UNIVERSIDAD COMPLUTENSE MADRID

Maria Zambrano grants to attract international talent

GOBIERNO DE ESPAÑA

Maria Zambrano

LHCb

FSI to enhance CP violation in charm decay

Patricia C. Magalhães

Complutense University of Madrid

p.magallhaes@cern.ch

Excited QCD 2022

23–29 Oct 2022
Sicily, italy

In collaboration with
Bediaga and Frederico
arxiv 2203.04056v2

CP asymmetry measurements

1956
Parity violation
T.D. Lee, C.N. Yang, C.S. Wu et al.

1963
Cabibbo Mixing
N. Cabibbo

1964
Strange particles: CPV in K meson decays
J.W. Cronin, V.L Fitch et al.

1973
CKM matrix
M. Kobayashi, T Maskawa

2001
Beauty particles: CPV in B^0 meson decays
Babar and Belle

2019
Charm particle: CPV in D^0 meson decays
LHCb

CP asymmetry measurements

1956
Parity violation
T.D. Lee, C.N. Yang,
C.S. Wu et al.

1963
Cabibbo Mixing
N. Cabibbo

1964
Strange particles: CPV in K meson decays
J.W. Cronin, V.L Fitch et al.

1973
CKM matrix
M. Kobayashi,
T Maskawa

2001
Beauty particles: CPV in B^0 meson decays
Babar and Belle

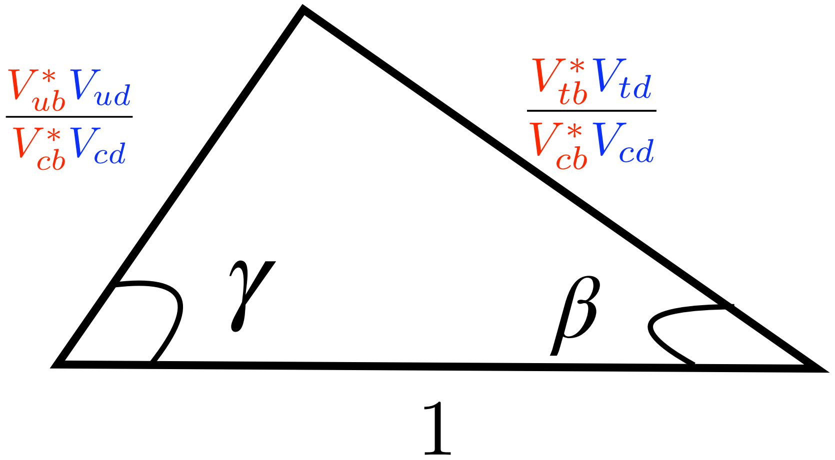
2019
Charm particle: CPV in D^0 meson decays
LHCb

2022
1st evidence of CPV in $D^0 \to \pi\pi$
LHCb

today

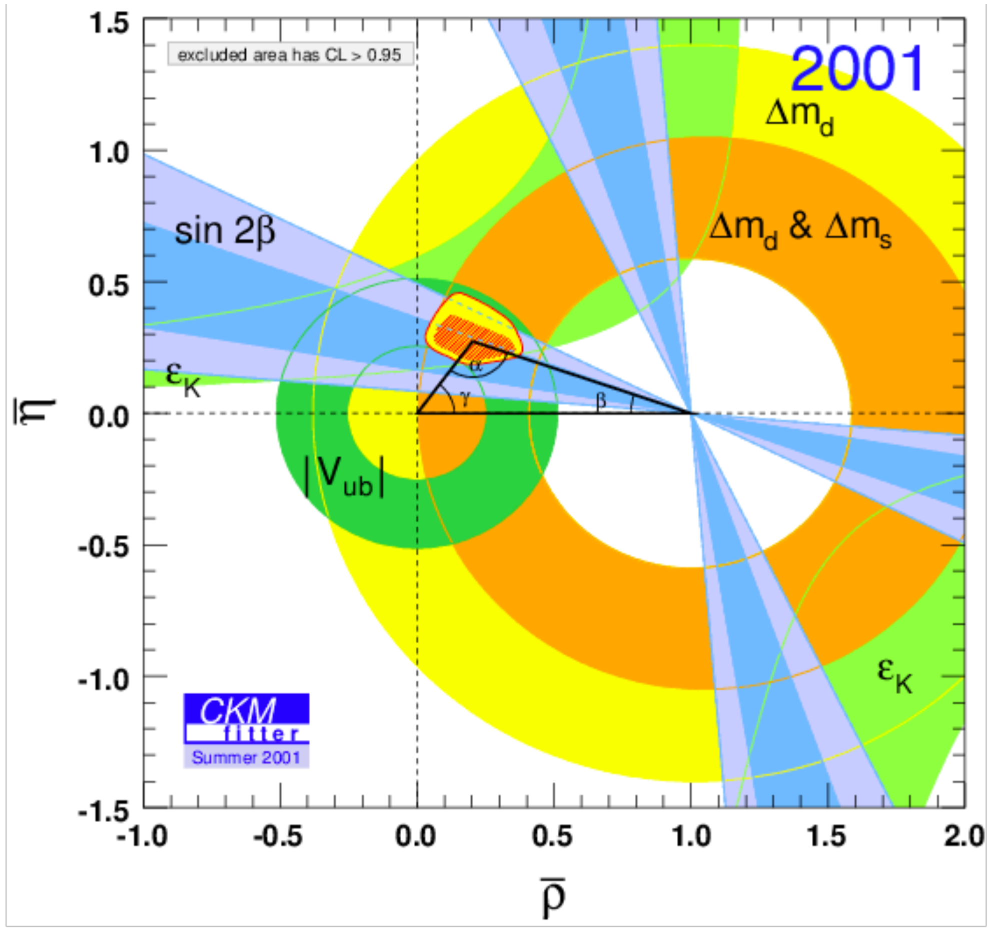
Unitary CKM matrix

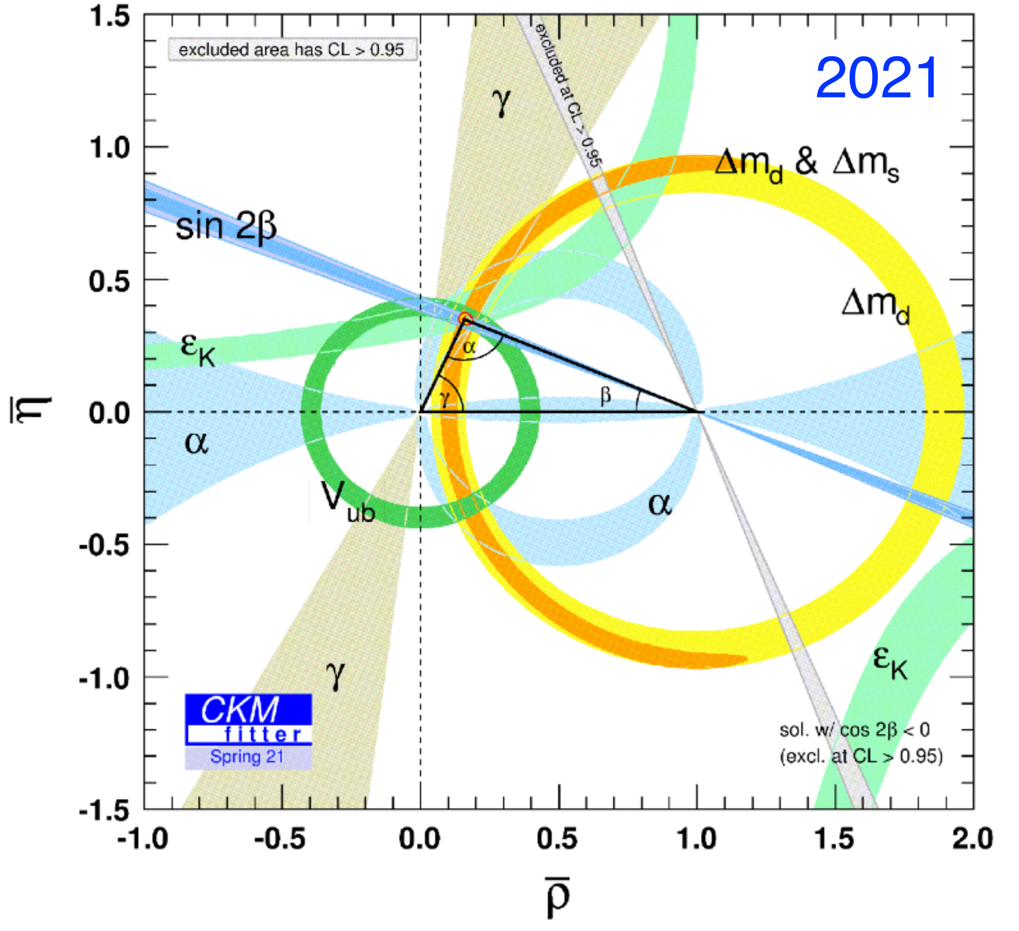
- $$V_{\rm CKM} = \begin{array}{cc} & \begin{array}{ccc} d & s & b \end{array} \\ \begin{array}{c} u \\ c \\ t \end{array} & \begin{pmatrix} 1-\frac{1}{2}\lambda^2 & \lambda & A\lambda^3(\rho-i\eta) \\ -\lambda & 1-\frac{1}{2}\lambda^2 & A\lambda^2 \\ A\lambda^3(1-\rho-i\eta) & -A\lambda^2 & 1 \end{pmatrix} \end{array} + \mathcal{O}(\lambda^4)$$



  - loads of CPV expected/found in B decays and not much in Kaon or D

- test Standard model limits





motivation

- $\Delta A_{CP}^{\mathrm{LHCb}} = A_{cp}(D^0 \to K^+K^-) - A_{cp}(D^0 \to \pi^+\pi^-) = -(1.54 \pm 0.29) \times 10^{-3}$

Phys. Rev. Lett.122, 211803 (2019)

motivation

- $\Delta A_{CP}^{\mathrm{LHCb}} = A_{cp}(D^0 \to K^+K^-) - A_{cp}(D^0 \to \pi^+\pi^-) = -(1.54 \pm 0.29) \times 10^{-3}$

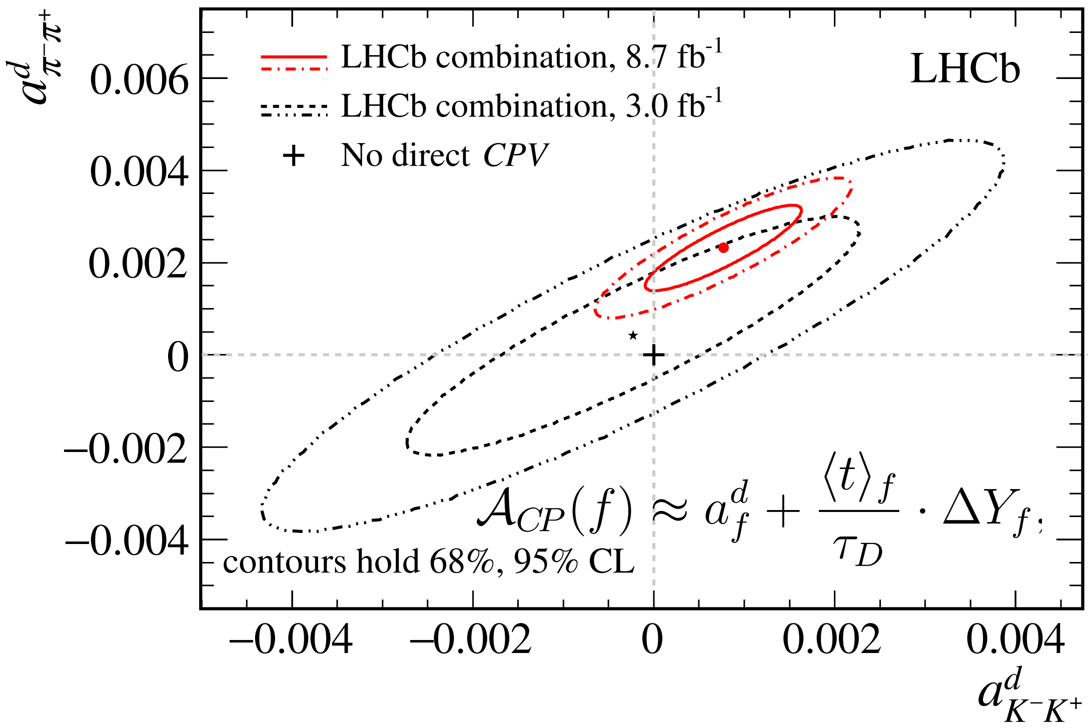
Phys. Rev. Lett.122, 211803 (2019)

➔ direct CP asymmetry observation

- $A_{CP}^{LHCb}(KK) = (0.77 \pm 0.57) \times 10^{-3}$

↳ $A_{CP}^{LHCb}(\pi\pi) = (2.32 \pm 0.61) \times 10^{-3}$

arXiv:2209.03179



motivation

- $\Delta A_{CP}^{\rm LHCb} = A_{cp}(D^0 \to K^+K^-) - A_{cp}(D^0 \to \pi^+\pi^-) = -(1.54 \pm 0.29) \times 10^{-3}$

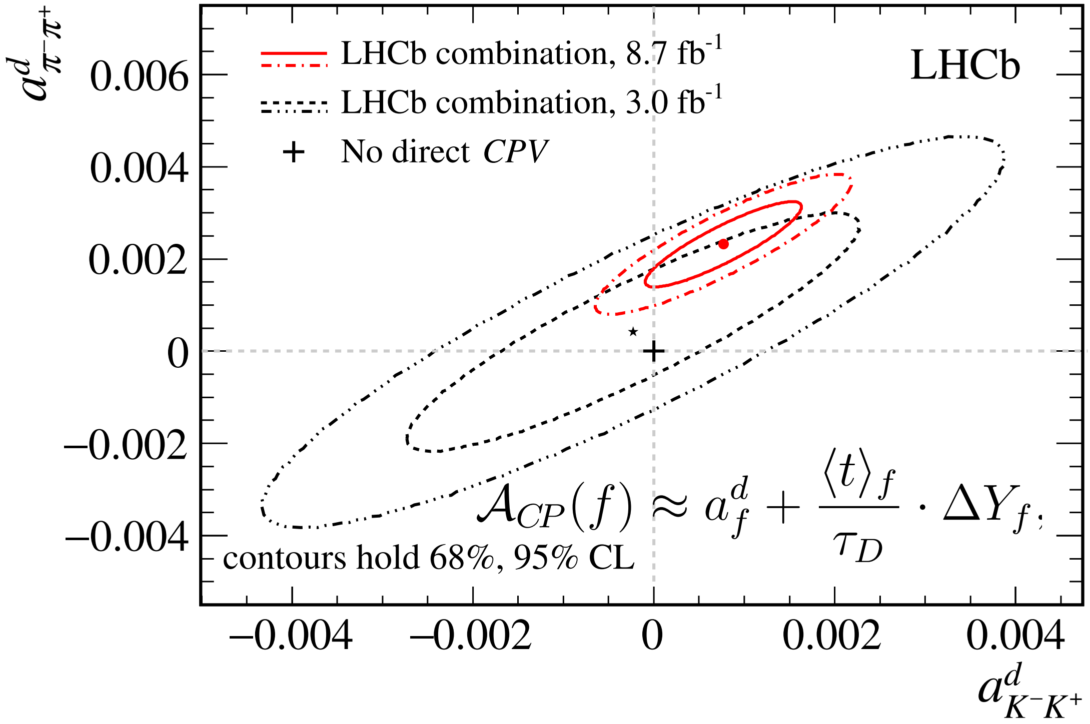
Phys. Rev. Lett.122, 211803 (2019)

→ direct CP asymmetry observation

- $A_{CP}^{LHCb}(KK) = (0.77 \pm 0.57) \times 10^{-3}$

↳ $A_{CP}^{LHCb}(\pi\pi) = (2.32 \pm 0.61) \times 10^{-3}$

arXiv:2209.03179



- QCD → LCSR predictions $A_{CP} \approx 10^{-4}$ (1 order magnitude bellow)

↳ new physics? nonperturbative effects?!

Khodjamirian, Petrov,
Phys. Lett. B 774, 235 (2017)

motivation

- $\Delta A_{CP}^{\mathrm{LHCb}} = A_{cp}(D^0 \to K^+K^-) - A_{cp}(D^0 \to \pi^+\pi^-) = -(1.54 \pm 0.29) \times 10^{-3}$

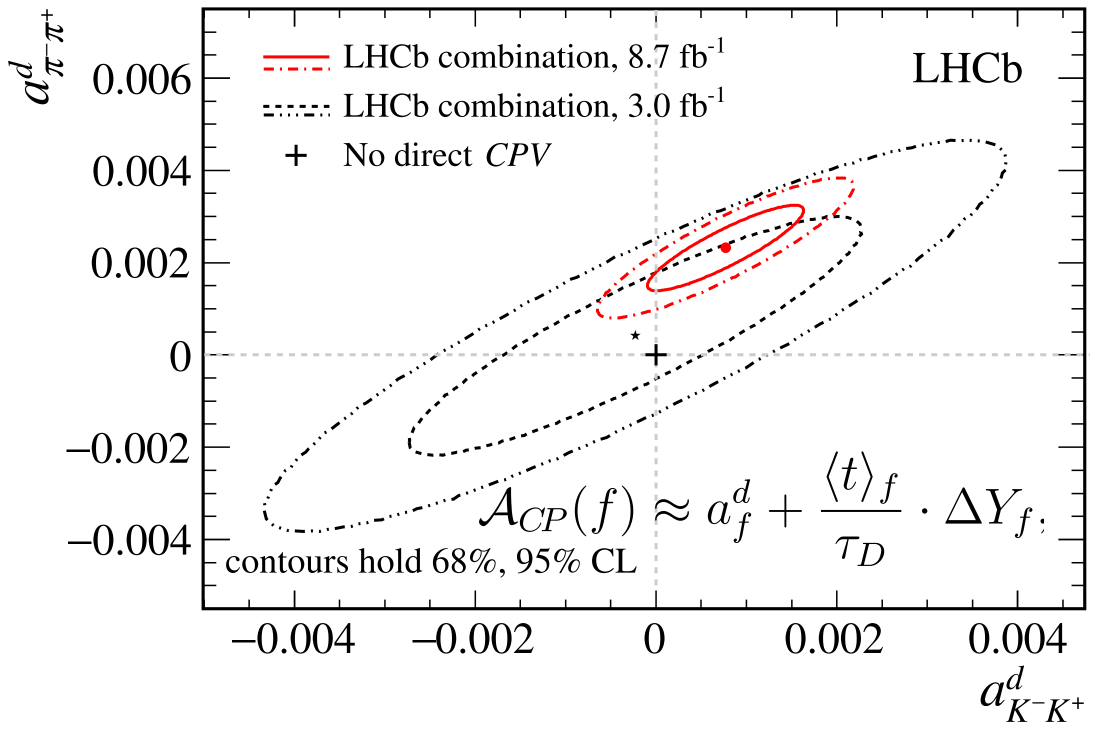
Phys. Rev. Lett.122, 211803 (2019)

➔ direct CP asymmetry observation

- $A_{CP}^{LHCb}(KK) = (0.77 \pm 0.57) \times 10^{-3}$

↳ $A_{CP}^{LHCb}(\pi\pi) = (2.32 \pm 0.61) \times 10^{-3}$

arXiv:2209.03179



- QCD ➔ LCSR predictions $A_{CP} \approx 10^{-4}$ (1 order magnitude bellow)

↳ new physics? nonperturbative effects?!

Khodjamirian, Petrov,
Phys. Lett. B 774, 235 (2017)

➔ CPV on $D \to hhh$?

➔ searches in many process at LHCb, BESIII, BeleII

↳ is expected soon with LHCb run II

direct CP violation

- 2 amplitudes: SAME final state, ≠ strong (δ_i) and weak (ϕ_i) phases

$$\langle f \,|\, T \,|\, M \rangle = A_1\, e^{i(\delta_1 + \phi_1)} + A_2\, e^{i(\delta_2 + \phi_2)}$$

$$\langle \bar{f} \,|\, T \,|\, \bar{M} \rangle = A_1\, e^{i(\delta_1 - \phi_1)} + A_2\, e^{i(\delta_2 - \phi_2)}$$

- weak phase → CKM
- strong phase → QCD

direct CP violation

- 2 amplitudes: SAME final state, $\neq$ strong (δ_i) and weak (ϕ_i) phases

$$\langle f\,|\,T\,|\,M\rangle = A_1\, e^{i(\delta_1+\phi_1)} + A_2\, e^{i(\delta_2+\phi_2)}$$

$$\langle \bar{f}\,|\,T\,|\,\bar{M}\rangle = A_1\, e^{i(\delta_1-\phi_1)} + A_2\, e^{i(\delta_2-\phi_2)}$$

- weak phase → CKM
- strong phase → QCD

- $$\boxed{\Delta\Gamma_{CP} = \Gamma(M \to f) - \Gamma(\bar{M} \to \bar{f}) = |\langle f\,|\,T\,|\,M\rangle|^2 - |\langle \bar{f}\,|\,T\,|\,\bar{M}\rangle|^2 = -4A_1A_2\sin(\delta_1-\delta_2)\,\sin(\phi_1-\phi_2)}$$

- $$A_{CP} = \frac{\Gamma(M \to f) - \Gamma(\bar{M} \to \bar{f})}{\Gamma(M \to f) + \Gamma(\bar{M} \to \bar{f})}$$

direct CP violation

- 2 amplitudes: SAME final state, $\neq$ strong (δ_i) and weak (ϕ_i) phases

$$\langle f\,|\,T\,|\,M\rangle = A_1\, e^{i(\delta_1+\phi_1)} + A_2\, e^{i(\delta_2+\phi_2)}$$

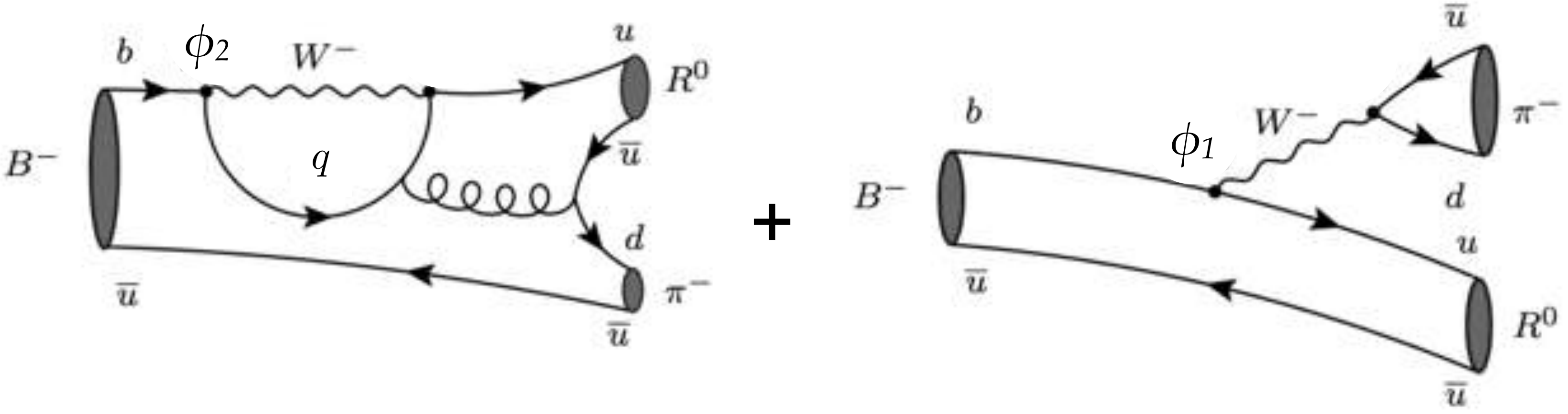
$$\langle \bar{f}\,|\,T\,|\,\bar{M}\rangle = A_1\, e^{i(\delta_1-\phi_1)} + A_2\, e^{i(\delta_2-\phi_2)}$$

- weak phase → CKM
- strong phase → QCD

- $$\boxed{\Delta\Gamma_{CP} = \Gamma(M\to f) - \Gamma(\bar{M}\to\bar{f}) = |\langle f\,|\,T\,|\,M\rangle|^2 - |\langle\bar{f}\,|\,T\,|\,\bar{M}\rangle|^2 = -4A_1A_2\sin(\delta_1-\delta_2)\,\sin(\phi_1-\phi_2)}$$

- $$A_{CP} = \frac{\Gamma(M\to f) - \Gamma(\bar{M}\to\bar{f})}{\Gamma(M\to f) + \Gamma(\bar{M}\to\bar{f})}$$

- BSS model Bander Silverman & Soni PRL 43 (1979) 242



direct CP violation

- 2 amplitudes: SAME final state, ≠ strong (δ_i) and weak (ϕ_i) phases

$$\langle f\,|\,T\,|\,M\rangle = A_1\, e^{i(\delta_1+\phi_1)} + A_2\, e^{i(\delta_2+\phi_2)}$$

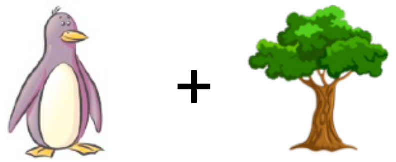
$$\langle \bar{f}\,|\,T\,|\,\bar{M}\rangle = A_1\, e^{i(\delta_1-\phi_1)} + A_2\, e^{i(\delta_2-\phi_2)}$$

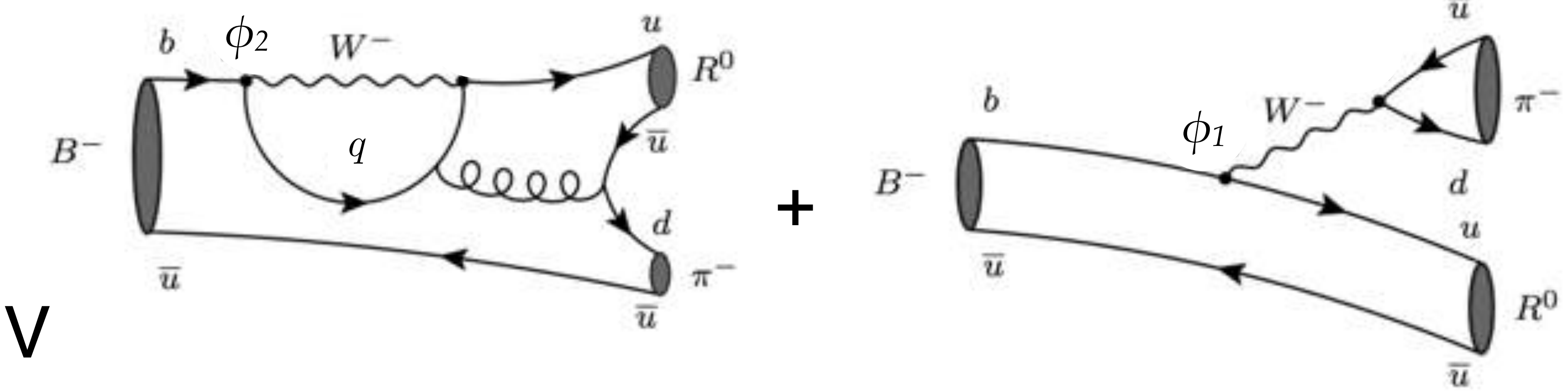
- weak phase → CKM
- strong phase → QCD

- $$\boxed{\Delta\Gamma_{CP} = \Gamma(M \to f) - \Gamma(\bar{M} \to \bar{f}) = |\langle f\,|\,T\,|\,M\rangle|^2 - |\langle \bar{f}\,|\,T\,|\,\bar{M}\rangle|^2 = -4A_1A_2\sin(\delta_1-\delta_2)\,\sin(\phi_1-\phi_2)}$$

- $$A_{CP} = \frac{\Gamma(M \to f) - \Gamma(\bar{M} \to \bar{f})}{\Gamma(M \to f) + \Gamma(\bar{M} \to \bar{f})}$$

- BSS model Bander Silverman & Soni PRL 43 (1979) 242





  - not enough for CPV

- hadronic interactions are natural sources of strong phase!

CPV on heavy meson decays

- CPV in $B^{\pm} \to h^{\pm}h^{-}h^{+}$

Run II 5.9 fb^{-1}

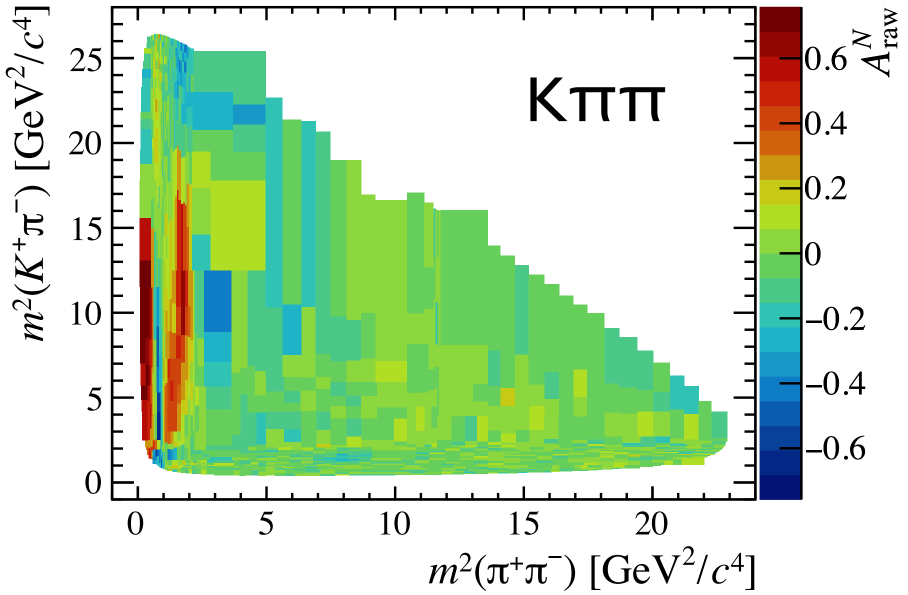
arXiv:2206.07622 PRD 2022 XX

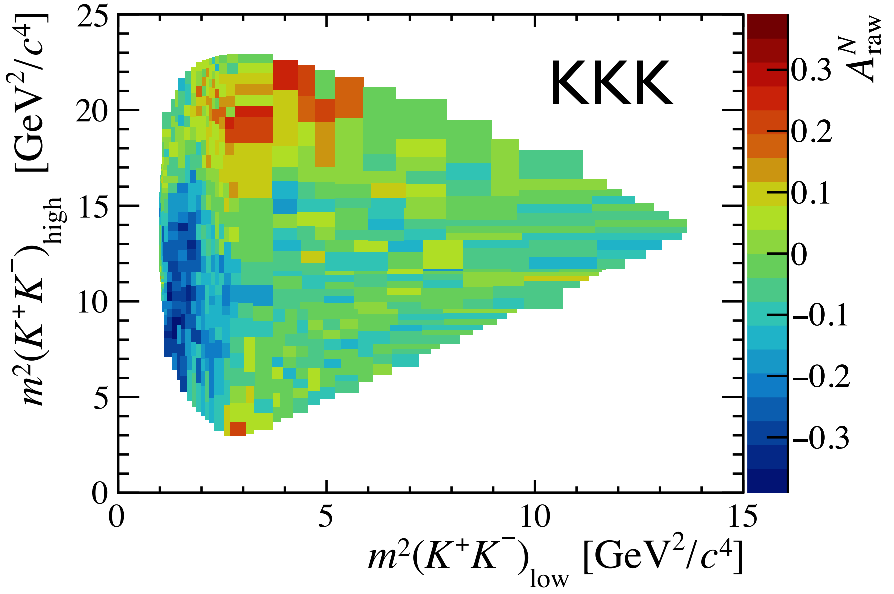
- integrated

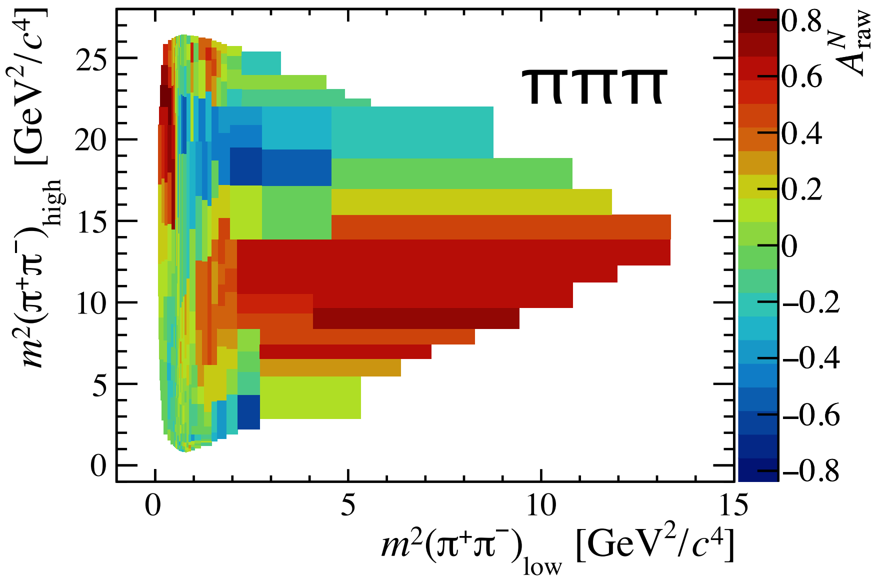
$$A_{CP}(B^{\pm} \to K^{\pm}\pi^{+}\pi^{-}) = +0.011 \pm 0.002,$$
$$A_{CP}(B^{\pm} \to K^{\pm}K^{+}K^{-}) = -0.037 \pm 0.002,$$
$$A_{CP}(B^{\pm} \to \pi^{\pm}\pi^{+}\pi^{-}) = +0.080 \pm 0.004,$$
$$A_{CP}(B^{\pm} \to \pi^{\pm}K^{+}K^{-}) = -0.114 \pm 0.007,$$

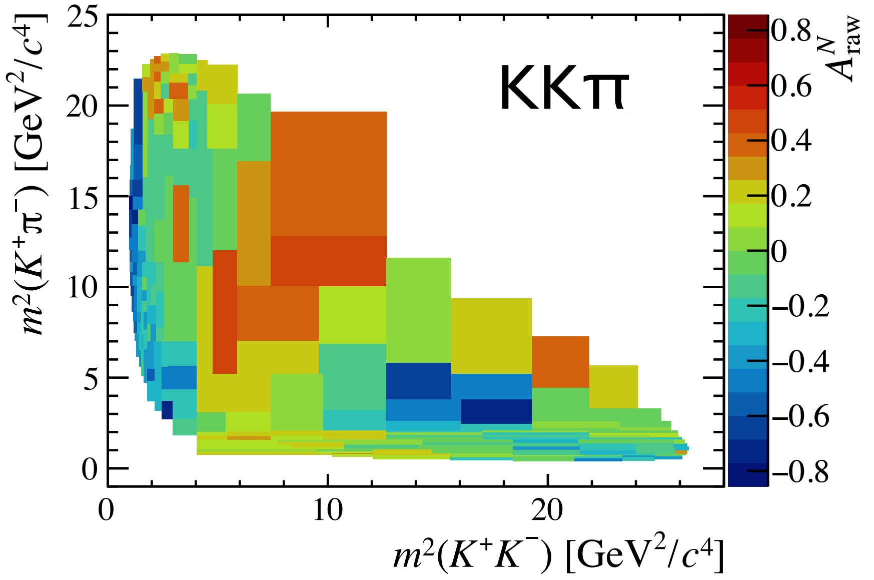
- massive localized Acp

- suggest dynamic effect



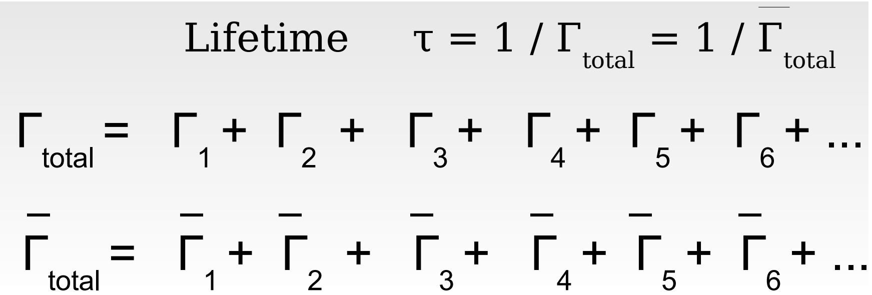






rescattering as a CPV mechanism

- CPT must be preserved



Lifetime $\tau = 1 / \Gamma_{\text{total}} = 1 / \overline{\Gamma}_{\text{total}}$

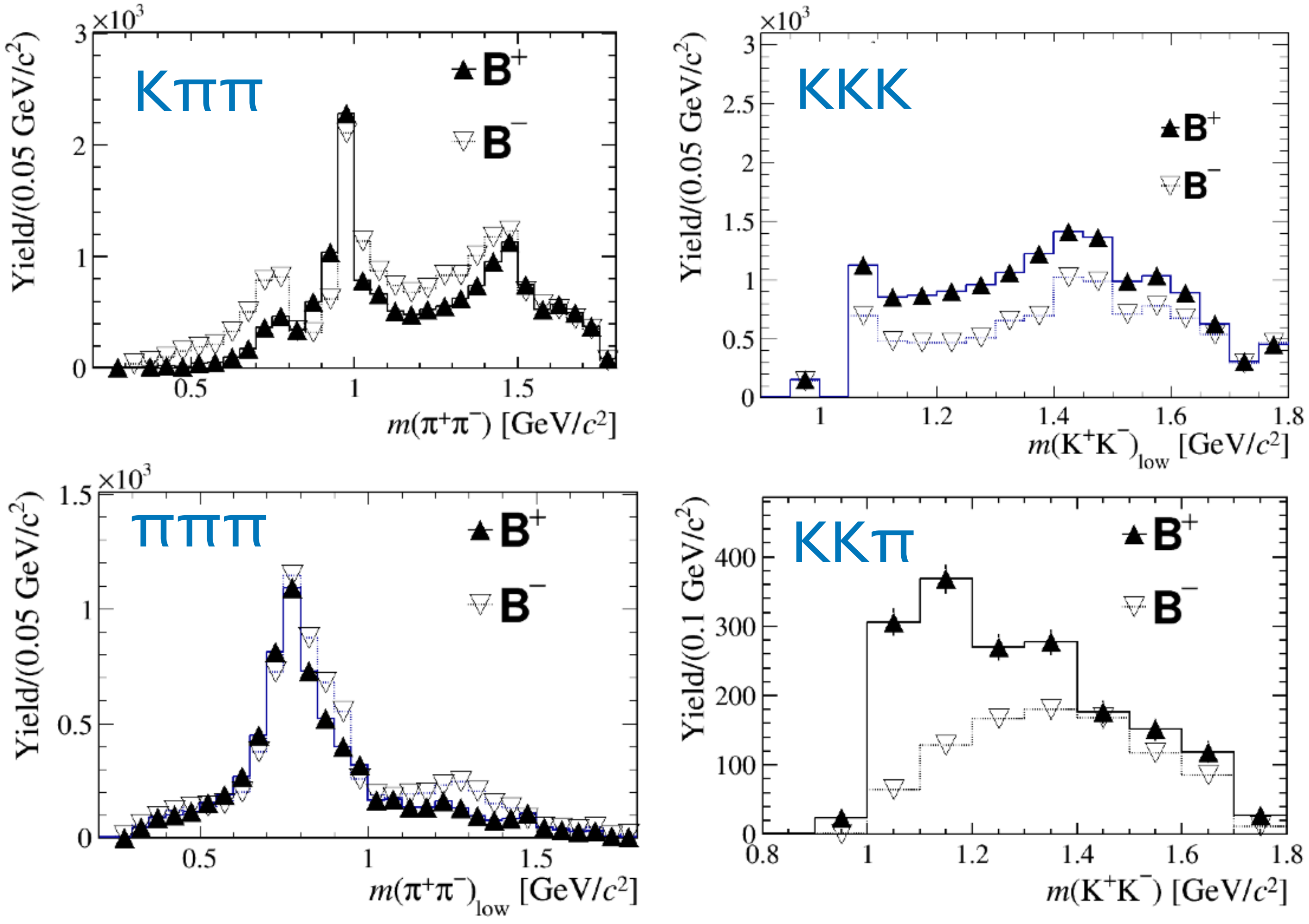
$$\Gamma_{\text{total}} = \Gamma_1 + \Gamma_2 + \Gamma_3 + \Gamma_4 + \Gamma_5 + \Gamma_6 + \ldots$$

$$\overline{\Gamma}_{\text{total}} = \overline{\Gamma}_1 + \overline{\Gamma}_2 + \overline{\Gamma}_3 + \overline{\Gamma}_4 + \overline{\Gamma}_5 + \overline{\Gamma}_6 + \ldots$$

$$\rightarrow \quad \sum \Delta\Gamma_{CP} = 0$$

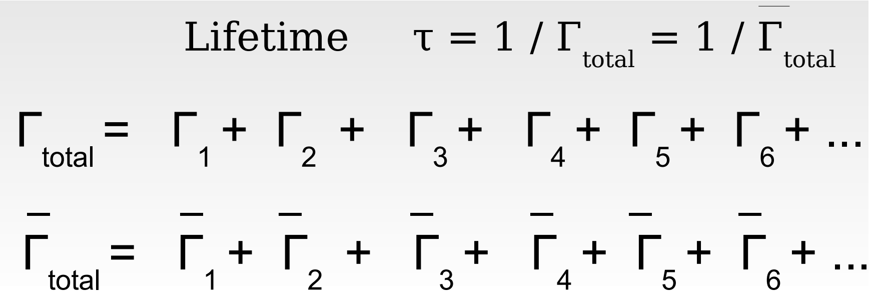
CPV in one channel should be compensated by another, same quantum #, with opposite sign

- LHCb run 1 projections



rescattering as a CPV mechanism

- CPT must be preserved



Lifetime $\tau = 1/\Gamma_{\text{total}} = 1/\overline{\Gamma}_{\text{total}}$

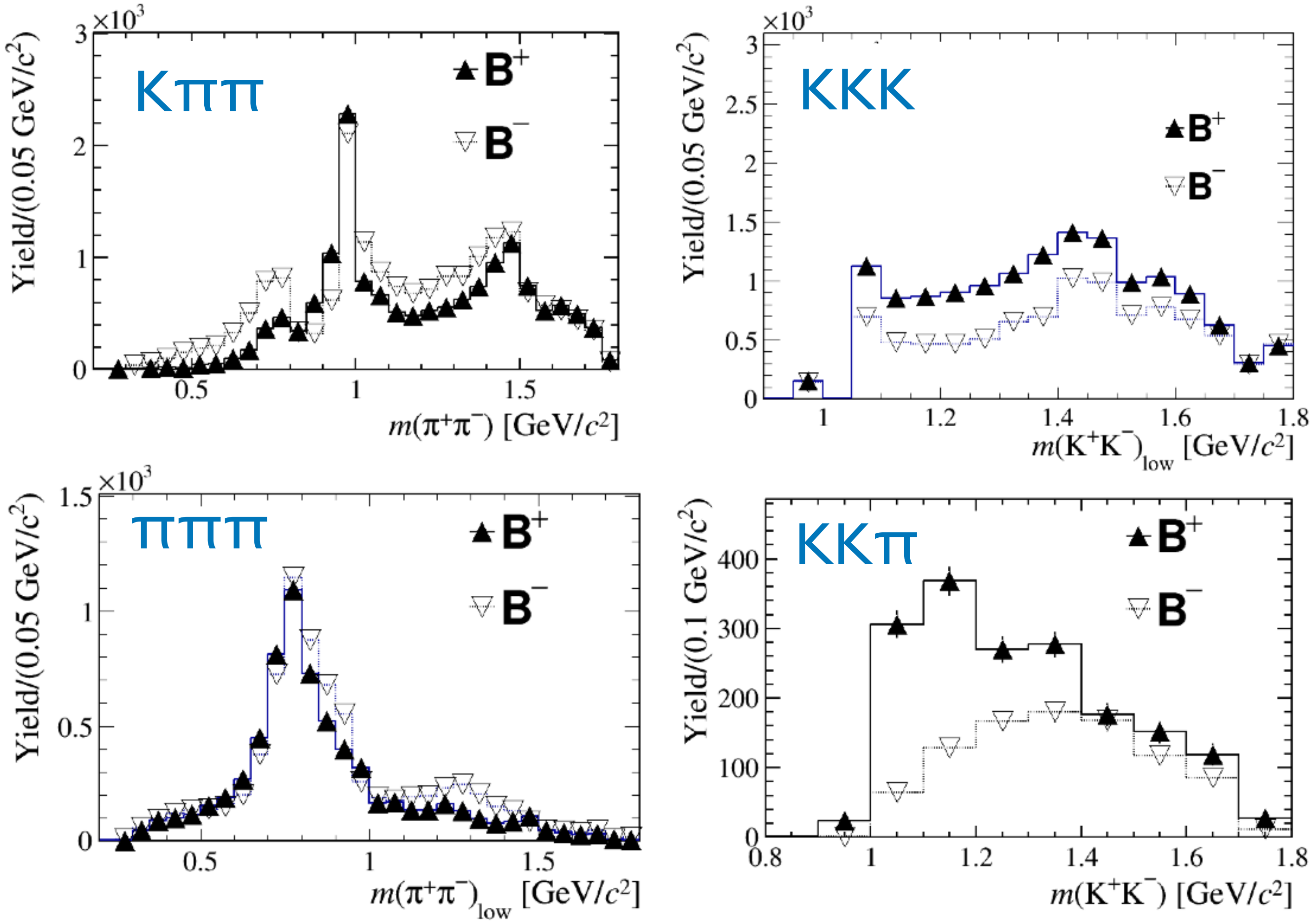
$\Gamma_{\text{total}} = \Gamma_1 + \Gamma_2 + \Gamma_3 + \Gamma_4 + \Gamma_5 + \Gamma_6 + \ldots$

$\overline{\Gamma}_{\text{total}} = \overline{\Gamma}_1 + \overline{\Gamma}_2 + \overline{\Gamma}_3 + \overline{\Gamma}_4 + \overline{\Gamma}_5 + \overline{\Gamma}_6 + \ldots$

→ $\sum \Delta\Gamma_{CP} = 0$

CPV in one channel should be compensated by another, same quantum #, with opposite sign

- LHCb run 1 projections



- rescattering $\boxed{\pi\pi \to KK}$

↳ CPV at [1 -1.6] GeV

Frederico, Bediaga, Lourenço
PRD89(2014)094013

rescattering as a CPV mechanism

- CPT must be preserved

Lifetime $\tau = 1 / \Gamma_{total} = 1 / \bar{\Gamma}_{total}$

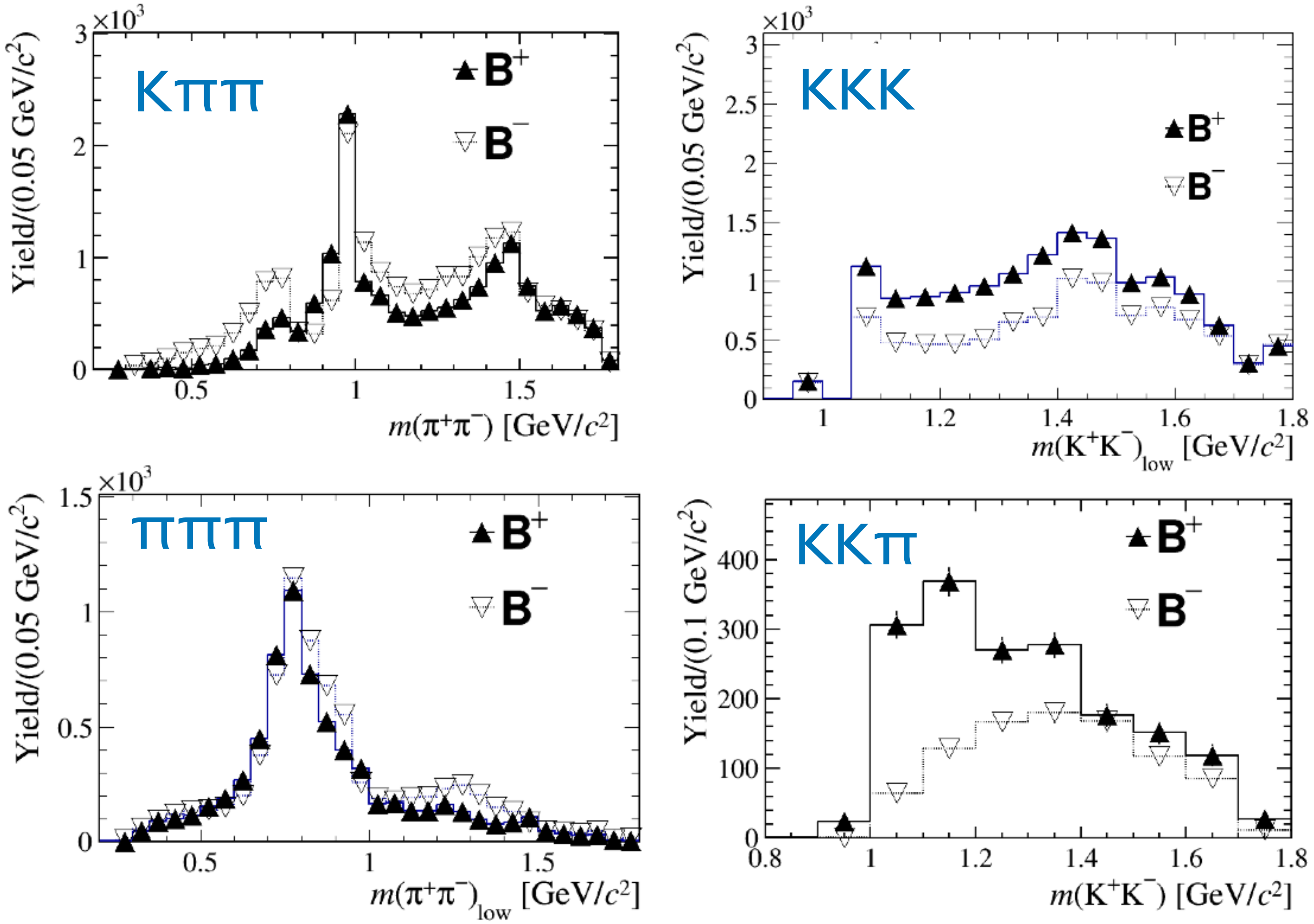
$$\Gamma_{total} = \Gamma_1 + \Gamma_2 + \Gamma_3 + \Gamma_4 + \Gamma_5 + \Gamma_6 + \ldots$$

$$\bar{\Gamma}_{total} = \bar{\Gamma}_1 + \bar{\Gamma}_2 + \bar{\Gamma}_3 + \bar{\Gamma}_4 + \bar{\Gamma}_5 + \bar{\Gamma}_6 + \ldots$$

→ $\sum \Delta\Gamma_{CP} = 0$

CPV in one channel should be compensated by another, same quantum #, with opposite sign

- LHCb run 1 projections



- rescattering $\boxed{\pi\pi \to KK}$

→ CPV at [1 -1.6] GeV

Frederico, Bediaga, Lourenço
PRD89(2014)094013

- implemented in LHCb amplitude analysis:

→ $B^{\pm} \to \pi^- \pi^+ \pi^{\pm}$ PRD101 (2020) 012006; PRL 124 (2020) 031801

→ $B^{\pm} \to \pi^{\pm} K^- K^+$ PRL 123 (2019) 231802

FSI as source of CP asymmetry in D decays

- how to explain the CPV in charm?

$$\Delta A_{CP}^{\mathrm{LHCb}} = A_{cp}(D^0 \to K^+K^-) - A_{cp}(D^0 \to \pi^+\pi^-) = -(1.54 \pm 0.29) \times 10^{-3}$$

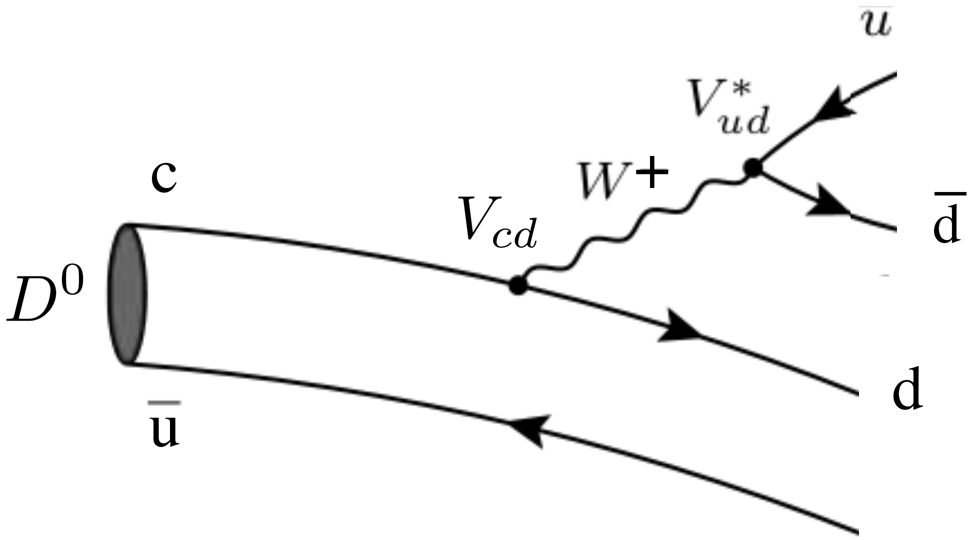
FSI as source of CP asymmetry in D decays

- how to explain the CPV in charm?

$$\Delta A_{CP}^{\text{LHCb}} = A_{cp}(D^0 \to K^+K^-) - A_{cp}(D^0 \to \pi^+\pi^-) \; = -(1.54 \pm 0.29) \times 10^{-3}$$

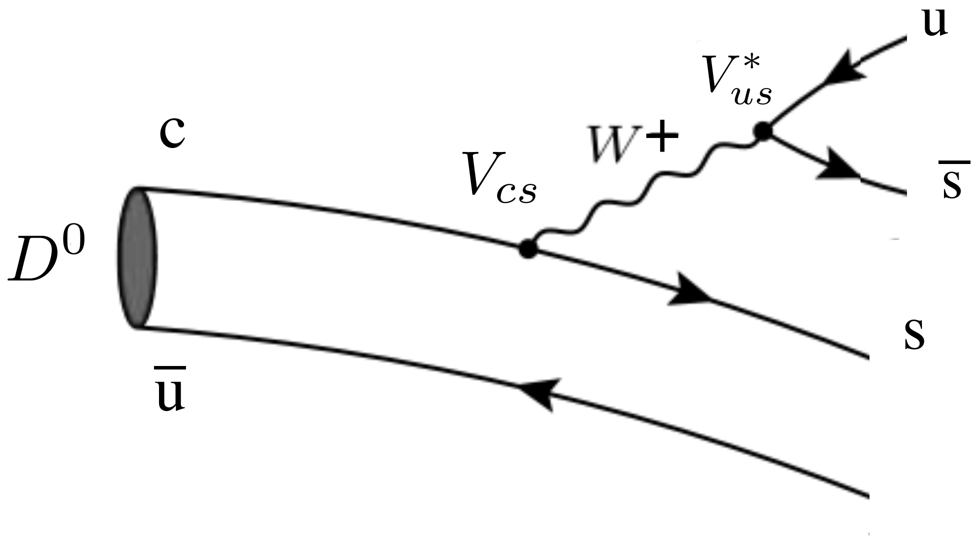
  - single cabibbo suppressed decays

$D^0 \to \pi^+\pi^-$



$V_{cd}V_{ud}^* \approx \lambda(1 - \lambda^4\, e^{i\delta})$

$D^0 \to K^+K^-$



$V_{cs}V_{us}^* \approx \lambda(1 - \lambda^2)$

  - weak phase in KK is 20 times smaller Lenz ,Wilkinson, Annu. Rev. Nucl. Part. Sci. 71, 59 (2021)

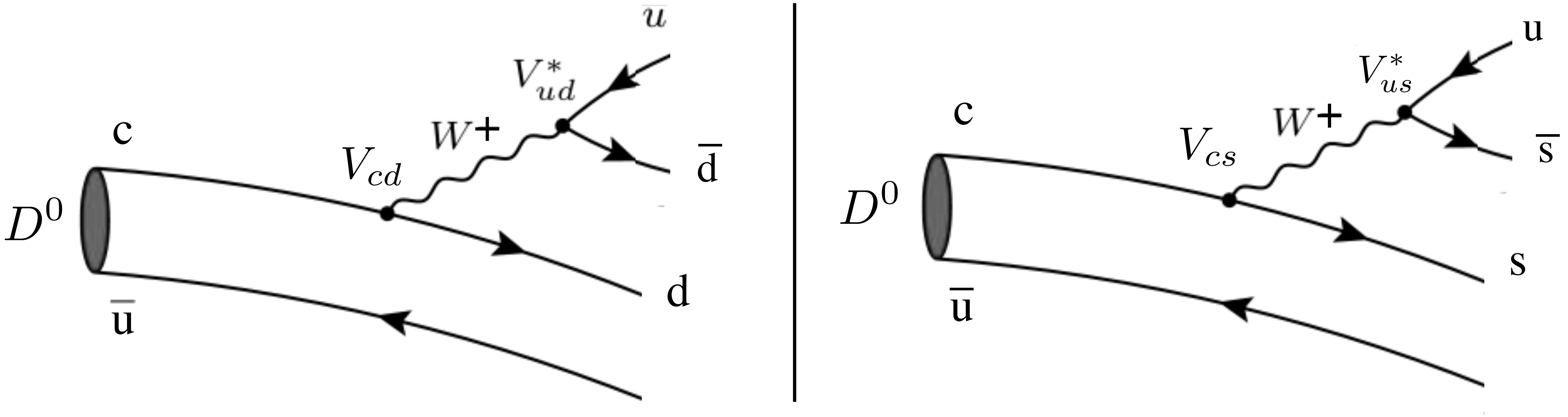
FSI as source of CP asymmetry in D decays

- how to explain the CPV in charm?

$$\Delta A_{CP}^{\rm LHCb} = A_{cp}(D^0 \to K^+K^-) - A_{cp}(D^0 \to \pi^+\pi^-) = -(1.54 \pm 0.29) \times 10^{-3}$$

  - single cabibbo suppressed decays

$D^0 \to \pi^+\pi^-$ $D^0 \to K^+K^-$



$V_{cd}V_{ud}^* \approx \lambda(1-\lambda^4\, e^{i\delta})$ $V_{cs}V_{us}^* \approx \lambda(1-\lambda^2)$

  - weak phase in KK is 20 times smaller

Lenz ,Wilkinson, Annu. Rev. Nucl. Part. Sci. 71, 59 (2021)

→ what about strong phases if not from penguin? hadronic FSI

Grossman, Schacht JHEP07 20 (2019); Schacht, Soni PLB825 136855 (2022).

Theoretical approaches to CPV on charm

QCD short-distance

- QCDF → how to calculate penguin contributions? call BSM effects

Chala, Lenz, Rusov, Scholtz, JHEP 07, 161 (2019).

- LCSR → QCD, model independent but predictions are 1 order magnitude bellow

Khodjamirian, Petrov, Phys. Lett. B 774, 235 (2017)

long distance effect:

- topological and group symmetry approach
  - with SU(3) breaking through FSI (fit agrees)
  - with resonances (fit agrees)

H.-Y. Cheng and C.-W. Chiang, PRD 100, 093002 (2019).
F. Buccella, A. Paul and P. Santorelli, PRD 99, 113001 (2019)
Franco, Mishima, Silvestrini JHEP05, 140 (2012)

Schacht and A. Soni, Phys. Lett. B 825, 136855 (2022).
Y. Grossman and S. Schacht, JHEP 07, 20 (2019)

- FSI with CPT (prediction agrees)

bediaga, Frederico, PCM
arxiv 2203.04056v2

FSI to enhance CPV

- D and $\bar{D}$ can decay to $\pi\pi$ and KK

arxiv 2203.04056v2

Bediaga, Frederico, PCM

$$\begin{tikzcd}
D^0 \arrow[rr, bend left, "V_{cs}V_{us}^*"] \arrow[r, bend right, "V_{cd}V_{ud}^*"'] & \pi^+\pi^- \arrow[r, bend right, "\delta_{\pi\pi\to KK}"'] & K^+K^-
\end{tikzcd}$$

→ known from 80's experiment

FSI to enhance CPV

arxiv 2203.04056v2

Bediaga, Frederico, PCM

- D and $\bar{D}$ can decay to $\pi\pi$ and KK

$$\begin{tikzcd} D^0 \arrow[r, bend right, "V_{cd}V^*_{ud}"'] \arrow[rr, bend left, "V_{cs}V^*_{us}"] & \pi^+\pi^- \arrow[r, bend right, "\delta_{\pi\pi\to KK}"'] & K^+K^- \end{tikzcd}$$

→ known from 80's experiment

- build amplitudes decays implying three constraints:
  - CPT invariance relates channels with same quantum numbers

$$\rightarrow \quad \sum \Delta\Gamma_{CP} = 0$$

  - Watson theorem relates the strong phase from the rescattering process to the decay amplitudes
  - the unitarity of the strong S-matrix.

FSI and CPT

- FSI in $D^0 \to \pi^+\pi^-$ and $D^0 \to K^+K^-$ can include multiple mesons
  - general S-matrix can mix many FSI states $S = \begin{pmatrix} S_{2M,2M} & S_{2M,3M} & S_{2M,4M} & \cdots \\ S_{3M,2M} & S_{3M,3M} & S_{3M,4M} & \cdots \\ S_{4M,2M} & S_{4M,3M} & S_{4M,4M} & \cdots \\ \cdots & \cdots & \cdots & \cdots \end{pmatrix}$

- assume only 2 coupled-channels to FSI: $\pi\pi$, $K\bar{K}$

$$\rightarrow \quad S_{2M,2M} = \begin{pmatrix} S_{\pi\pi,\pi\pi} & S_{\pi\pi,KK} \\ S_{KK,\pi\pi} & S_{KK,KK} \end{pmatrix}$$

$$\boxed{\begin{array}{l} S_{\pi\pi,\pi\pi} = \eta\, \mathrm{e}^{2i\delta_{\pi\pi}} \quad S_{KK,KK} = \eta\, \mathrm{e}^{2i\delta_{KK}} \\ S_{\pi\pi,KK} = S_{KK,\pi\pi} = \imath\sqrt{1-\eta^2}\, \mathrm{e}^{\imath(\delta_{\pi\pi}+\delta_{KK})} \end{array}}$$

FSI and CPT

- FSI in $D^0 \to \pi^+\pi^-$ and $D^0 \to K^+K^-$ can include multiple mesons
  - general S-matrix can mix many FSI states $S = \begin{pmatrix} S_{2M,2M} & S_{2M,3M} & S_{2M,4M} & \cdots \\ S_{3M,2M} & S_{3M,3M} & S_{3M,4M} & \cdots \\ S_{4M,2M} & S_{4M,3M} & S_{4M,4M} & \cdots \\ \cdots & \cdots & \cdots & \cdots \end{pmatrix}$

- assume only 2 coupled-channels to FSI: $\pi\pi$, $K\bar{K}$

→ $S_{2M,2M} = \begin{pmatrix} S_{\pi\pi,\pi\pi} & S_{\pi\pi,KK} \\ S_{KK,\pi\pi} & S_{KK,KK} \end{pmatrix}$

$$\boxed{\begin{array}{lclcl} S_{\pi\pi,\pi\pi} & = & \eta\, e^{2i\delta_{\pi\pi}} & S_{KK,KK} & = \eta\, e^{2i\delta_{KK}} \\ S_{\pi\pi,KK} & = & S_{KK,\pi\pi} & = & \imath\sqrt{1-\eta^2}\, e^{\imath(\delta_{\pi\pi}+\delta_{KK})} \end{array}}$$

  - two pions cannot go to three pions due to G-parity
  - ignore four pion coupling to the 2M channel based on 1/Nc counting
  - ignore $\eta\eta$ channel once their coupling to the $\pi\pi$ channel are suppressed with respect to $K\bar{K}$.

FSI and CPT

- FSI in $D^0 \to \pi^+\pi^-$ and $D^0 \to K^+K^-$ can include multiple mesons
  - general S-matrix can mix many FSI states $S = \begin{pmatrix} S_{2M,2M} & S_{2M,3M} & S_{2M,4M} & \cdots \\ S_{3M,2M} & S_{3M,3M} & S_{3M,4M} & \cdots \\ S_{4M,2M} & S_{4M,3M} & S_{4M,4M} & \cdots \\ \cdots & \cdots & \cdots & \cdots \end{pmatrix}$

- assume only 2 coupled-channels to FSI: $\pi\pi$, $K\bar{K}$

$$\rightarrow \quad S_{2M,2M} = \begin{pmatrix} S_{\pi\pi,\pi\pi} & S_{\pi\pi,KK} \\ S_{KK,\pi\pi} & S_{KK,KK} \end{pmatrix}$$

$$\boxed{\begin{array}{l} S_{\pi\pi,\pi\pi} = \eta\, e^{2i\delta_{\pi\pi}} \qquad S_{KK,KK} = \eta\, e^{2i\delta_{KK}} \\ S_{\pi\pi,KK} = S_{KK,\pi\pi} = \imath\sqrt{1-\eta^2}\, e^{\imath(\delta_{\pi\pi}+\delta_{KK})} \end{array}}$$

  - two pions cannot go to three pions due to G-parity
  - ignore four pion coupling to the 2M channel based on 1/Nc counting
  - ignore $\eta\eta$ channel once their coupling to the $\pi\pi$ channel are suppressed with respect to $K\bar{K}$.

- CPT constraint restricted to the two-channels: $\sum_{f=(\pi\pi,KK)} (|\mathcal{A}_{D^0\to f}|^2 - |\mathcal{A}_{\bar{D}^0\to f}|^2) = 0$

Decay amplitudes

- dressing the weak tree topology with FSI
  - → penguin are suppressed

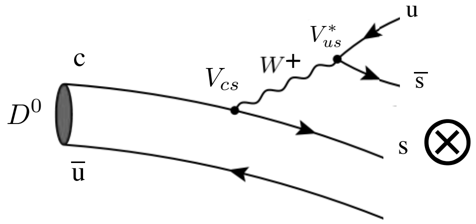
$$\begin{tikzcd}
D^0 \arrow[r, bend right, "V_{cd}V_{ud}^*"'] \arrow[rr, bend left, "V_{cs}V_{us}^*"] & \pi^+\pi^- \arrow[r, bend right, "\delta_{\pi\pi\to KK}"'] & K^+K^-
\end{tikzcd}$$

Decay amplitudes

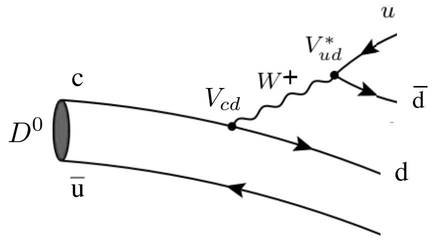
- dressing the weak tree topology with FSI
  - → penguin are suppressed

$$\begin{tikzcd} D^0 \arrow[rr, bend left, "V_{cs}V_{us}^*"] \arrow[r, bend right, "V_{cd}V_{ud}^*"'] & \pi^+\pi^- \arrow[r, bend right, "\delta_{\pi\pi\to KK}"'] & K^+K^- \end{tikzcd}$$

- $D^0 \to KK$



$\otimes$ **$KK \to KK$** +

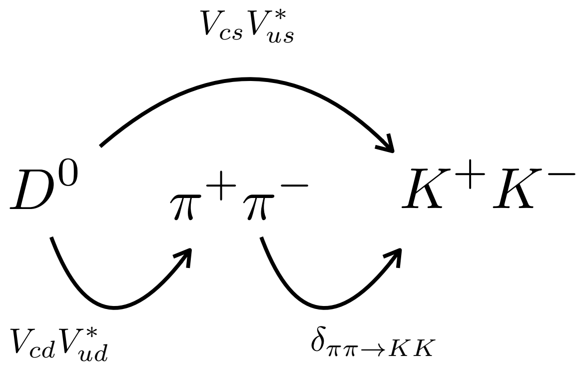


$\otimes$ **$\pi\pi \to KK$**

→ $$\mathcal{A}_{D^0\to KK} = \eta\, e^{2i\delta_{KK}}\, V_{cs}^* V_{us}\, a_{KK} + i\sqrt{1-\eta^2}\, e^{i(\delta_{\pi\pi}+\delta_{KK})}\, V_{cd}^* V_{ud}\, a_{\pi\pi}$$

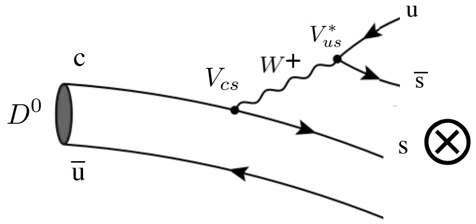
→ $\mathcal{A}_{\bar{D}^0\to f}$ same with CKM cc.

Decay amplitudes

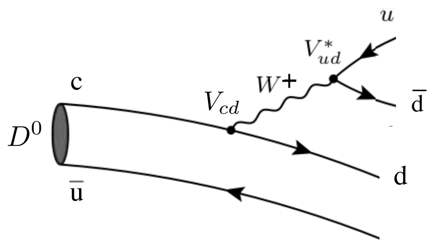


- dressing the weak tree topology with FSI
  - → penguin are suppressed

- $D^0 \to KK$



$\otimes$ $KK \to KK$ +

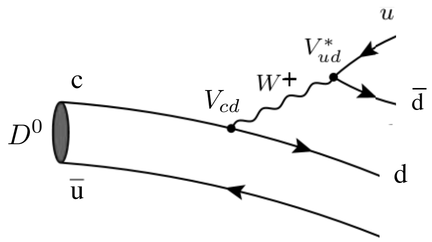


$\otimes$ $\pi\pi \to KK$

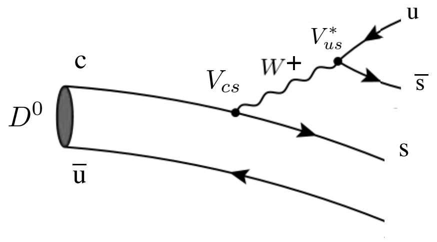
→ $$\mathcal{A}_{D^0\to KK} = \eta\, \mathrm{e}^{2i\delta_{KK}}\, V^*_{cs}V_{us}\, a_{KK} + i\sqrt{1-\eta^2}\, \mathrm{e}^{i(\delta_{\pi\pi}+\delta_{KK})}\, V^*_{cd}V_{ud}\, a_{\pi\pi}$$

→ $\mathcal{A}_{\bar{D}^0\to f}$ same with CKM cc.

- $D^0 \to \pi\pi$



$\otimes$ $\pi\pi \to \pi\pi$ +



$\otimes$ $KK \to \pi\pi$

→ $$\mathcal{A}_{D^0\to \pi\pi} = \eta\, \mathrm{e}^{2i\delta_{\pi\pi}}\, V^*_{cd}V_{ud}\, a_{\pi\pi} + i\sqrt{1-\eta^2}\, \mathrm{e}^{i(\delta_{\pi\pi}+\delta_{KK})}\, V^*_{cs}V_{us}\, a_{KK}$$

- a_{KK} and $a_{\pi\pi}$ do not carry any strong phases (real)

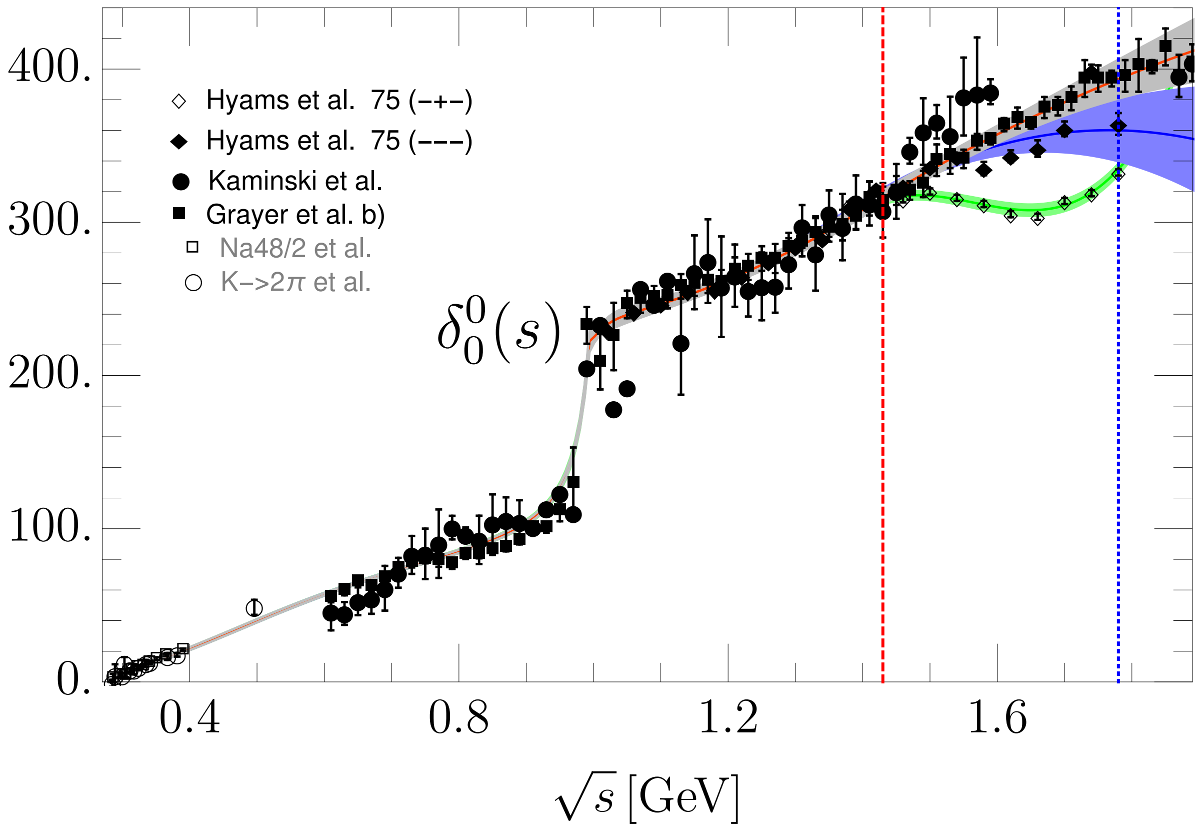
Watson theorem

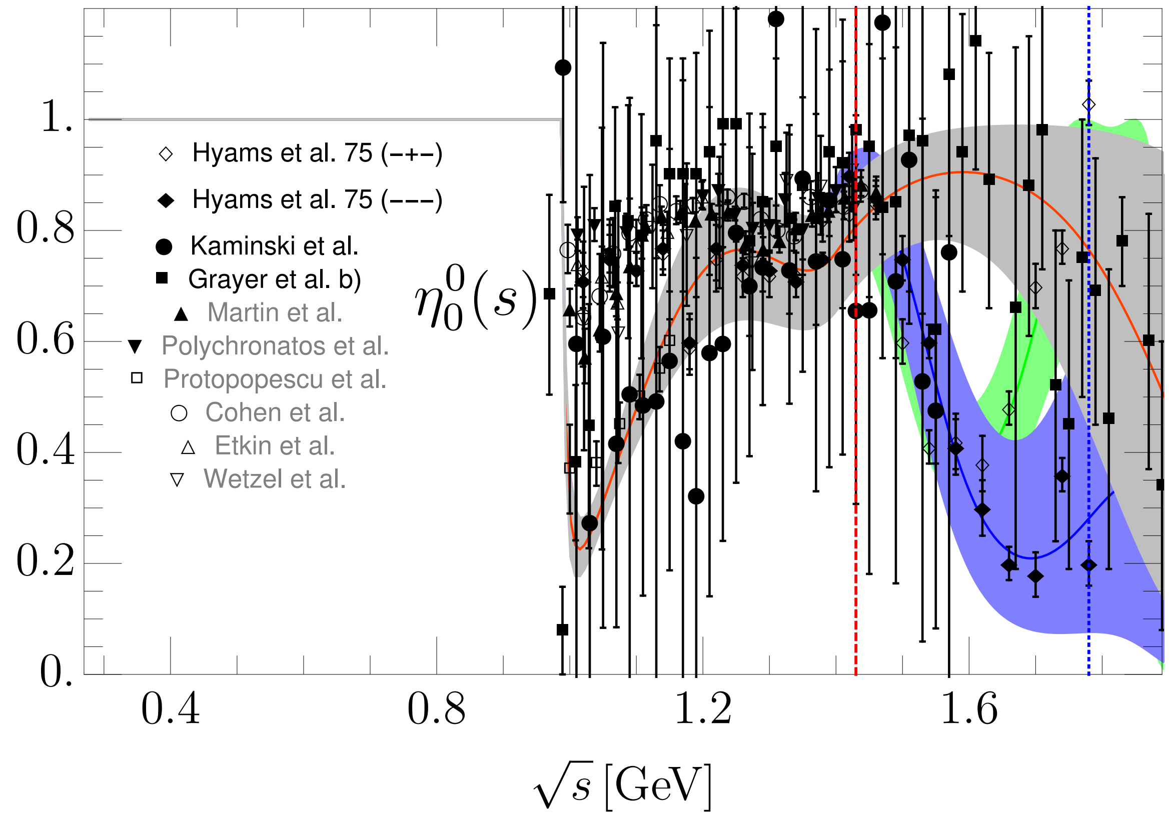
- $\delta_{\pi\pi}$, δ_{KK} and $\delta_{\pi\pi\to KK}$ are the same independent of the initial process

→ we can use CERN-Munich data from 80's

Longacre et al., Phys. Lett. B 177, 223 (1986)
Hyams et al., Nucl. Phys. B 100, 205 (1975)
Ochs, J. Phys. G 40, 043001 (2013)

- $\pi\pi \to \pi\pi$





Pelaez, Rodas, Elvira *Eur.Phys.J.C* 79 (2019) 12, 1008

amplitude $\hat{f}_l(s) = \left[\frac{\eta_l e^{2i\delta_l} - 1}{2i}\right].$

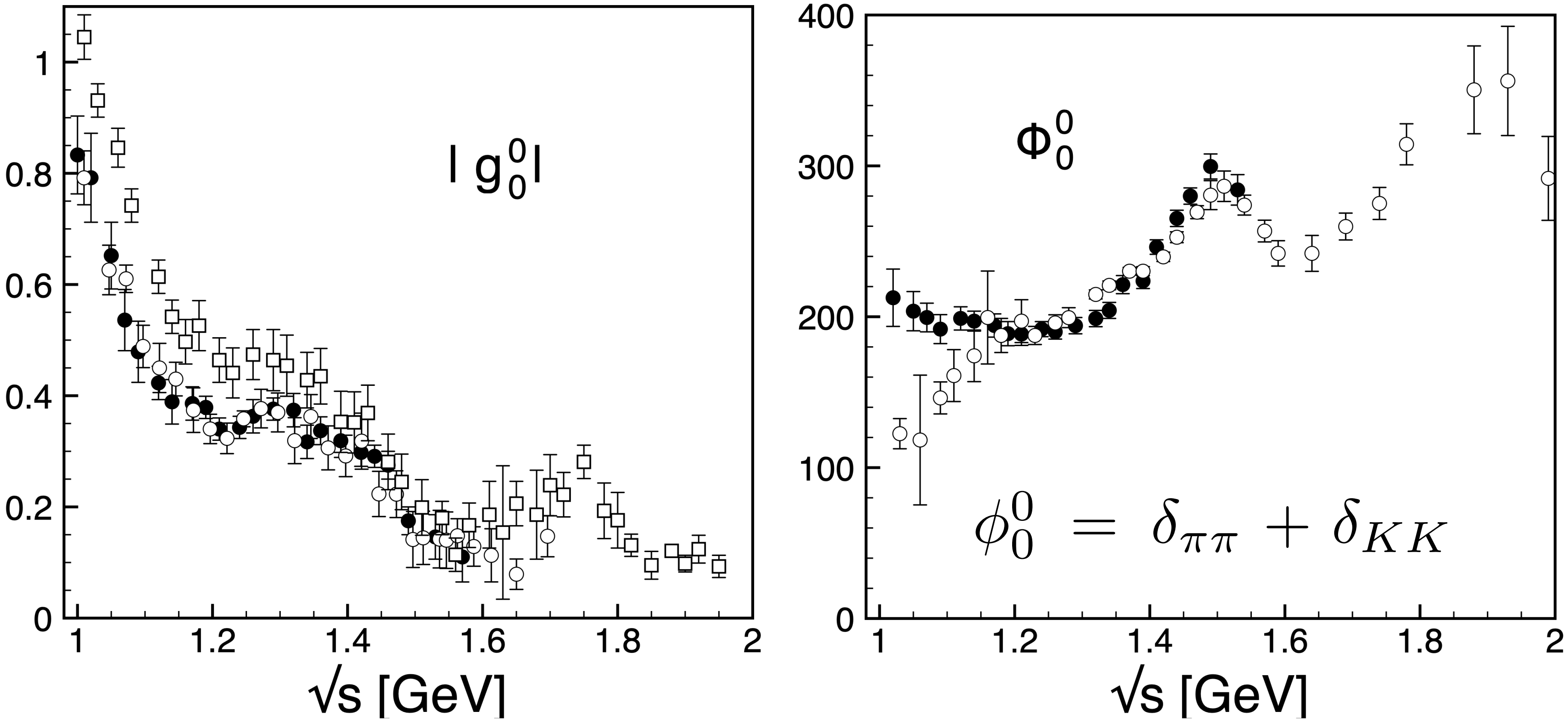
→ elasticity drops dramatically near $K\bar{K}$ → strongly couple

Watson theorem

- $\pi\pi \to KK$

$$\rightarrow \quad S_{\pi\pi,KK}(s) \;=\; \imath\sqrt{1-\eta^2}\,\mathrm{e}^{\imath(\delta_{\pi\pi}+\delta_{KK})} = i\,4\sqrt{\frac{q_\pi q_K}{s}}|g_0^0(s)|\,e^{i\phi_0^0(s)}\,\Theta(s-4m_K^2)$$

Pelaez and Rodas, Eur. Phys. J. C 78, 897 (2018)



Cohen et al., Phys. Rev. D 22, 2595 (1980)
Etkin et al., Phys. Rev. D 25, 1786 (1982)

- Pelaez parametrization @ M_D^2:

$$|g_0^0(M_D^2)| \approx 0.125 \pm 0.025 \quad \rightarrow \quad \sqrt{1-\eta^2} \;\approx\; 0.229 \pm 0.046 \quad \rightarrow \quad \eta \;\approx\; 0.973$$

$$\phi_0^0 = \delta_{\pi\pi} + \delta_{KK} \approx 343^{\mathrm{o}} \pm 8^{\mathrm{o}}$$

Partial decay widths

- $\Delta\Gamma_f = \Gamma\left(D^0 \to f\right) - \Gamma(\bar{D}^0 \to f)$

$$\mathcal{A}_{D^0\to\pi\pi} = \eta\, \mathrm{e}^{2i\delta_{\pi\pi}}\, V_{cd}^* V_{ud}\, a_{\pi\pi} + i\sqrt{1-\eta^2}\, \mathrm{e}^{i(\delta_{\pi\pi}+\delta_{KK})}\, V_{cs}^* V_{us}\, a_{KK}$$

$$\mathcal{A}_{D^0\to KK} = \eta\, \mathrm{e}^{2i\delta_{KK}}\, V_{cs}^* V_{us}\, a_{KK} + i\sqrt{1-\eta^2}\, \mathrm{e}^{i(\delta_{\pi\pi}+\delta_{KK})}\, V_{cd}^* V_{ud}\, a_{\pi\pi}$$

Partial decay widths

- $\Delta\Gamma_f = \Gamma\left(D^0 \to f\right) - \Gamma(\bar{D}^0 \to f)$

$$\mathcal{A}_{D^0\to\pi\pi} = \eta\, \mathrm{e}^{2i\delta_{\pi\pi}}\, V_{cd}^* V_{ud}\, a_{\pi\pi} + i\sqrt{1-\eta^2}\, \mathrm{e}^{i(\delta_{\pi\pi}+\delta_{KK})}\, V_{cs}^* V_{us}\, a_{KK}$$

$$\mathcal{A}_{D^0\to KK} = \eta\, \mathrm{e}^{2i\delta_{KK}}\, V_{cs}^* V_{us}\, a_{KK} + i\sqrt{1-\eta^2}\, \mathrm{e}^{i(\delta_{\pi\pi}+\delta_{KK})}\, V_{cd}^* V_{ud}\, a_{\pi\pi}$$

→ $\Delta\Gamma_{\pi\pi} = -\Delta\Gamma_{KK} = 4\,\mathrm{Im}[V_{cs}V_{us}^* V_{cd}^* V_{ud}]\, a_{\pi\pi}\, a_{KK}\, \eta\sqrt{1-\eta^2}\cos\phi$

  - $\phi = \delta_{KK} - \delta_{\pi\pi}$
  - the sign of $\Delta\Gamma_f$ is determined by the CKM elements and the S-wave phase-shifts

Partial decay widths

- $\Delta\Gamma_f = \Gamma\left(D^0 \to f\right) - \Gamma(\bar{D}^0 \to f)$

$$\mathcal{A}_{D^0\to\pi\pi} = \eta\, \mathrm{e}^{2i\delta_{\pi\pi}}\, V_{cd}^* V_{ud}\, a_{\pi\pi} + i\sqrt{1-\eta^2}\, \mathrm{e}^{i(\delta_{\pi\pi}+\delta_{KK})}\, V_{cs}^* V_{us}\, a_{KK}$$

$$\mathcal{A}_{D^0\to KK} = \eta\, \mathrm{e}^{2i\delta_{KK}}\, V_{cs}^* V_{us}\, a_{KK} + i\sqrt{1-\eta^2}\, \mathrm{e}^{i(\delta_{\pi\pi}+\delta_{KK})}\, V_{cd}^* V_{ud}\, a_{\pi\pi}$$

→ $$\Delta\Gamma_{\pi\pi} = -\Delta\Gamma_{KK} = 4\,\mathrm{Im}[V_{cs} V_{us}^* V_{cd}^* V_{ud}]\, \underline{a_{\pi\pi}\, a_{KK}}\, \eta\sqrt{1-\eta^2}\cos\phi$$

- $\phi = \delta_{KK} - \delta_{\pi\pi}$
- the sign of $\Delta\Gamma_f$ is determined by the CKM elements and the S-wave phase-shifts
- need to quantify $a_{\pi\pi}$ and a_{KK}:

at D^0 mass $\sqrt{1-\eta^2} << 1$ →

$$\Gamma_{\pi\pi} \approx \eta^2 |V_{cd}^* V_{ud}|^2\, a_{\pi\pi}^2$$
$$\Gamma_{KK} \approx \eta^2 |V_{cs}^* V_{us}|^2 a_{KK}^2$$

$Br[D \to f] = \Gamma_f / \Gamma_{total}$

we can use experimental input

Partial decay widths

- $\Delta\Gamma_f = \Gamma\left(D^0 \to f\right) - \Gamma(\bar{D}^0 \to f)$

$$\mathcal{A}_{D^0\to\pi\pi} = \eta\, \mathrm{e}^{2i\delta_{\pi\pi}}\, V_{cd}^* V_{ud}\, a_{\pi\pi} + i\sqrt{1-\eta^2}\, \mathrm{e}^{i(\delta_{\pi\pi}+\delta_{KK})}\, V_{cs}^* V_{us}\, a_{KK}$$

$$\mathcal{A}_{D^0\to KK} = \eta\, \mathrm{e}^{2i\delta_{KK}}\, V_{cs}^* V_{us}\, a_{KK} + i\sqrt{1-\eta^2}\, \mathrm{e}^{i(\delta_{\pi\pi}+\delta_{KK})}\, V_{cd}^* V_{ud}\, a_{\pi\pi}$$

→ $\Delta\Gamma_{\pi\pi} = -\Delta\Gamma_{KK} = 4\,\mathrm{Im}[V_{cs} V_{us}^* V_{cd}^* V_{ud}]\, \underline{a_{\pi\pi}\, a_{KK}}\, \eta\sqrt{1-\eta^2}\cos\phi$

  - $\phi = \delta_{KK} - \delta_{\pi\pi}$
  - the sign of $\Delta\Gamma_f$ is determined by the CKM elements and the S-wave phase-shifts
  - need to quantify $a_{\pi\pi}$ and a_{KK}:

at D^0 mass $\sqrt{1-\eta^2} << 1$ → $\Gamma_{\pi\pi} \approx \eta^2 |V_{cd}^* V_{ud}|^2\, a_{\pi\pi}^2$, $\Gamma_{KK} \approx \eta^2 |V_{cs}^* V_{us}|^2 a_{KK}^2$

$Br[D \to f] = \Gamma_f / \Gamma_{total}$

we can use experimental input

- $A_{CP}(f) = \dfrac{\Gamma\left(D^0 \to f\right) - \Gamma(\bar{D}^0 \to f)}{\Gamma\left(D^0 \to f\right) + \Gamma(\bar{D}^0 \to f)}$ $= \Delta\Gamma_f / 2\Gamma_f$

Final values for A_{CP}

- $$A_{CP}(f) \approx \pm 2\,\frac{\mathrm{Im}[V_{cs}V_{us}^{*}V_{cd}^{*}V_{ud}]}{|V_{cs}V_{us}^{*}V_{cd}^{*}V_{ud}|}\,\eta^{-1}\sqrt{1-\eta^{2}}\,\cos\phi\left[\frac{\mathrm{Br}(D^{0}\to K^{+}K^{-})}{\mathrm{Br}(D^{0}\to\pi^{+}\pi^{-})}\right]^{\pm\frac{1}{2}}$$

  $+\ \pi\pi$
  $-\ KK$

  - $\mathrm{Br}(D^{0}\to\pi^{+}\pi^{-}) = (1.455\pm 0.024)\times 10^{-3}$
    $\mathrm{Br}(D^{0}\to K^{+}K^{-}) = (4.08\pm 0.06)\times 10^{-3}$ PDG

  - $$\frac{\mathrm{Im}[V_{cs}V_{us}^{*}V_{cd}^{*}V_{ud}]}{|V_{cs}V_{us}^{*}V_{cd}^{*}V_{ud}|} = (6.02\pm 0.32)\times 10^{-4}$$ PDG

  - $\cos\phi$: $\phi = \delta_{KK} - \delta_{\pi\pi} \quad = (\delta_{KK} + \delta_{\pi\pi}) - 2\delta_{\pi\pi}$

Final values for A_{CP}

- $$A_{CP}(f) \approx \pm 2 \frac{\mathrm{Im}[V_{cs}V_{us}^{*}V_{cd}^{*}V_{ud}]}{|V_{cs}V_{us}^{*}V_{cd}^{*}V_{ud}|} \, \eta^{-1} \sqrt{1-\eta^{2}} \, \cos\phi \left[\frac{\mathrm{Br}(D^{0} \to K^{+}K^{-})}{\mathrm{Br}(D^{0} \to \pi^{+}\pi^{-})} \right]^{\pm\frac{1}{2}}$$

$+ \ \pi\pi$

$- \ KK$

  - $\mathrm{Br}(D^{0} \to \pi^{+}\pi^{-}) = (1.455 \pm 0.024) \times 10^{-3}$ PDG
    $\mathrm{Br}(D^{0} \to K^{+}K^{-}) = (4.08 \pm 0.06) \times 10^{-3}$
  - $\frac{\mathrm{Im}[V_{cs}V_{us}^{*}V_{cd}^{*}V_{ud}]}{|V_{cs}V_{us}^{*}V_{cd}^{*}V_{ud}|} = (6.02 \pm 0.32) \times 10^{-4}$ PDG
  - $\cos\phi$: $\phi = \delta_{KK} - \delta_{\pi\pi} = \underbrace{(\delta_{KK} + \delta_{\pi\pi})}_{\nearrow \ \phi_0^0} - 2\delta_{\pi\pi}$

Final values for A_{CP}

- $$A_{CP}(f) \approx \pm 2 \frac{\mathrm{Im}[V_{cs}V_{us}^{*}V_{cd}^{*}V_{ud}]}{|V_{cs}V_{us}^{*}V_{cd}^{*}V_{ud}|}\ \eta^{-1}\sqrt{1-\eta^{2}}\ \cos\phi \left[\frac{\mathrm{Br}(D^{0}\to K^{+}K^{-})}{\mathrm{Br}(D^{0}\to \pi^{+}\pi^{-})}\right]^{\pm\frac{1}{2}}$$

$+\ \pi\pi$
$-\ KK$

  - $\mathrm{Br}(D^{0}\to \pi^{+}\pi^{-}) = (1.455 \pm 0.024)\times 10^{-3}$ PDG
    $\mathrm{Br}(D^{0}\to K^{+}K^{-}) = (4.08 \pm 0.06)\times 10^{-3}$

  - $\dfrac{\mathrm{Im}[V_{cs}V_{us}^{*}V_{cd}^{*}V_{ud}]}{|V_{cs}V_{us}^{*}V_{cd}^{*}V_{ud}|} = (6.02 \pm 0.32)\times 10^{-4}$ PDG

ϕ_0^0

  - $\cos\phi$: $\phi = \delta_{KK} - \delta_{\pi\pi} = (\delta_{KK} + \delta_{\pi\pi}) - 2\delta_{\pi\pi}$

from $\pi\pi$ and $\pi\pi \to KK$ data: $\cos\phi = 0.99 \pm 0.18$

Final values for A_{CP}

- $$A_{CP}(f) \approx \pm 2 \frac{\mathrm{Im}[V_{cs}V_{us}^{*}V_{cd}^{*}V_{ud}]}{|V_{cs}V_{us}^{*}V_{cd}^{*}V_{ud}|} \, \eta^{-1}\sqrt{1-\eta^{2}} \, \cos\phi \left[\frac{\mathrm{Br}(D^{0}\to K^{+}K^{-})}{\mathrm{Br}(D^{0}\to \pi^{+}\pi^{-})} \right]^{\pm\frac{1}{2}}$$

$+$ $\pi\pi$

$-$ KK

  - $\mathrm{Br}(D^{0}\to \pi^{+}\pi^{-}) = (1.455 \pm 0.024)\times 10^{-3}$
    $\mathrm{Br}(D^{0}\to K^{+}K^{-}) = (4.08 \pm 0.06)\times 10^{-3}$ PDG

  - $$\frac{\mathrm{Im}[V_{cs}V_{us}^{*}V_{cd}^{*}V_{ud}]}{|V_{cs}V_{us}^{*}V_{cd}^{*}V_{ud}|} = (6.02 \pm 0.32)\times 10^{-4}$$ PDG

  - $\cos\phi$: $\phi = \delta_{KK} - \delta_{\pi\pi} = (\delta_{KK} + \delta_{\pi\pi}) - 2\delta_{\pi\pi}$

    ϕ_0^0

from $\pi\pi$ and $\pi\pi \to KK$ data: $\cos\phi = 0.99 \pm 0.18$

- $$\boxed{\begin{aligned} A_{CP}(\pi\pi) &= (1.99 \pm 0.37)\times 10^{-3}\sqrt{\eta^{-2}-1} \\ A_{CP}(KK) &= -(0.71 \pm 0.13)\times 10^{-3}\sqrt{\eta^{-2}-1} \end{aligned}}$$

as a function of inelasticity

Predictions for ΔA_{CP}

- $\Delta A_{CP}^{th} = -(2.70 \pm 0.50) \times 10^{-3} \sqrt{\eta^{-2} - 1}$.

Predictions for ΔA_{CP}

$$\boxed{\Delta A_{CP}^{\mathrm{LHCb}} = -(1.54 \pm 0.29) \times 10^{-3}}$$

- $\Delta A_{CP}^{th} = -(2.70 \pm 0.50) \times 10^{-3} \sqrt{\eta^{-2} - 1}$

Predictions for ΔA_{CP}

$$\boxed{\Delta A_{CP}^{\mathrm{LHCb}} = -(1.54 \pm 0.29) \times 10^{-3}}$$

- $\Delta A_{CP}^{th} = -(2.70 \pm 0.50) \times 10^{-3} \sqrt{\eta^{-2} - 1}$

- from $\pi\pi \to KK$ data (only one set) → $\eta \approx 0.973 \pm 0.011$

$$\boxed{\Delta A_{CP}^{th} = -(0.64 \pm 0.18) \times 10^{-3}}$$

3σ

→ largest theoretical prediction within SM without relying on fitting parameters

→ systematic uncertainties are unknown in η → error is underestimated

Predictions for ΔA_{CP}

$$\Delta A_{CP}^{\mathrm{LHCb}} = -(1.54 \pm 0.29) \times 10^{-3}$$

- $\Delta A_{CP}^{th} = -(2.70 \pm 0.50) \times 10^{-3} \sqrt{\eta^{-2} - 1}$

- from $\pi\pi \to KK$ data (only one set) → $\eta \approx 0.973 \pm 0.011$

$$\boxed{\Delta A_{CP}^{th} = -(0.64 \pm 0.18) \times 10^{-3}}$$

3σ

→ largest theoretical prediction within SM without relying on fitting parameters

→ systematic uncertainties are unknown in η → error is underestimated

- Alternatively one can assume all inelasticity in $\pi\pi \to \pi\pi$ is due to KK

→ more precise data (Grayer) → $\eta = 0.78 \pm 0.08$

$$\boxed{\Delta A_{CP}^{th} = -(2.17 \pm 0.70) \times 10^{-3}}$$

1σ

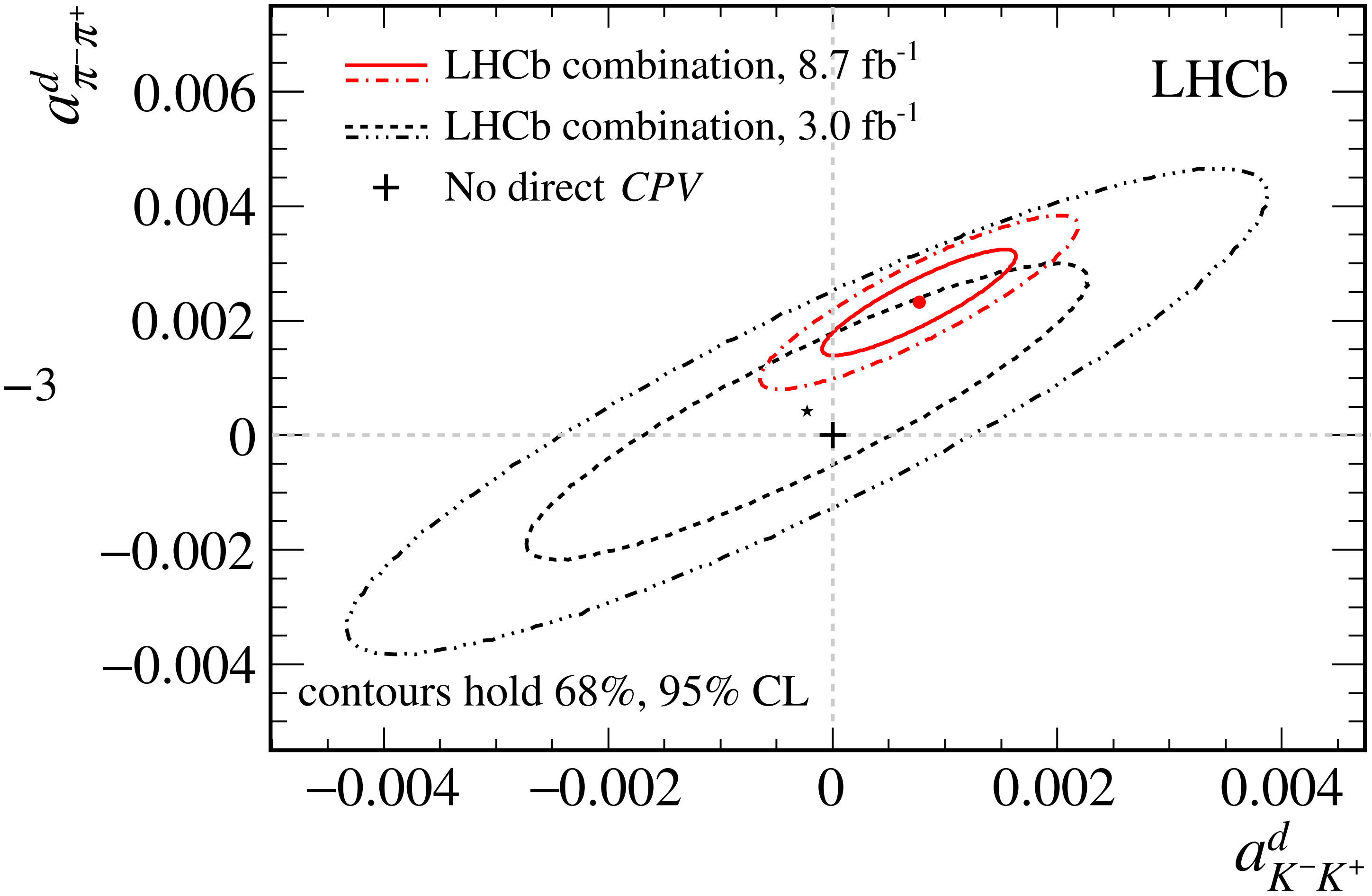
Predictions for $A_{CP}(hh)$

- direct CP asymmetry observation

$$A_{CP}^{LHCb}(KK) = (0.77 \pm 0.57) \times 10^{-3}$$

$$\hookrightarrow A_{CP}^{LHCb}(\pi\pi) = (2.32 \pm 0.61) \times 10^{-3}$$

arXiv:2209.03179



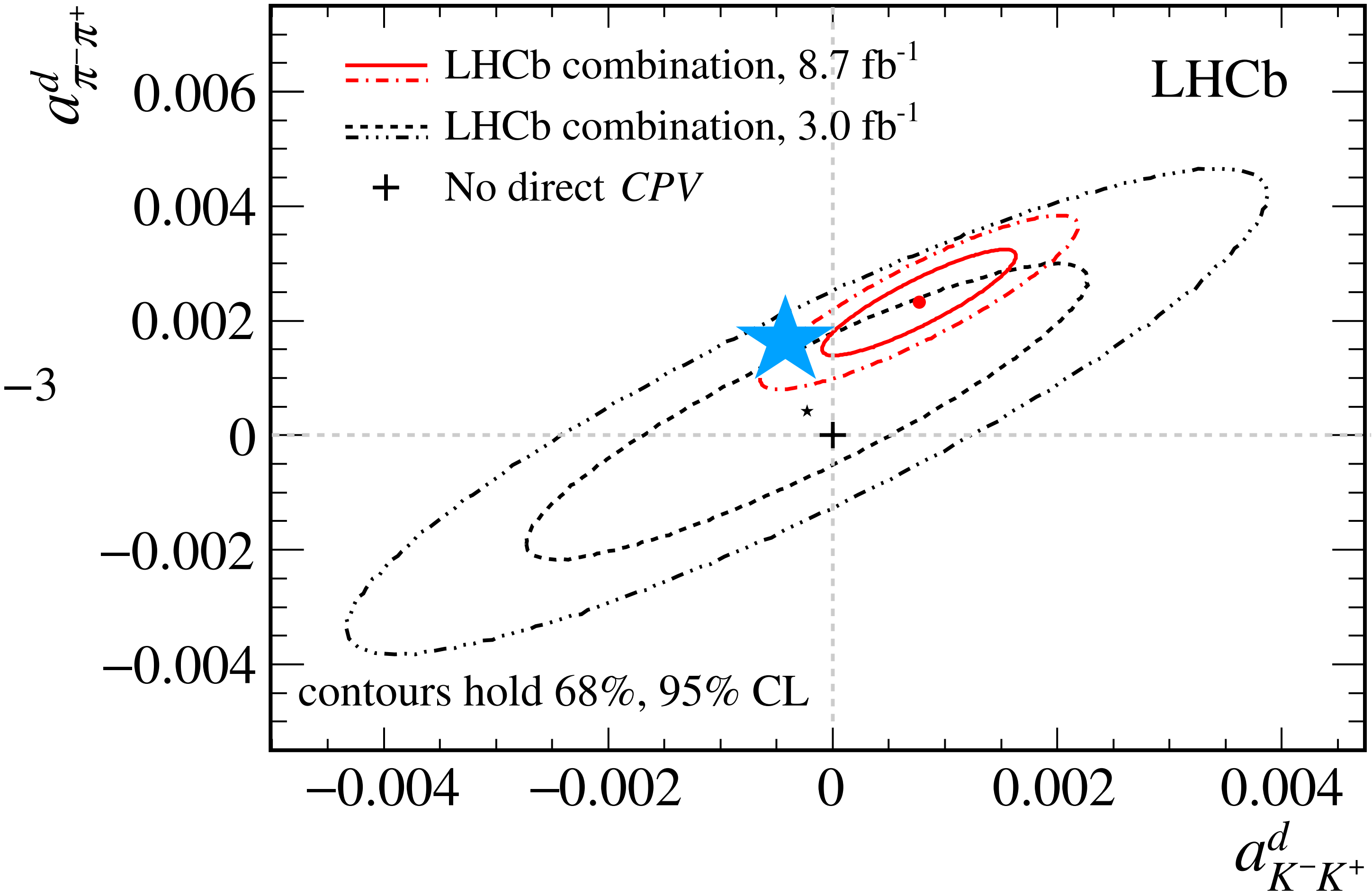
Predictions for $A_{CP}(hh)$

- direct CP asymmetry observation

$$A^{LHCb}_{CP}(KK) = (0.77 \pm 0.57) \times 10^{-3}$$

$$\hookrightarrow A^{LHCb}_{CP}(\pi\pi) = (2.32 \pm 0.61) \times 10^{-3}$$

arXiv:2209.03179



- with $\eta = 0.78 \pm 0.08$

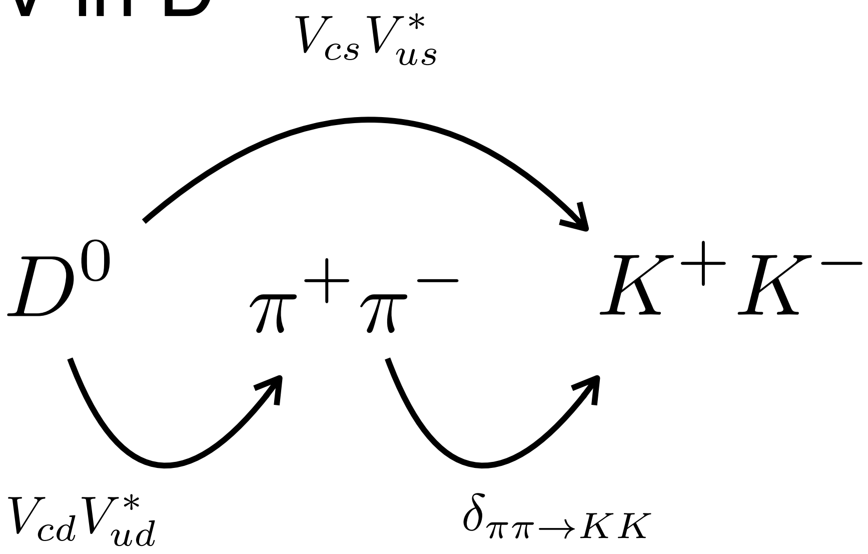
$$A_{CP}(KK) = -(0.57 \pm 0.18) \times 10^{-3}$$ 2σ

$$A_{CP}(\pi\pi) = (1.60 \pm 0.51) \times 10^{-3}$$ 1σ

Final remarks

- hadronic FSI (and their strong phases)are crucial to explain CP violation in B and D decays

- we proposed a mechanism that can explain CPV in D
  - coupling $\pi\pi \leftrightarrow K\bar{K}$ in a CPT invariant framework
  - still room to add 2nd order effects



→ predicted ΔA_{CP} which is compatible with LHCb

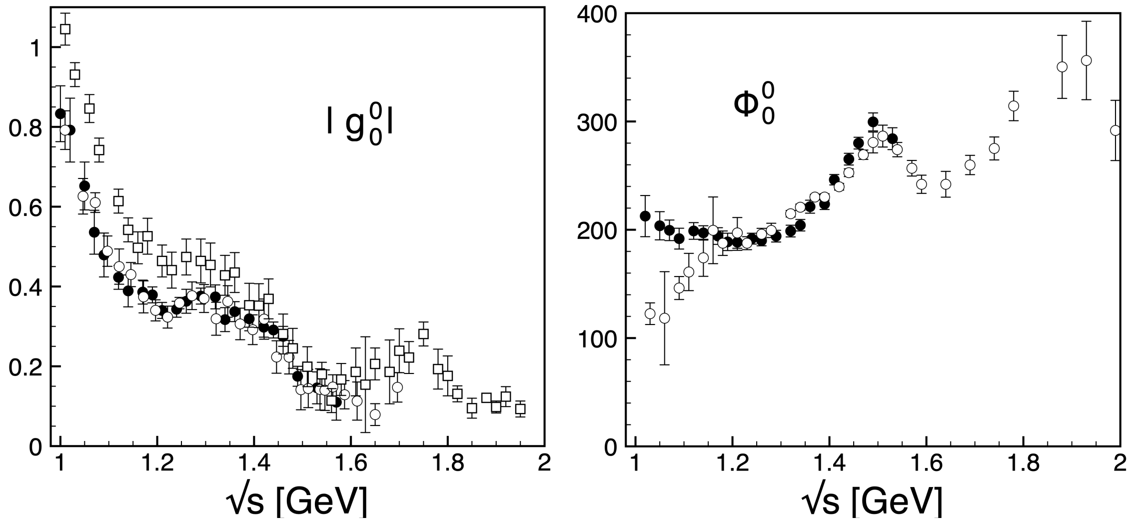
- new measurement for $A_{CP}(hh)$ from LHCb
  - agrees with our predictions with 2σ

↳ we still need more data to fully understood it

Final remarks

- In 3-body decays this effect will be bigger and phase-space distributed

↳ $D^+ \rightarrow \pi^+\pi^-\pi^+$ and $D^+ \rightarrow \pi^+K^-K^+$ have exactly the same Weak vertex

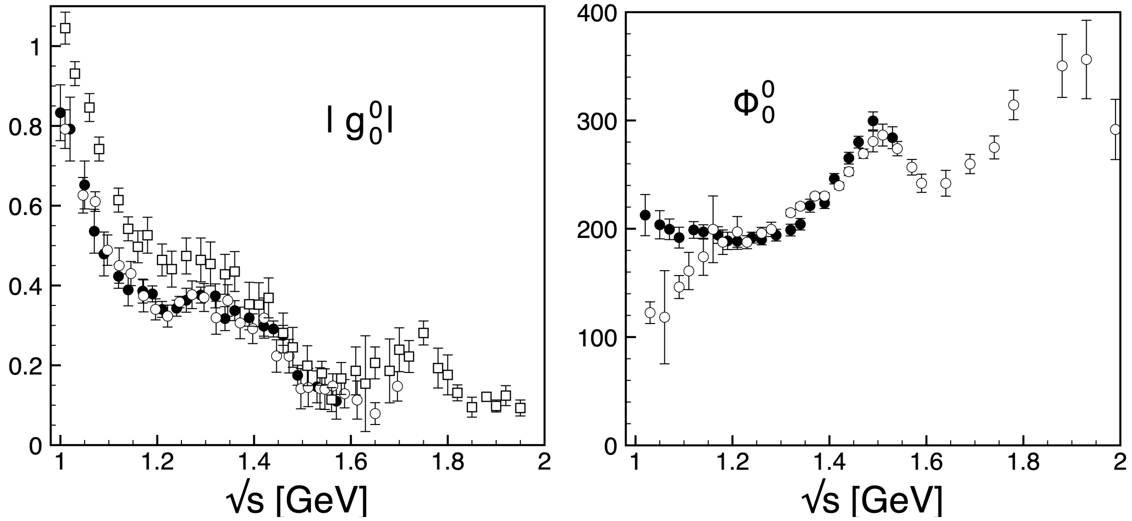


- expected CPV in run II analysis

Final remarks

- In 3-body decays this effect will be bigger and phase-space distributed

↪ $D^+ \to \pi^+\pi^-\pi^+$ and $D^+ \to \pi^+K^-K^+$ have exactly the same Weak vertex



- expected CPV in run II analysis

thank you!

obrigada!!

#forabolsonaro

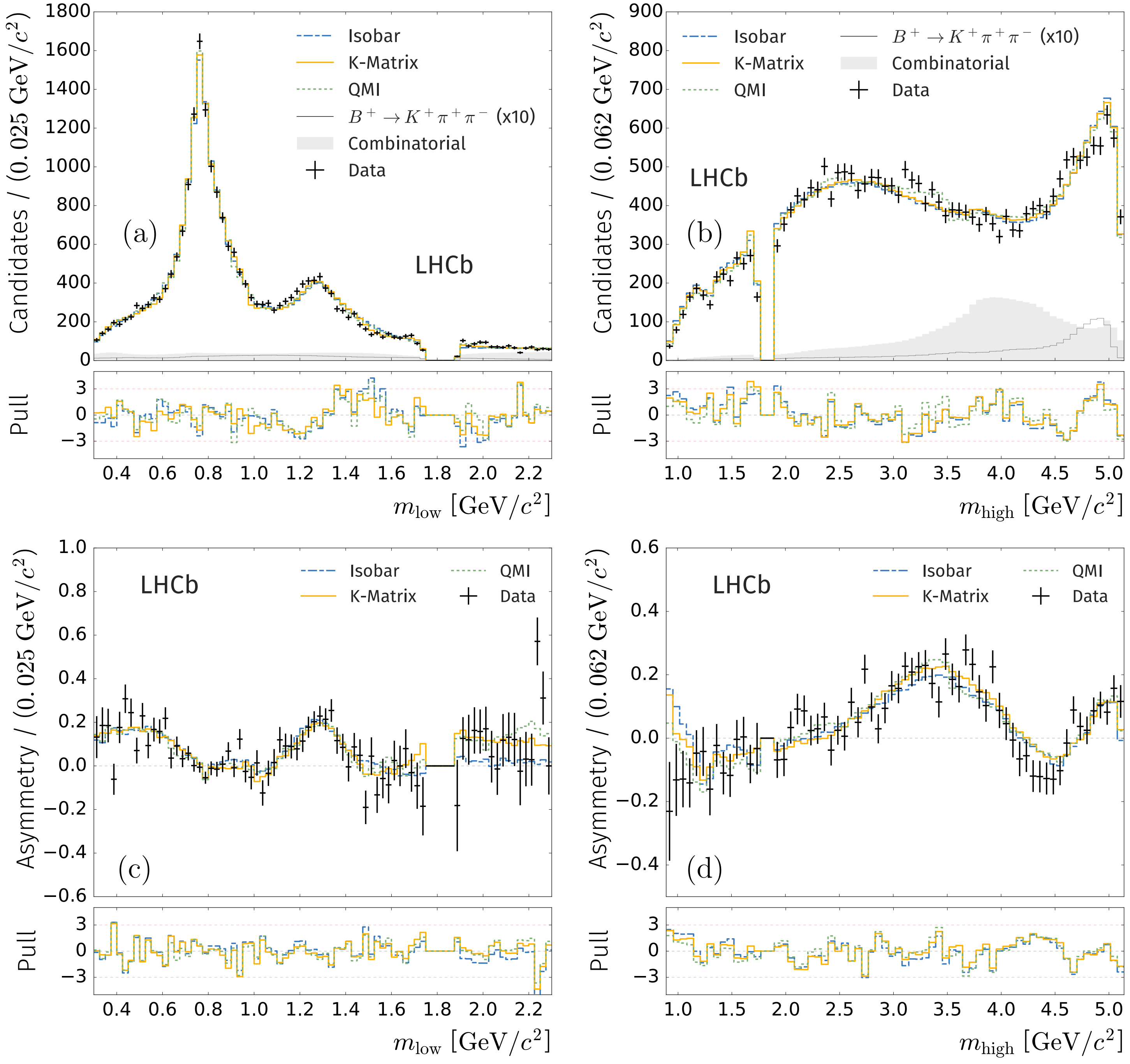
Backup slides

CPV: amplitude analysis

- $B^{\pm} \to \pi^{-}\pi^{+}\pi^{\pm}$
  - $(\pi^{-}\pi^{+})_{S-Wave}$ 3 different model:
    - ↪ σ as BW (!) + rescattering;
    - ↪ P-vector K-Matrix;
    - ↪ binned freed lineshape (QMI);

PRD101 (2020) 012006; PRL 124 (2020) 031801

Contribution	Fit fraction (10^{-2})	A_{CP} (10^{-2})	B^{+} phase (°)	B^{-} phase (°)
Isobar model				
$\rho(770)^0$	$55.5 \pm 0.6 \pm 2.5$	$+0.7 \pm 1.1 \pm 1.6$	—	—
$\omega(782)$	$0.50 \pm 0.03 \pm 0.05$	$-4.8 \pm 6.5 \pm 3.8$	$-19 \pm 6 \pm 1$	$+8 \pm 6 \pm 1$
$f_2(1270)$	$9.0 \pm 0.3 \pm 1.5$	$+46.8 \pm 6.1 \pm 4.7$	$+5 \pm 3 \pm 12$	$+53 \pm 2 \pm 12$
$\rho(1450)^0$	$5.2 \pm 0.3 \pm 1.9$	$-12.9 \pm 3.3 \pm 35.9$	$+127 \pm 4 \pm 21$	$+154 \pm 4 \pm 6$
$\rho_3(1690)^0$	$0.5 \pm 0.1 \pm 0.3$	$-80.1 \pm 11.4 \pm 25.3$	$-26 \pm 7 \pm 14$	$-47 \pm 18 \pm 25$
S-wave	$25.4 \pm 0.5 \pm 3.6$	$+14.4 \pm 1.8 \pm 2.1$	—	—
Rescattering	$1.4 \pm 0.1 \pm 0.5$	$+44.7 \pm 8.6 \pm 17.3$	$-35 \pm 6 \pm 10$	$-4 \pm 4 \pm 25$
σ	$25.2 \pm 0.5 \pm 5.0$	$+16.0 \pm 1.7 \pm 2.2$	$+115 \pm 2 \pm 14$	$+179 \pm 1 \pm 95$
K-matrix				
$\rho(770)^0$	$56.5 \pm 0.7 \pm 3.4$	$+4.2 \pm 1.5 \pm 6.4$	—	—
$\omega(782)$	$0.47 \pm 0.04 \pm 0.03$	$-6.2 \pm 8.4 \pm 9.8$	$-15 \pm 6 \pm 4$	$+8 \pm 7 \pm 4$
$f_2(1270)$	$9.3 \pm 0.4 \pm 2.5$	$+42.8 \pm 4.1 \pm 9.1$	$+19 \pm 4 \pm 18$	$+80 \pm 3 \pm 17$
$\rho(1450)^0$	$10.5 \pm 0.7 \pm 4.6$	$+9.0 \pm 6.0 \pm 47.0$	$+155 \pm 5 \pm 29$	$-166 \pm 4 \pm 51$
$\rho_3(1690)^0$	$1.5 \pm 0.1 \pm 0.4$	$-35.7 \pm 10.8 \pm 36.9$	$+19 \pm 8 \pm 34$	$+5 \pm 8 \pm 46$
S-wave	$25.7 \pm 0.6 \pm 3.0$	$+15.8 \pm 2.6 \pm 7.2$	—	—
QMI				
$\rho(770)^0$	$54.8 \pm 1.0 \pm 2.2$	$+4.4 \pm 1.7 \pm 2.8$	—	—
$\omega(782)$	$0.57 \pm 0.10 \pm 0.17$	$-7.9 \pm 16.5 \pm 15.8$	$-25 \pm 6 \pm 27$	$-2 \pm 7 \pm 11$
$f_2(1270)$	$9.6 \pm 0.4 \pm 4.0$	$+37.6 \pm 4.4 \pm 8.0$	$+13 \pm 5 \pm 21$	$+68 \pm 3 \pm 66$
$\rho(1450)^0$	$7.4 \pm 0.5 \pm 4.0$	$-15.5 \pm 7.3 \pm 35.2$	$+147 \pm 7 \pm 152$	$-175 \pm 5 \pm 171$
$\rho_3(1690)^0$	$1.0 \pm 0.1 \pm 0.5$	$-93.2 \pm 6.8 \pm 38.9$	$+8 \pm 10 \pm 24$	$+36 \pm 26 \pm 46$
S-wave	$26.8 \pm 0.7 \pm 2.2$	$+15.0 \pm 2.7 \pm 8.1$	—	—



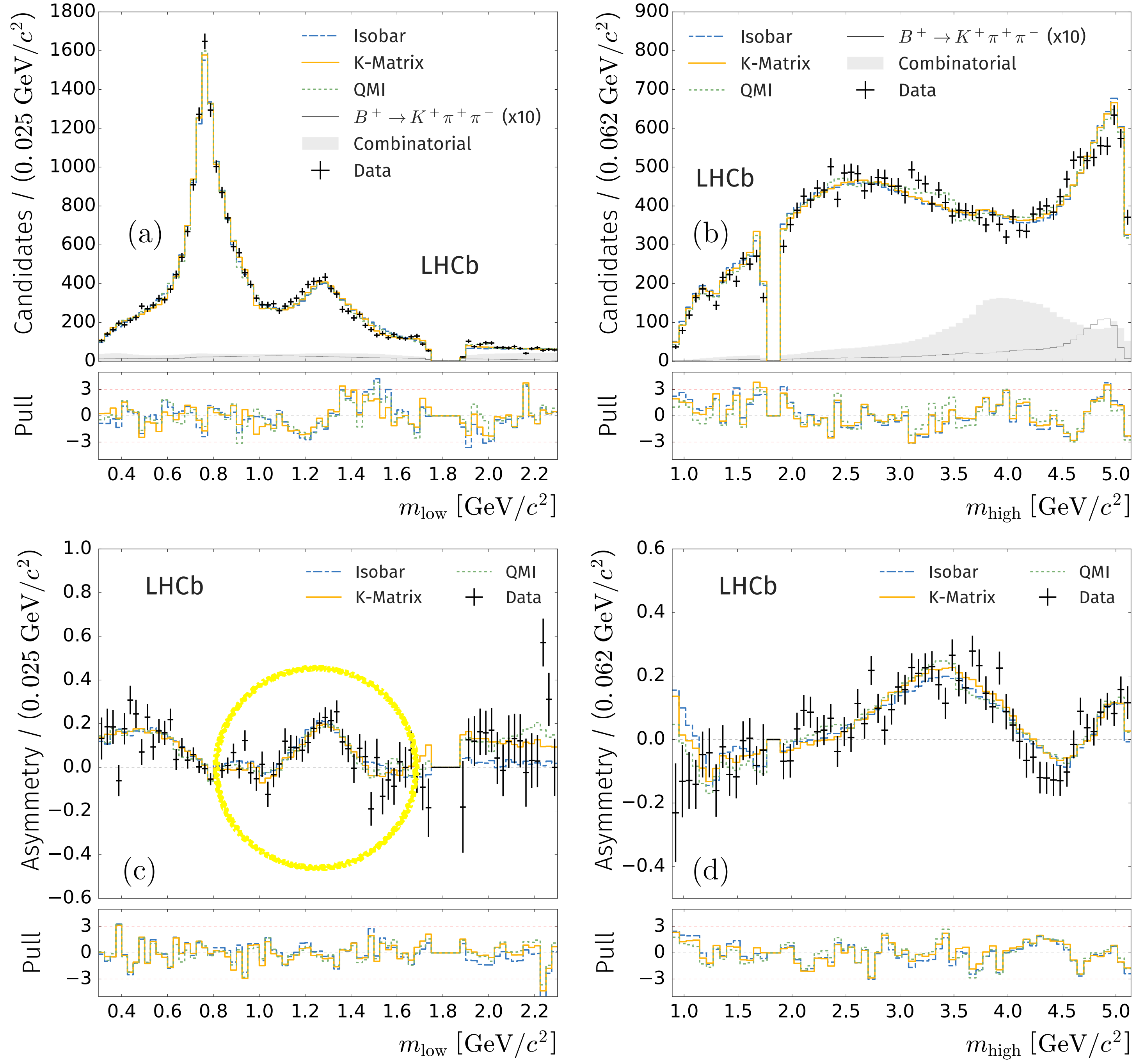
- $B^{\pm} \to \pi^{\pm}K^{-}K^{+}$

PRL 123 (2019) 231802

Contribution	Fit Fraction(%)	A_{CP}(%)	Magnitude (B^+/B^-)	Phase[°] (B^+/B^-)
$K^*(892)^0$	$7.5 \pm 0.6 \pm 0.5$	$+12.3 \pm 8.7 \pm 4.5$	$0.94 \pm 0.04 \pm 0.02$ $1.06 \pm 0.04 \pm 0.02$	0 (fixed) 0 (fixed)
$K_0^*(1430)^0$	$4.5 \pm 0.7 \pm 1.2$	$+10.4 \pm 14.9 \pm 8.8$	$0.74 \pm 0.09 \pm 0.09$ $0.82 \pm 0.09 \pm 0.10$	$-176 \pm 10 \pm 16$ $136 \pm 11 \pm 21$
Single pole	$32.3 \pm 1.5 \pm 4.1$	$-10.7 \pm 5.3 \pm 3.5$	$2.19 \pm 0.13 \pm 0.17$ $1.97 \pm 0.12 \pm 0.20$	$-138 \pm 7 \pm 5$ $166 \pm 6 \pm 5$
$\rho(1450)^0$	$30.7 \pm 1.2 \pm 0.9$	$-10.9 \pm 4.4 \pm 2.4$	$2.14 \pm 0.11 \pm 0.07$ $1.92 \pm 0.10 \pm 0.07$	$-175 \pm 10 \pm 15$ $140 \pm 13 \pm 20$
$f_2(1270)$	$7.5 \pm 0.8 \pm 0.7$	$+26.7 \pm 10.2 \pm 4.8$	$0.86 \pm 0.09 \pm 0.07$ $1.13 \pm 0.08 \pm 0.05$	$-106 \pm 11 \pm 10$ $-128 \pm 11 \pm 14$
Rescattering	$16.4 \pm 0.8 \pm 1.0$	$-66.4 \pm 3.8 \pm 1.9$	$1.91 \pm 0.09 \pm 0.06$ $0.86 \pm 0.07 \pm 0.04$	$-56 \pm 12 \pm 18$ $-81 \pm 14 \pm 15$
$\phi(1020)$	$0.3 \pm 0.1 \pm 0.1$	$+9.8 \pm 43.6 \pm 26.6$	$0.20 \pm 0.07 \pm 0.02$ $0.22 \pm 0.06 \pm 0.04$	$-52 \pm 23 \pm 32$ $107 \pm 33 \pm 41$

CPV: amplitude analysis

- $B^{\pm} \to \pi^{-}\pi^{+}\pi^{\pm}$
  - $(\pi^{-}\pi^{+})_{S-Wave}$ 3 different model:
    - ↪ σ as BW (!) + rescattering;
    - ↪ P-vector K-Matrix;
    - ↪ binned freed lineshape (QMI);



PRD101 (2020) 012006; PRL 124 (2020) 031801

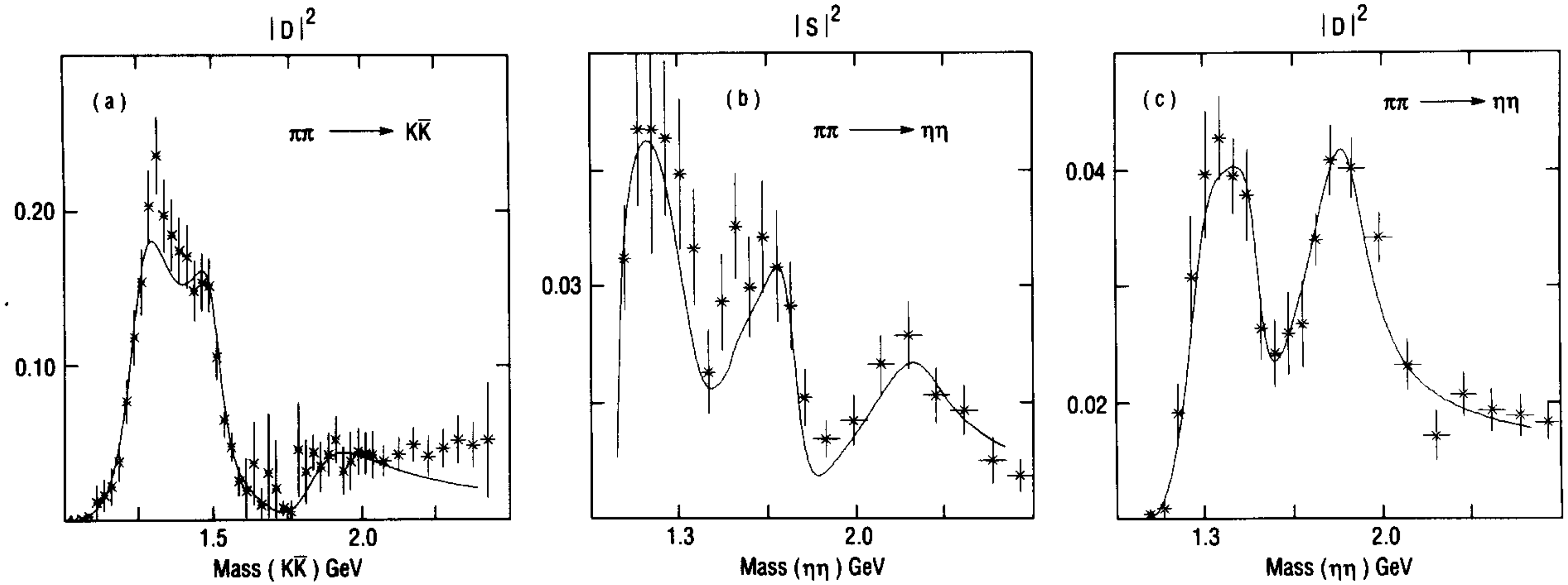
Contribution	Fit fraction (10^{-2})	A_{CP} (10^{-2})	B^+ phase (°)	B^- phase (°)
Isobar model				
$\rho(770)^0$	$55.5 \pm 0.6 \pm 2.5$	$+0.7 \pm 1.1 \pm 1.6$	—	—
$\omega(782)$	$0.50 \pm 0.03 \pm 0.05$	$-4.8 \pm 6.5 \pm 3.8$	$-19 \pm 6 \pm 1$	$+8 \pm 6 \pm 1$
$f_2(1270)$	$9.0 \pm 0.3 \pm 1.5$	$+46.8 \pm 6.1 \pm 4.7$	$+5 \pm 3 \pm 12$	$+53 \pm 2 \pm 12$
$\rho(1450)^0$	$5.2 \pm 0.3 \pm 1.9$	$-12.9 \pm 3.3 \pm 35.9$	$+127 \pm 4 \pm 21$	$+154 \pm 4 \pm 6$
$\rho_3(1690)^0$	$0.5 \pm 0.1 \pm 0.3$	$-80.1 \pm 11.4 \pm 25.3$	$-26 \pm 7 \pm 14$	$-47 \pm 18 \pm 25$
S-wave	$25.4 \pm 0.5 \pm 3.6$	$+14.4 \pm 1.8 \pm 2.1$	—	—
Rescattering	$1.4 \pm 0.1 \pm 0.5$	$+44.7 \pm 8.6 \pm 17.3$	$-35 \pm 6 \pm 10$	$-4 \pm 4 \pm 25$
σ	$25.2 \pm 0.5 \pm 5.0$	$+16.0 \pm 1.7 \pm 2.2$	$+115 \pm 2 \pm 14$	$+179 \pm 1 \pm 95$
K-matrix				
$\rho(770)^0$	$56.5 \pm 0.7 \pm 3.4$	$+4.2 \pm 1.5 \pm 6.4$	—	—
$\omega(782)$	$0.47 \pm 0.04 \pm 0.03$	$-6.2 \pm 8.4 \pm 9.8$	$-15 \pm 6 \pm 4$	$+8 \pm 7 \pm 4$
$f_2(1270)$	$9.3 \pm 0.4 \pm 2.5$	$+42.8 \pm 4.1 \pm 9.1$	$+19 \pm 4 \pm 18$	$+80 \pm 3 \pm 17$
$\rho(1450)^0$	$10.5 \pm 0.7 \pm 4.6$	$+9.0 \pm 6.0 \pm 47.0$	$+155 \pm 5 \pm 29$	$-166 \pm 4 \pm 51$
$\rho_3(1690)^0$	$1.5 \pm 0.1 \pm 0.4$	$-35.7 \pm 10.8 \pm 36.9$	$+19 \pm 8 \pm 34$	$+5 \pm 8 \pm 46$
S-wave	$25.7 \pm 0.6 \pm 3.0$	$+15.8 \pm 2.6 \pm 7.2$	—	—
QMI				
$\rho(770)^0$	$54.8 \pm 1.0 \pm 2.2$	$+4.4 \pm 1.7 \pm 2.8$	—	—
$\omega(782)$	$0.57 \pm 0.10 \pm 0.17$	$-7.9 \pm 16.5 \pm 15.8$	$-25 \pm 6 \pm 27$	$-2 \pm 7 \pm 11$
$f_2(1270)$	$9.6 \pm 0.4 \pm 4.0$	$+37.6 \pm 4.4 \pm 8.0$	$+13 \pm 5 \pm 21$	$+68 \pm 3 \pm 66$
$\rho(1450)^0$	$7.4 \pm 0.5 \pm 4.0$	$-15.5 \pm 7.3 \pm 35.2$	$+147 \pm 7 \pm 152$	$-175 \pm 5 \pm 171$
$\rho_3(1690)^0$	$1.0 \pm 0.1 \pm 0.5$	$-93.2 \pm 6.8 \pm 38.9$	$+8 \pm 10 \pm 24$	$+36 \pm 26 \pm 46$
S-wave	$26.8 \pm 0.7 \pm 2.2$	$+15.0 \pm 2.7 \pm 8.1$	—	—

- $B^{\pm} \to \pi^{\pm}K^{-}K^{+}$ PRL 123 (2019) 231802

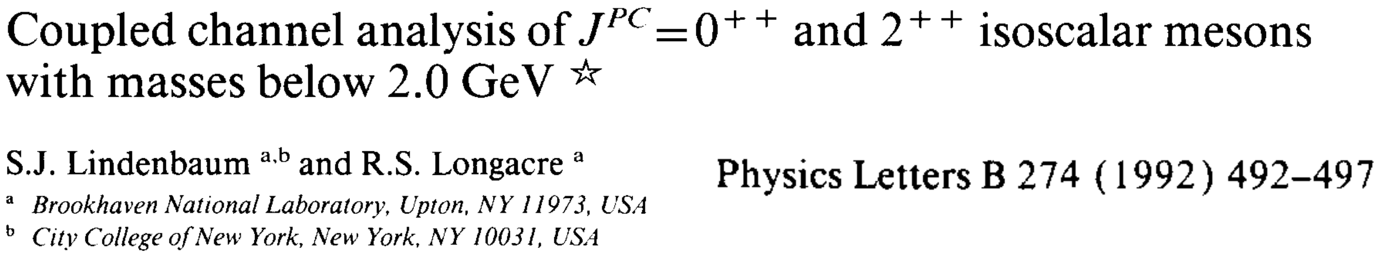
Contribution	Fit Fraction(%)	A_{CP}(%)	Magnitude (B^+/B^-)	Phase[^{o}] (B^+/B^-)
$K^*(892)^0$	$7.5 \pm 0.6 \pm 0.5$	$+12.3 \pm 8.7 \pm 4.5$	$0.94 \pm 0.04 \pm 0.02$ $1.06 \pm 0.04 \pm 0.02$	0 (fixed) 0 (fixed)
$K_0^*(1430)^0$	$4.5 \pm 0.7 \pm 1.2$	$+10.4 \pm 14.9 \pm 8.8$	$0.74 \pm 0.09 \pm 0.09$ $0.82 \pm 0.09 \pm 0.10$	$-176 \pm 10 \pm 16$ $136 \pm 11 \pm 21$
Single pole	$32.3 \pm 1.5 \pm 4.1$	$-10.7 \pm 5.3 \pm 3.5$	$2.19 \pm 0.13 \pm 0.17$ $1.97 \pm 0.12 \pm 0.20$	$-138 \pm 7 \pm 5$ $166 \pm 6 \pm 5$
$\rho(1450)^0$	$30.7 \pm 1.2 \pm 0.9$	$-10.9 \pm 4.4 \pm 2.4$	$2.14 \pm 0.11 \pm 0.07$ $1.92 \pm 0.10 \pm 0.07$	$-175 \pm 10 \pm 15$ $140 \pm 13 \pm 20$
$f_2(1270)$	$7.5 \pm 0.8 \pm 0.7$	$+26.7 \pm 10.2 \pm 4.8$	$0.86 \pm 0.09 \pm 0.07$ $1.13 \pm 0.08 \pm 0.05$	$-106 \pm 11 \pm 10$ $-128 \pm 11 \pm 14$
Rescattering	$16.4 \pm 0.8 \pm 1.0$	$-66.4 \pm 3.8 \pm 1.9$	$1.91 \pm 0.09 \pm 0.06$ $0.86 \pm 0.07 \pm 0.04$	$-56 \pm 12 \pm 18$ $-81 \pm 14 \pm 15$
$\phi(1020)$	$0.3 \pm 0.1 \pm 0.1$	$+9.8 \pm 43.6 \pm 26.6$	$0.20 \pm 0.07 \pm 0.02$ $0.22 \pm 0.06 \pm 0.04$	$-52 \pm 23 \pm 32$ $107 \pm 33 \pm 41$

$\eta\eta$ coupling to ππ

- coupling of $\pi\pi \rightarrow KK$ in D wave is bigger than $\eta\eta$ in S-wave



- $\sim M_D$ (1.864) mass



Coupled channel analysis of $J^{PC}=0^{++}$ and 2^{++} isoscalar mesons with masses below 2.0 GeV ☆

S.J. Lindenbaum [a,b] and R.S. Longacre [a]

[a] *Brookhaven National Laboratory, Upton, NY 11973, USA*
[b] *City College of New York, New York, NY 10031, USA*

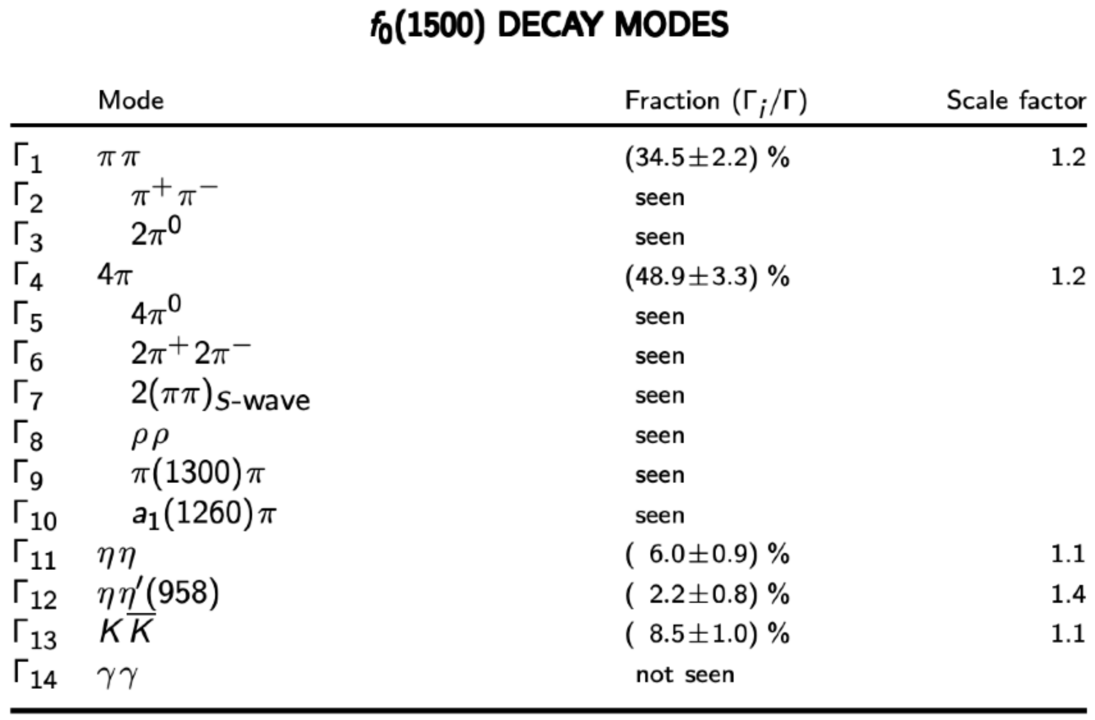
Physics Letters B 274 (1992) 492–497

- ignore $\eta\eta$ channel once their coupling to the $\pi\pi$ channel are suppressed with respect to $K\bar{K}$.

4π coupling to ππ

- although the $D^0 \to 4\pi$ decays have a large branching fraction, there is no compelling experimental evidence that 4π is strongly coupled to $\pi\pi$ at M_{D_0}

- $f_0(1500)$ decays in bot channels

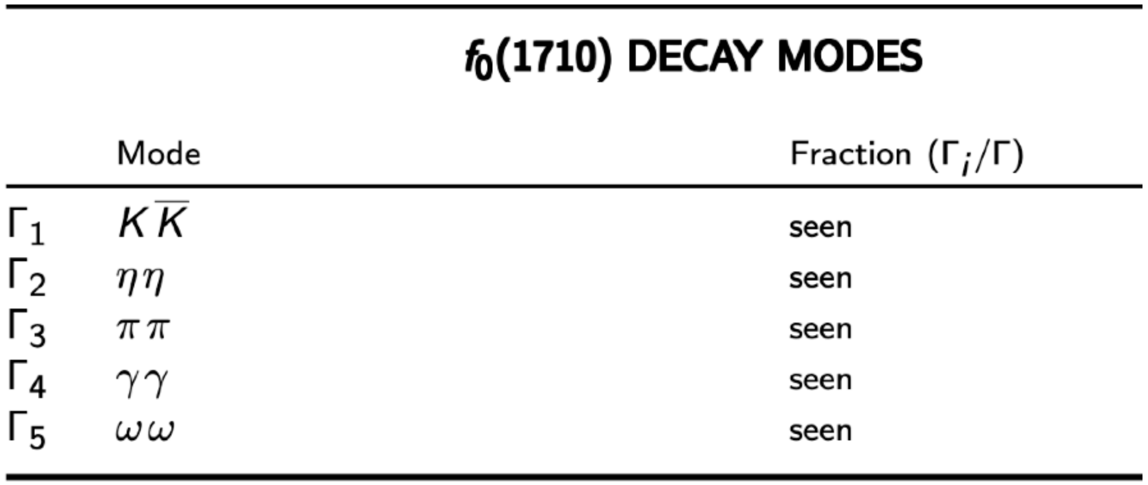


$f_0(1500)$ DECAY MODES

	Mode	Fraction (Γ_i/Γ)	Scale factor
Γ_1	$\pi\pi$	(34.5±2.2) %	1.2
Γ_2	$\pi^+\pi^-$	seen	
Γ_3	$2\pi^0$	seen	
Γ_4	4π	(48.9±3.3) %	1.2
Γ_5	$4\pi^0$	seen	
Γ_6	$2\pi^+2\pi^-$	seen	
Γ_7	$2(\pi\pi)_{S\text{-wave}}$	seen	
Γ_8	$\rho\rho$	seen	
Γ_9	$\pi(1300)\pi$	seen	
Γ_{10}	$a_1(1260)\pi$	seen	
Γ_{11}	$\eta\eta$	(6.0±0.9) %	1.1
Γ_{12}	$\eta\eta'(958)$	(2.2±0.8) %	1.4
Γ_{13}	$K\overline{K}$	(8.5±1.0) %	1.1
Γ_{14}	$\gamma\gamma$	not seen	

PDG

- The nearest $f_0(1710)$ resonance have no observation of four pions reported.



$f_0(1710)$ DECAY MODES

	Mode	Fraction (Γ_i/Γ)
Γ_1	$K\overline{K}$	seen
Γ_2	$\eta\eta$	seen
Γ_3	$\pi\pi$	seen
Γ_4	$\gamma\gamma$	seen
Γ_5	$\omega\omega$	seen

$cos\phi$

- we don't have data from KK scattering !
  - we can use $\pi\pi$ and $KK \to \pi\pi$ data: $\delta_{KK} - \delta_{\pi\pi} = \phi_0^0 - 2\delta_{\pi\pi} = (\delta_{KK} + \delta_{\pi\pi}) - 2\delta_{\pi\pi}$
  - CERN-Munich data (revised Ochs)

$\sqrt{s}$ [GeV]	$\cos\phi$
1.58	0.989 ± 0.149
1.62	0.994 ± 0.105
1.66	0.999 ± 0.040
1.70	0.987 ± 0.160
1.74	0.999 ± 0.048
1.78	0.999 ± 0.037
1.846	0.987 ± 0.175

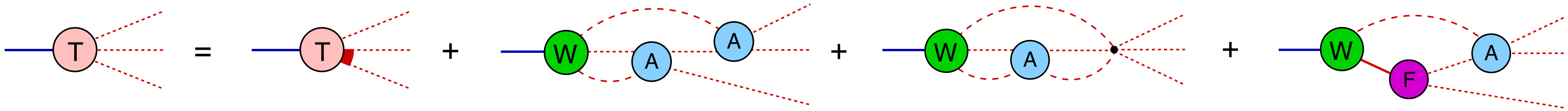
$\rightarrow \cos(\delta_{KK} - \delta_{\pi\pi}) \lesssim 1$

$m_D^2 \rightarrow$ 1.846

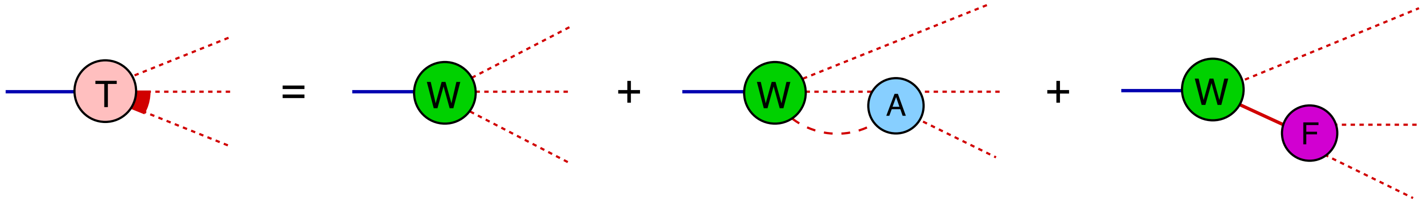
→ **Pelaez parametrization**

Full story in 3-body decay

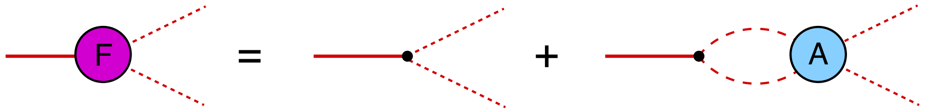
- Any 3-body decay amplitude



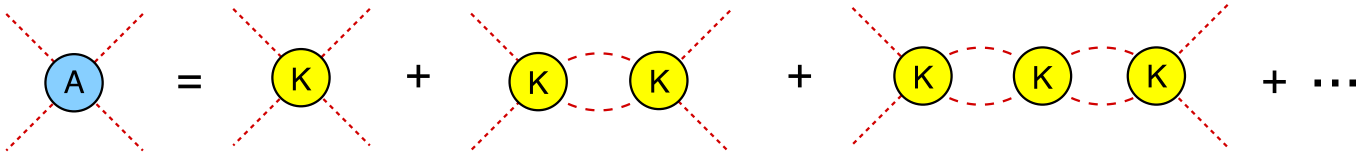
(2+1) approach



Form factor

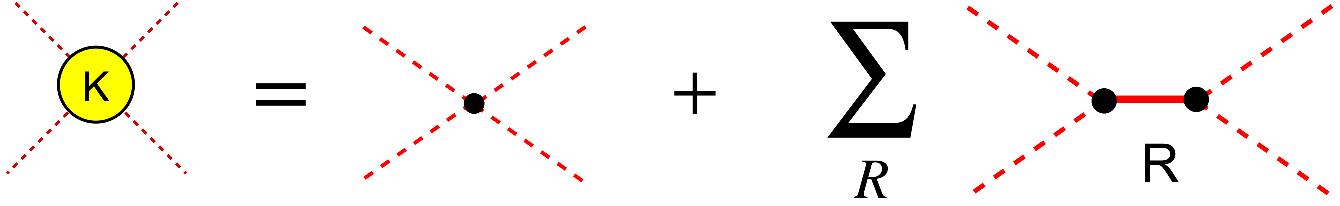


meson-meson



MAGALHAES, A.dos Reis, Robilotta
PRD 102, 076012 (2020)

- kernel should includes all the mm dynamics



- Unitarized amplitude should includes all channels with the same (J,I)