Theoretical study of heavy-quark diffusion in the quark-gluon plasma

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Outline

1. Heavy quark diffusion coefficient
2. LO & NLO HTL results
3. Gribov quantization
4. Gribov confinement scenario in deconfined phase
5. Diffusion coefficient with Gribov propagator
6. Conclusion

1. Heavy quark diffusion coefficient
2.
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HQ diffusion coefficient

- When a heavy-quark passes through a thermal medium, it losses its kinetic energy due the collision as well as due to the radiation.
- Heavy quark diffusion coefficient is related with the collisional energy loss and momentum boardaning of the heavy-quark
- The momentum of the heavy quark evolves according to the Langevin equations as

$$\frac{dp_i}{dt} = \xi_i(t) - \eta_D p_i, \quad \left\langle \xi_i(t)\xi_j\left(t'\right)\right\rangle = \kappa\delta_{ij}\delta\left(t - t'\right)$$

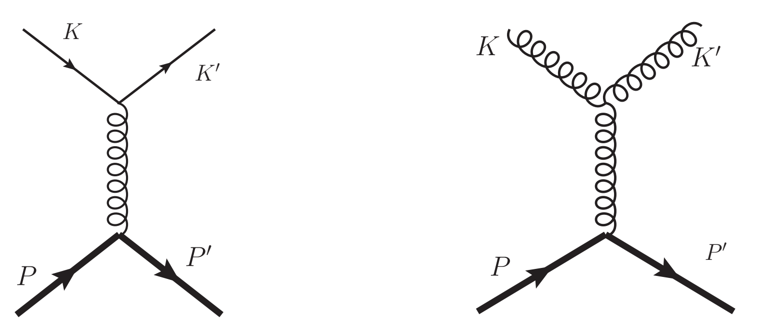
- The diffusion constant in space, D_s, can be found by starting a particle at $x = 0$ at $t = 0$ and finding the mean-squared position at a later time,

$$\langle x_i(t)x_j(t)\rangle = 2Dt\delta_{ij} \quad \rightarrow \quad 6D_s t = \left\langle x^2(t)\right\rangle.$$

The relation between position and momentum $x_i(t) = \int_0^t dt' \frac{p_i(t')}{M}$, we have

$$6D_s t = \int_0^t dt_1 \int_0^t dt_2 \frac{1}{M^2} \langle p(t_1)\, p(t_2) \rangle = \frac{6Tt}{M\eta_D}$$

$$\Rightarrow D_s = \frac{T}{M\eta_D} = \frac{2T^2}{\kappa}.$$

- Momentum diffusion κ from the t-channel diagram of $qH \to qH$ and $gH \to gH$ scattering.
- 

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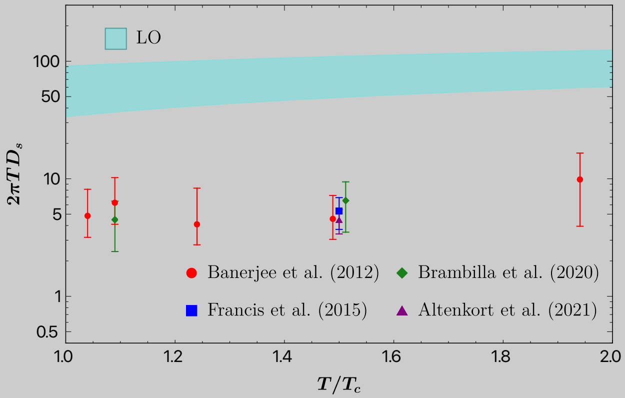
$$3\kappa = \frac{1}{16M^2}\int \frac{d^3\boldsymbol{k}}{(2\pi)^4 k k'}\int q^2 dq \int_{-1}^{1} d\cos\theta_{\boldsymbol{kq}}\,\delta\left(k'-k\right) q^2 \times\left[|\mathcal{M}|^2_{\text{quark}}\, n_F(k)\left[1-n_F\left(k'\right)\right] + |\mathcal{M}|^2_{\text{gluon}}\, n_B(k)\left[1+n_B\left(k'\right)\right]\right].$$

Leading order HTL

PRC 71, 064904 (2005), Moore & Teaney

$$
\begin{aligned}
3\kappa &= \frac{C_F g^4}{4\pi^3} \int_0^\infty k^2 dk \int_0^{2k} q\ dq \frac{q^2}{(q^2+m_D^2)^2} \\
&\quad \times \left[N_f n_F(k)[1-n_F(k)] \left(2 - \frac{q^2}{2k^2}\right) + N_c n_B(k)[1+n_B(k)] \left(2 - \frac{q^2}{k^2} + \frac{q^4}{4k^4}\right) \right] \\
&= \frac{C_F g^4}{18\pi} \left[\left(N_c + \frac{N_f}{2}\right) \left[\ln \frac{2T}{m_D} + \frac{1}{2} - \gamma_E + \frac{\zeta'(2)}{\zeta(2)} \right] + \frac{N_f}{2} \ln 2 \right]
\end{aligned}
$$

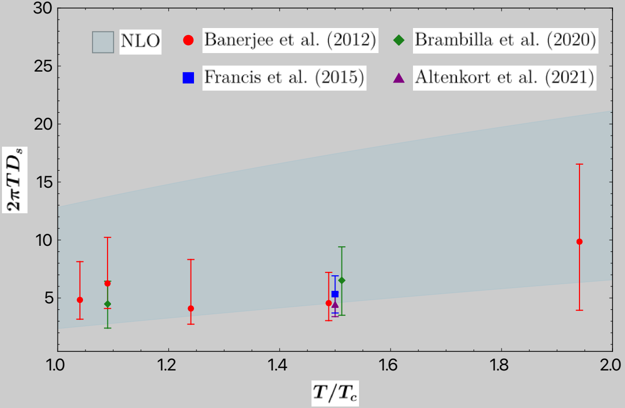
PRC 71, 064904 (2005), Moore & Teaney



Next-to-Leading order HTLL

PRL 100, 052301 (2008), Caron-Huot,& Moore

$$3\kappa = \frac{C_F g^4}{18\pi}\left[\left(N_c + \frac{N_f}{2}\right)\left[\ln\frac{2T}{m_D} + \frac{1}{2} - \gamma_E + \frac{\zeta'(2)}{\zeta(2)}\right] + \frac{N_f}{2}\ln 2 + 2.3302\,\frac{N_c m_D}{T}\right)\right]$$



Motivation

- The gluon and ghost propagators with Gribov quantization show an IR suppression and enhancement, respectively, as compared to the Faddeev-Popov case.
- These results heuristically encompass desirable features of confinement: the IR-suppressed gluon propagator indicates gluon confinement at large distance, and the IR-enhanced ghost is responsible for confinement.
- It was shown by Zwanziger (PRL94, 182301 (2005)) in a phenomenological way that a free gas of Gribov quasiparticles qualitatively captures the nonperturbative features of the lattice equation of state.

- In covariant gauge, the gluon propagator is

$$D^{ab}_{\mu\nu}(K) = -\frac{\delta_{ab}}{K^2}\left[g_{\mu\nu} - (1-\xi)\,\frac{K_\mu K_\nu}{K^2}\right]$$

- Faddeev-Popo action with ghost field

$$\begin{aligned} S &= S_{YM} + S_{GF} + S_{ghost} \\ &= S_{YM} + \int d^4x \left(\bar{c}^a \partial^\mu \left(D_\mu c\right)^a - \frac{1}{2\xi}\left(\partial_\mu A^{\mu a}\right)^2\right) \end{aligned}$$

- Gribov demonstrated for the first time in 1978 that the gauge condition proposed by Faddeev and Popov is not ideal.

- In the Gribov quantization, the YM partition function in Euclidean space reads

$$Z = \int_\Omega \mathcal{D}A(x)V(\Omega)\delta(\partial\cdot A)\,\det[-\partial\cdot D(A)]e^{-S_{\rm YM}}$$

The restriction of the integration to the Gribov region is realized by inserting a function $V(\Omega)$ into the partition function, where

$$V(\Omega) = \theta[1-\sigma(0)] = \int_{-i\infty+\epsilon}^{+i\infty+\epsilon}\frac{d\beta}{2\pi i\beta}e^{\beta[1-\sigma(0)]}$$

represents the no-pole condition. Here, $1-\sigma(P)$ is the inverse of the ghost dressing function $Z_G(P)$.

- The integration variable β is identified as the Gribov mass parameter γ_G after some redefinition.

- Gribov's gluon propagator in the Landau gauge reads

$$D_A(P) = \delta^{ab} \frac{P^2}{P^4 + m_{\mathrm{G}}^4} \left(\delta^{\mu\nu} - \frac{P^\mu P^\nu}{P^2} \right)$$

- The ghost propagator in the Landau gauge

$$D_c(P) = \delta^{ab} \frac{1}{1 - \sigma(P)} \cdot \frac{1}{P^2},$$

- The inverse of the ghost dressing function is

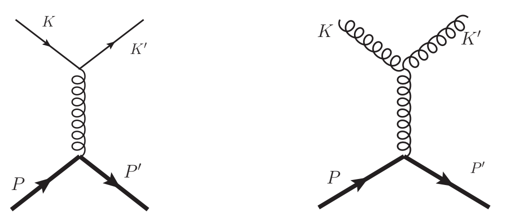
$$Z_G^{-1} \equiv [1 - \sigma(P)] = \frac{N_{\mathrm{c}} g^2}{128\pi^2} \left[-5 + \left(3 - \frac{\gamma_G^4}{P^4} \right) \ln \left(1 + \frac{P^4}{\gamma_G^4} \right) + \frac{\pi P^2}{\gamma_G^2} + 2 \left(3 - \frac{P^4}{\gamma_G^4} \right) \frac{\gamma_G^2}{P^2} \arctan \frac{P^2}{\gamma_G^2} \right]$$

Applicability of Gribov confinement scenario

Ref: D. Zwanziger,PRD76, 125014 (2007)

- Long-distance behavior of the color-Coulomb potential $V_{\rm coul}(R) \sim \sigma_{\rm coul}\, R$, $\sigma_{\rm coul} \sim 3\sigma$ and σ being the physical string tension between a pair of external quarks.
- It was also found numerically that the long-distance behavior of $V_{\rm coul}\,(R)$ is consistent with a linear increase, $\sigma_{\rm coul}\ > 0$, above the phase transition temperature, $T > T_c$, where σ vanishes.
- Investigation of the temperature dependence of $\sigma_{\rm coul}$ revealed that in the deconfined phase, the Coulomb string tension increases with T, which is consistent with a magnetic mass $\sigma^{1/2}_{\rm coul}(T) \sim g_s^2(T)\, T$.
- Thus, from the numerical evidence one can say that the Gribov parameter is nonzero in the deconfined phase also.

t-channel heavy quark scattering



$$|\mathcal{M}|^2_{\text{gluon}} = 16\mathrm{N_c} C_F g^4 M^2 k^2$$
$$\times\,(1+\cos^2\theta_{kk'})\frac{q^4}{(q^4+\gamma_G^4}$$

$$|\mathcal{M}|^2_{\text{quark}} = 16 N_f C_F g^4 M^2 k^2$$
$$\times\,(1+\cos\theta_{kk'})\frac{q^4}{(q^4+\gamma_G^4)^2}$$

$$\cos\theta_{kk'} = 1 - \frac{(QM)^2}{2(K\cdot P)^2} = 1 - \frac{q^2}{2k^2}\; viz\;((K\cdot P)\sim Mk, \mathrm{Q}-\text{purely spatial}) \tag{1}$$

$$3\kappa = \frac{C_F g^4}{4\pi^3} \int_0^\infty k^2 dk \int_0^{2k} q dq \frac{q^6}{(q^4+\gamma_G^4)^2} \times \left[N_f\, n_F(k)[1-n_F(k)] \left(2 - \frac{q^2}{2k^2}\right) \right.$$
$$\left. + N_c\, n_B(k)[1+n_B(k)] \left(2 - \frac{q^2}{k^2} + \frac{q^4}{4k^4}\right) \right]$$

Fixing γ_G perturbatively with $(N_f = 3)$

Analytic form of γ_G, in the limit $T \to \infty$, Phys. Rev. D 88, 076008

$$\gamma_G(T) = \frac{d-1}{d} \frac{\mathrm{N_c}}{4\sqrt{2}\pi} g^2(T) T \ , \tag{2}$$

d is dimension of space time(here, 4), $g(T)$ is the running coupling, in perturbative limit(one loop):

$$\frac{g^2(T)}{4\pi} = \frac{6\pi}{(11\mathrm{N_c} - 2\mathrm{N_f}) \ln\left(\frac{\Lambda}{\lambda_{\overline{\mathrm{MS}}}}\right)} \ , \tag{3}$$

where

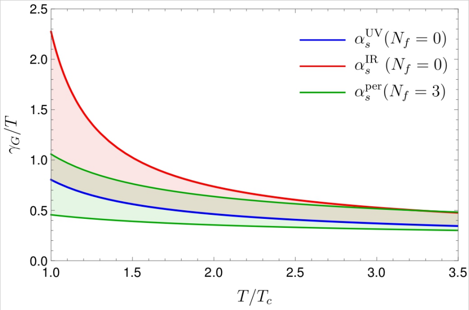
- $\pi T \leq \Lambda \leq 4\pi T$, and
- $\Lambda_{\overline{\mathrm{MS}}} = 0.176$ GeV, (lattice result) .

Fixing γ_G from LQCD fitted coupling

PRD 88, 076008

$$\alpha_S(T/T_c) \equiv \frac{g^2(T/T_c)}{4\pi} = \frac{6\pi}{11\ \mathrm{N_c}\ \ln[c(T/T_c)]}\ ,$$

$c = 1.43$ for IR and $c = 2.97$ for UV $\rightarrow \alpha^{IR}_{T=T_c} = 1.59$ and $\alpha^{UV}_{T=T_c} = 0.524$. The fitted parameter values corresponding to the coupling data extracted from the large distance (IR) and the short distance (UV) behaviour of the heavy quark free energy .



Result and conclusion

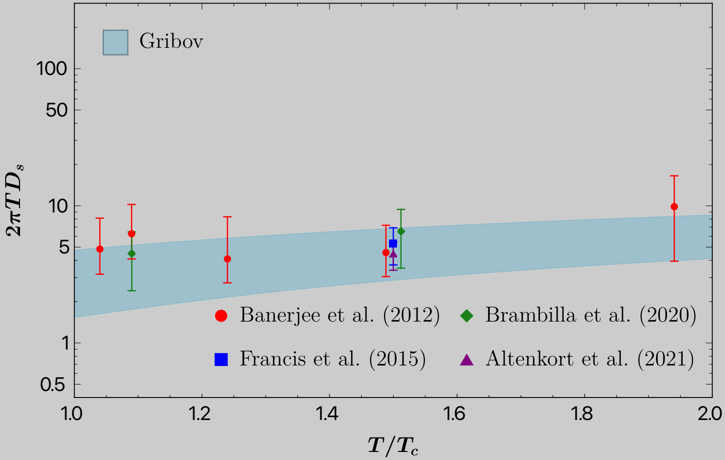


Figure: Plot of $(2\pi T)D_S$ vs (T/T_c). For LO and NLO 2 loop coupling has been taken with $T_c/\Lambda_{\overline{MS}} = 1.15$.

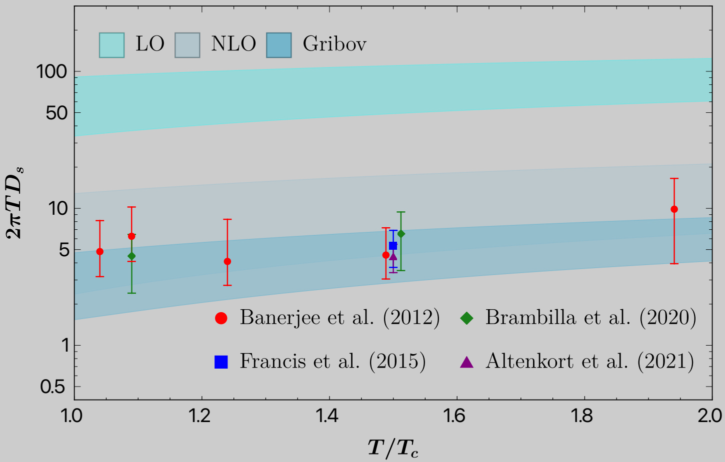


Figure: Plot of $(2\pi T)D_S$ vs (T/T_c). For LO and NLO 2 loop coupling has been taken with $T_c/\Lambda_{\overline{MS}} = 1.15$.

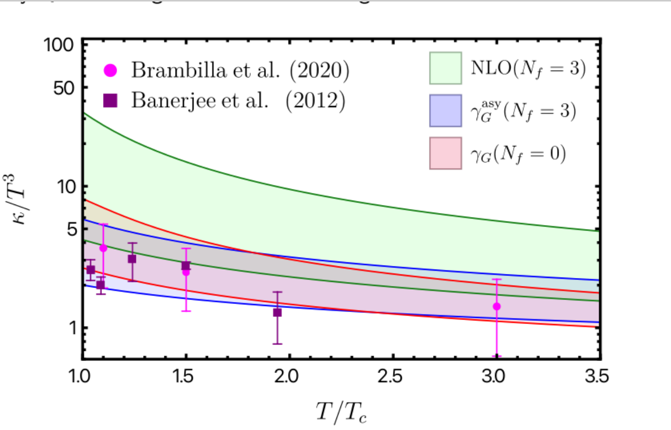


Figure: Plot κ/T^3 vs (T/T_c). For NLO, one loop coupling has been taken with $\Lambda_{\overline{\mathrm{MS}}} = 0.176$ GeV.

Conclusion

- We have discussed existing LO and NLO HTLpt results for heavy quark diffusion coefficient.
- We have discussed the motivation to include Gribov quantization to the estimation of heavy-quark diffusion coefficients.
- We have also discussed our recent results for the heavy-quark diffusion rate in Gribov Plasma.

Thank you for your attention.