\alpha)-2p\,\gamma_G(g,\alpha) -2f\,\gamma_F(g,\alpha)\right]\Gamma^{(n,p,f_k)}=0, \quad (3.12)$$

in the full and effective theories, respectively, result in the relation

$$\frac{1}{2}\mu\frac{d}{d\mu}\ln z_A=\tilde{\gamma}_A(\tilde{g},\tilde{\alpha})-\gamma_A(g,\alpha) \quad (3.13)$$

and similar ones for z_G and z_F. Here the anomalous dimension of the gauge field calculated in the full (effective) theory is denoted by $\tilde{\gamma}_A(\gamma_A)$. Similarly defined are the anomalous dimensions of the ghost and fermionic fields.

Let us now move on to the effective Higgs boson interaction (3.3). The problem is to find the set of

local operators O_i possessing appropriate dimensions and quantum numbers which are necessary to describe the Higgs boson interaction of the full theory at low energies, and the corresponding (unrenormalized) coefficients C^i_0. This type of problem has been studied earlier by Kazama and Yao [14]. They have shown that heavy particle effects are calculable by a finite set of local operators with their coefficients, into which all the large mass dependence is factored out to all orders in the perturbation theory. They have also pointed out that the symmetry due to Becchi-Rouet-Stora transformation [15] puts restrictions on the form of the relevant local operators. The method used by Kazama and Yao is also vital in our case. All the $\tilde{M}_j$-dependence is contained in the coefficients as in the case of Kazama and Yao. The problem to find the operators O_i is reduced to finding a complete set of local operators with appropriate quantum numbers that close under renormalization [7], and in fact we find

$$O_1=-\tfrac{1}{4}G^a_{\mu\nu}G^{\mu\nu}_a, \quad (3.14)$$

$$O_2=\{D^{ab}_\mu G^{\mu\nu}_b+g_0 f^{abc}\partial^\nu\bar{c}^b c^c +g_0\sum_l\bar{\psi}\gamma^\nu T^a\psi\}A^a_\nu+\partial^\mu\bar{c}^a D^{ab}_\mu c^b, \quad (3.15)$$

$$O_3=\{D^{ab}_\mu\partial^\mu\bar{c}^b\}c^a, \quad (3.16)$$

$$O_4=\bar{\psi}(i\gamma^\mu\partial_\mu+g_0\gamma^\mu T^a A^a_\mu)\psi, \quad (3.17)$$

$$O_5=i\partial_\mu(\bar{\psi}\gamma^\mu\psi). \quad (3.18)$$

Here we recall for later purpose the classification of operators into two classes, classes I and II, according to Deans and Dixon [8]: the class I consists of gauge invariant operators and the class II of gauge variant operators and operators which are gauge invariant but vanish on application of the equations of motion. In this classification the gauge invariant operator (3.14) belongs to class I, the others (3.15)–(3.18) to class II.

The Higgs boson decay amplitude is given by the Green's function with a single insertion of the scalar current, i.e. $\tilde{\Gamma}^{(n,p,f_k)}(J_H)$ in the full theory and $\Gamma^{(n,p,f_k)}(O_i^{(R)})$ in the effective theory. They are related by

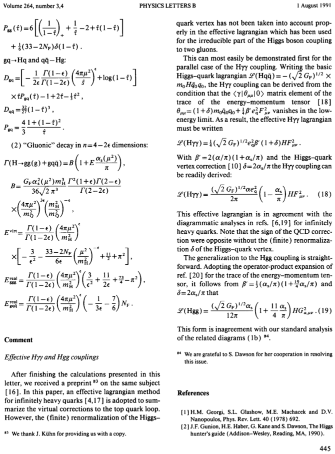
$$\tilde{\Gamma}^{(n,p,f_k)}(J_H) =(z_A)^{n/2}(z_G)^p(z_F)^f\sum_i C^i\,\Gamma^{(n,p,f_k)}(O_i^{(R)}) \quad (3.19)$$

with an error of order $1/\tilde{M}_j^2$. On the right hand side of (3.19) the renormalization of the operators O_i and the coefficient functions is to be performed in the MS scheme

$$O_i^{(R)}=\sum_j Z_{ij}O_j, \quad (3.20)$$

Figure 4: Effective Gauge Theory and the Effect of Heavy Quarks in Higgs Boson Decays, Inami, Kubota, Okada

EFT



Volume 264, number 3,4 PHYSICS LETTERS B 1 August 1991

$$P_{gg}(\hat\tau)=6\left[\left(\frac{1}{1-\hat\tau}\right)_+ +\frac{1}{\hat\tau}-2+\hat\tau(1-\hat\tau)\right]$$
$$+\tfrac{1}{6}(33-2N_F)\delta(1-\hat\tau)\,.$$

gq→Hq and $q\bar q$–Hg:

$$D_{gq}=\left[-\frac{1}{2\epsilon}\frac{\Gamma(1-\epsilon)}{\Gamma(1-2\epsilon)}\left(\frac{4\pi\mu^2}{\hat s}\right)^{\epsilon}+\log(1-\hat\tau)\right]$$
$$\times\hat\tau P_{gq}(\hat\tau)-1+2\hat\tau-\tfrac{1}{3}\hat\tau^2\,,$$
$$D_{q\bar q}=\tfrac{32}{27}(1-\hat\tau)^3\,,$$
$$P_{gq}=\frac{4}{3}\frac{1+(1-\hat\tau)^2}{\hat\tau}\,.$$

(2) "Gluonic" decay in $n=4-2\epsilon$ dimensions:

$$\Gamma(H\to gg(g)+gq\bar q)=B\left(1+E\frac{\alpha_s(\mu^2)}{\pi}\right),$$
$$B=\frac{G_F\alpha_s^2(\mu^2)m_H^3}{36\sqrt{2}\,\pi^3}\frac{\Gamma^2(1+\epsilon)\Gamma(2-\epsilon)}{\Gamma(2-2\epsilon)}$$
$$\times\left(\frac{4\pi\mu^2}{m_Q^2}\right)^{3\epsilon}\left(\frac{m_H^2}{m_Q^2}\right)^{-\epsilon},$$
$$E^{\rm virt}=\frac{\Gamma(1-\epsilon)}{\Gamma(1-2\epsilon)}\left(\frac{4\pi\mu^2}{m_H^2}\right)^{\epsilon}$$
$$\times\left[-\frac{3}{\epsilon^2}-\frac{33-2N_F}{6\epsilon}\left(\frac{\mu^2}{m_H^2}\right)^{-\epsilon}+\tfrac{11}{2}+\pi^2\right],$$
$$E^{\rm real}_{ggg}=\frac{\Gamma(1-\epsilon)}{\Gamma(1-2\epsilon)}\left(\frac{4\pi\mu^2}{m_H^2}\right)^{\epsilon}\left(\frac{3}{\epsilon^2}+\frac{11}{2\epsilon}+\tfrac{73}{4}-\pi^2\right),$$
$$E^{\rm real}_{gq\bar q}=\frac{\Gamma(1-\epsilon)}{\Gamma(1-2\epsilon)}\left(\frac{4\pi\mu^2}{m_H^2}\right)^{\epsilon}\left(-\frac{1}{3\epsilon}-\frac{7}{6}\right)N_F\,.$$

Comment

Effective Hγγ and Hgg couplings

After finishing the calculations presented in this letter, we received a preprint [#3] on the same subject [16]. In this paper, an effective lagrangian method for infinitely heavy quarks [4,17] is adopted to summarize the virtual corrections to the top quark loop. However, the (finite) renormalization of the Higgs–quark vertex has not been taken into account properly in the effective lagrangian which has been used for the irreducible part of the Higgs boson coupling to two gluons.

This can most easily be demonstrated first for the parallel case of the Hγγ coupling. Writing the basic Higgs–quark lagrangian $\mathscr{L}(Hq\bar q)=-(\sqrt{2}\,G_F)^{1/2}\times m_0H\bar q_0q_0$, the Hγγ coupling can be derived from the condition that the $\langle\gamma|\theta_{\mu\mu}|0\rangle$ matrix element of the trace of the energy–momentum tensor [18] $\theta_{\mu\mu}=(1+\delta)m_0\bar q_0q_0+\frac{1}{4}\beta' e_q^2F_{\mu\nu}^2$ vanishes in the low-energy limit. As a result, the effective Hγγ lagrangian must be written

$$\mathscr{L}(H\gamma\gamma)=\tfrac{1}{4}(\sqrt{2}\,G_F)^{1/2}e_q^2\beta'(1+\delta)HF_{\mu\nu}^2\,.$$

With $\beta'=2(\alpha/\pi)(1+\alpha_s/\pi)$ and the Higgs–quark vertex correction [10] $\delta=2\alpha_s/\pi$ the Hγγ coupling can be readily derived:

$$\mathscr{L}(H\gamma\gamma)=\frac{(\sqrt{2}\,G_F)^{1/2}\alpha e_q^2}{2\pi}\left(1-\frac{\alpha_s}{\pi}\right)HF_{\mu\nu}^2\,. \qquad (18)$$

This effective lagrangian is in agreement with the diagrammatic analyses in refs. [6,19] for infinitely heavy quarks. Note that the sign of the QCD correction were opposite without the (finite) renormalization δ of the Higgs–quark vertex.

The generalization to the Hgg coupling is straightforward. Adopting the operator-product expansion of ref. [20] for the trace of the energy–momentum tensor, it follows from $\beta'=\frac{1}{3}(\alpha_s/\pi)(1+\frac{19}{4}\alpha_s/\pi)$ and $\delta=2\alpha_s/\pi$ that

$$\mathscr{L}(Hgg)=\frac{(\sqrt{2}\,G_F)^{1/2}\alpha_s}{12\pi}\left(1+\frac{11}{4}\frac{\alpha_s}{\pi}\right)HG^2_{a,\mu\nu}\,. \qquad (19)$$

This form is inagreement with our standard analysis of the related diagrams (1b) [#4].

#3 We thank J. Kühn for providing us with a copy.

#4 We are grateful to S. Dawson for her cooperation in resolving this issue.

References

[1] H.M. Georgi, S.L. Glashow, M.E. Machacek and D.V. Nanopoulos, Phys. Rev. Lett. 40 (1978) 692.

[2] J.F. Gunion, H.E. Haber, G. Kane and S. Dawson, The Higgs hunter's guide (Addison–Wesley, Reading, MA, 1990).

Figure 5: Production of Higgs bosons in proton colliders. QCD correction, Djouadi

Spinor-helicity

2 Scalar plus gluon amplitudes

Here we consider amplitudes with a single spin-0, gauge singlet, h, and two or three gluons. We neglect quarks throughout this discussion. Tree-level amplitudes with a single Higgs and any number of gluons were derived in [9, 10], assuming the SM dimension-5 top-loop operator

$$\frac{1}{\Lambda} h\, G^{\mu\nu} G_{\mu\nu}\,. \tag{2.1}$$

Some of our analysis reproduces these known results. Our aim however, is to generalize these results beyond the operator Eq. (2.1), to any possible higher-dimension operator, suppressed by the appropriate power of a single scale Λ. The contribution of dimension-7 operators was inferred from Lorentz symmetry considerations in [18, 19].

2.1 The scalar plus two gluon amplitude $\mathcal{M}(h;gg)$

We start with the single scalar, 2-gluon amplitude, $\mathcal{M}\big(h; g^{a,h_1}(p_1) g^{b,h_2}(p_2)\big)$. The most general ansatz for this amplitude is,

$$\mathcal{M}\Big(h; g^{a,h_1}(p_1)\, g^{b,h_2}(p_2)\Big) = \delta^{ab}\, [12]^n\, f_{-\ell}\,(s_{12};\Lambda)\,, \tag{2.2}$$

where δ^{ab} is a color factor, h_1, h_2 are the gluon helicities, n is an integer, $f_{-\ell}$ is an analytic function of mass dimension $-\ell$, and $s_{12} = (p_1+p_2)^2 = m^2$. Since h is a scalar, the only little group weights are carried by the gluon spinor products. We then have,

$$n = 2h_1 = 2h_2\,, \tag{2.3}$$

which immediately sets

$$\mathcal{M}\left(h; g^+ g^-\right) = 0\,. \tag{2.4}$$

The only relevant amplitude to consider is then $\mathcal{M}\,(h;++)$ (with $\mathcal{M}\,(h;--)$ determined by a parity transformation). Then $n = 2$, and since the amplitude has mass dimension 1, $\ell = 1$ and

$$\mathcal{M}\Big(h; g^{a+}(p_1) g^{b+}(p_2)\Big) = \delta^{ab}\, [12]^2\, f_{-1}\left(m^2,\Lambda^2\right) = \delta^{ab}\, \frac{1}{\Lambda}\, [12]^2\, \tilde{f}\left(\frac{m^2}{\Lambda^2}\right), \tag{2.5}$$

where $\tilde{f}$ is dimensionless. Note that Eq. (2.3), combined with the mass dimension of the amplitude, gives a selection rule relating the sum of the gluon helicities to the dimension of the coupling which generates the amplitude (see also [14]). Specifically, here

$$|h_1 + h_2| = l + 1\,, \tag{2.6}$$

with $l = 1$.

At tree-level, the function $\tilde{f}$ can be written as a power series in m^2. No negative power of m^2 can appear, since the amplitude must vanish for $m \to 0$. The amplitude is therefore given by,

$$\mathcal{M}\Big(h; g^{a+}(p_1) g^{b+}(p_2)\Big) = \delta^{ab}\, \frac{[12]^2}{\Lambda} \sum_{n=0}^{\infty} c_n \left(\frac{m^2}{\Lambda^2}\right)^n \equiv \delta^{ab}\, \frac{c_5^{hgg}}{\Lambda}\, [12]^2\,, \tag{2.7}$$

where we rescaled the infinite series into the coefficient c_5^{hgg}. This is indeed the most general three-point amplitude for one massive scalar and two massless vectors [20, 21].

We can now make contact with the EFT calculation. The lowest order operators mediating scalar decay to two spin-1 particles are dimension-5. In a CP-conserving theory, there is only a single such operator, namely Eq. (2.1), in accord with the single real coefficient c_5^{hgg} at this order. Operators of higher dimension which contribute to the amplitude still contain two powers of the field-strength G, but an even power of derivatives. Since we consider a purely gluonic theory with no quarks, the EOM is $D^\mu G_{\mu\nu} = 0$. Using this and integration by parts, there is only a single independent operator at each order in Λ, with the derivatives acting on h and giving powers of m^2. In this case, this series merely gives a rescaling of c_5^{hgg}.

Figure 6: Effective Field Theory Amplitudes the On-Shell Way: Scalar and Vector Couplings to Gluons, Shadmi, Weiss

UV divergent and UV finite integrals

UV divergent integrals $\longrightarrow$ BDI's classified in all orders of perturbation

- Do not need to be evaluated to compute physical observables (do not contain any physics)
- Subtracted via renormalization

$$
\begin{aligned}
I_{quad}^{\nu_1\ldots\nu_{2r}}(\mu^2) &= \int_k \frac{k^{\nu_1}\ldots k^{2r}}{(k^2-\mu^2)^{r+1}} \qquad I_{quad} = \frac{1}{(k^2-\mu^2)} \\
I_{log}^{\nu_1\ldots\nu_{2r}}(\mu^2) &= \int_k \frac{k^{\nu_1}\ldots k^{2r}}{(k^2-\mu^2)^{r+2}} \qquad I_{log} = \frac{1}{(k^2-\mu^2)^2}
\end{aligned}
\tag{43}
$$

Using surface terms, [Batista et al., 2018]:

$$
\Gamma_0^{\mu\nu} = \int_k \frac{\partial}{\partial_\mu}\frac{k^\nu}{(k^2-\mu^2)^2} = 4\Big(\frac{g_{\mu\nu}}{4} I_{log}(\mu^2) - I_{log}^{\mu\nu}(\mu^2)\Big) = 0 \longrightarrow \text{Gauge theories}
\tag{44}
$$

UV finite integrals $\longrightarrow$ (**IR divergent or IR finite**)

- Information about physics
- Evaluated with *Package-X* of software *Mathematica*

[Batista et al., 2018] Batista, Y. R., Hiller, B., Cherchiglia, A., and Sampaio, M. (2018).

Supercurrent anomaly and gauge invariance in the N =1 supersymmetric Yang-Mills theory.

, 98(2):025018.

[Cherchiglia et al., 2011] Cherchiglia, A. L., Sampaio, M., and Nemes, M. C. (2011).

Systematic Implementation of Implicit Regularization for Multiloop Feynman Diagrams.

International Journal of Modern Physics A, 26(15):2591–2635.

[Denner et al., 2011] Denner, A., Heinemeyer, S., Puljak, I., Rebuzzi, D., and Spira, M. (2011).

Standard model Higgs-boson branching ratios with uncertainties.

European Physical Journal C, 71:1753.

[Dixon, 2013] Dixon, L. J. (2013).

A brief introduction to modern amplitude methods.

arXiv e-prints, page arXiv:1310.5353.

[Djouadi et al., 1991] Djouadi, A., Spira, M., and Zerwas, P. M. (1991).

Production of Higgs bosons in proton colliders: QCD corrections.

Phys. Lett. B, 264:440–446.

[Gnendiger et al., 2017] Gnendiger, C., Signer, A., Stöckinger, D., Broggio, A., Cherchiglia, A. L., Driencourt-Mangin, F., Fazio, A. R., Hiller, B., Mastrolia, P., Peraro, T., Pittau, R., Pruna, G. M., Rodrigo, G., Sampaio, M., Sborlini, G., Bobadilla, W. J. T., Tramontano, F., Ulrich, Y., and Visconti, A. (2017).

To {d}, or not to {d}: recent developments and comparisons of regularization schemes.

European Physical Journal C, 77(7):471.

[Kauffman et al., 1997] Kauffman, R. P., Desai, S. V., and Risal, D. (1997).

Production of a Higgs boson plus two jets in hadronic collisions.

Physical Review D, 55(7):4005–4015.

[Kinoshita, 1962] Kinoshita, T. (1962).

Mass singularities of Feynman amplitudes.

Journal of Mathematical Physics, 3(4).

[Lee and Nauenberg, 1964] Lee, T. D. and Nauenberg, M. (1964).

Degenerate Systems and Mass Singularities.

Phys. Rev., 133:B1549–B1562.

[Sampaio et al., 2006] Sampaio, M., Scarpelli, A. P. B., Ottoni, J. E., and Nemes, M. C. (2006).

Implicit Regularization and Renormalization of QCD.

International Journal of Theoretical Physics, 45(2):436–457.

[Torres Bobadilla et al., 2021] Torres Bobadilla, W. J., Sborlini, G. F. R., Banerjee, P., Catani, S., Cherchiglia, A. L., Cieri, L., Dhani, P. K., Driencourt-Mangin, F., Engel, T., Ferrera, G., Gnendiger, C., Hernández-Pinto, R. J., Hiller, B., Pelliccioli, G., Pires, J., Pittau, R., Rocco, M., Rodrigo, G., Sampaio, M., Signer, A., Signorile-Signorile, C., Stöckinger, D., Tramontano, F., and Ulrich, Y. (2021).

May the four be with you: novel IR-subtraction methods to tackle NNLO calculations.

European Physical Journal C, 81(3):250.